

HYDRODYNAMICS
And
MAGNETO HYDRODYNAMICS

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PRELIMINARY REMARKS

Concerning hydrodynamics & magnetohydro dynamics in nature, where no one applies external electrical potentials.

Basic Point:

The dynamics of gases and magnetized plasmas is described by the equations of Newton and Maxwell

Consider the large-scale bulk motion of gases, plasmas, and magnetic field.

Consider the necessary and sufficient conditions for applicability of hydrodynamics (HD) to the bulk motion within a cloud of independently moving particles.

Imagine an infinite space filled with particles, each particle moving freely and independently along a straight path with its own arbitrary constant velocity u_i .

$$x_i(t) = x_i(0) + u_i t.$$

The initial mean particle density and/or the distribution of individual particle velocities is nonuniform on a scale L , so that the subsequent particle density $N(x_k, t)$ varies with time on the same scale.

What are the necessary and sufficient conditions that the bulk motion v_i is described by the continuum hydrodynamic equations?

An obvious necessary condition is that there are enough particles that the local mean particle density $N(x_i, t)$ is statistically well defined.

The local particle density $N(x_i, t)$ is defined as the mean over some small scale l ($\ll L$).

Hence, it is necessary that

$$Nl^3 \gg 1,$$

where l is chosen sufficiently small that the difference equations on the grid spacing l provide a good approximation to the differential equations. For most purposes it is sufficient that $l \leq 10^{-3} L$.

Smaller l may be needed to treat the build up of shock fronts and singular current sheets, of course.

So, given enough particles, the local density is well defined, and, therefore, the local bulk velocity, momentum density, kinetic energy density, etc, are all statistically well defined.

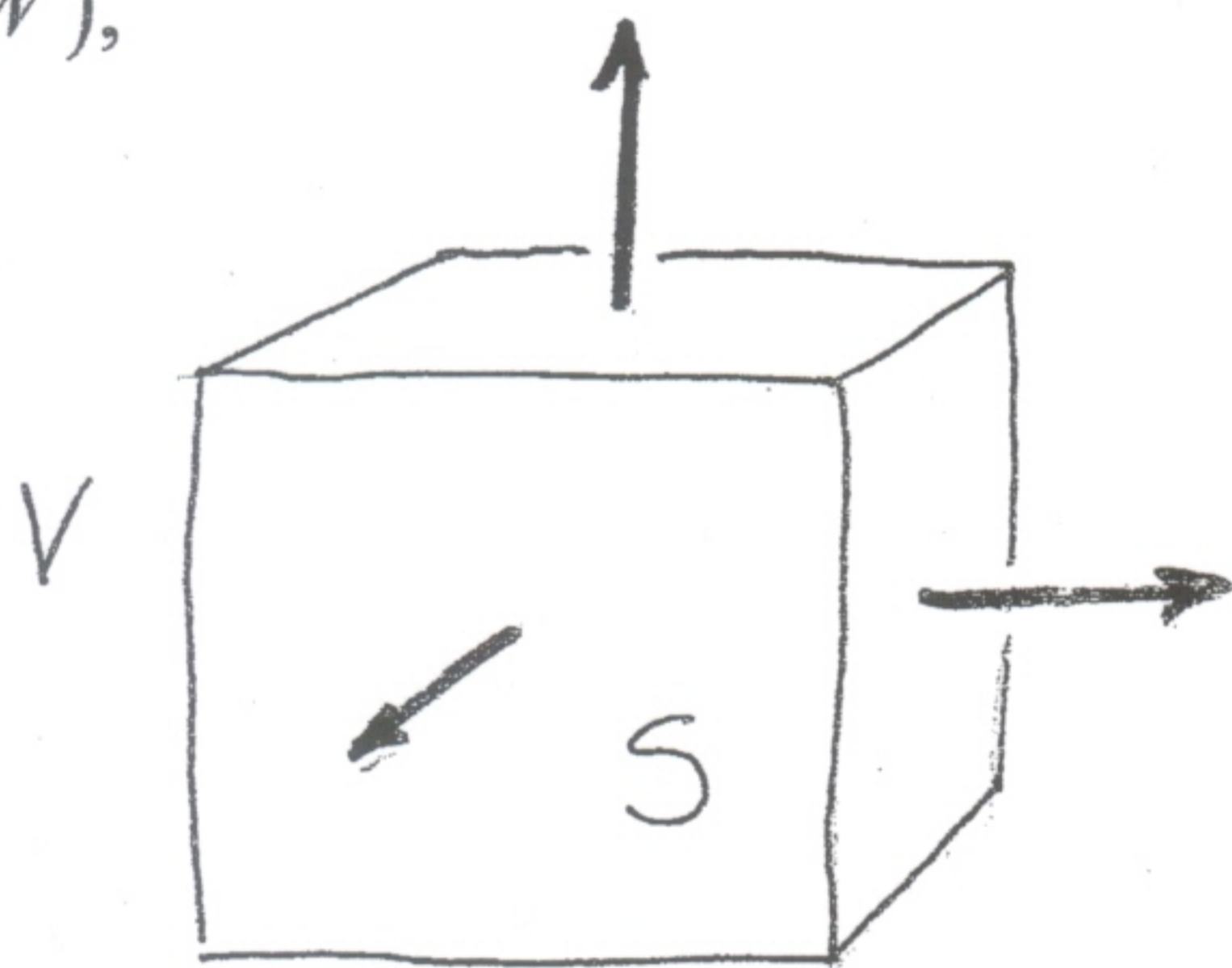
THEN ASSUME THAT THERE IS CONSERVATION OF PARTICLES, MOMENTUM, AND ENERGY.

As we shall see, the familiar equations of HD are the result.

Note that the time derivative of the density W of some conserved quantity is equal to the negative divergence of the flux of W .

$$\int dV \frac{\partial W}{\partial t} = - \int dS \cdot \mathbf{v}W$$
$$= - \int dV \nabla \cdot (\mathbf{v}W),$$

by Gauss's theorem.



Hence

$$\frac{\partial W}{\partial t} = -\nabla \cdot (\mathbf{v}W).$$

This is the mathematical statement needed to treat the three conserved quantities: particles, momentum, and energy.

Let u_i = velocity of individual particle with

$$u_i = v_i + w_i$$

v_i = local mean bulk velocity.

w_i = thermal velocity relative to mean.

Compute mean over local volume $V = l^3$

$$N = \frac{1}{V} \sum_v, Nv_i = \frac{1}{V} \sum_v u_i = \frac{1}{V} \sum_v (v_i + w_i)$$

$$\sum_v w_i = 0$$

Particle density N , particle flux $\sum_{\nu} N u_i = N v_i$

$$\frac{\partial N}{\partial t} = -\frac{\partial}{\partial x_k} N v_k, \quad \frac{dN}{dt} = -N \frac{\partial v_k}{\partial x_k}$$

Momentum density

$$\frac{1}{V} \sum_{\nu} M u_i = N M v_i$$

Flux of momentum density

$$\frac{1}{V} \sum_{\nu} M u_i u_j = \frac{1}{V} \sum_{\nu} M v_i v_j + \frac{1}{V} \sum_{\nu} M w_i w_j$$

Flux of momentum density

$$N M v_i v_j + p_{ij}, \quad p_{ij} = \frac{1}{V} \sum_{\nu} M w_i w_j$$

p_{ij} is pressure tensor = flux of momentum density carried by thermal motions.

The time rate of change of the momentum density is equal to the negative divergence of the flux of momentum density.

$$\frac{\partial}{\partial t} NMv_i = -\frac{\partial}{\partial x_j} NMv_i v_j - \frac{\partial}{\partial x_i} p_{ij}$$

Using the equation for conservation of particles, this reduces to the familiar Euler equation,

$$NM \left(\frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} \right) = -\frac{\partial p_{ij}}{\partial x_j},$$

i.e Newton's equation of motion, recognizing that the momentum flux p_{ij} is equivalent to a force.

If an external force F_i (dynes/cm³) is introduced, the momentum equation becomes

$$NM \left(\frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} \right) = -\frac{\partial p_{ij}}{\partial x_j} + F_i$$

