Introduction to Solar Radiative Transfer: II Detailed Radiative Processes

Han Uitenbroek
National Solar Observatory/Sacramento Peak
Sunspot NM, USA

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Overview

- Spectral lines in atoms, ions, and molecules

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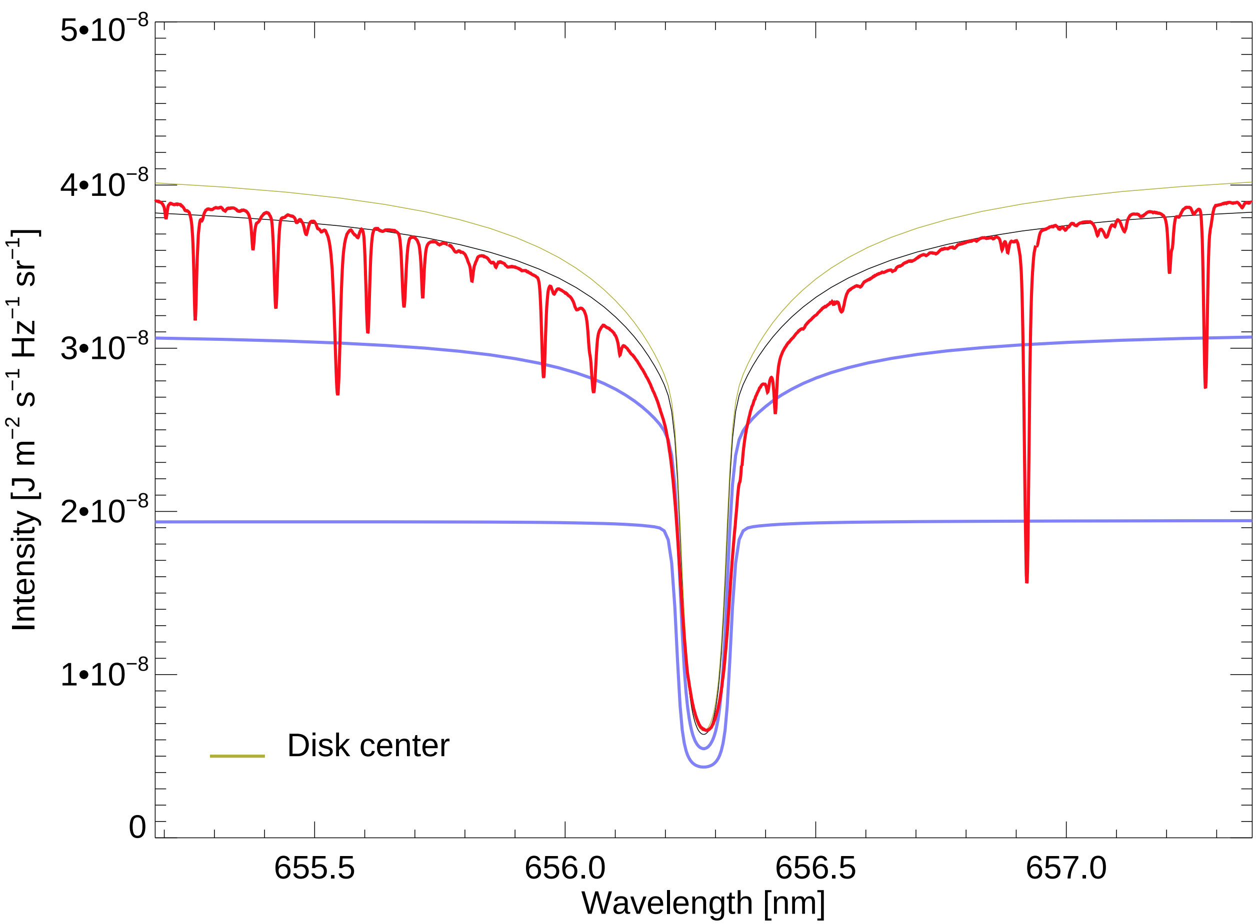
- Molecular lines

Hα Spectral Line



Termdiagram and Transitions in Hydrogen

$$\Delta E = h\nu = \frac{hc}{\lambda}$$

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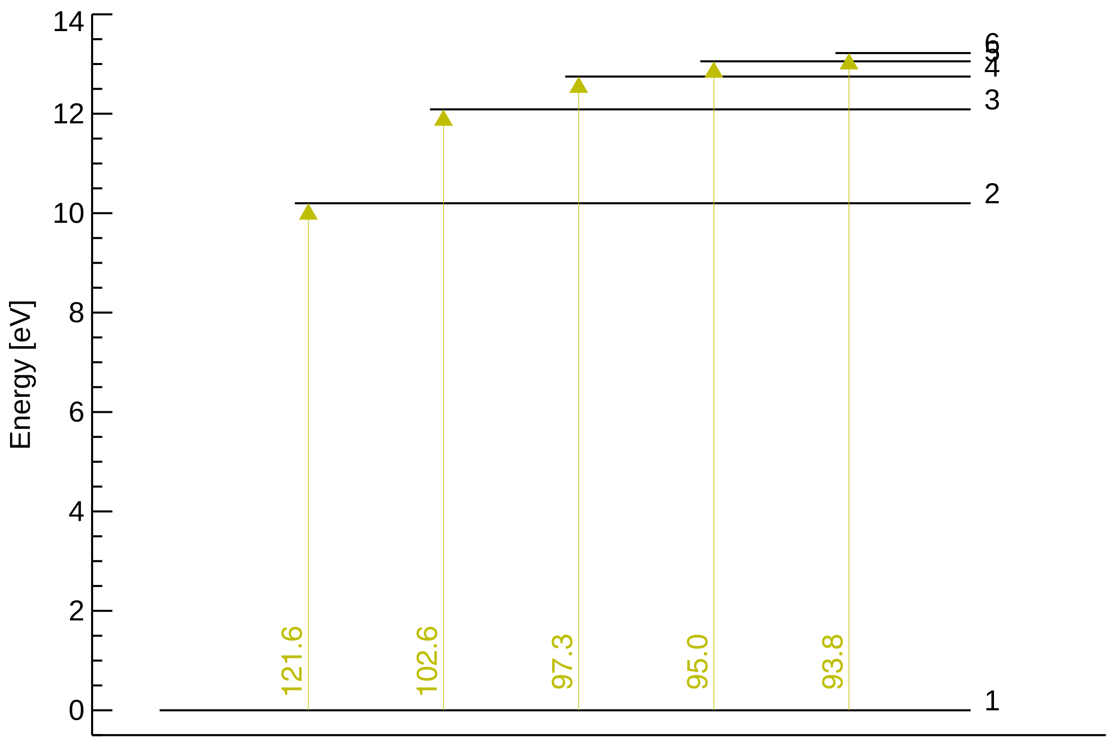

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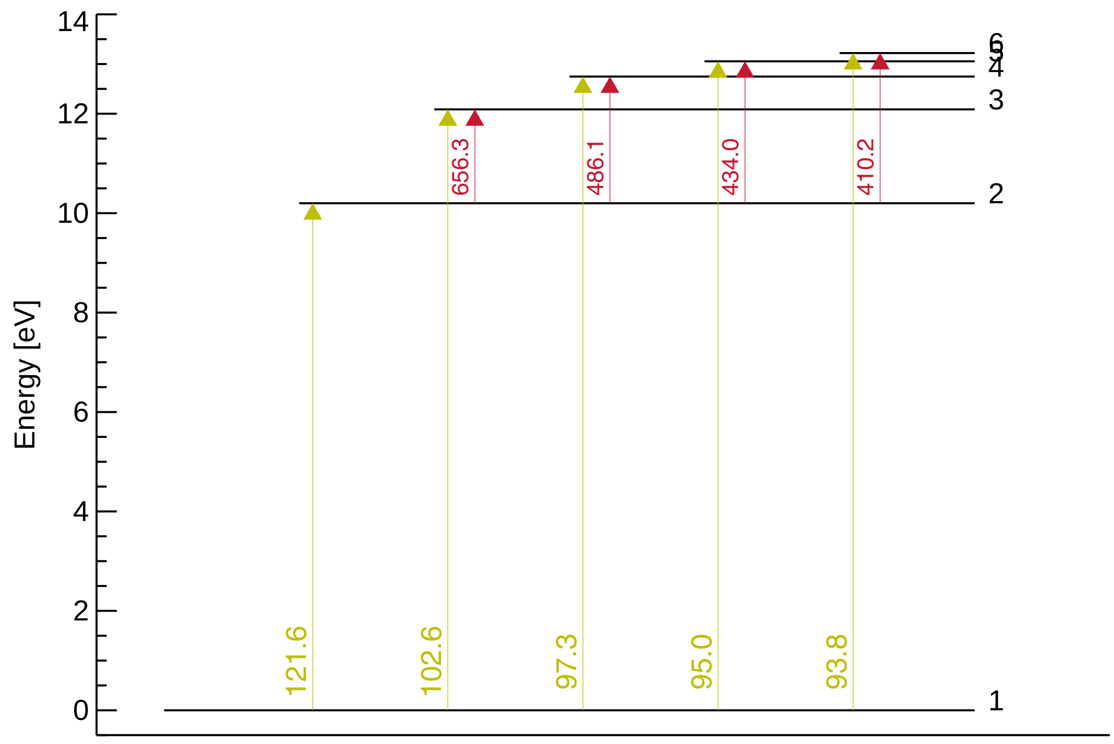

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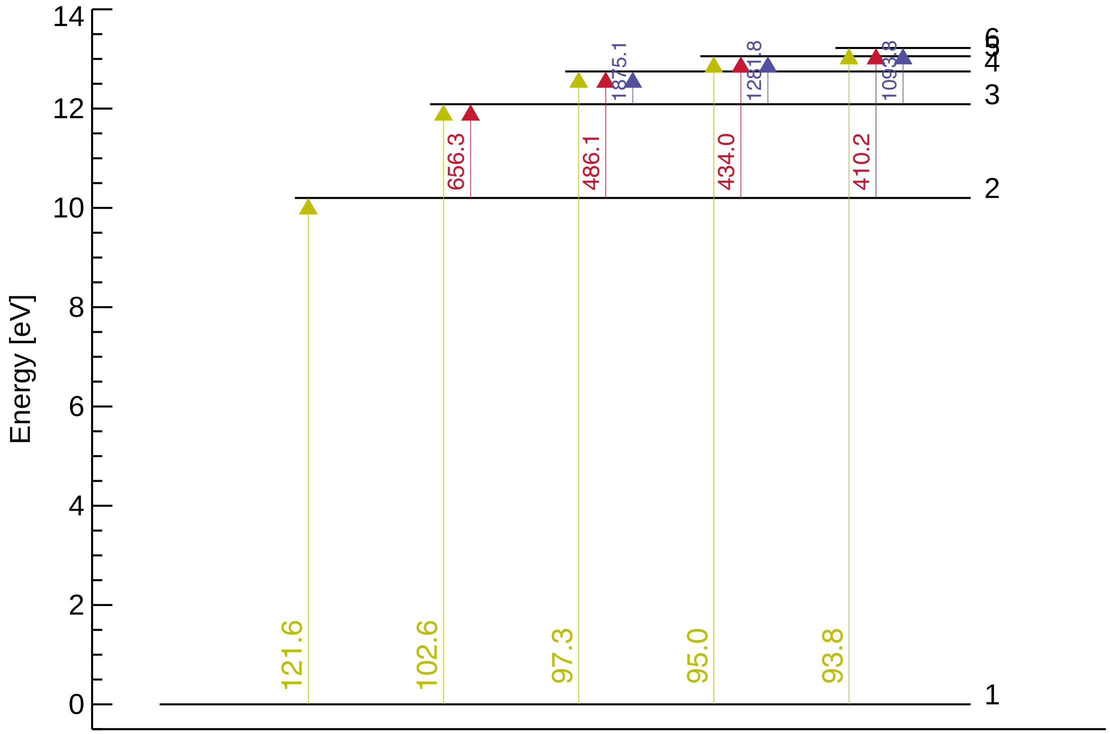

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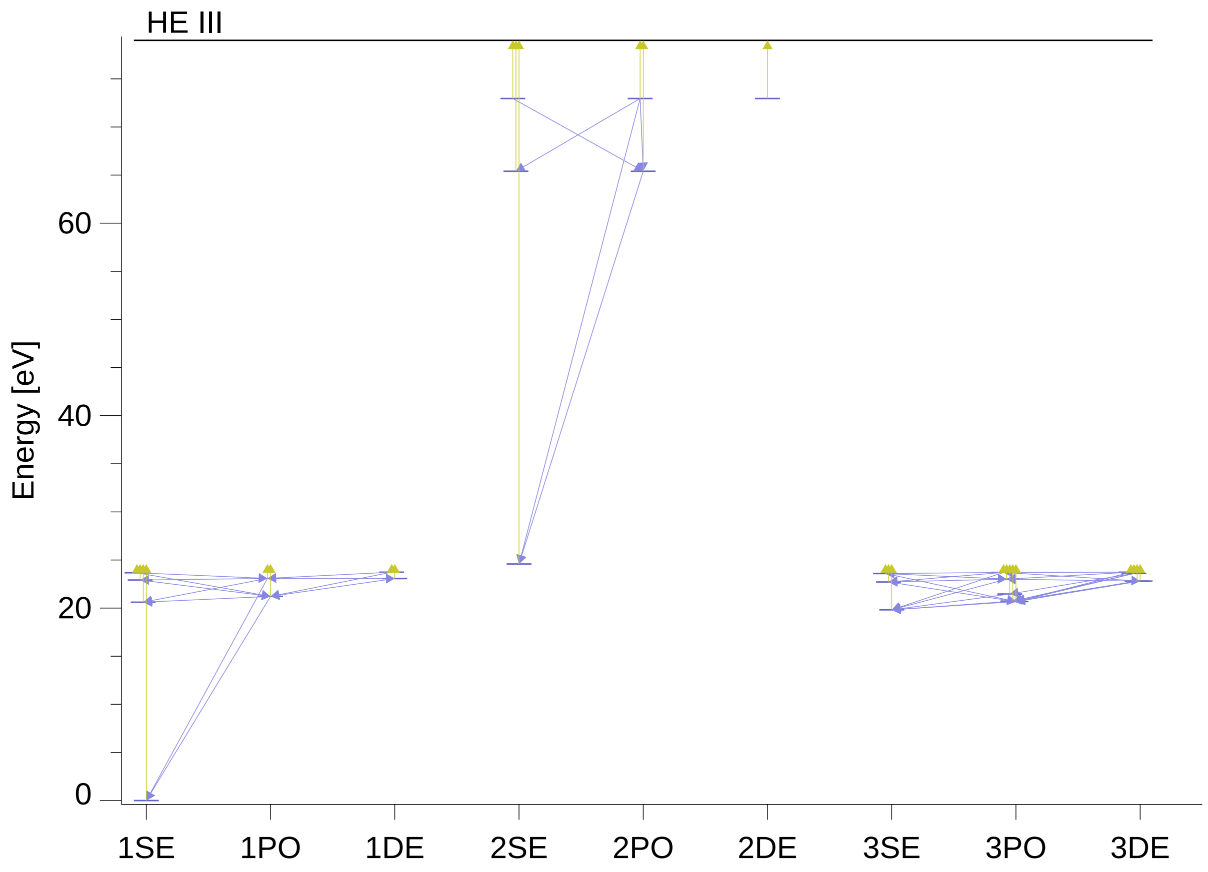
Termdiagram of Helium with Three Ionization Stages



Einstein Relations for Bound-Bound Transitions

Spontaneous emission $j \rightarrow i$:

$$j_\nu^{\rm spont} = n_j (A_{ji} h\nu_{ij}/4\pi)\psi_\nu \tag{1}$$

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$$j_\nu^{\rm stim} = n_j (B_{ji} h\nu_{ij}/4\pi)\psi_\nu I_\nu, \qquad A_{ji} = (2h\nu^3/c^2)B_{ji} \tag{2}$$

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Absorption $i \rightarrow j$:

$$\alpha_\nu I_\nu = n_i (B_{ij} h\nu_{ij}/4\pi)\varphi_\nu I_\nu, \qquad g_i B_{ij} = g_j B_{ji} \tag{3}$$

Collisional Bound-Bound Transitions

Maxwellian velocity distribution

$$f(v)\mathrm{d}v = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp(-mv^2/2kT)\, 4\pi v^2 \mathrm{d}v \qquad (4)$$

Collisional excitation by electrons:

$$n_i C_{ij}(T) = n_i N_e \int_{v_0}^{\infty} \sigma_{ij}(v) v f(v) \mathrm{d}v, \quad (1/2) m v_0^2 = h\nu_0 \qquad (5)$$

Collisional deexcitation by electrons:

$$g_j C_{ji} = g_i C_{ij} e^{\Delta E_{ji}/kT} \qquad (6)$$

Equilibrium Distributions

Boltzmann distribution:

$$\left[\frac{n_{r,j}}{n_{r,i}}\right]_{\mathrm{LTE}} = \frac{g_{r,j}}{g_{r,i}} e^{-\Delta E_{ji}/kT} \quad (7)$$

Saha distribution:

$$\left[\frac{n_{r+1,0}}{n_{r,0}}\right]_{\mathrm{LTE}} = \frac{1}{N_e} \frac{2g_{r+1,0}}{g_{r,0}} \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_r/kT} \quad (8)$$

Partition function:

$$U_r \equiv \sum_i g_{r,i} e^{-\Delta_{r,i}/kT} \quad (9)$$

Local Thermodynamic Equilibrium (LTE)

- Consider an isolated enclosure in steady state, containing a homogeneous medium. We can expect this medium to have the same temperature everywhere, and the radiation field to be isotropic. Otherwise, the material would be in violation with the second law of thermodynamics.

- The energy absorbed and emitted by an element of the medium must be equal:

$$j_\nu = \alpha_\nu I_\nu, \qquad \mathrm{(Kirchhoff's\,law)}$$

- The radiation field in an enclosure in strict Thermodynamic Equilibrium is given by the Planck function $B_\nu(T)$

$$j_\nu^{\mathrm{TE}} = \alpha_\nu^{\mathrm{TE}} B_\nu(T), \qquad \mathrm{(Kirchhoff - Planck)}$$

Einstein-Milne Relations for Bound-Free Transitions

Absorption:

$$\alpha_\nu^{\rm bf} I_\nu = n_i \sigma^{\rm bf}(\nu) I_\nu \tag{10}$$

Spontaneous emission:

$$j_\nu^{\rm bf} = n_j N_e \Phi_{ij}(T) \left(\frac{2h\nu^3}{c^2}\right) e^{-h\nu/kT} \sigma^{\rm bf}(\nu) \tag{11}$$

$$\Phi_{ij}(T) = \frac{g_i}{2g_j} \left(\frac{h^2}{2\pi mkT}\right)^{3/2} e^{(E_j - E_i)/kT}, \qquad \mathrm{Saha-Boltzmann}$$

Stimulated emission:

$$j_\nu^{\rm bf} = n_j N_e \Phi_{ij}(T) e^{-h\nu/kT} \sigma^{\rm bf}(\nu) I_\nu \tag{12}$$

Continuum processes

- Free–free transitions

- H$^-$ bound–free and free-free
  $\mathrm{H}^- + h\nu \rightleftharpoons \mathrm{H} + e(v)$
  $\mathrm{H} + e(v) + h\nu \rightleftharpoons \mathrm{H} + e(v')$

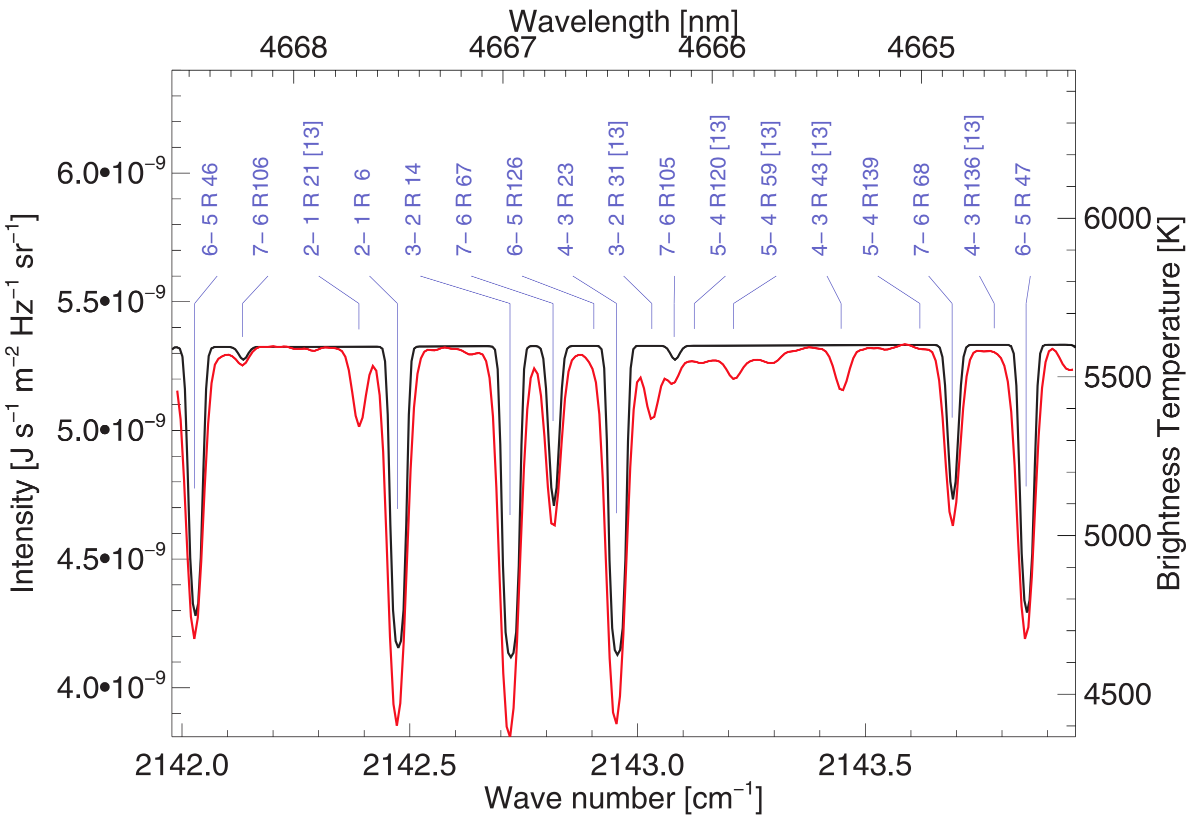
- Thomson scattering

$$\alpha_e^T = N_e \sigma_e = N_e \frac{8\pi}{3 m_e^4 c^2} \frac{q_e^4}{(4\pi\epsilon_0)^2}$$

- Rayleigh scattering

$$\alpha^R(\omega) = \sigma_e f_{ij} \omega^4 / (\omega_{ij}^2 - \omega^2)^2$$

CO lines in the Infrared



Concentration of Molecules

Chemical equilibrium:

$$\frac{n_A n_B}{n_{AB}} = \left(\frac{2\pi m_{AB} kT}{h^2}\right)^{3/2} e^{-D/kT} \left(\frac{U_A U_B}{Q_{AB}}\right) \tag{13}$$

$$m_{AB} = \frac{m_A m_B}{m_A + m_B} \tag{14}$$

$$Q_{AB} = Q_{\rm rot} Q_{\rm vib} Q_{\rm el} \tag{15}$$

Molecular lines

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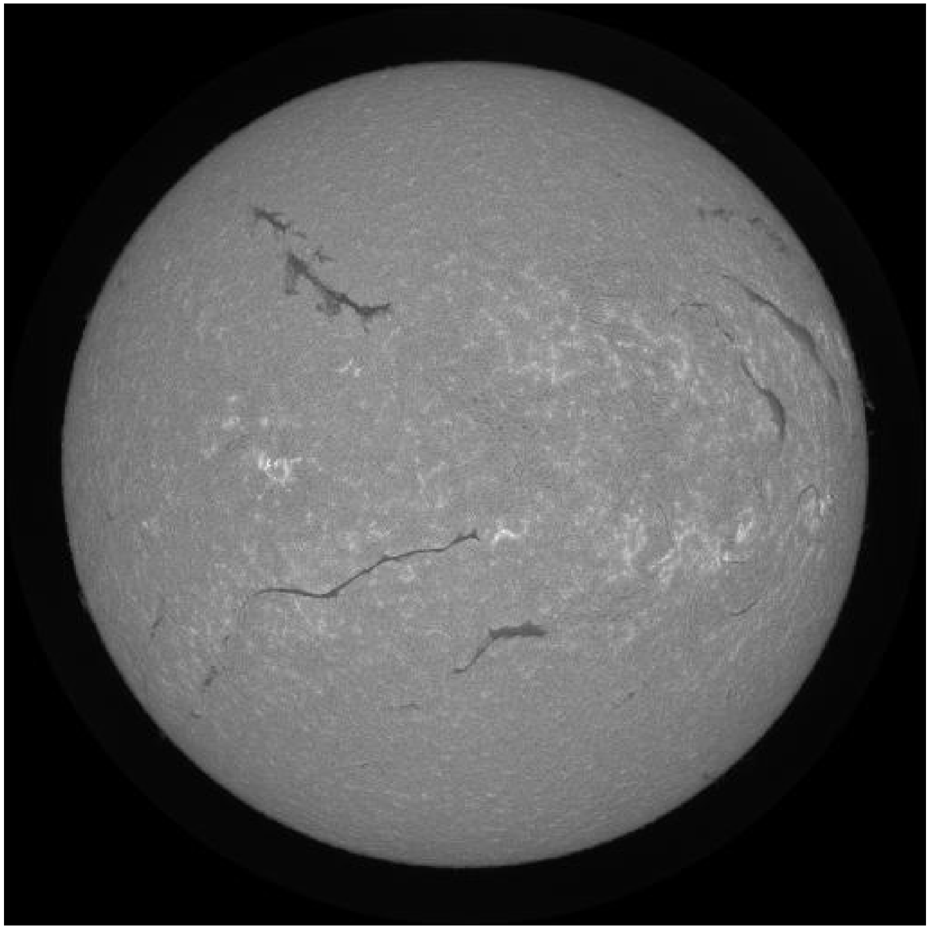
- Molecules have more degrees of freedom: rotation, vibration
- Hence there are many more molecular levels and transitions between them
- Molecular lines are grouped in bands, because energy differences between different electronic states are generally much larger than between different vibration states within one electronic state, and these are typically larger again than energy differences between rotational states in one vibrational state. Example

End Part II

The Sun in the light of Hα



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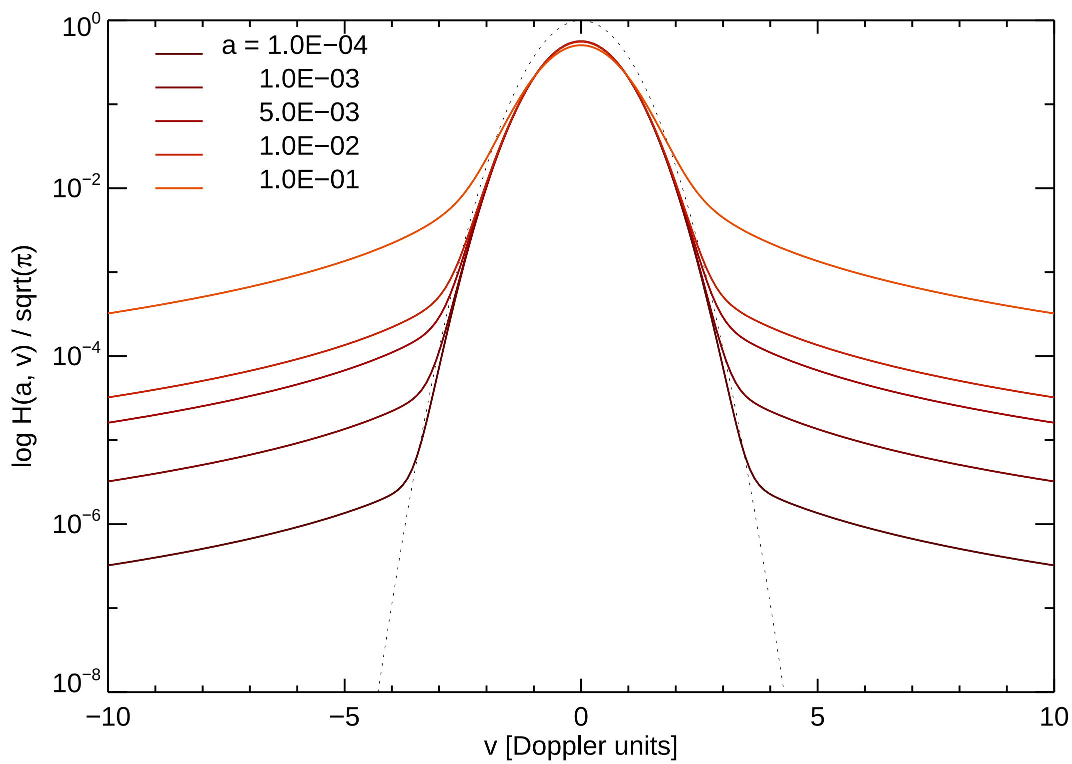
Voigt Functions



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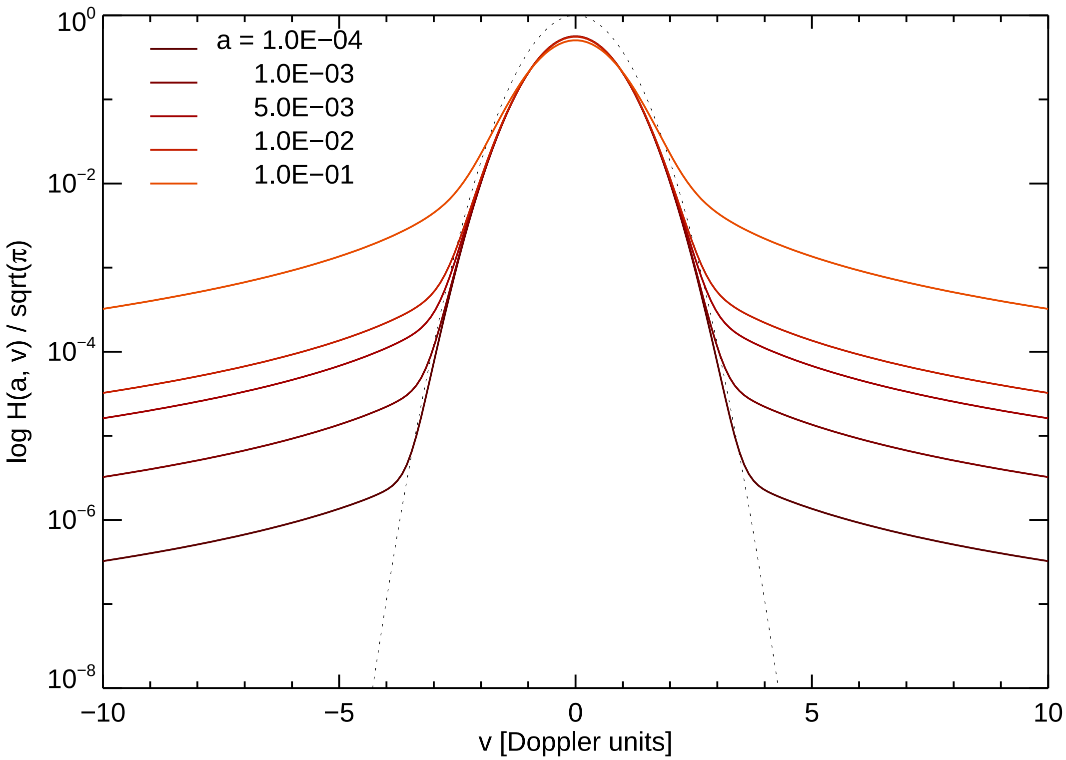
Voigt Functions



$$\psi(\nu - \nu_0) = \frac{H(a,v)}{\sqrt{\pi}\Delta\nu_D}$$

$$\Delta\nu_D \equiv \frac{\nu_0}{c}\sqrt{\frac{2kT}{m}}$$

$$a = \frac{\Gamma}{4\pi\Delta\nu_D}$$

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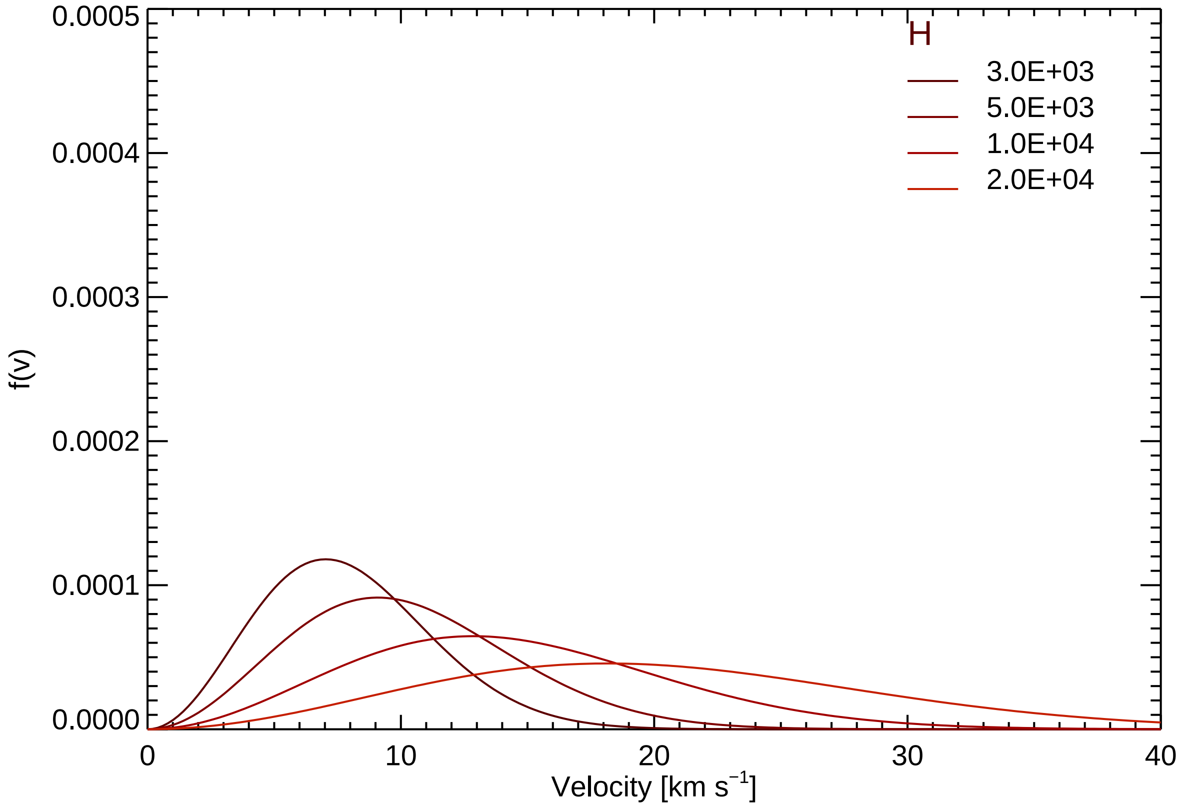
Maxwellian Velocity Distribution



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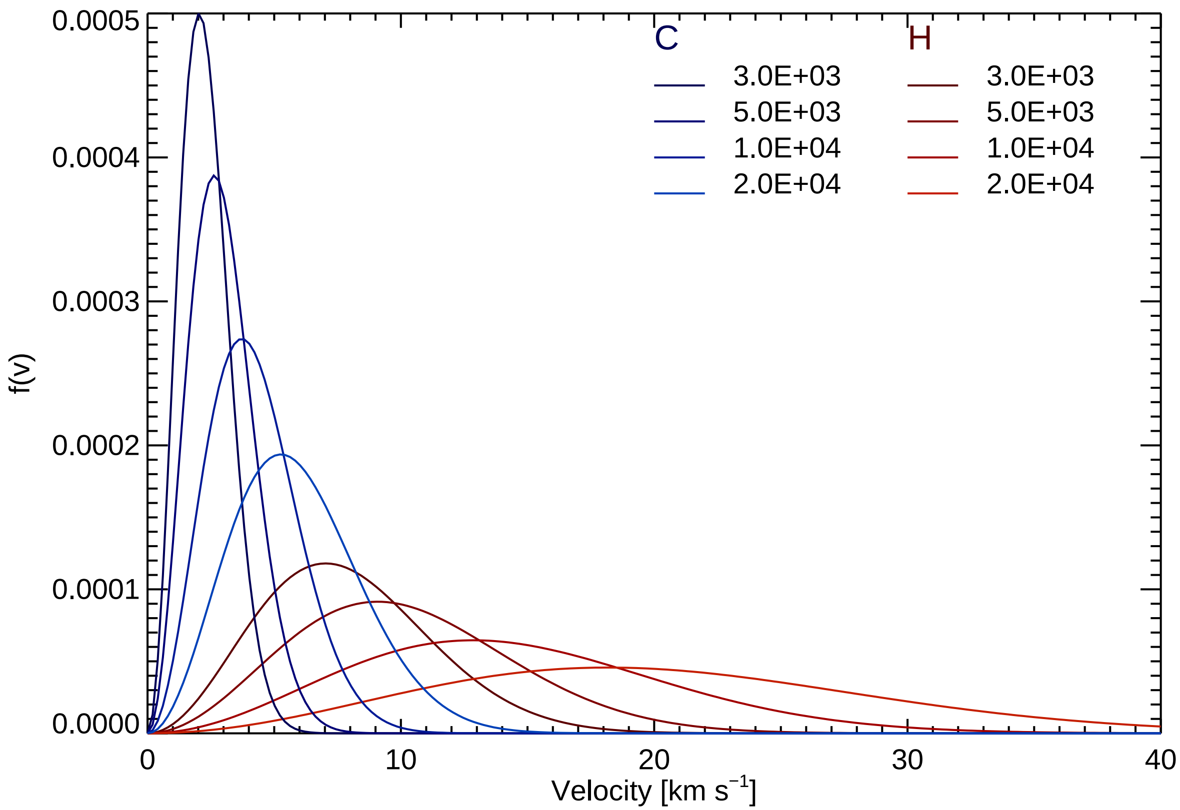

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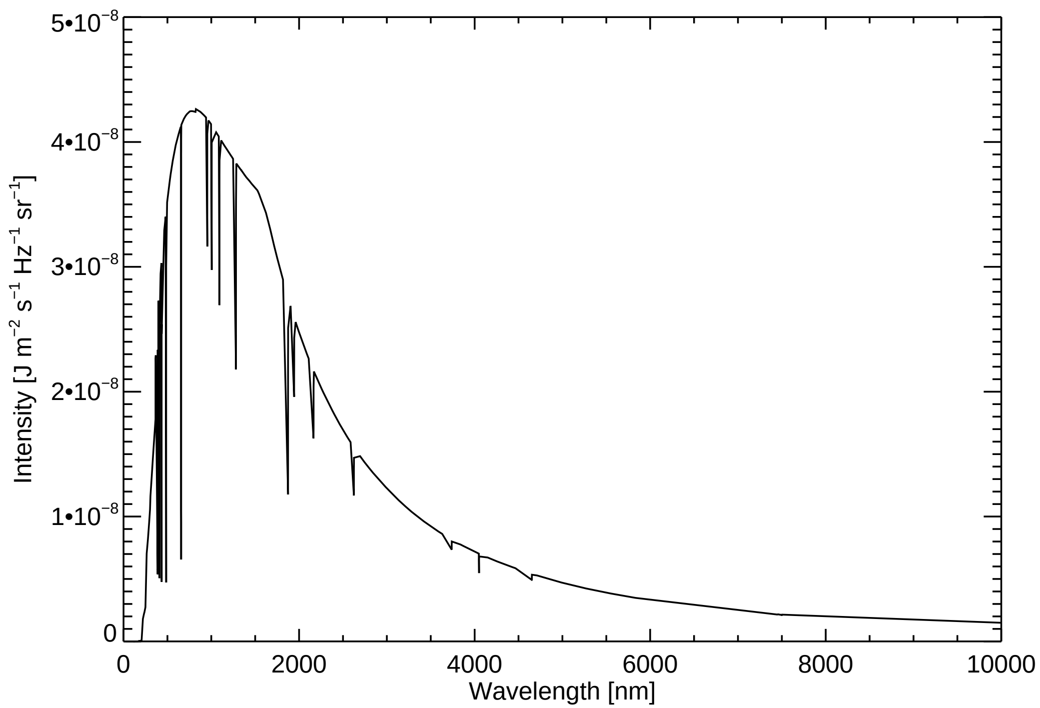
The Solar Spectrum and Surface Temperature



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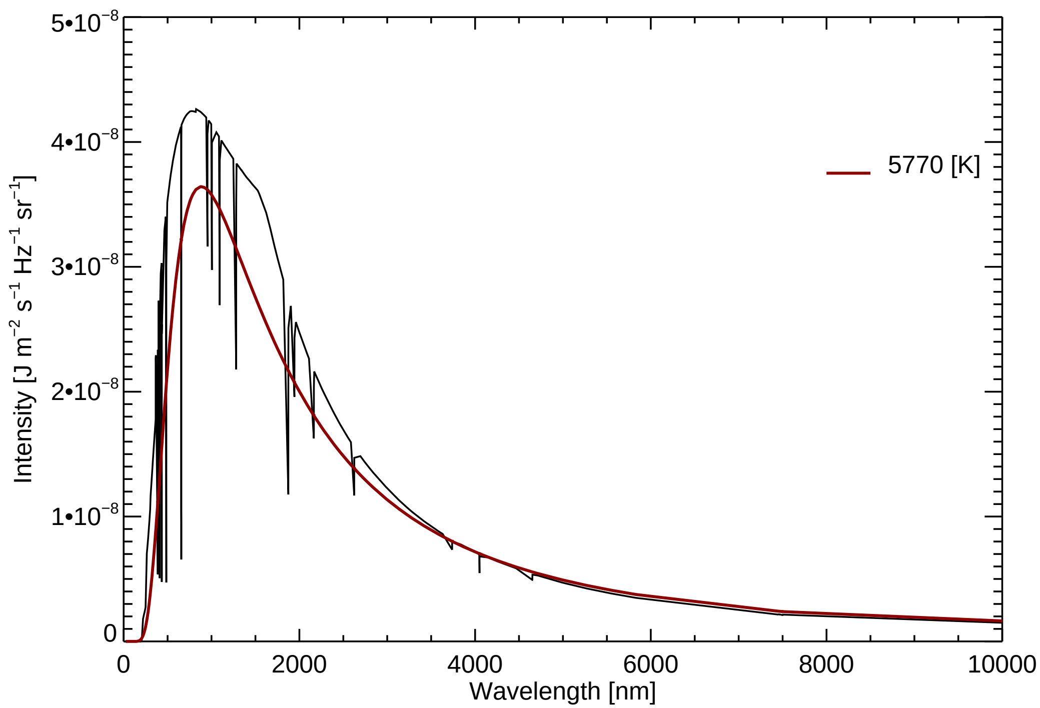

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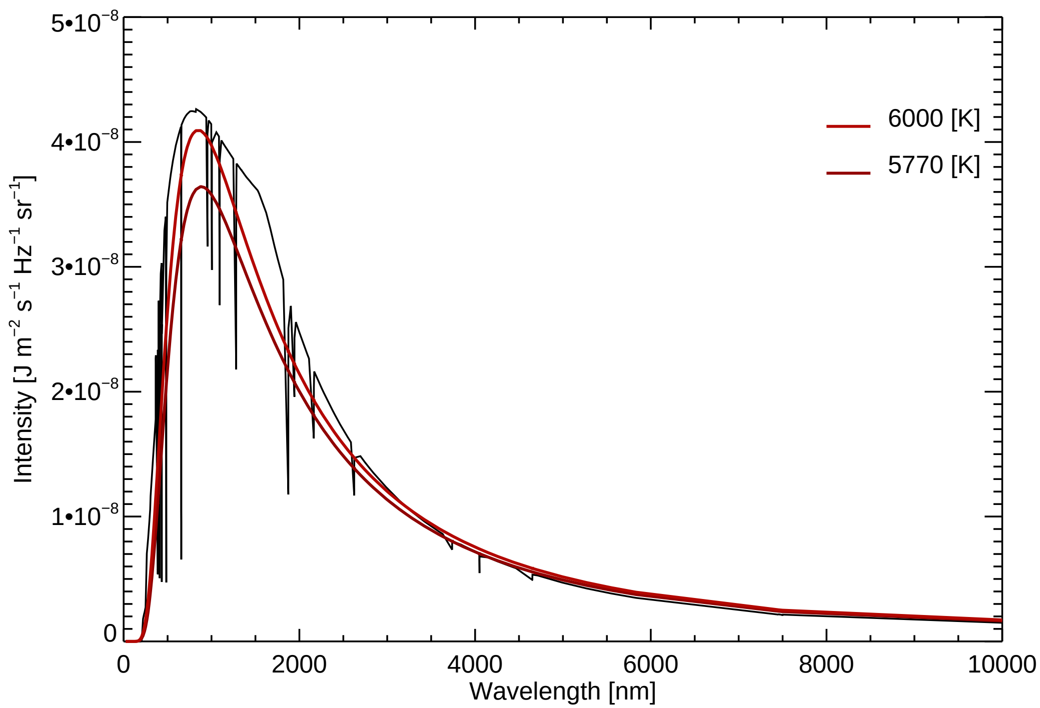

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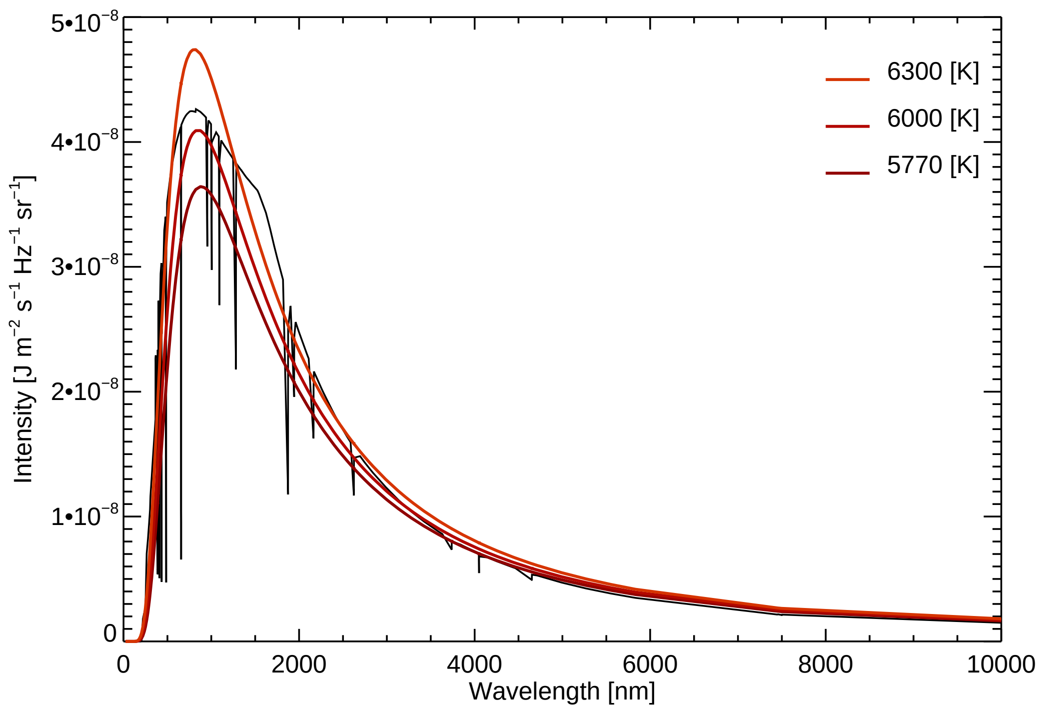

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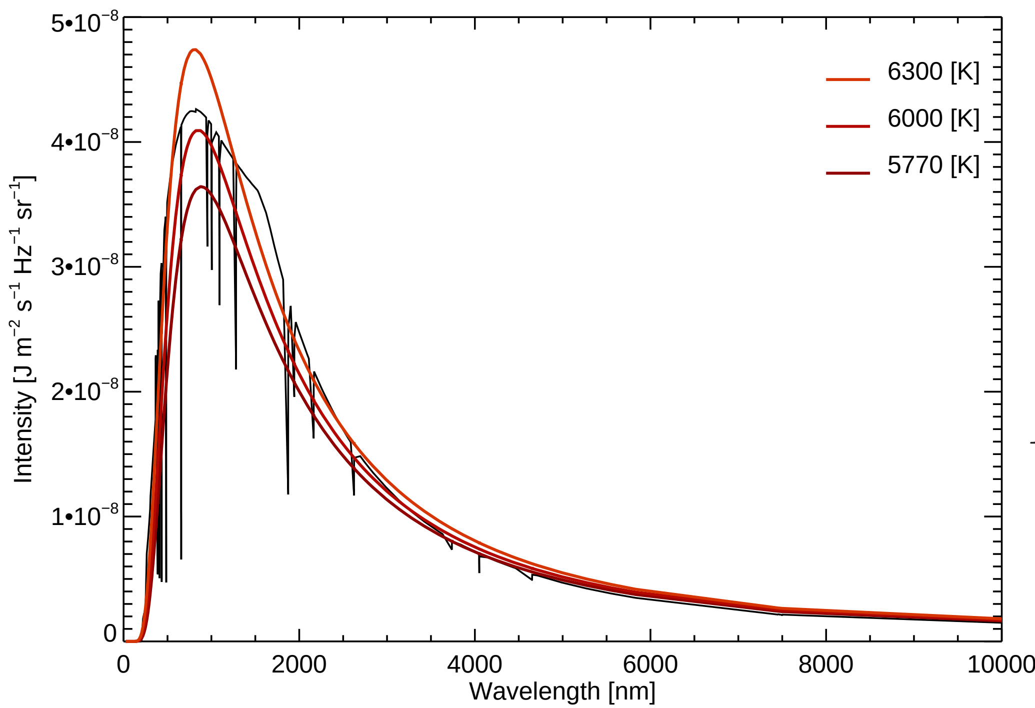

$$B_\nu(T) \equiv \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

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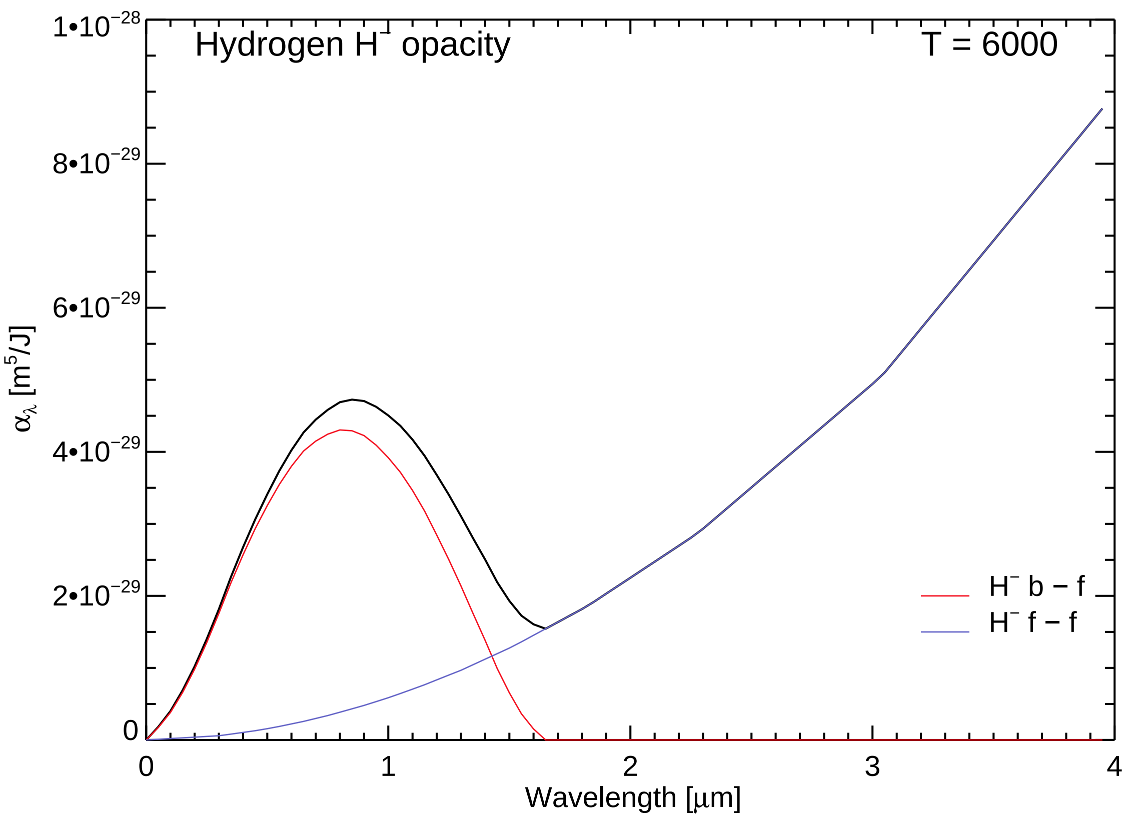
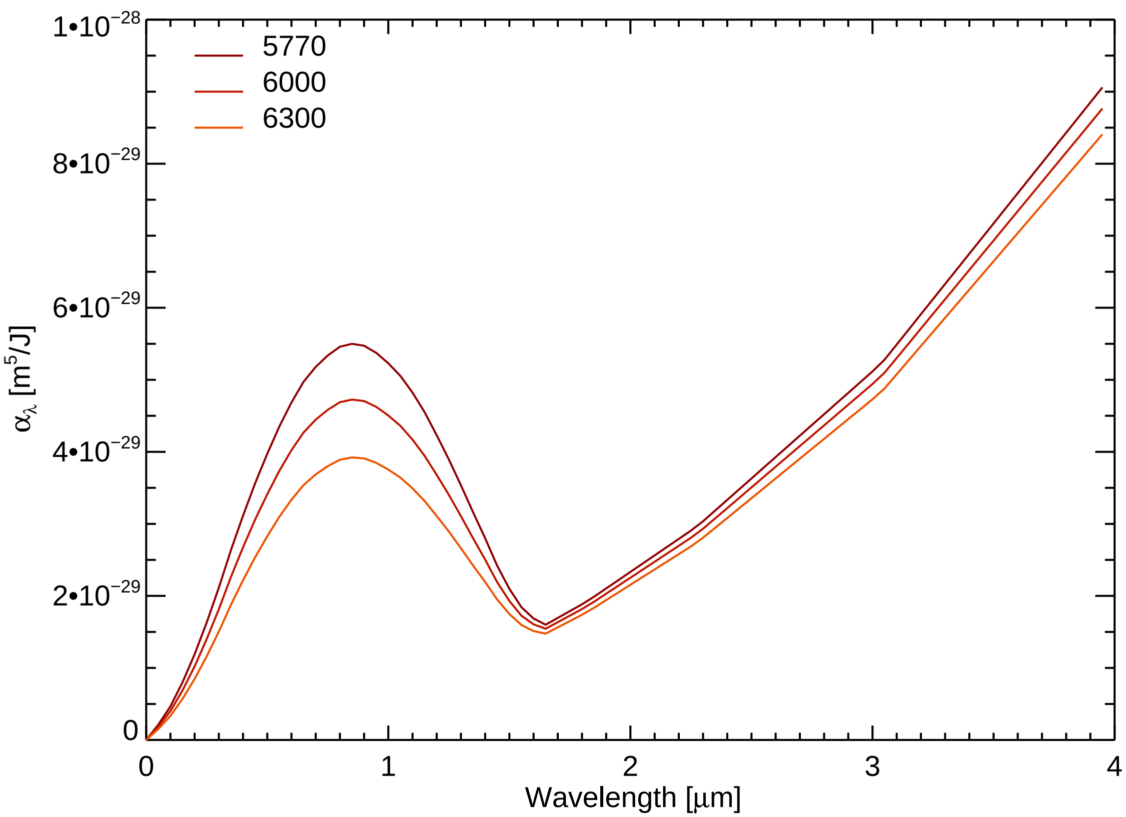
H^- Opacity



$$\Delta E^{\mathrm{bf}} = 0.754 \text{ eV}$$

H^- Opacity Changes with Temperature



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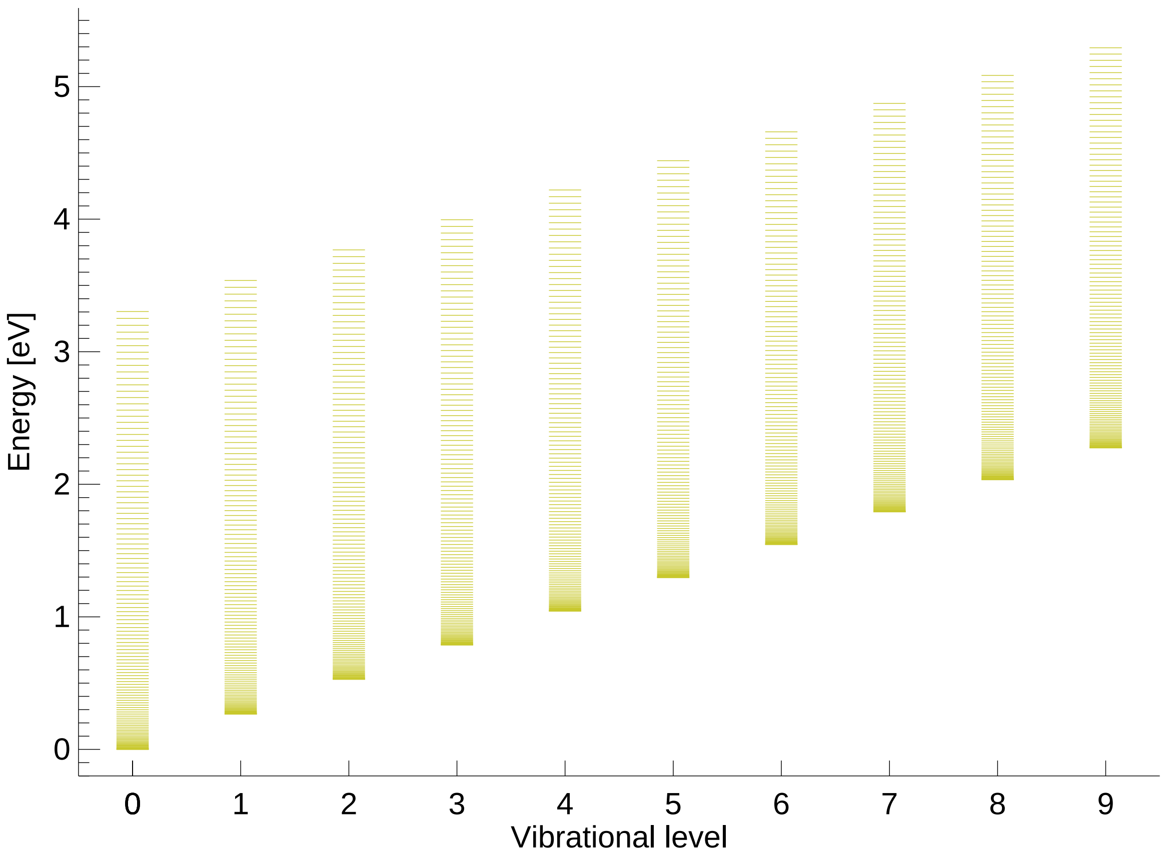
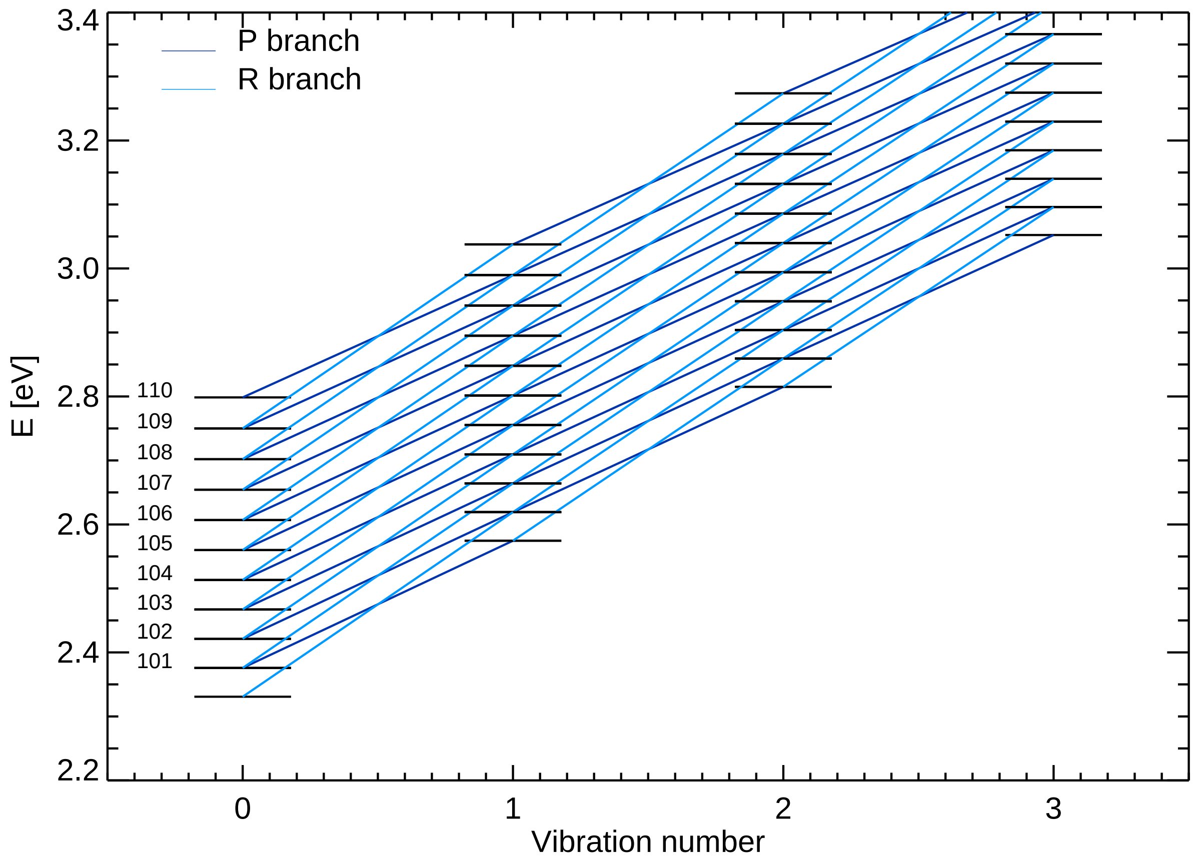
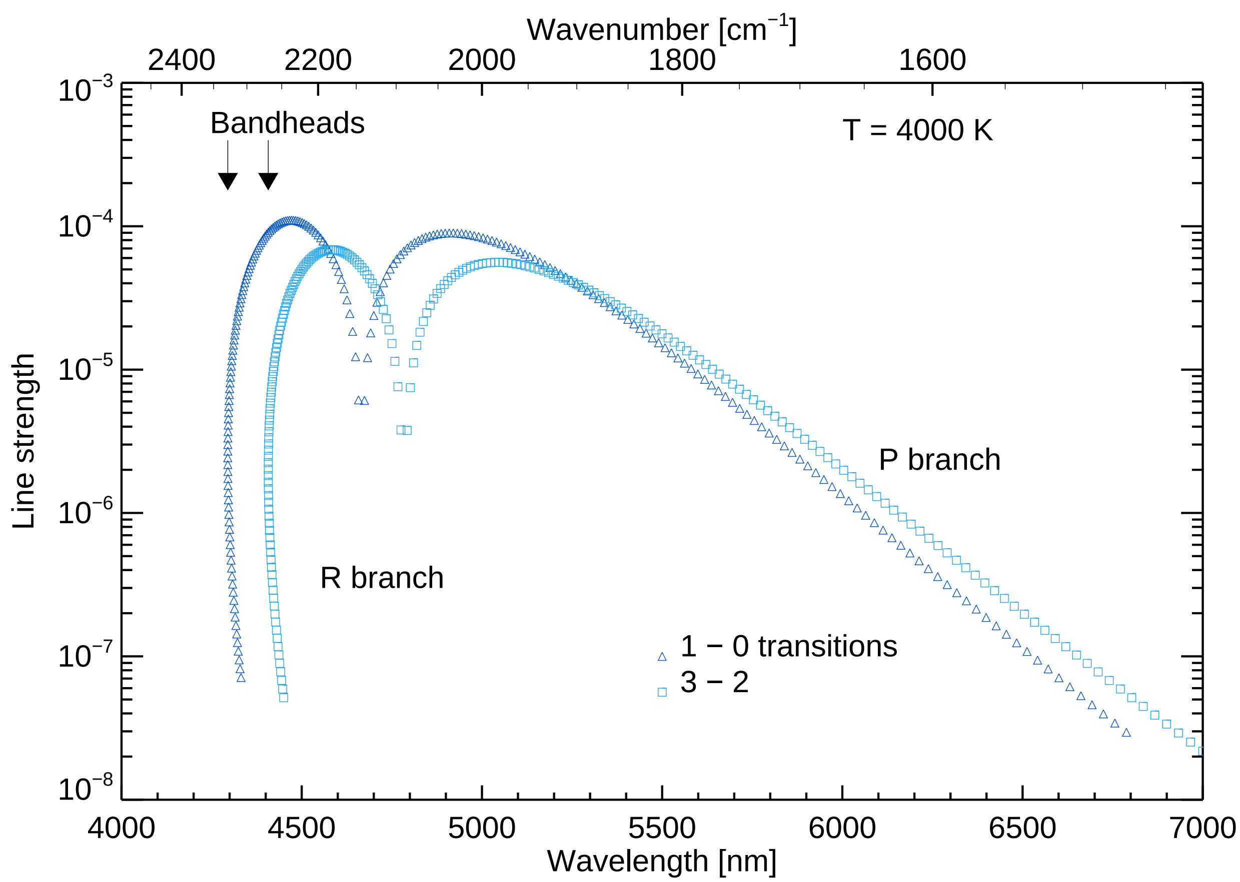
Energy levels of the CO Molecule



Vibration-Rotation Transitions in CO



Molecular Lines are grouped in Bands



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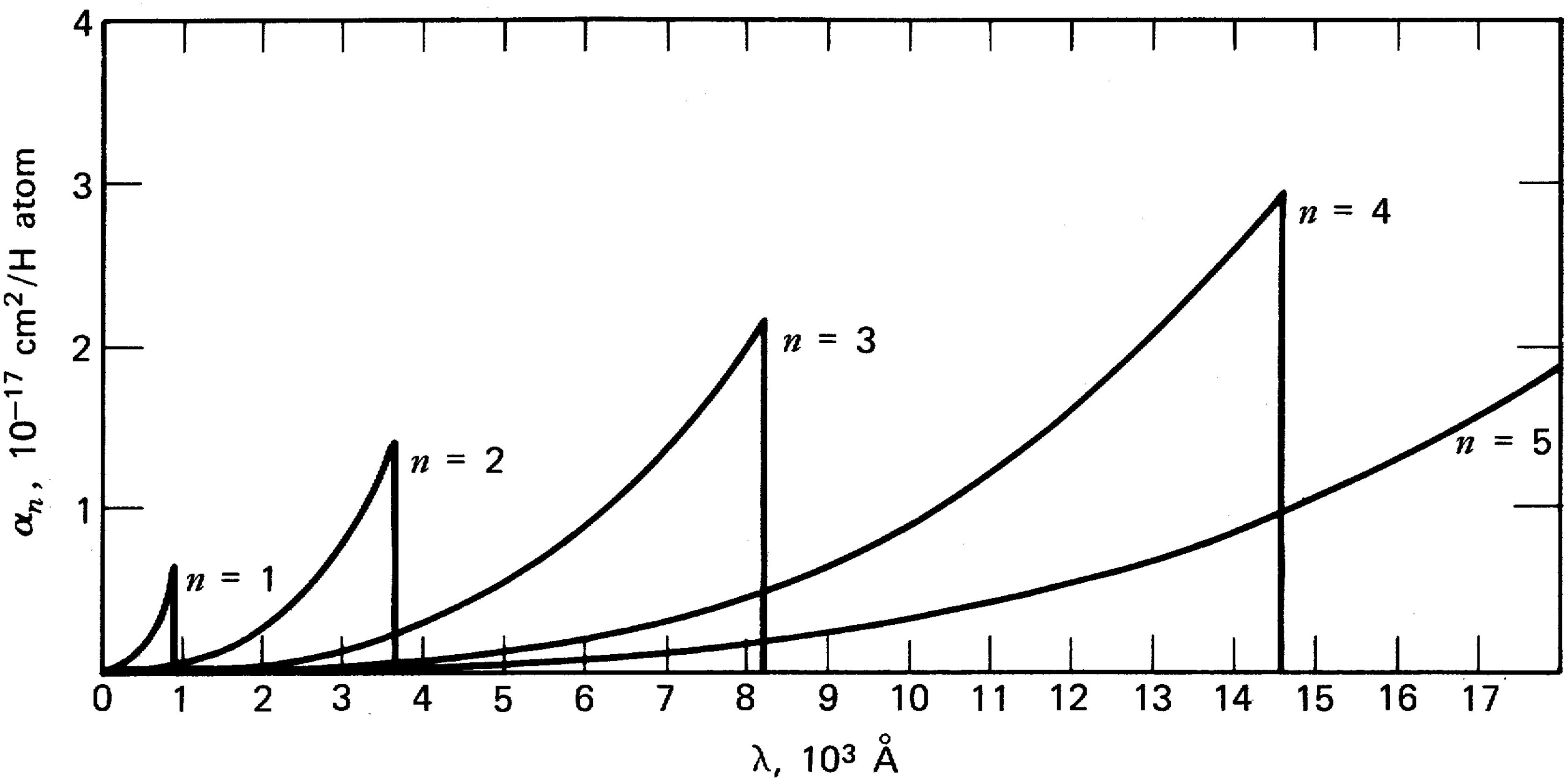
Bound–Free Cross Sections of Hydrogen



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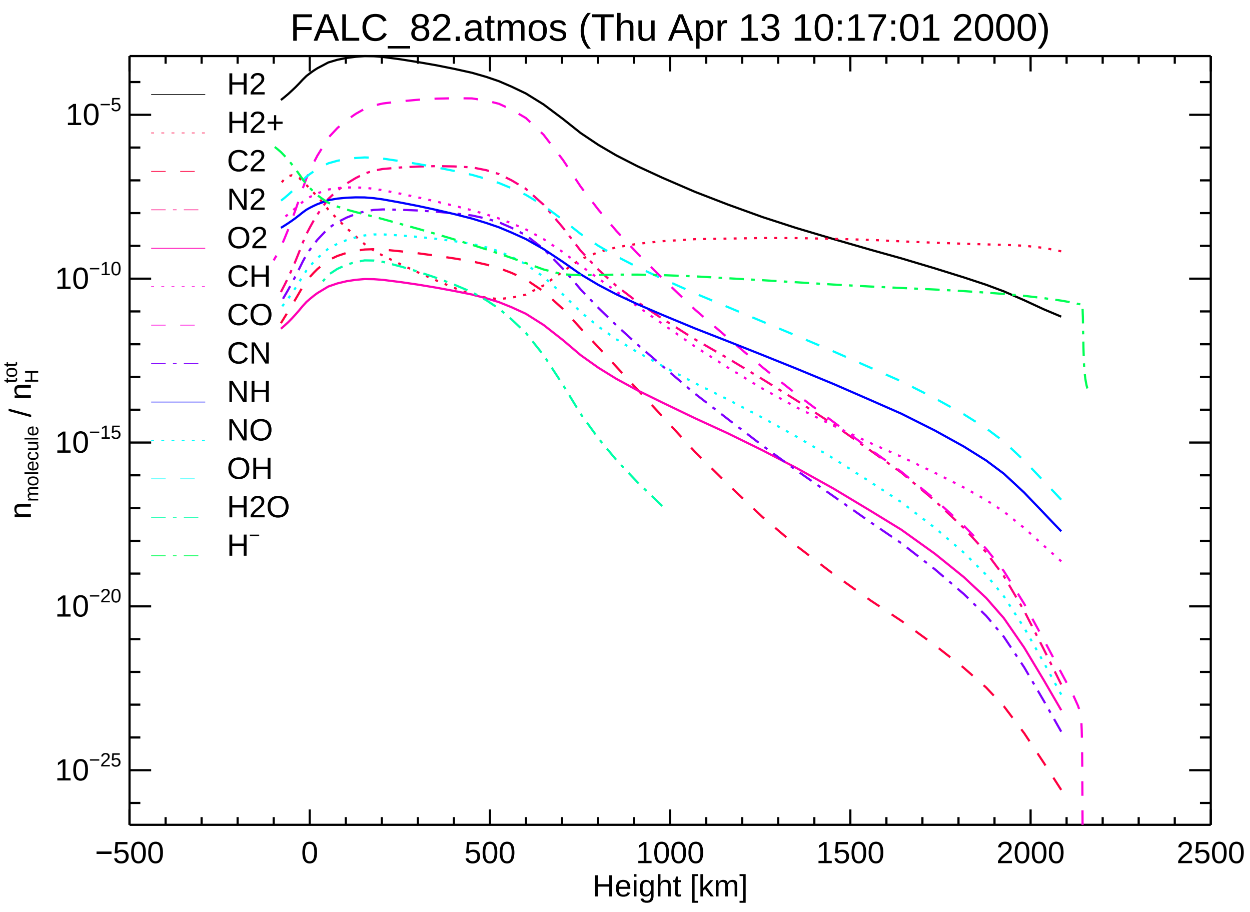
Molecules in the Sun



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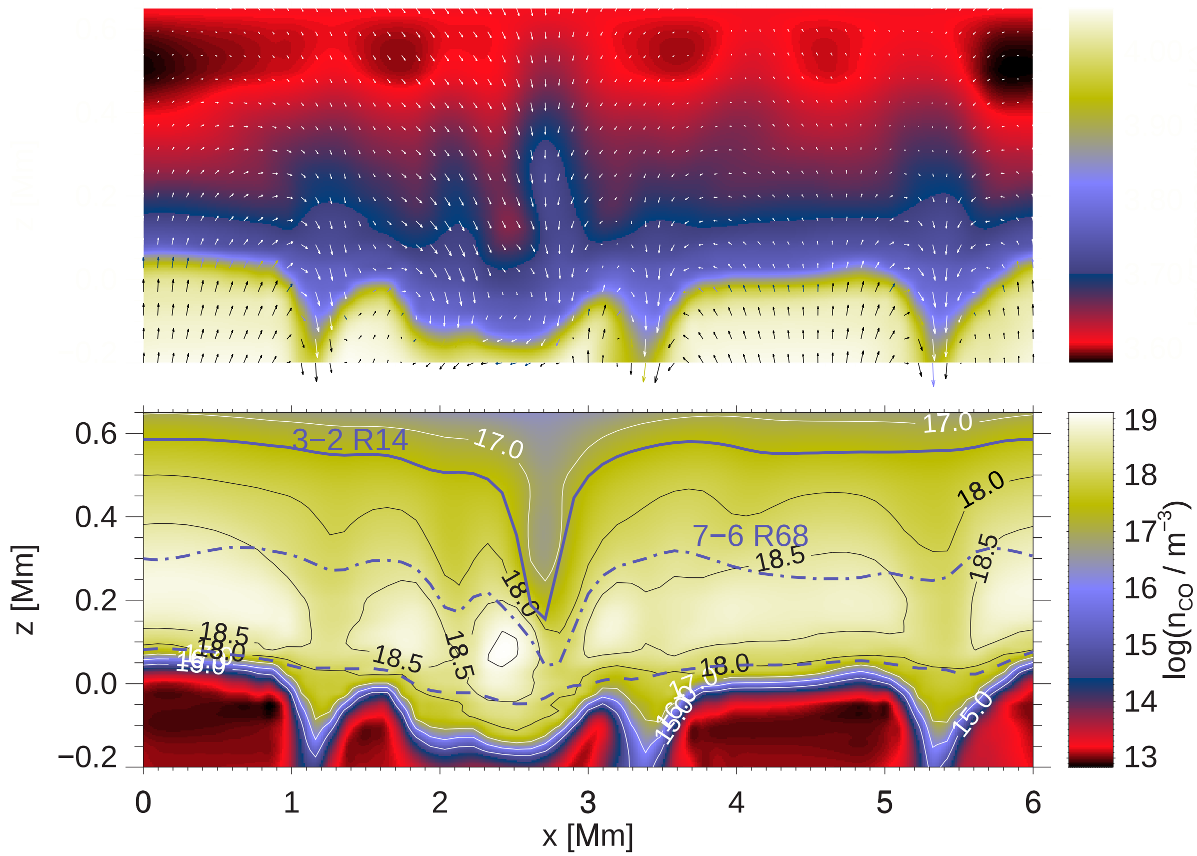

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