PLUTO'S HELIOCENTRIC ORBIT*

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We review the current state of knowledge regarding Pluto's heliocentric orbital motion. Pluto's orbit is unusually eccentric and inclined to the ecliptic, and overlaps the orbit of Neptune. Consequently, Pluto suffers significant planetary perturbations. The current uncertainties in Pluto's orbital parameters and their implications for its long-term dynamical evolution are reviewed. Numerical integrations of increasingly long times indicate that Pluto exists in a dynamical niche consisting of several resonances which ensures its macroscopic stability over timescales comparable to the age of the Solar system. In particular, the 3:2 orbital period resonance with Neptune protects it from close encounters with the giant planets. Furthermore, Pluto's motion is formally chaotic, with a Lyapunov timescale of $\mathcal{O}(10^7)$ years. The extent and character of this dynamical niche is described. The emplacement of Pluto in this niche requires some dissipative mechanism in the early history of the Solar system. We discuss some plausible scenarios for the origin of this unusual orbit.

I. INTRODUCTION

The heliocentric motion of Pluto is of great interest for several reasons. First, Pluto's orbit departs very significantly in character from the usual well-separated, near-circular and co-planar orbits of the major planets of the Solar system. During one complete revolution about the Sun [in a period of 248 years at a mean distance of about 40 astronomical units (AU)], Pluto's heliocentric distance changes by almost 20 AU from perihelion to aphelion, and the planet makes excursions of 8 AU above and 13 AU below the plane of the ecliptic (see Figure 1). For approximately two decades in its orbital period, Pluto is closer to the

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Sun than Neptune. Furthermore, Pluto is accompanied in its orbit about the Sun by a large satellite, Charon; the large mass ratio of Charon to Pluto makes this truly a binary planet. The origin and dynamical stability of this binary planet in a very peculiar orbit in the outer reaches of the planetary system is a fascinating question in Solar system dynamics and may hold clues to planet formation processes in the outer Solar system. Pluto's orbital history is also of importance for the geophysical and climate evolution of this system (cf. chapters in *BULK PROPERTIES* and *ATMOSPHERES*).

Pluto's* orbital period oscillates about a mean value which is exactly 3/2 that of Neptune. Owing to this orbital resonance and Pluto's large eccentricity and inclination, the usual analytical methods of celestial mechanics have been of limited use in determining the long-term motion of Pluto under the influence of perturbations from the giant planets. Therefore, most studies of Pluto's orbital dynamics have involved numerical integrations of increasingly long times. The enormous increase in computing speed facilitated by digital computers and faster numerical integration algorithms in recent years now allows the exploration of planetary dynamics over billion year timescales with relative ease. We now know that Pluto's long-term motion exhibits a rich variety of dynamical phenomena: the strong mean motion resonance with Neptune, several resonances and near-resonances with the secular motions of the giant planets, as well as evidence of deterministic chaos. The latter is especially curious, because numerical simulations also suggest that over timescales comparable to the age of the Solar system, Pluto is secure from macroscopically large changes in its orbital parameters. This complex dynamics has recently motivated two plausible scenarios for the origin of such an orbit. It is likely that Pluto formed in an ordinary near-circular, co-planar orbit beyond Neptune and was transported to its current peculiar orbit by dynamical processes in the early history of the Solar system. These new theories are a striking departure from the early speculations that Pluto may be an escaped Neptunian satellite.

This chapter reviews the current state of knowledge about the orbit of Pluto and is organized as follows. In Section II we describe the history of Pluto's orbit determination and discuss the quality of the present ephemerides and prospects for improvement in the future. In Section III we describe Pluto's long-term orbital dynamics. In Section IV we discuss the mechanisms that determine Pluto's orbital stability. Section V is a review of the theories for the origin of Pluto's orbit. In Section VI we provide a summary of the chapter and indicate avenues for future studies.

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* For brevity, we will refer to the heliocentric motion of the center-of-mass of the Pluto-Charon binary as simply that of 'Pluto'.

II. CURRENT ORBIT

Although Pluto was discovered in 1930, there exist prediscovery photographs that provide its positions back to 1914. Thus Pluto has been observed for nearly 80 years, or 1/3 of its orbital period. Recently updated osculating, heliocentric elements in the J2000 coordinate system are listed in Table 1, and use a Sun/Pluto mass ratio of 135,000,000 (Beletic *et al.* 1989). They are based on approximately 900 astrometric positions observed over nearly eight decades. In addition to the six Keplerian elements (semimajor axis a, eccentricity e, inclination i, node Ω, argument of perihelion ω, and mean anomaly M), some auxiliary quantities (mean motion n, orbital period, perihelion distance q, and aphelion distance Q) are also listed. These elements are affected by short-period planetary perturbations. For example, if the short-period effects are removed, the average (over a few centuries) orbital period is 248 yr. Pluto's most recent perihelion passage occurred on 5th September 1989 with a heliocentric distance of 29.6556 AU.

Orbit determination for observation times less than an orbit period gives best accuracies along the observed arc of the orbit and degraded accuracies elsewhere. The difficulties stemming from an incomplete orbit are complicated further by systematic star catalog errors. Pluto's orbit suffers from nonuniform accuracy as shown by the uncertainties in Table 1: the semimajor axis is less well known than the perihelion distance, the perihelion direction and mean anomaly are coarser than the other angular elements (but the mean longitude, $\lambda = \omega + \Omega + M$, is known nearly an order of magnitude better, 0.00015°), and the error ellipse for the pole direction is elongated by 2-to-1.

The uneven orbit accuracy shows up in predictions of Pluto's future position. Predictions only a decade beyond the last observation are noticeably in error (Seidelmann *et al.* 1980, Standish 1994). At present the least well known coordinate is the radial distance with an uncertainty which exceeds 10,000 km. A future spacecraft mission to Pluto would benefit from high accuracies for the ephemeris. Lower accuracies result in pointing and arrival time uncertainties. To maintain the highest accuracy in the future, it is necessary to make positional observations and to update the orbit regularly.

III. LONG TERM EVOLUTION

Like the orbit of the innermost planet Mercury, the orbit of distant Pluto is distinguished from that of the other planets by the magnitude of its eccentricity and inclination. Figure 1 shows the orbits of the five outer planets. The extent of both radial and out-of-ecliptic-plane excursions of Pluto far exceeds those of all other major planets.

Table 1: Pluto's orbital elements*

epoch	MJD 40400.0
a [AU]	39.77445 ± 41
e	0.2533182 ± 55
i [deg]	17.13487 ± 3
Ω [deg]	110.28631 ± 19
ω [deg]	112.98240 ± 130
M [deg]	331.37659 ± 130
n [deg/day]	0.00392914 ± 6
period [yr]	250.8502 ± 40
q [AU]	29.69886 ± 11
Q [AU]	49.85004 ± 73

* from the DE 245 planetary and lunar ephemeris, by Standish, Newhall and Williams (1993, personal communication); uncertainties are from the solution covariance matrix.

Pluto's perihelion distance is smaller than Neptune's mean heliocentric distance — indeed its present perihelion (29.7 AU) is slightly smaller than Neptune's (29.8 AU, both with short-period variations removed). The question naturally arises whether close approaches between Pluto and Neptune prevent orbital stability. The large eccentricity and inclination and its Neptune-crossing orbit make Pluto a difficult subject for studies by analytical perturbation theory. Consequently, numerical integrations have dominated the studies of Pluto's orbit evolution. The length of these integrations is limited by the speed of available computers and integration methods. Over the past three decades the succession of integrations with longer and longer times and more realistic physical models is testimony to the improvement in computer speed and innovative numerical integration algorithms. A listing of these numerical integrations is given in Table 2.

The variations of Pluto's orbital elements over 40,000 years and 8 million years are plotted in Figures 2 and 3, respectively.* The orbital variations are due to the gravitational effects of the other planets, and it is evident that the perturbations occur on several different timescales.

Planetary perturbations can be divided into short- and long-period effects. The short-period perturbations depend on the positions of the bodies in their orbits, *i.e.* on the mean anomalies or mean longitudes. The longer-period effects, commonly called secular perturbations, include the secular motions of nodes and perihelia and long-period variations in nodes, perihelia, eccentricities, and inclinations. Pluto exhibits resonances with both types of perturbations.

A description of the long-term dynamics of Pluto's orbit is a tale of resonances. A resonance is associated with some repetitive geometrical pattern of motion that arises from a low-integer commensurability of some pair of frequencies. (For example, Pluto's average orbital period is 3/2 Neptune's; as a result, the relative orbital phases of Pluto and Neptune recur periodically.) This causes the perturbative forces to act in nearly the same phase at each repetition of the geometrical pattern. Mathematically, this situation leads to a serious problem as the usual linear perturbation theory for the analysis of orbital perturbations breaks down due to the notorious problem of "small divisors" (see, for example, Brown and Shook 1933). Each resonance (or each periodic perturbation) has an associated "resonance angle" which is made up of a linear combination of angular orbital parameters. The motion of a pendulum is commonly used as an analogy for resonances. For resonant motion the resonance angle oscillates (librates) about some value

* The plots in Figures 2–6 were obtained from direct N-body numerical integrations of the five outer planets' motion using recently updated planetary parameters and initial conditions provided by Myles Standish. The integrations were performed using a mixed variable symplectic integrator (Wisdom and Holman 1991).

Table 2: Numerical Integrations of Pluto

Authors	Pub Date	Span	Comp Time
Cohen & Hubbard	1965	120 kyr	3 d
Cohen *et al.*	1967	300 kyr	
Williams & Benson	1971	4.5 Myr	1 hr
Cohen *et al.*	1973	1 Myr	
Kinoshita & Nakai	1984	5 Myr	4 hr
Milani *et al.*	1986	9.3 Myr	
Applegate *et al.*	1986	217 Myr	14 d
Sussman & Wisdom	1988	845 Myr	
Richardson & Walker	1988	1 Myr	65 d
Milani *et al.*	1989	100 Myr	
Quinn *et al.*	1991	3 Myr	65 d
Wisdom & Holman	1991	1 Byr	14 d
Sussman & Wisdom	1992	100 Myr	40 d
Nakai & Kinoshita	1994	1.3 Byr	
Kinoshita & Nakai	1995	5.5 Byr	110 d
Levison & Stern	1995	100 Myr	
Nakai & Kinoshita	1995	11.2 Byr	

— like a swinging pendulum — and its averaged time derivative vanishes; for nonresonant motion this angle circulates – like a pendulum rotating over the top. Stronger resonances have shorter libration periods and broader libration regions (*i.e.* a larger range of orbital elements which will allow libration). The periods of nonresonant circulation and resonant libration will appear in a Fourier analyses of the perturbed orbital parameters. [See Malhotra (1994) for a recent more detailed review of resonances in Solar system dynamics.]

A. The 3:2 Resonance

Cohen & Hubbard (1965) integrated the five outer planets for 120,000 yr and discovered that the orbit of Pluto is locked in a 3:2 mean motion resonance (commensurability) with Neptune. During every five centuries, Pluto makes two revolutions and Neptune three, and the two planets pass one another once. After five centuries the geometric pattern nearly repeats (see Figure 4). A resonance angle, ϕ, can be defined using the mean longitudes of Pluto and Neptune, λ and λ_N respectively, and the longitude of Pluto's perihelion, $\varpi = \Omega + \omega$,

$$\phi = 3\lambda - 2\lambda_N - \varpi$$

Cohen & Hubbard found that this argument librates about 180° with an amplitude of 76° and a period of 19,700 yr. (These numbers have been revised in more recent integrations with improved planetary parameters and numerical models; see below and section III-D.) The importance of the libration about 180° can be seen by writing the resonance argument as

$$\phi = M - 2(\lambda_N - \lambda),$$

where $M = \lambda - \varpi$ is Pluto's mean anomaly. For Pluto to be at perihelion ($M = 0$) while passing Neptune ($\lambda \approx \lambda_N$), the resonance argument, ϕ, would need to approach zero. Thus the libration of ϕ about 180° prohibits very close approaches between Neptune and Pluto and causes Pluto's conjunctions with Neptune (i.e., the configuration when the two planets share the same heliocentric longitude) to be closer to Pluto's aphelion than perihelion. Another way to understand the resonance protection is to note that the libration of ϕ about 180° means that at perihelion ($M = 0$), Pluto's mean longitude is near 90° away from Neptune's longitude, thereby avoiding conjunctions of the two planets when Pluto crosses the orbit of Neptune. This is shown in Figure 4 in a coordinate system rotating with Neptune's mean angular velocity.

Cohen & Hubbard showed that over approximately five-century cycles the distance between Pluto and Neptune has three minima, the smallest of them (18 AU) occurs when the planets have similar heliocentric longitudes and Pluto is near aphelion. The other two minima occur closer to Pluto's perihelion (at the small loops in Figure 4), but the longitudes of the two planets are very different and the distances are larger. Figure 5 illustrates how the distance between Pluto and Neptune changes during the 20 kyr libration. It is interesting to note that Pluto makes closer (and more frequent) approaches to Uranus than to Neptune (see Figure 6). However, the Pluto-Uranus distance varies so rapidly in successive close approaches that the Uranian short-period perturbations are periodic over only a few thousand years and do not accumulate significantly over longer time scales.

Subsequent 300 kyr and 1 Myr integrations (Cohen *et al.* 1967, 1973) revised the libration amplitude of ϕ to 80° and slightly shortened the libration period. Even longer numerical integrations since these original studies have confirmed the 3:2 resonance libration and the protection it provides against close approaches with Neptune (see Figure 7). These integrations find slightly different values of the libration amplitude and period of ϕ, and are discussed in more detail in section III-D below.

B. The Argument-of-perihelion Libration

The major planets exhibit sizable "secular" variations on time scales from 46 kyr to 2 Myr. These variations are not associated with the fast time scale of the orbit periods, but with the much slower precession of the perihelia and nodes. During Cohen & Hubbard's original 120 kyr integration the argument of Pluto's perihelion, ω, moved only 1.4°. Because the present perihelion and aphelion are 16° out of the plane of the ecliptic, the possibility remained that the 3:2 libration would not survive for times comparable to either the circulation of the perihelion or the secular perturbations. Even if the 3:2 resonance remained locked in libration, the possibility existed that the closest approach distance would be reduced when the encounter point got closer to the ecliptic plane. But commenting on the very slow argument-of-perihelion motion during 120 kyr, Brouwer (1966) suggested another possibility: ω might librate rather than circulate.

Kozai (1962) had shown that in the circular restricted three body problem, stationary and librating solutions for ω were possible for large inclinations of the test particle. [For a given mean motion, the stationary-ω solution lies on a curve in the eccentricity-inclination (e, i) plane, and belongs to a class of periodic orbits of the third kind in the three-dimensional restricted three body problem (see e.g., Jefferys & Standish 1966, 1972).] An early attempt (Hori & Giacaglia 1968) to analytically compute Pluto's orbit evolution based on three-body theory (Sun, Neptune, Pluto) failed to find the ω libration. However, subsequent work using semianalytic techniques and multiple perturbing planets did confirm the ω libration (Nacozy & Diehl 1974, 1978a,b).

It turns out that in the Sun-Neptune-Pluto three-body system, Pluto's eccentricity and inclination lie far away from the stationary ω curve in the (e, i) plane, but inclusion of the secular effects of the other giant planets in the perturbation potential for Pluto shifts this curve significantly, so that the argument of perihelion rate vanishes near Pluto's observed eccentricity and inclination.

A numerical integration of 4.5 Myr undertaken by Williams & Benson (1971) prior to Nacozy & Diehl's analytical work had already determined that the argument of perihelion librated around 90° with a 4 Myr period and a 24° amplitude. In the early days of digital computing, these authors had to use numerical averaging techniques to keep the computer usage modest. Thus, their numerical integration was in essence the same as the subsequent analytic solution of Nacozy & Diehl. The first direct numerical integration (no averaging techniques or semianalytic treatment) with a long enough span to confirm the ω libration was that of Kinoshita & Nakai (1984) who integrated the five outer planets for a time span of 5 Myr. They found that the ω libration had a 3.8 Myr period and an amplitude of 23°.

The 4 Myr periodicity of Pluto's argument of perihelion is accompanied by corresponding oscillations of the eccentricity and inclination, as expected from secular perturbation theory (Kozai 1962). The eccentricity and inclination variations maintain a phase difference of 180° from each other, and a 90° phase difference from ω. The peak-to-peak variation of Pluto's inclination is 2° while the eccentricity varies by 0.05. The phase relationships ensure that the extrema of these e and i variations (one a maximum and the other a minimum) occur when $\omega = 90°$. In other words, when the aphelion or perihelion contracts towards the Sun, its latitude increases.

The libration of ω keeps the closest approach point out of the plane of Neptune's orbit making the minimum distance larger than it would otherwise be. Both, the 3:2 mean motion resonance libration and the argument of perihelion libration tend to make the cumulative perturbations smaller in magnitude than they would otherwise be. The minimum distance between the two planets is 17 AU.

Longer integrations have corroborated the above facts further, and also found that the ω libration is modulated by a longer period of about 34 million years which causes its amplitude to vary between approximately 17 degrees and 27 degrees (cf. Applegate *et al.* 1986). The 34 Myr period is associated with yet another resonance, as discussed in the next section.

C. Other resonances

Williams & Benson (1971) had suggested that Pluto might exhibit two other "secular" resonances involving Pluto and Neptune's node and perihelion precession rates and Pluto's argument of perihelion libration, namely, (i) a 1:1 commensurability between the libration period of ω and circulation period of Pluto's node referred to the longitude of Neptune's node, $\Omega - \Omega_N$, and (ii) a 3:1 commensurability between the circulation periods of $(\Omega_N - \Omega)$ and $(\varpi - \varpi_N)$ that would cause the argument $(\varpi - \varpi_N) - 3(\Omega_N - \Omega)$ to be in libration. However, their 4.5 Myr integration was not long enough to confirm this possibility. Since that work, much longer integrations have been performed (100 Myr—11.2 Byr; see Table 2). Long period variations of Pluto's orbital elements were reported by Applegate *et al.* (1986) in their integration. Some of these were subsequently identified by Milani *et al.* (1989) with the secular resonances proposed by Williams & Benson.

Applegate *et al.* (1986) used a special purpose computer — the *Digital Orrery* — to numerically integrate the orbits of the outer planets (the four giant planets plus a zero-mass test particle representing Pluto) for approximately 217 Myr centered on the present epoch. This was a leap by a factor of ~ 40 over the longest outer planet integrations prior to that time. Pluto's orbit was determined to be stable, and the previously known librations of ϕ and ω were preserved over this time span. Some interesting new features also emerged. The ω libration was found to be modulated with a 34.4 Myr period, its amplitude varying between $\sim 17^\circ$ and $\sim 27^\circ$ (Figure 8). Because the variation of Pluto's i and e are strongly coupled through the ω libration, both these orbital elements also exhibit significant modulation. The 3.8 Myr oscillations of i were found to be strongly modulated by the 34 Myr period, and there were indications of even longer period variations; the Poincaré variable $h = e \sin \varpi$ was reported to exhibit a strong modulation with a 27 Myr period, as well as a 137 Myr period.

Sussman & Wisdom (1988) extended the Digital Orrery integration to a time span of 845 Myr. This integration confirmed all the above features in Pluto's motion. They also reported a 150 Myr periodicity in the variations of Pluto's inclination, and indications of an even longer period of approximately 600 Myr. In addition, they found evidence of chaotic behavior (see below).

An independent 100 Myr numerical integration of the outer planets was performed by the LONGSTOP project (see Nobili 1988 for a review of this work). In a thorough paper on the analysis of their numerical solution for Pluto, Milani *et al.* (1989) confirmed the 1:1 commensurability between the libration period of ω and the circulation period of $(\Omega - \Omega_N)$. They referred to this as a "super-resonance", and identified the 34.5 Myr modulations with its libration period. This resonance has a geometrical consequence when considering the inclination of Pluto's orbit: the phase of the 3.8 Myr, 2° peak-to-peak variation in the inclination with respect to the invariable plane is synchronized with the precession of Neptune's orbit plane, so that Pluto's inclination with respect to Neptune's orbit plane has only a 1° peak-to-peak variation.

Milani *et al.* also attempted to determine the origin of the longer period perturbations and the signature of chaos detected in the Orrery calculations. They identified a second "super-resonance": the difference of the longitudes of perihelion of Pluto and Neptune, $(\varpi - \varpi_N)$, circulates with a period of 1.267 Myr, very close to 1/3 of the 3.8 Myr circulation period of $(\Omega_N - \Omega)$. (Owing to the existence of the first super-resonance, this guarantees a commensurability with the ω libration also.) In the LONGSTOP integration, the combination of angles, $(\varpi - \varpi_N) - 3(\Omega_N - \Omega)$, was close to but not locked in resonance. This judgement was based on less than one-half cycle of the resonance angle. However, in the Orrery integration, the average rate of this angle was indistinguishable from zero within numerical resolution. Milani *et al.* have suggested that the origin of the chaos in the Orrery calculation could be this super-resonance.

More recently, a 100 million year integration has been published by Levison & Stern (1995), and a long 5.5 billion year integration has been done by Kinoshita & Nakai (1995). The 20 kyr, 3.8 Myr, and 34 Myr librations in Pluto's orbit are confirmed. Kinoshita & Nakai detected chaotic behavior, but it was not strong enough to be obvious in plots of the orbital elements, which showed similar behavior throughout the length of the integration. Nakai & Kinoshita (1995) added a 5.7 Byr backward integration to the 5.5 Byr forward integration for a total of 11.2 Byr. Both of these spans [as well as an earlier 1.3 Byr integration (Nakai *et al.* 1992, Nakai & Kinoshita 1994)] show libration of the argument $(\varpi - \varpi_N) - 3(\Omega_N - \Omega)$ about 180°. For the 11.2 Byr interval, the libration period is 590 Myr and the amplitude is about 100°.

D. Comparison of integrations

The accuracy of Pluto's orbital elements has improved with time, and our knowledge of the outer planet masses benefited greatly from the Voyager flybys. The influence of mass correction and orbit uncertainties on the long-term orbit evolution will be considered here. We will mostly discuss the 217 Myr integration by Applegate *et al.* (1986) and the 100 Myr integration of Milani *et al.* (1989) as these are the longest that have been analyzed for their fundamental frequencies and amplitudes; but we will also note comparisons, when appropriate, with the $5.5 Byr$ integrations by Kinoshita & Nakai (1995) which used recently updated planetary parameters and initial conditions.

Most of the integrations have used a Neptune mass which is 0.51% larger than the Voyager value; the value used by Milani *et al.* is 0.10% smaller than Voyager's. To a good approximation the 3:2 libration frequency scales as the square root of Neptune's mass. When corrected to the Voyager value, the libration periods from both Milani *et al.* and

Applegate *et al.* are within 6 yr of 19,912 yr.

The 3:2 libration amplitude is very sensitive to the measured mean motion. This libration causes Pluto's mean motion to oscillate by 0.6% about its mean value. Consequently, the relative amplitude and phase uncertainty is greater than the relative mean motion uncertainty by two orders-of-magnitude. For the mean motion uncertainty given in Table 1, the libration amplitude uncertainty is $\sim 0.1°$. Kinoshita & Nakai (1995) give a libration amplitude of 81.2°. From the saturation distance between two initially neighboring orbits given by Nakai & Kinoshita (1995), one can infer an amplitude of 82°. However, owing to a small coupling with the ω libration, the 3:2 resonance libration center oscillates by $\sim 3-4°$ about 180° (Williams & Benson 1971, Milani *et al.* 1989). Therefore, the maximum deviation of the resonance angle ϕ from 180° is larger than the libration amplitude. For the peak deviation, Milani *et al.* give 84°, Nakai & Kinoshita (1994) give 84.9°, while Applegate *et al.* and Nakai & Kinoshita (1995) give 86°. [Levison & Stern (1995) erroneously attribute this small oscillation of the libration center to "a random variation" of the libration amplitude.]

The libration period for the argument of perihelion, ω, is near 3.8 Myr. Applegate *et al.* give 3.796 Myr and Milani *et al.* find 3.783 Myr. The 0.3% difference is plausibly due to mass differences since the libration frequency depends upon the square root of a linear combination of all of the outer planet masses, but the coefficients of the linear combination are not known. Assuming that the libration frequency is dominated by Neptune's mass gives the right size correction, but the wrong sign.

Because the ω libration is resonant with the difference in the nodes of Neptune and Pluto, $(\Omega_N - \Omega)$, consideration of the sensitivity of the node precession rate to outer planet masses should permit reconciliation. This can only be done approximately, but the approximation from Laplace-Lagrange theory is better known than that for the ω libration. For Pluto's average nodal rate, both integrations correct to $-0.3502''$/yr. The average longitude of perihelion rate will be the same (owing to the ω libration). For Neptune's average nodal rate, the discrepancy between the two integrations is made worse by adding a mass correction ($-0.6921''$/yr for Applegate *et al.* and $-0.6930''$/yr for Milani *et al.*). When corrected for mass, the periods for a full cycle of $(\Omega_N - \Omega)$ are 3.791 Myr and 3.781 Myr, and these should match the ω libration periods.

Neptune's average longitude of perihelion rate is also of interest. Here the agreement is good and the mass-corrected value is $0.6730''$/yr. The difference in the longitudes of perihelion of Pluto and Neptune, $(\varpi - \varpi_N)$, makes a complete circulation in 1.267-Myr, very close to 1/3 of the periods of the ω libration and the circulation of $(\Omega_N - \Omega)$. For the argument $(\varpi - \varpi_N) - 3(\Omega_N - \Omega)$, Milani *et al.* concluded that their integration gave circulation (with rate $0.005''$/yr), not libration. On the basis of a near-zero rate in the Applegate *et al.* integration, Milani *et al.* suggested that this resonance was present in that integration, and further suggested that it might be linked to chaotic behavior. Different masses and initial conditions in the two integrations may be the reason for this apparent conflict. Because the long libration period of this resonance (590 Myr in Nakai & Kinoshita 1995) implies a narrow resonance width, strong sensitivity to masses and initial conditions is to be expected. Kinoshita & Nakai (1995) and Nakai & Kinoshita (1995) found a librating argument using recent (DE245) masses and initial conditions, so we do not need to go through the exercise of "mass-correcting" the earlier results. It may be that Milani *et al.* did not find libration because their integration span was less than half of the libration period and would have been on the more linear portion of the sinusoidal curve. We note that the rate on the same segment of Kinoshita & Nakai's curve is $0.004''$/yr.

The difficulty in accounting for the differences in these rates in the two integrations is due to some combination of the approximations used for the mass sensitivities, the resonances, different initial conditions for the planets, and modeling. Improved planetary masses and initial conditions are now available and their use by Kinoshita & Nakai is a welcome development.

As a final point, we note that in all the integrations the average ω rate is zero, but its libration (with a 3.8 Myr period) causes the rate to vary with time. The average ω libration amplitude from the long integrations is between 21° and 22°, and the 34 Myr modulation causes it to vary between 17° and 27°. There should be a small influence of the difference in the assumed Neptune mass, approximately 0.1°, on the amplitude.

IV. DYNAMICAL STABILITY AND CHAOS

The dominant perturbations on Pluto's motion arise from Neptune. The simplest dynamical model for analyzing these perturbations is to consider the planar, restricted three-body problem consisting of the Sun and Neptune as the massive primaries in circular orbits about their center of mass, and Pluto as a massless test particle. This is a reasonable starting point because Pluto's mass is only $\sim 10^{-4}$ that of Neptune, and Neptune's eccentricity and inclination are both very small. Furthermore, this model has the important advantage that the structure of the phase space can be visualized in a 2-D surface-of-section. In such a picture, quasiperiodic (i.e. secularly stable) motion appears as points that lie on a smooth, closed curve, while chaotic (or secularly unstable) motion appears as points that fill up a 2-D region (cf. Henon

1983). One such surface-of-section is given in Figure 9 which shows the structure of the phase space in the vicinity of the 3:2 Neptune-Pluto resonance. In obtaining this surface-of-section, the value of the Jacobi integral was set equal to that for the observed Pluto but with its inclination suppressed. [The plot shown is actually a *pseudo*-surface-of-section, as the dynamical variables, ϕ (the resonance angle), and the semimajor axis, a, are not canonical variables. See Malhotra (1996) for more details.] It is obvious from this figure that stable librations are possible only in a narrow region of the phase space. The approximate half-width of this stable resonance region in terms of semimajor axis is only $\Delta a \approx 0.5AU$. In terms of the resonance angle, ϕ, librations with amplitude greater than $\sim 130°$ are chaotically unstable on very short timescales, $\mathcal{O}(10^5)yr$. The ϕ libration amplitude of Pluto inferred from direct numerical integrations is $\sim 82°$, well inside the stable region. Thus, within this approximate model, Pluto's motion is bounded and stable for all time.

The question naturally arises whether the motion remains stable in the realistic case in which Pluto has a non-zero inclination, Neptune's orbit is not on a fixed circle but has a small eccentricity and inclination, and the perturbations of the other planets are also included. It can be argued that the third degree of freedom — *i.e.* Pluto's inclination — by itself will not make the orbit unstable as it actually decreases the magnitude of the perturbations on Pluto. However, taking account of the non-circular orbit of Neptune and the perturbations of the other planets introduces new dynamical features whose effects on Pluto's long-term orbital stability are more difficult to analyze. The most significant of these is the ω libration described in the previous section. As noted there, the high inclination of Pluto's orbit together with the ω libration helps to keep Pluto's perihelion out of the ecliptic plane, and therefore helps reduce the magnitude of the planetary perturbations. Other, weaker resonances (i.e. the "super-resonances" in Milani *et al.* 1989) that have been identified in the long term numerical integrations of the outer planets also have the effect of increasing the closest approach distance between Pluto and Neptune. On the other hand, it is well known that resonance regions are accompanied by chaotic zones in phase space. (This is evident in the surface-of-section shown in Figure 9.) The relevant question, therefore, is whether the dynamical protection mechanisms remain robust for a sufficiently wide range of initial conditions and parameters that encompass those of the actual Solar system. This question has been addressed in several numerical studies in recent years and we discuss it in detail below.

A. Lyapunov exponent

Chaotic solutions of a dynamical system are characterized by an extreme sensitivity to initial conditions which is most directly measured by the maximal Lyapunov exponent, Γ. Γ measures the rate of exponential divergence of two trajectories in phase space that initially are arbitrarily close to each other:

$$\Gamma = \lim_{d(0)\to 0} \lim_{t\to\infty} \frac{\ln[d(t)/d(0)]}{t}.$$

Here $d(0)$ is the initial separation in phase space and $d(t)$ the separation at time t. In a regular region of phase space, Γ is zero; in a chaotic region it is finite and positive. In practice, in numerical experiments one determines the so-called finite-time maximal Lyapunov exponent,

$$\gamma = \frac{\ln[d(t)/d(0)]}{t},$$

where $d(0)$ is small but non-zero. Then, at increasingly large t, γ asymptotically approaches Γ. The associated timescale for chaotic divergence of orbits is $T_L = \Gamma^{-1}$.

The Lyapunov exponent, γ, for Pluto's motion has now been determined in several long numerical integrations. In the first of these (Sussman & Wisdom 1988), the orbits of the four massive outer planets and a massless "Pluto" were integrated for a period of 845 Myr using a special purpose computer (the *Digital Orrery*) and the multistep Stormer integrator. The Lyapunov timescale, T_L, was found to be 20 Myr (see Figure 10). The same model was integrated for 1 Byr using a symplectic mapping method (Wisdom & Holman 1991). In this work, Pluto's Lyapunov exponent was reported to be "consistent with" that obtained in Sussman & Wisdom (1988). More recently, several different numerical experiments were reported in Sussman & Wisdom (1992). In one of these, all nine planets were integrated for 100 Myr; in the other experiments, only the four outer planets plus a massless Pluto, were integrated for time periods ranging from 250 Myr to 1 Byr. Each of these runs yielded a positive Lyapunov exponent for Pluto, with Lyapunov timescale between 10 Myr and 20 Myr. In other independent calculations, a Lyapunov timescale of 18 Myr was found in a 1.3 Byr integration by Nakai *et al.* (1992), and about 20 Myr in two integrations of 5.5 Byr and 11.2 Byr (Kinoshita & Nakai 1995, Nakai & Kinoshita 1995). The value of the Lyapunov exponent obtained is evidently somewhat sensitive to the integration method and step-size used, as well as to slight differences in the modeling and in planetary masses and initial conditions.

The chaotic character of a dynamical system also manifests itself in the power spectrum of its dynamical variables. For regular (quasiperiodic) motion, the power spectrum has discrete lines composed of linear combinations of the fundamental frequencies of the system. However, for irregular (chaotic) motion, the power spectrum has a broadband component. In their 845 Myr integration, Sussman & Wisdom (1988) reported just the latter type of spectrum for Pluto's $h = e \sin \varpi$, thus providing corroboration for the chaotic character of their numerical solution for Pluto's orbit.

B. Stable chaos?

The determination of a positive Lyapunov exponent for a dynamical system is usually a quantitative confirmation of chaos that is readily apparent in the time evolution of its dynamical variables. However, Pluto's orbit has now been integrated for more than 500 times its Lyapunov time, and yet no obvious chaotic behavior is to be found in the evolution of its orbital elements.

Could Pluto's motion be a case of "stable chaos"? An example of "stable chaos" has been reported recently in a numerical study of the long term evolution of asteroid 522 Helga (Milani & Nobili 1992). This asteroid has a Lyapunov time of only 6900 years, yet its orbit remains narrowly confined for more than 1000 times its Lyapunov timescale. Other examples are described in Gladman (1993) where chaotic orbits are found to be bounded for times as long as 10^5 Lyapunov times! Perhaps this should not come as a complete surprise: a positive Lyapunov exponent is a measure of a local instability only; it does not necessarily imply large-scale chaotic behavior.

All the long integrations to date show no large-scale instability for Pluto's motion on billion-year timescales. In their 845 Myr integration, Sussman & Wisdom (1988) reported that the divergence of two initially nearby Plutos saturates at a distance of $\sim$45 AU. It was pointed out by Milani *et al.* (1989) that this saturation should be expected if the different Plutos remain in approximately the same orbits, but simply diverge in phase while preserving the libration of ϕ with an amplitude near 80°. This point has been confirmed recently in Kinoshita & Nakai (1995) who found by direct calculation that the deviations of mean longitude and mutual distance of two initially nearby Plutos saturate (after approximately 420 Myr) at 70° and 44 AU, respectively, while the deviation of the resonance angle saturates at exactly twice its amplitude of libration. This suggests that the chaos detected by Sussman & Wisdom does not affect the stability of the 3:2 resonance libration. Milani *et al.* have argued that the origin of the chaos is one of the weaker super-resonances and that the positive Lyapunov exponent indicates that Pluto may be near a chaotic orbit associated with that resonance. Kinoshita & Nakai (1995) and Levison & Stern (1995) have integrated a variety of orbits similar to Pluto. Some of these did not show libration of the first super-resonance and few showed libration of the second one. Absence of these two resonances did not cause obvious chaotic instability in the orbital evolution. However, quite dramatic changes were caused by increasing the amplitude of the 3:2 libration: when the amplitude of ϕ was increased above $\sim 110°$, the ω libration was destroyed and the orbital elements showed chaotic variations. These results indicate that the macroscopic stability of Pluto's orbit is determined primarily by the 3:2 resonance lock.

Thus it is reasonable to conclude that, if Pluto does indeed live in a chaotic zone, that zone is exceedingly narrow. Whether it is connected to a larger chaotic zone which would allow large scale chaotic changes in its orbit [such as happens to asteroids near the 3:1 Kirkwood Gap (Wisdom 1988)] remains an open question. In this context, we note that Lecar *et al.* (1992) have found an empirical correlation between the Lyapunov timescale and the timescale for macroscopic instability for asteroidal orbits. A similar correlation was found for orbits in the Kuiper Belt (Levison & Duncan 1993). If such a relation applies to Pluto-like orbits, it suggests that Pluto's orbit is macroscopically stable for timescales of $\sim 10^{11}$ yr. However, it is prudent to be cautious in this matter, for the physical causes of these correlations are not yet understood, and it is not known whether Pluto belongs in the class of objects where this relation applies.

V. ORIGIN OF PLUTO'S ORBIT

According to the accepted paradigm for the origin of the Solar system, the planets accumulated in a flattened disk of dust and gas orbiting the young Sun approximately 4.5 Byr ago. Internal dissipative processes efficiently damped the random non-circular and out-of-plane motions of the forming planets, and, as a result, the major planets move on nearly circular and co-planar orbits. Mercury and Pluto are the striking exceptions to this general rule, Pluto being the more extreme case.

The earliest speculation about the origin of Pluto was a suggestion by Lyttleton (1936) that Pluto may have been a satellite of Neptune which escaped into a heliocentric orbit due to a rare catastrophic event. The main observations that led to this suggestion were that Pluto's orbit crosses that of Neptune, and that Neptune itself possesses a large satellite, Triton, similar in brightness to Pluto. Various means of accomplishing such escape have been considered in the literature (Lyttleton 1936, Horedt 1974, Harrington & van Flandern 1979, Farinella *et al.* 1979, Dormand & Woolfson 1980). This hypothesis has fallen out of favor in recent years as a result of the recognition of the dynamical constraints imposed by the existence of the 3:2 orbital resonance with Neptune, and also the improved knowledge about the characteristics of the Pluto-Charon system which strongly support the formation of both these bodies in heliocentric orbit. The detailed arguments are reviewed in the chapter by Stern, McKinnon & Lunine.

If Pluto's formation were similar to that of the other planets, it would have formed in a near-circular, low-inclination orbit about the Sun. Indeed, it may have been one of many small icy planets that formed in the outer planetary region (Stern 1991). Its peculiar orbit must then be explained as a result of post-formation dynamical processes. Pluto's long term dynamical stability is owed primarily to the protection afforded by a sufficiently small libration amplitude about the 3:2 Neptune resonance. Its emplacement in this very narrow stable region in phase space strongly suggests the role of some dissipative mechanism in its early dynamical evolution. Here we summarize two scenarios proposed recently that appear promising. In both these scenarios, Pluto formed in a low-e, low-i orbit beyond Neptune, and outside the 3:2 resonance; and both require a dissipative process to evolve Pluto into its resonant Neptune-crossing orbit.

A. Resonance capture

One of us (Malhotra 1993a) has proposed that Pluto may have been captured into the 3:2 Neptune resonance during the late stages of planet formation, when Neptune's orbit expanded as a result of angular momentum exchange with residual planetesimals. Resonant phase-locking as a result of some slow dissipative process is a phenomenon well-known in nature. In the Solar system, orbit-orbit resonances (as well as spin-orbit resonances) are commonly found amongst the satellites of the giant planets. Capture into an orbit-orbit resonance occurs when the orbits of two bodies approach each other through some dissipative process. The origin of orbital resonances amongst planetary satellites, thought to be due to tidal friction, has been extensively studied in the literature (see Peale 1986 and Malhotra 1994 for reviews).

The mechanism for the capture of Pluto into the exterior 3:2 Neptune resonance proposed by Malhotra (1993a) is summarized as follows. The giant planets' gravitational perturbations cleared out their interplanetary regions by scattering the unaccreted mass of planetesimals. Some fraction of this mass now resides in the Oort Cloud of comets in a roughly isotropic distribution surrounding the planetary system (e.g. Weissman 1990), but most has been lost from the planetary system. A planetesimal scattered outward gains angular momentum, while one scattered inward loses angular momentum at the expense of the planets. The back reaction of planetesimal scattering on the planets caused the planetary orbits to evolve. Consider the evolution of a planetesimal swarm in the vicinity of Neptune. The mean specific angular momentum and energy of the swarm is initially approximately equal to that of Neptune. At first, a small fraction of the planetesimals would be accreted, and of the remaining, approximately equal numbers would be scattered inward as outward. To first order, these cause no net change in Neptune's orbit. However, the subsequent fate of the inward and outward scattered planetesimals is not symmetrical. Most of the inwardly scattered objects enter the zones of influence of the inner Jovian planets (Uranus, Saturn and Jupiter). Of those scattered outward, some are lifted into wide, Oort Cloud orbits while others return to be reaccreted or rescattered; a fraction of the latter is again rescattered inward where the inner Jovian planets control the dynamics. In particular, Jupiter, due to its large mass, is very effective in causing a systematic loss of planetesimal mass by ejection into Solar system escape orbits. Therefore, as Jupiter preferentially removes the inward scattered Neptune-zone planetesimals, the planetesimal population encountering Neptune at later times is increasingly biased towards objects with specific angular momentum and energy larger than Neptune's. Encounters with this planetesimal population produce a negative drag on Neptune which causes Neptune to gain orbital energy and angular momentum; as a result, its orbit expands. Jupiter is in effect the source of this angular momentum and energy; however, owing to its much larger mass, its orbit shrinks by only a small amount. This effect was first found in numerical simulations by Fernandez & Ip (1984).

If the above phenomenon did occur in the late stages of planet formation, it has profound consequences for the dynamical history of Pluto (and, indeed, for any primordial small bodies in the outer Solar system). If Pluto were initially in a nearly circular and co-planar orbit beyond the orbit of Neptune, then as Neptune's orbit expanded, its exterior orbital resonances approached Pluto. In particular, if Pluto's initial orbital radius were such that the 3:2 resonance was the first major Neptune resonance to sweep by, Pluto would be captured into this resonance. Capture into resonance would be certain if Pluto's initial eccentricity were smaller than ~ 0.03; the capture probability would be smaller for higher initial eccentricities (10% for $e_{\text{initial}} \approx 0.15$). [See Henrard & Lemaitre (1983) and Borderies & Goldreich (1984) for the formulas for capture probability.] In the subsequent evolution, as Neptune's orbit continued to expand, the resonant perturbations increased Pluto's orbital eccentricity. Malhotra derives the following relation between Pluto's eccentricity and Neptune's semimajor axis:

$$e^2_{\text{final}} - e^2_{\text{initial}} \approx \frac{1}{3} \ln \left(\frac{a_{\text{N,final}}}{a_{\text{N,initial}}} \right).$$

This equation shows that Pluto's current eccentricity would have been produced by its capture into the 3:2 resonance when Neptune's semimajor axis was approximately 25 AU ($\sim$5 AU less than its current value). By implication, Pluto's initial orbital radius was $\sim$33AU. This equation also indicates a rather weak dependence of the final eccentricity on the initial e. We emphasize that in this scenario, Pluto is initially not in a Neptune-crossing orbit. As the evolution within the resonance forces the high eccentricity on Pluto, the libration also provides protection against close approaches during the entire evolution.

The above analysis takes account of the perturbations of only Neptune on Pluto, and explains the origin of Pluto's 3:2 resonance lock with Neptune and its high orbital eccentricity. This was confirmed in numerical simulations presented by Malhotra (1993a). In those simulations, the orbits of the outer planets were integrated self-consistently (except that Pluto was treated as a massless 'test particle') with a model where the giant planet orbits evolve adiabatically. In subsequent work, using a larger number of 'test Plutos' and longer integration times, Malhotra (1995a,b) has shown that the model with migrating giant planets which sweeps the trans-Neptune Solar system with mean motion resonances is able to account for all the major dynamical properties of Pluto's orbit, *i.e.* the observed eccentricity, the libration amplitude of ϕ, as well as the high inclination and the argument-of-perihelion libration. However, the inclination amplification to values as large as Pluto's observed inclination was found in only a small fraction, $\lesssim 10\%$, of the 'test Plutos'. [This explains why it was not found in the first results presented in Malhotra (1993a).] The importance of the multi-planet perturbations arises from the special circumstance that there exists a nodal secular resonance in the neighborhood of the 3:2 Neptune resonance (Knezevic *et al.* 1989). The amplification of the inclination is probably due this secular resonance; the argument-of-perihelion libration is also due to secular effects (see section III-B).

The formation of the Pluto-Charon binary pair is not addressed in Malhotra's model. However, a pre-existing binary formed in a low-e, low-i nonresonant orbit would undergo the same resonance capture and subsequent evolution outlined above.

B. Chaos + collisions

One scenario for the origin of Pluto's orbit that has been around in a general way is the Darwinian survival-of-the-fittest: namely, that Pluto was one of a swarm of similar small bodies which were continuously scattered by their mutual collisions into and out of the 3:2 resonance with Neptune; Pluto simply happened to be the one that survived to the present time in its protected orbit, whereas the other bodies were removed by collisions with the giant planets. The existence of Triton, Pluto and Charon lends support to the idea that there were other similar ice-dwarf planets in the outer Solar system (Stern 1991).

Numerical calculations by Applegate *et al.* (1986), Kinoshita & Nakai (1984), and Olson-Steel (1988) have indicated that, given the current masses and orbits of the giant planets, most Pluto-like orbits near the 3:2 Neptune resonance exhibit large scale chaotic variations over very short timescales, $\sim 10^5$yr. [The origin of this behavior is discussed in Malhotra (1996).] Recent numerical integrations by Holman & Wisdom (1993), Levison & Duncan (1993) and Levison & Stern (1995) have further explored the orbital dynamics near the 3:2 Neptune resonance.

In these studies, it was found that test particles placed in initially low-eccentricity, low-inclination orbits just outside the 3:2 Neptune resonance evolve rapidly (on $\sim 10^6$yr timescales) into orbits with high eccentricity and inclination similar to that of Pluto. In particular, with some fine-tuning of initial conditions (initial semimajor axis in the narrow range $39.48AU \leq a \leq 39.65AU$), Levison & Stern find that a small fraction — approximately 5% — of these orbits exhibit librations about the 3:2 resonance, but with large, chaotically varying amplitude. The dynamical lifetime of such orbits before a close encounter with Neptune is typically short, $\mathcal{O}(10^7)$ yr, although in a few cases, the time to first Neptune encounter can be several hundred million years.

Levison & Stern suggest that a "Pluto" on such a chaotic orbit may be nudged into the stable 3:2 resonance libration region (with a final libration amplitude of $\phi \lesssim 90°$) by means of gravitational scattering interactions or inelastic collisions with other primordial Kuiper Belt objects. They considered two mechanisms for stabilizing the orbit: (i) damping the resonance libration amplitude to a stable value by a slow diffusion of orbital elements due to gravitational interactions of Pluto with a large number of small bodies, and (ii) a single giant impact knocking Pluto (or a pre-existing Pluto-Charon binary) into a stable orbit; they also considered the possibility of forming the Pluto-Charon binary in such an orbit-stabilizing impact. From their numerical modeling, they concluded that the first mechanism was not efficient unless the Kuiper Belt were several orders of magnitude more massive (during the orbit-stabilization epoch) than its estimated mass at the present epoch. For the second, they found that a single giant impact with a Charon-sized impactor can, in principle, stabilize Pluto's orbit and simultaneously produce a binary with properties similar to the Pluto-Charon binary. Both scenarios were found to be able to account for the major dynamical properties of Pluto's heliocentric orbit. Because Levison & Stern's scenarios require a series of probabilistic events, their cumulative probability needs to be evaluated.

VI. SUMMARY AND FUTURE DIRECTIONS

The heliocentric orbit of the Pluto-Charon pair has been observed for nearly one-third of its orbit period. Ephemerides of the pair (collectively called "Pluto" in this chapter) fit the known observations well.

However, one must bear in mind that the span of observation is still much less than an orbit period, and there is a need for future positional observations and orbit fits. A spacecraft mission to Pluto would need ephemerides of high accuracy.

Pluto's orbit is the most eccentric and inclined of the major planets (Figure 1). At perihelion it ventures closer to the Sun than Neptune, seemingly violating the well-spaced, hierarchical pattern of the other planets that is generally associated with long-term orbital stability. Investigation of the dynamical evolution of this configuration shows a surprisingly complex behavior involving four resonances.

1. Every 494 years Pluto orbits the Sun twice while Neptune orbits three times. The resonant perturbations from Neptune cause Pluto's orbital period to librate about 248 yr and the resonance argument (cf. Eq. 1) to librate about 180° with an amplitude of 82°. The libration period is 19,912 yr. This orbital resonance lock prevents Pluto from passing close to Neptune; when the two planets have the same heliocentric longitude, Pluto is closer to its aphelion than its perihelion (Figure 4). This resonance is so effective at keeping the two planets apart that Pluto approaches Uranus more closely than Neptune. The 3:2 resonance is a strong stabilizing influence on Pluto's orbit. Indeed, Pluto-like orbits just outside of the resonance libration region display obvious chaotic behavior in only a few million years.
2. Pluto's argument of perihelion does not precess through 360°; rather, it librates about a value of 90°. The libration period is 3.8 Myr and its amplitude averages 21°. Thus, Pluto's perihelion and aphelion never cross Neptune's orbit plane. This also helps keep the two planets apart.
3. The difference between Pluto and Neptune's nodal longitudes circulates every 3.8 Myr — the same period as that of the argument-of-perihelion libration. The frequencies of these two angles are locked in a 1:1 commensurability, and this resonance has a libration period of 34 Myr. The argument of perihelion librations, as well as the eccentricity and inclination variations, are modulated by this period.
4. The difference of the longitudes of perihelion of Neptune and Pluto circulates in 1.267 Myr — one-third of the period of the argument of perihelion libration, and also of the circulation period of the node difference. This weak resonance has a 590 Myr libration period.

That Pluto's orbit is chaotic is indicated by the positive Lyapunov exponent determined in several different long numerical integrations. However, the integrations which indicate the presence of chaos do not show obvious erratic changes in the orbital elements that one might expect with the short Lyapunov time of $\mathcal{O}(10^7)$yr. Indeed, all the evidence suggests that the protection accorded Pluto by the 3:2 Neptune resonance is robust over billion-year timescales. There remains a degree of uncertainty in the magnitude of the Lyapunov timescale, as values differing by factors of a few have been obtained in different numerical integrations. The origin of the chaos remains unknown, as well. The fourth resonance described above is one possible, but unproven, source of the chaos; it does illustrate an important point: the magnitude of the chaotic orbital perturbations may be small and bounded, or, if unbounded, may require times much longer than the age of the Solar system to threaten Pluto's dynamical state. Nevertheless, a Lyapunov timescale of $\mathcal{O}(10^7)$yr implies a relatively short horizon of predictability for its exact position and velocity.

Is it significant that small differences in modeling or integration methods result in Lyapunov exponents differing by a factor of several? If Pluto's orbit is chaotic, what is the origin of the chaos, and how is it manifested in its orbital element evolution? Because Pluto's chaotic motion may be associated with a narrow chaotic zone, it is desirable to eliminate the uncertainties in past numerical integrations due to (now known) errors in planetary masses and orbital initial conditions. The Voyager flybys of the outer planets have provided a much improved set of outer planet masses and continued analyses of positional data sets has provided compatible ephemerides for the planetary initial conditions. Future numerical integrations should take advantage of these improvements. The recent work by Kinoshita and Nakai is a good beginning. Detailed studies of the dynamics of each of Pluto's resonances may provide clues to the underlying causes of the large-Lyapunov-exponent-without-large-scale-chaotic-behavior.

What role do the various resonances play in the origin and continued existence of the Pluto-Charon system? Because some of the resonances are protective in that they increase the minimum distance between Pluto and Neptune, is it possible that a dynamical "survival of the fittest" has left objects only in protected orbits? Or, instead, has some dissipative process in Solar system history caused these (and perhaps also other) bodies to be swept into the resonances? We have discussed two origin scenarios proposed recently (Malhotra 1993a,1995b; Levison & Stern 1995). Both these scenarios suggest the formation of Pluto in an initially non-resonant, nearly circular and co-planar orbit in the outer reaches of the Solar system, beyond the orbit of Neptune.

In the scenario proposed by Malhotra (1993a), the adiabatic evolution of the giant planet orbits during the late stages of their formation caused the trans-Neptune region to be swept by orbital resonances; Pluto was captured into the 3:2 orbital resonance with Neptune and its eccentricity was pumped up to a Neptune-crossing value during that evolution. Secular resonant effects amplified its inclination, while other non-resonant secular effects account for the argument-of-perihelion libration. The magnitude of the radial migration of Neptune (as well as of the other giant planets) implied in this theory has other theoretical and observational consequences. Malhotra (1995b) has examined in detail the efficiency of resonance capture and its implications for the evolution of other small bodies in the trans-Neptune region. A major consequence of her model for the origin of Pluto's orbit is that the distribution of small objects in the Kuiper Belt would be highly non-uniform, with high concentrations of objects trapped in the major orbital resonances with Neptune. The 3:2 and the 2:1 Neptune resonances (located at $a \approx 39.4$ AU and $a \approx 47.8$ AU) are the most efficient at sweeping up mass, but other resonances, notably the 4:3 and the 5:3 (located at 36.5 AU and 42.3 AU, respectively) also have significant capture probabilities. Objects swept up in these resonances during the radial migration of the giant planets would have had their orbital eccentricities pumped up to values typically in the range 0.1–0.3. Thus, this scenario is falsifiable by means of a census of orbital distribution of objects in the Kuiper Belt. The discovery of several small objects in recent and ongoing observational surveys of the outer Solar system with orbits that are consistent with the predictions of this model may lend support to this model of the origin of Pluto's orbit (Jewitt & Luu 1995; Cochran *et al.* 1995, Marsden 1995). A conclusive evaluation must await a robust determination of orbits of a sufficiently large and unbiased sample of the Kuiper Belt population.

The second scenario that we have discussed for the origin of Pluto's orbit is based on the observation that given the current configuration of the planetary system, there is a narrow zone just outside the exterior 3:2 Neptune resonance where circular, zero-inclination orbits are unstable, and the perturbations of the planets amplify the eccentricity and inclination to values similar to that of Pluto's orbit (Holman & Wisdom 1993, Levison & Duncan 1993). However, objects in such orbits remain chaotic and eventually suffer a destructive close encounter with Neptune on timescales ranging from 10^7 to 10^9 yr. Levison & Stern (1995) propose that gravitational scattering and/or inelastic collisions between the proto-Pluto and other Kuiper Belt objects could have knocked Pluto from an initially chaotic orbit in the vicinity of the 3:2 Neptune resonance into the stable libration region, possibly simultaneously forming the Pluto-Charon binary. Their numerical modeling also shows that the major dynamical properties of Pluto's heliocentric orbit as well as the Pluto-Charon binary can be accounted for in this manner.

Much work remains to be done to establish the details of each of these models. With regard to Malhotra's model, improved calculations of the orbital evolution of the giant planets during the late stages of their formation are called for. This is important both for establishing the fundamental assumption in her model of the outward resonance sweeping in the trans-Neptune region, and also for making quantitative comparisons with forthcoming observational data on the mass distribution in the Kuiper Belt. With regard to Levison & Stern's model, the plausibility of their scenario remains to be evaluated considering the following assumptions of their model: (*i*) Pluto's initial low-e, low-i orbit is required to be in a very narrow range of a; (*ii*) their model relies upon the current configuration of the giant planets; is the e, i excitation mechanism sufficiently insensitive to the evolution of the outer planet masses and orbits during their formation? and (*iii*) the probability of a Charon-forming impact during the period of time after Pluto becomes Neptune-crossing and before it has a close encounter with Neptune needs to be evaluated.

Finally, we should note that mean motion resonance locks are vulnerable to collisional disruption, as discussed in some detail in Malhotra (1993b). The strength of the Neptune-Pluto 3:2 resonance can therefore yield estimates of impactor masses in the outer Solar system. The current 3:2 Neptune-Pluto resonance lock has a high probability of being destroyed by an impact on Neptune of mass $\sim 10^{27}$ g (Malhotra 1993a), and by an impact on Pluto-Charon of mass similar to Charon's mass [see Hahn & Ward (1995) for an analytic derivation, Levison & Stern (1995) for an estimate from numerical simulations]. The former provides an upper limit for Neptune impactors post-dating the formation of the Pluto-Neptune resonance, while the latter shows that the formation of the Pluto-Charon binary by a giant impact post-dating the resonance capture is unlikely as it has a high probability of dislodging these bodies from the stable libration region. Therefore, it is likely that Pluto and Charon were transported together (as a binary planet) to their current orbit.

The fascinating dynamical complexity of Pluto's orbital evolution could not have been guessed at its first sighting as a moving point of light in 1930. A large number of questions invites further study of the Pluto-Charon pair. The orbit of this pair of bodies at the edge of the planetary system may hold unanticipated clues to the dynamical evolution of the Solar system.

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FIGURES

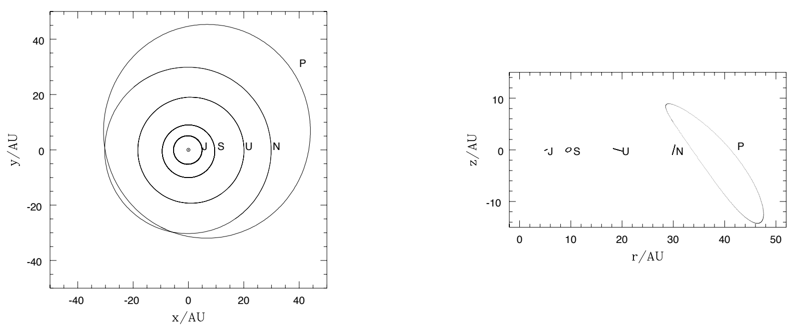

Figure 1. The orbits of the outer planets in a heliocentric reference frame: (a) projection in the plane of the ecliptic; (b) projection in cylindrical polar coordinates (r is the distance from the Sun projected in the ecliptic, and z is the distance above the ecliptic).

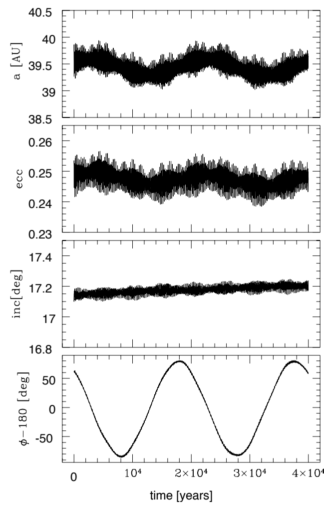

Figure 2. The variation of Pluto's orbital elements over 40,000 years: the semimajor axis, eccentricity, inclination are shown in the upper three panels. The bottom panel shows the libration of the resonance angle with a period of about 20,000 years. The 20,000 year periodic variations are evident in the semimajor axis and eccentricity, but not the inclination. The latter is only very weakly affected by Pluto's 3:2 mean motion resonance with Neptune.

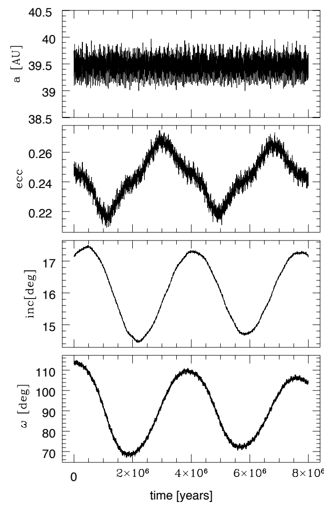

Figure 3. The variation of Pluto's orbital elements over 8 million years: the semimajor axis, eccentricity, inclination are shown in the upper three panels. The bottom panel shows the libration of the argument-of-perihelion with a period of about 3.8 million years. The 3.8 million year periodic variations are evident in the eccentricity and the inclination, but not the semimajor axis. The latter is almost unaffected because the argument-of-perihelion libration involves only secular perturbations.

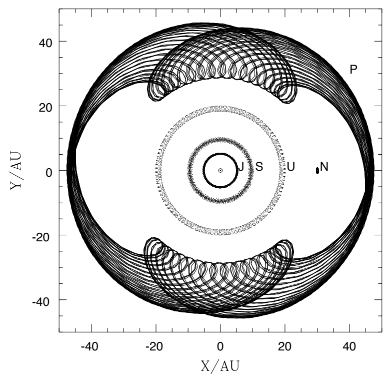

Figure 4. The orbits of the outer planets for 40,000 years in a reference frame co-rotating with the mean motion of Neptune. In this reference frame two orbits of Pluto trace out a complete loop in $\sim$500 yr. This figure visualizes the effects of the most important resonance of Pluto's motion: the 3:2 resonance libration which causes Pluto's heliocentric longitude conjunctions with Neptune to librate about aphelion with a period of about 20,000 yr; its perihelion librates 90° away from Neptune with that same period.

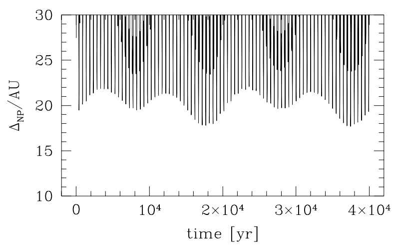

Figure 5. The distance between Neptune and Pluto: their closest approach distance (near aphelion) varies between $\sim$17 AU and $\sim$22 AU. (The synodic period of Neptune and Pluto is $\sim$500 yr.)

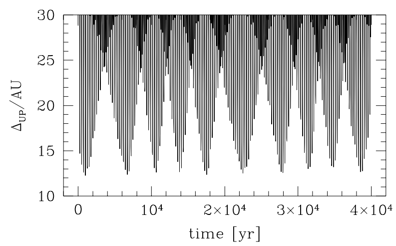

Figure 6. The distance between Uranus and Pluto: their closest approach distance varies between $\sim$12 AU and $\sim$26 AU. (The synodic period of Uranus and Pluto is $\sim$130 yr.)

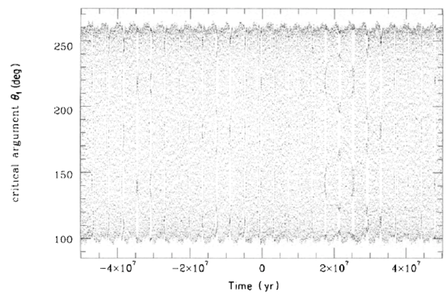

Figure 7. The resonance argument, $\phi = 3\lambda - 2\lambda_N - \varpi$ (in degrees) as a function of time, in years, from the present. (Reproduced, with permission, from Milani *et al.* 1989).

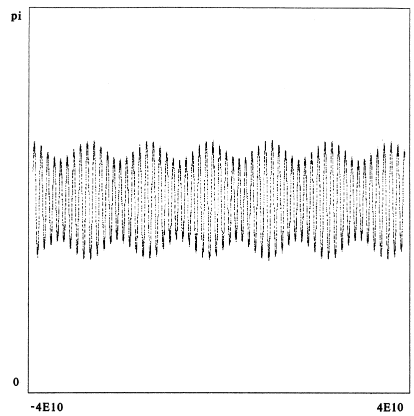

Figure 8. Pluto's argument of perihelion, ω, for 214 Myr. The abscissa is time, in days. The 3.8 Myr libration of ω is modulated with a 34 Myr period. (Reproduced, with permission, from Applegate *et al.* 1986).

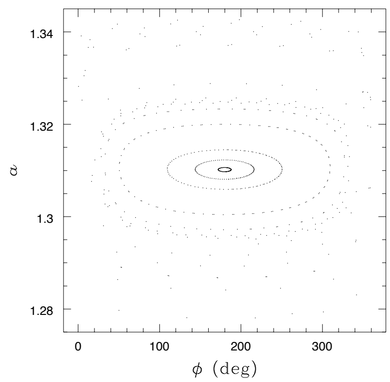

Figure 9. A (pseudo)surface-of-section in the circular planar restricted 3-body model for the Neptune-Pluto 3:2 resonance. The dynamical variables plotted here are the resonance angle, ϕ, and the semimajor axis a (the latter in units of the Sun-Neptune distance). The stable libration region is surrounded by a large chaotic zone. (Adapted from Malhotra 1996.)

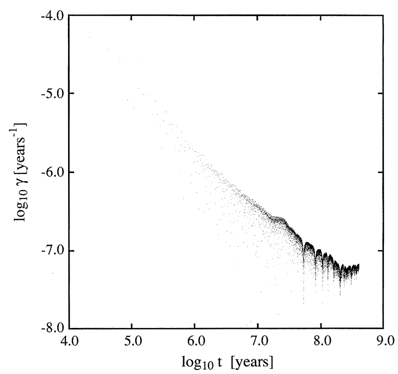

Figure 10. The finite-time Lyapunov exponent of Pluto, γ, as a function of time. In this log-log plot, convergence to a positive Lyapunov exponent (for a chaotic trajectory) is indicated by a leveling off; for a regular trajectory, this trace would approach a straight line with slope -1. (Reproduced, with permission, from Sussman & Wisdom 1988).