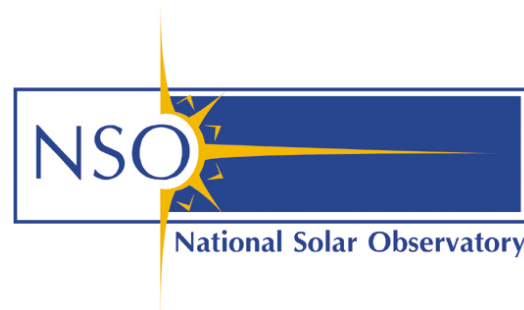


Introduction to Solar Radiative Transfer: II Detailed Radiative Processes

Han Uitenbroek
National Solar Observatory/Sacramento Peak
Sunspot NM, USA



Summer School Sunspot, June 11 2009

Overview

- Local Thermodynamic Equilibrium

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- Spectral lines in atoms, ions, and molecules

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- Non-Local Thermodynamic Equilibrium
- Doppler shift, Zeeman splitting

Local Thermodynamic Equilibrium (LTE)

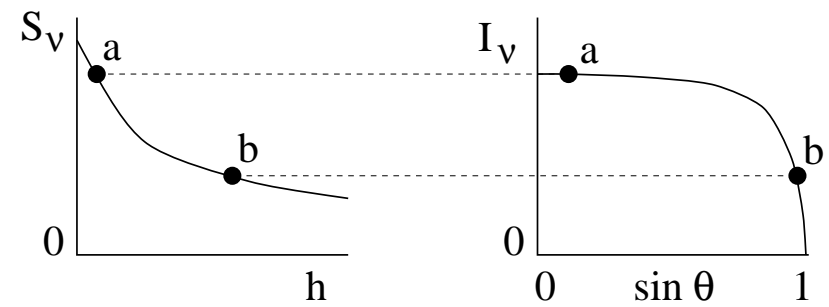
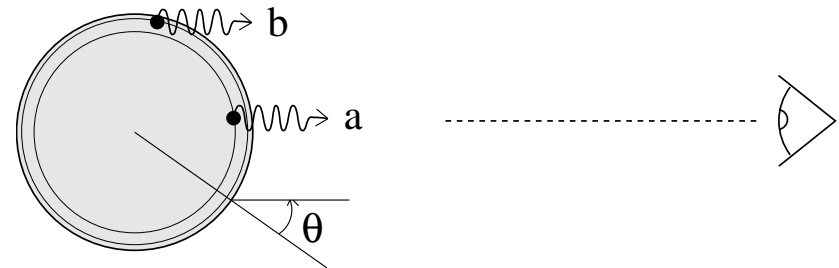
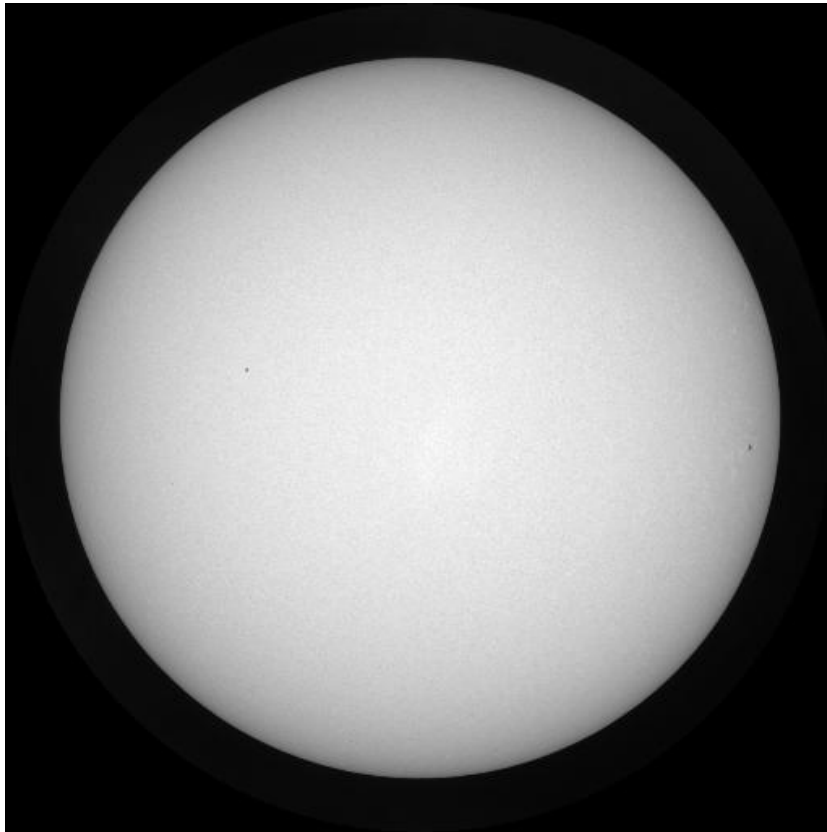
- Consider a medium in equilibrium in an isolated enclosure. The second law of thermodynamics tells us that this medium must have the same temperature and composition everywhere, and that its radiation field is isotropic.
- The energy absorbed by an infinitesimal volume of the medium must be equal to what it absorbs at **every** frequency:

$$j_\nu = \alpha_\nu I_\nu, \quad (\text{Kirchhoff's law})$$

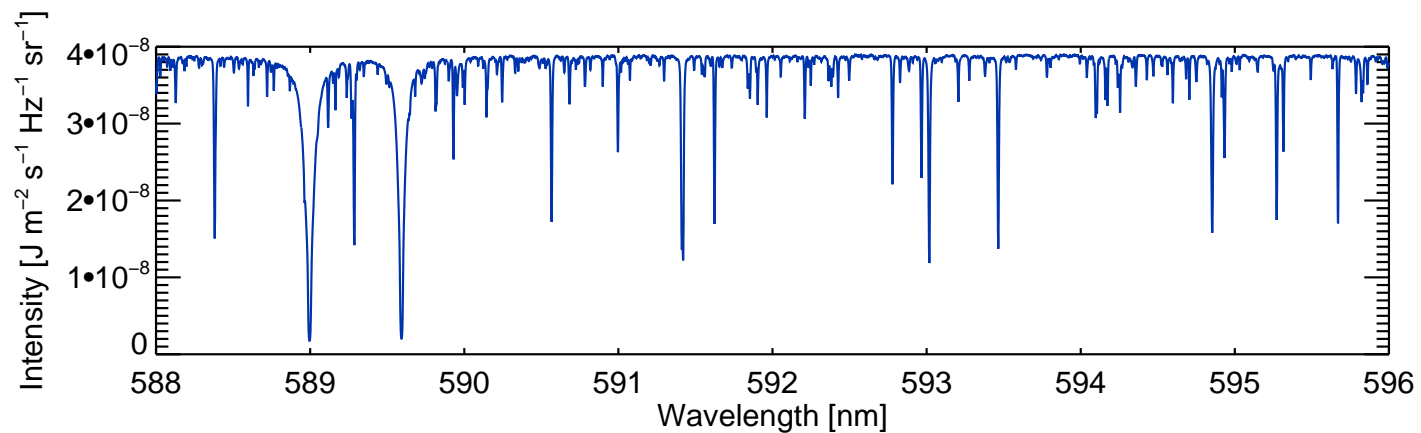
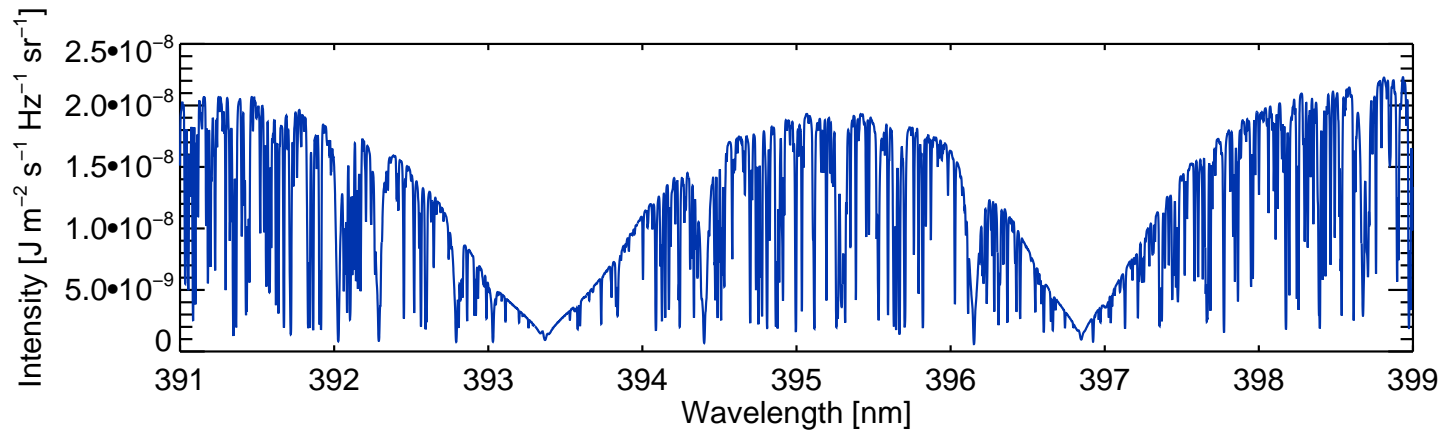
- The radiation field in an enclosure in strict Thermodynamic Equilibrium is given by the **Planck function** $B_\nu(T)$

$$j_\nu^{\text{TE}} = \alpha_\nu^{\text{TE}} B_\nu(T), \quad (\text{Kirchhoff – Planck})$$

Basic Radiative Transfer: Limb Darkening



Absorption lines also tell us temperature decreases with height

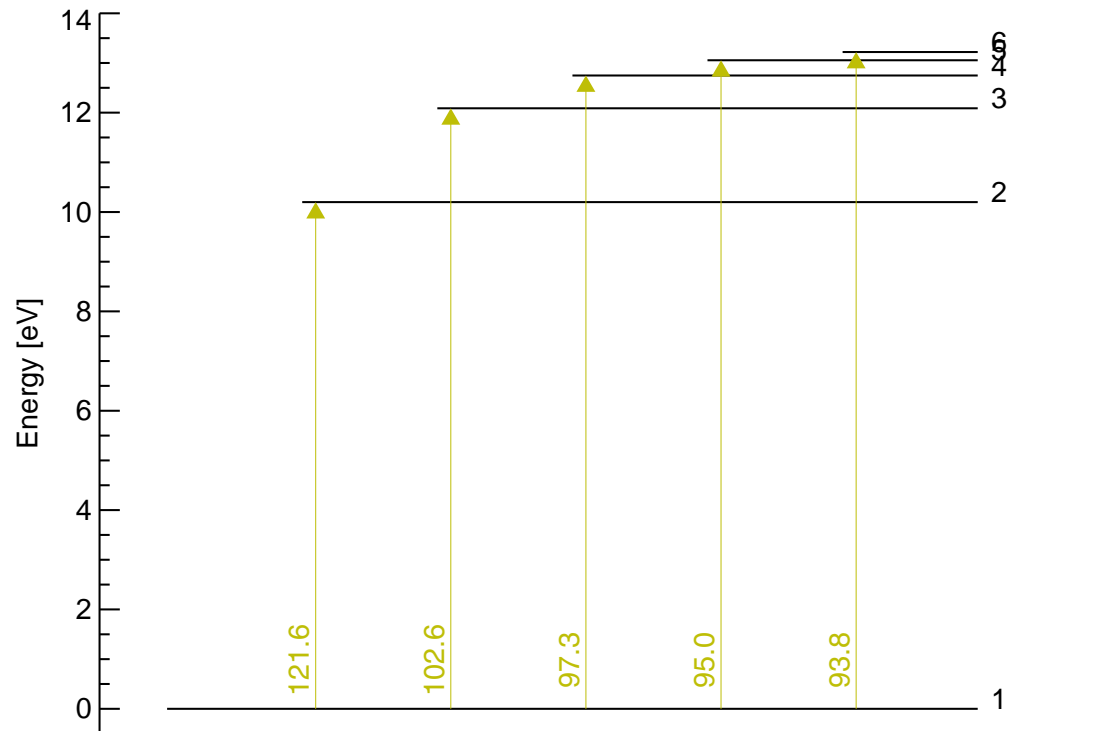


Termdiagram and Transitions in Hydrogen



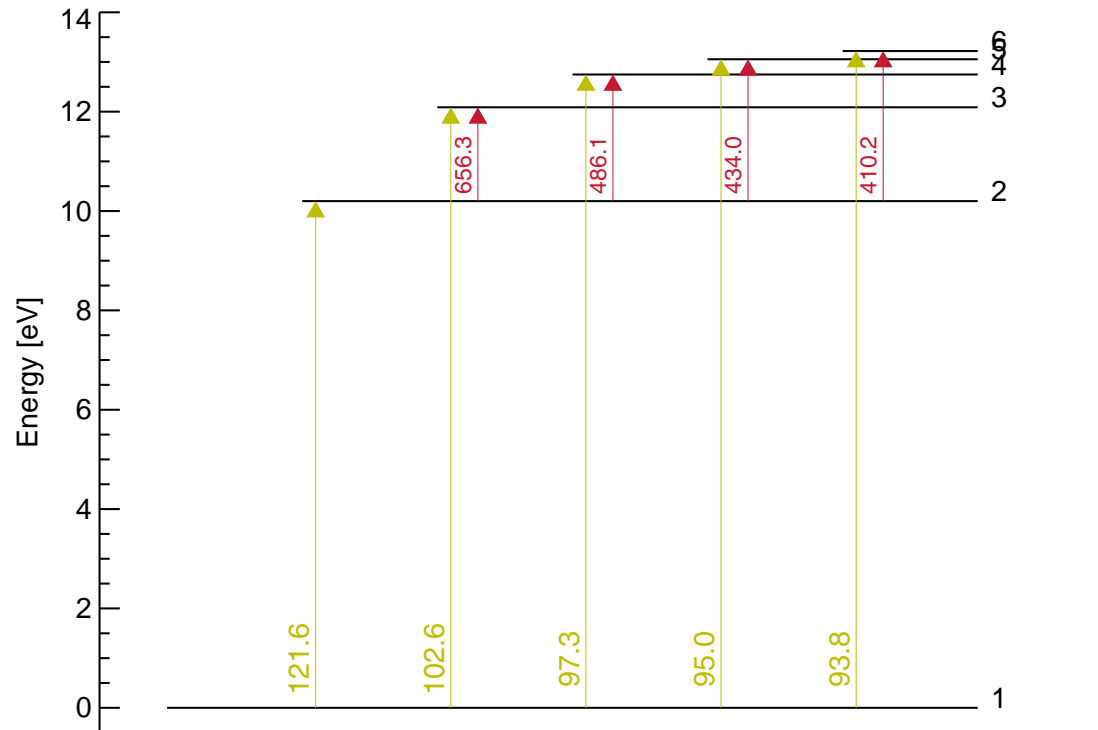
$$\Delta E = h\nu = \frac{hc}{\lambda}$$

Termdiagram and Transitions in Hydrogen



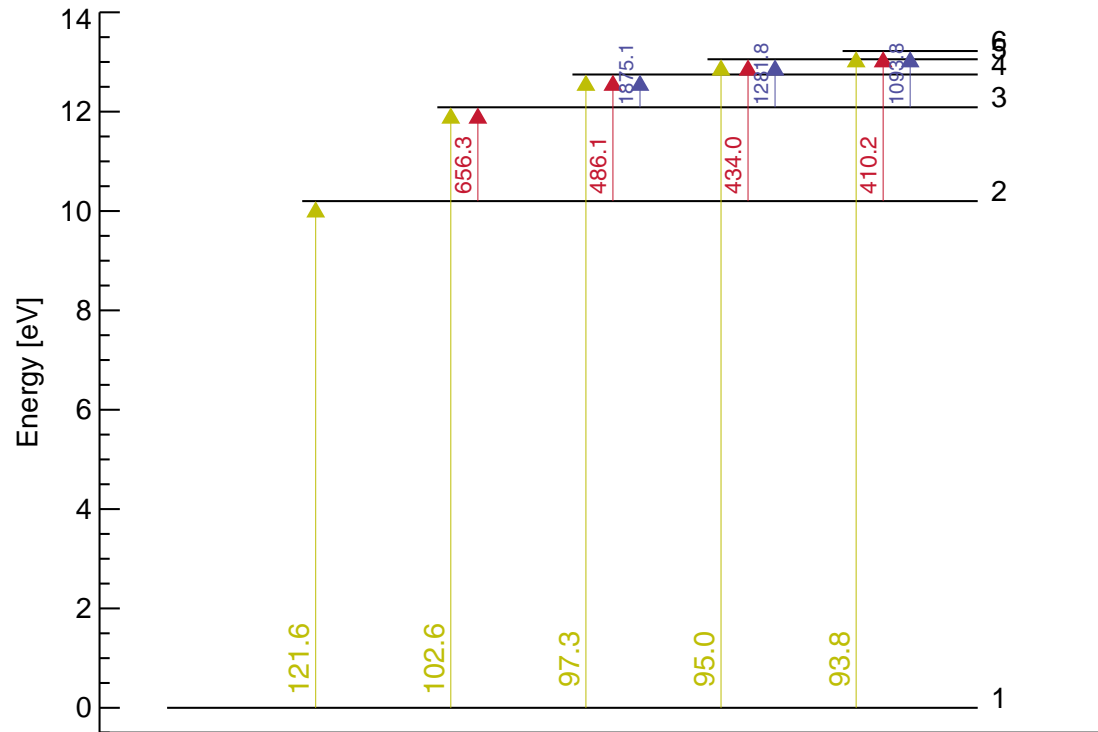
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Termdiagram and Transitions in Hydrogen



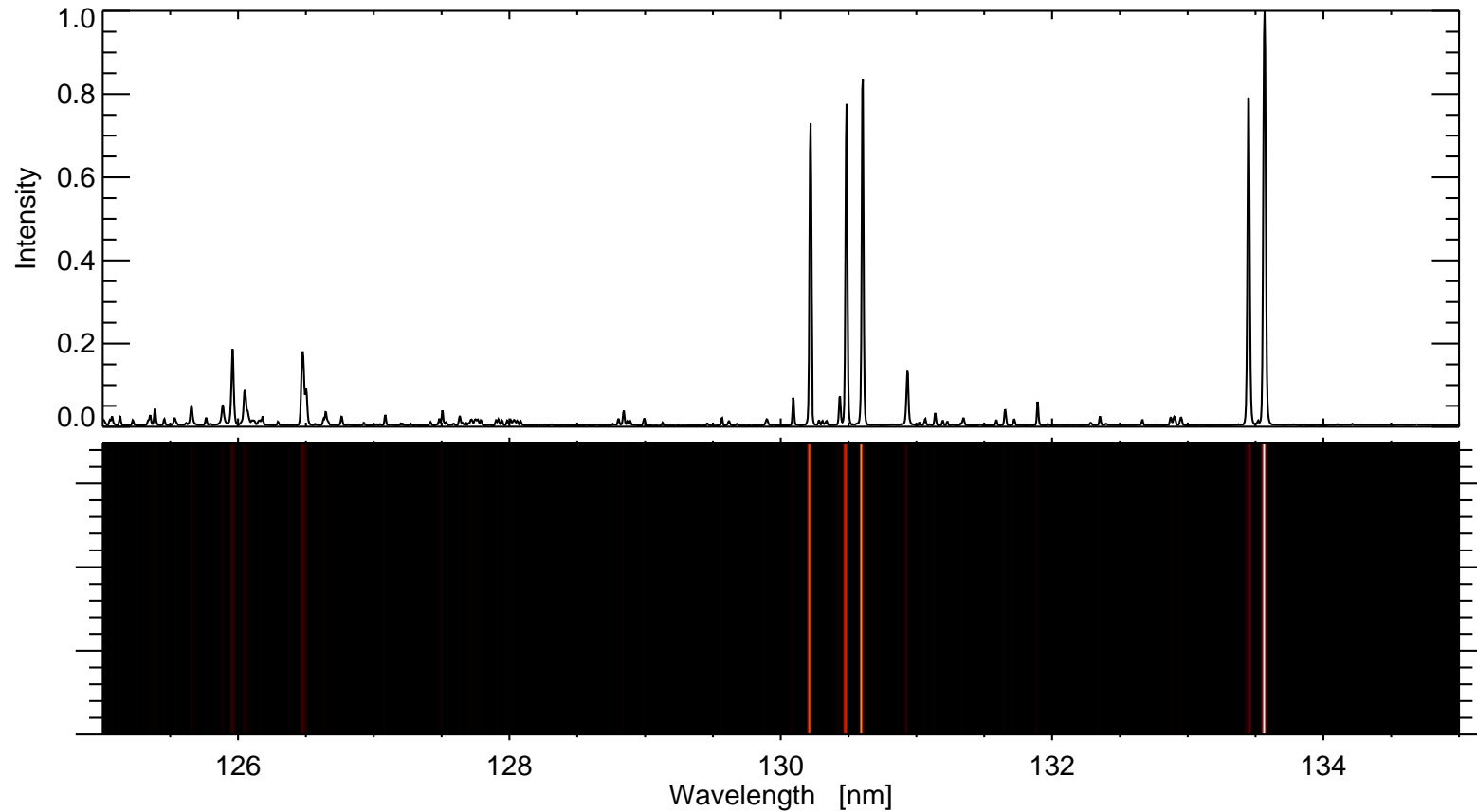
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Termdiagram and Transitions in Hydrogen

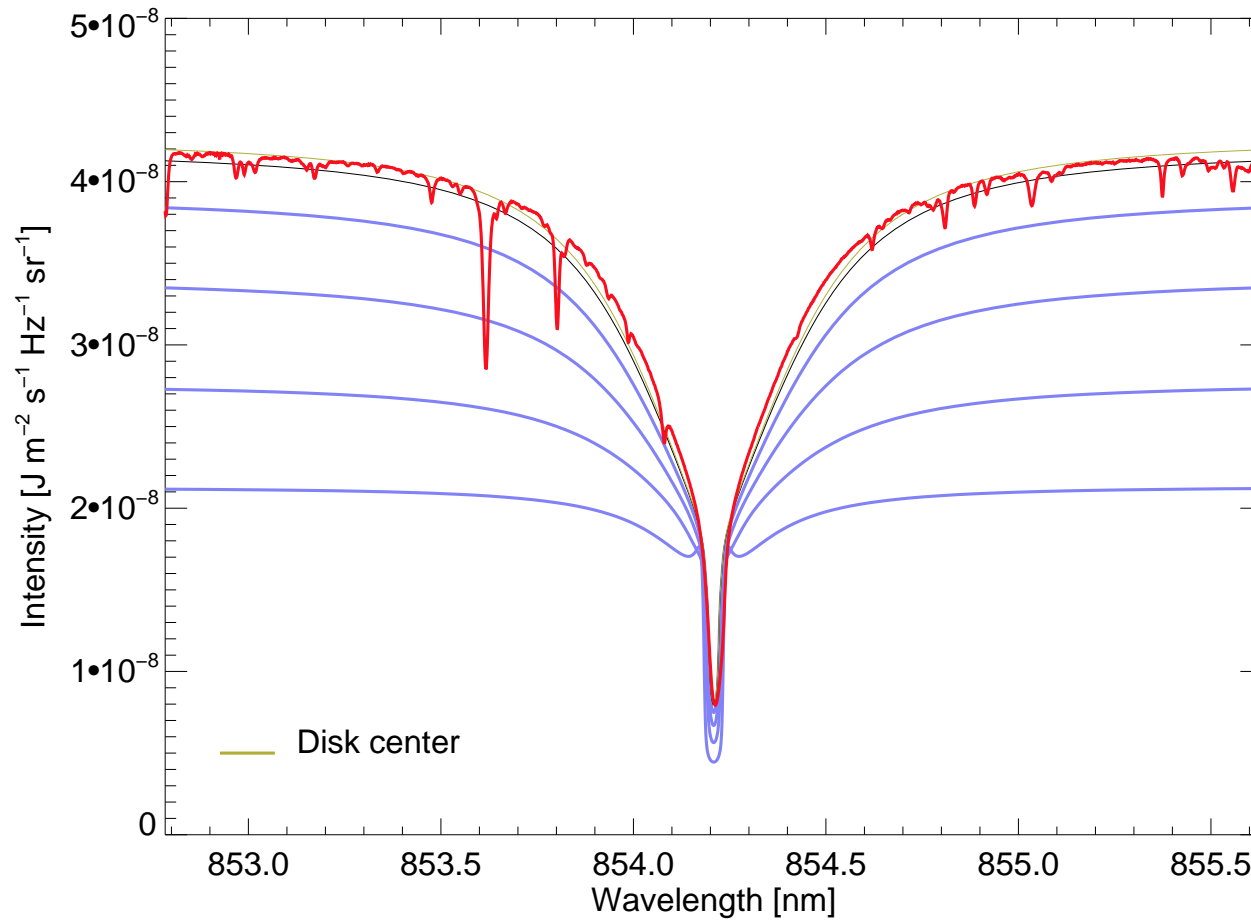


$$\Delta E = h\nu = \frac{hc}{\lambda}$$

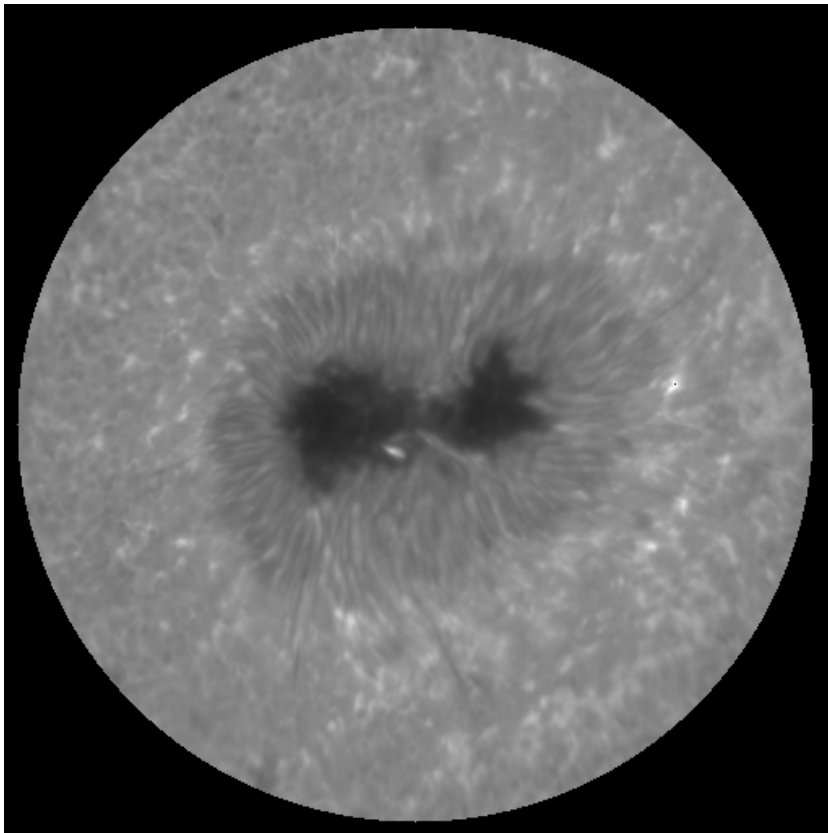
Emission lines tell us temperature is increasing with height



The Ca II 854.21 nm line

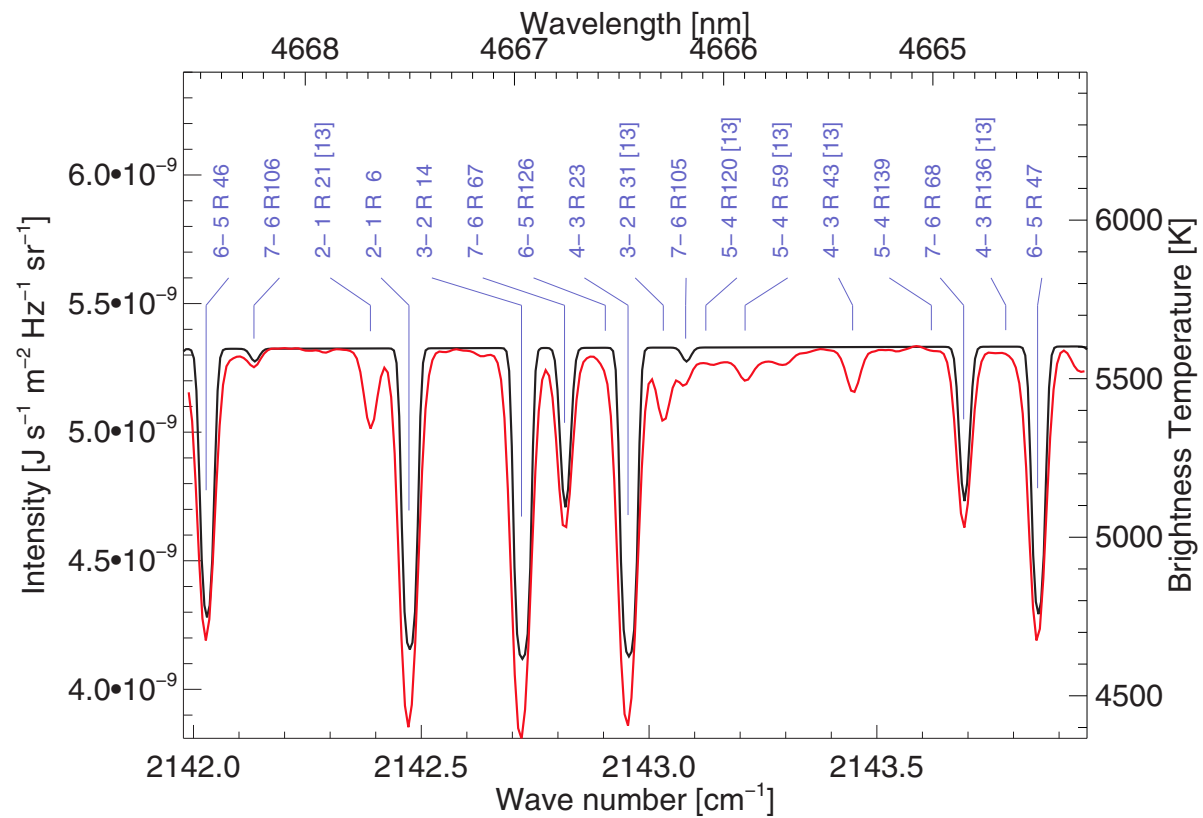


Tuning through the line, sampling different heights



Tritschler & Uitenbroek, IBIS DST

CO lines in the Infrared



Concentration of Molecules

Chemical equilibrium:

$$\frac{n_A n_B}{n_{AB}} = \left(\frac{2\pi m_{AB} kT}{h^2} \right)^{3/2} e^{-D/kT} \left(\frac{U_A U_B}{Q_{AB}} \right) \quad (1)$$

$$m_{AB} = \frac{m_A m_B}{m_A + m_B} \quad (2)$$

$$Q_{AB} = Q_{\text{rot}} Q_{\text{vib}} Q_{\text{el}} \quad (3)$$

Molecular lines

- Molecules have more degrees of freedom: rotation, vibration

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- Molecules have more degrees of freedom: **rotation, vibration**
- Hence there are many more molecular levels and transitions between them
- Molecular lines are grouped in **bands**, because energy differences between different electronic states are generally much larger than between different vibration states within one electronic state, and these are typically larger again than energy differences between rotational states in one vibrational state. **Example**

Continuum processes

- Free–free transitions
- H^- bound–free and free–free
 $\text{H}^- + h\nu \rightleftharpoons \text{H} + e(\nu)$
 $\text{H} + e(\nu) + h\nu \rightleftharpoons \text{H} + e(\nu')$
- Thomson scattering

$$\alpha_e^T = N_e \sigma_e = N_e \frac{8\pi}{3m_e^4 c^2} \frac{q_e^4}{(4\pi\epsilon_0)^2}$$

- Rayleigh scattering

$$\alpha^R(\omega) = \sigma_e f_{ij} \omega^4 / (\omega_{ij}^2 - \omega^2)^2$$

Einstein Relations for Bound-Bound Transitions

Spontaneous emission $j \rightarrow i$:

$$j_{\nu}^{\text{spont}} = n_j (A_{ji} h \nu_{ij} / 4\pi) \varphi_{\nu} \quad (4)$$

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Stimulated emission $j \rightarrow i$:

$$j_{\nu}^{\text{stim}} = n_j (B_{ji} h\nu_{ij} / 4\pi) \psi_{\nu} I_{\nu}, \quad A_{ji} = (2h\nu^3 / c^2) B_{ji} \quad (5)$$

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Absorption $i \rightarrow j$:

$$\alpha_{\nu} I_{\nu} = n_i (B_{ij} h \nu_{ij} / 4\pi) \varphi_{\nu} I_{\nu}, \quad g_i B_{ij} = g_j B_{ji} \quad (6)$$

Source Function for Spectral Line

$$\frac{dI_\nu}{ds} = [n_j A_{ji} \varphi_\nu + n_j B_{ji} \varphi_\nu I_\nu - n_i B_{ij} \varphi_\nu I_\nu] (h\nu_{ij}/4\pi) \quad (7)$$

$$\frac{dI_\nu}{ds} = [n_j A_{ji} - (n_i B_{ij} - n_j B_{ji}) I_\nu] (h\nu_{ij} \varphi_\nu / 4\pi)$$

Opacity and source function:

$$\chi_\nu^l = \left(\frac{n_i B_{ij} h \nu_{ij}}{4\pi} \right) \varphi_\nu \left(1 - \frac{n_j B_{ji}}{n_i B_{ij}} \right) \quad (8)$$

$$S_\nu^l = \frac{n_j A_{ji}}{n_i B_{ij} - n_j B_{ji}}$$

Collisional Bound-Bound Transitions

Collisional excitation by electrons:

$$n_i C_{ij}(T) = n_i N_e \int_{v_0}^{\infty} \sigma_{ij}(v) v f(v) dv, \quad (1/2) m v_0^2 = h \nu_0 \quad (9)$$

Maxwellian velocity distribution

$$f(v) dv = \left(\frac{m}{2\pi kT} \right)^{3/2} \exp(-mv^2/2kT) 4\pi v^2 dv \quad (10)$$

Statistical Equilibrium Equations

$$\frac{dn_i}{dt} = \sum_{j \neq i} n_j P_{ji} - n_i \sum_{j \neq i} P_{ij} = 0 \quad (11)$$

$$P_{ij} \equiv C_{ij} + R_{ij}$$

Radiative rates:

$$R_{ij} = \int \oint \frac{B_{ij}}{4\pi} I_\nu \varphi_\nu d\Omega d\nu \quad (12)$$

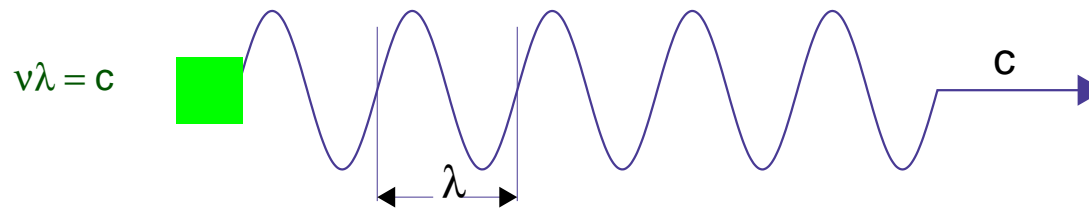
$$R_{ji} = A_{ji} + \int \oint \frac{B_{ij}}{4\pi} I_\nu \varphi_\nu d\Omega d\nu$$

Numerical Non-LTE Radiative Transfer

$$\frac{dn_i}{dt} = \sum_{j \neq i} n_j (R_{ji} + C_{ji}) - n_i \sum_{j \neq i} (R_{ij} + C_{ij}) = 0 \quad (13)$$

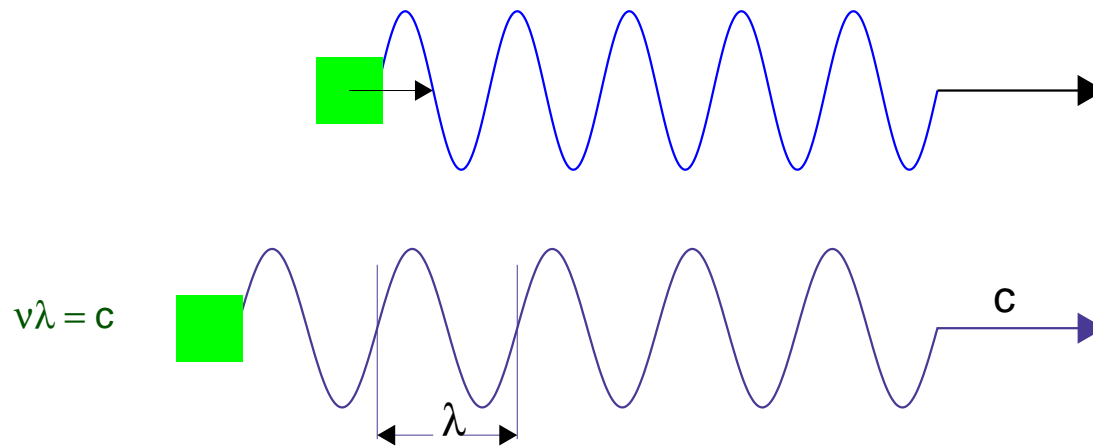
$$\frac{1}{\chi_\nu^{\text{tot}}} \frac{dI_\nu}{ds} = \frac{j_\nu^{\text{tot}}}{\chi_\nu^{\text{tot}}} - I_\nu = S_\nu^{\text{tot}} - I_\nu \quad (14)$$

Shift of wavelength due to motion: Doppler shift



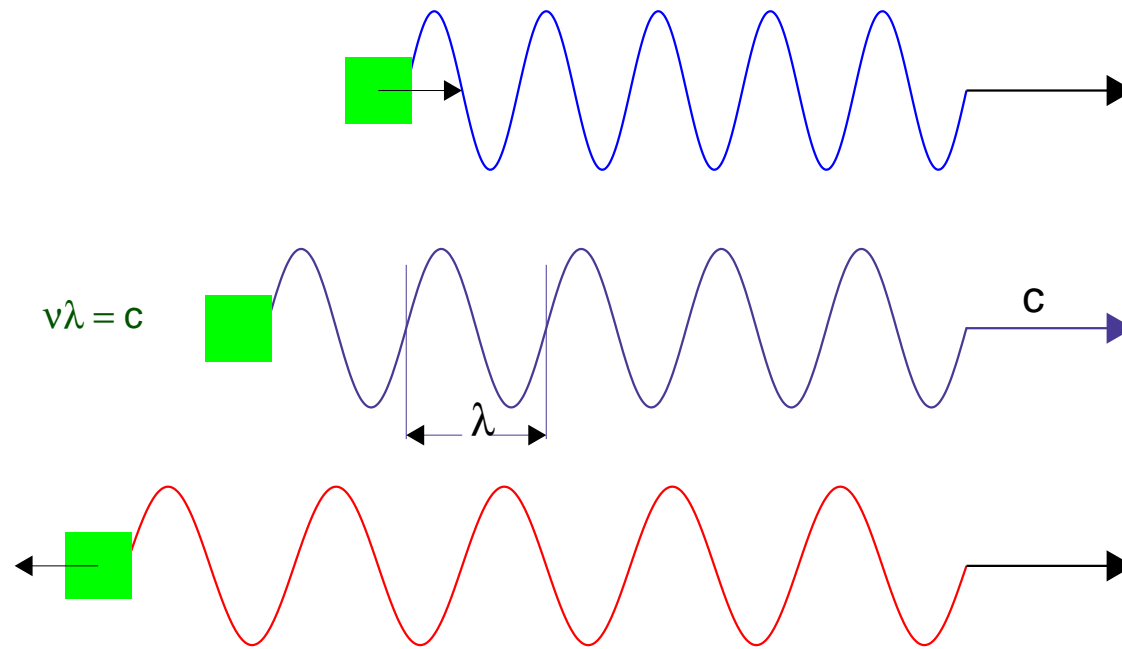
$$\Delta\lambda = \frac{v}{c}\lambda_0$$

Shift of wavelength due to motion: Doppler shift



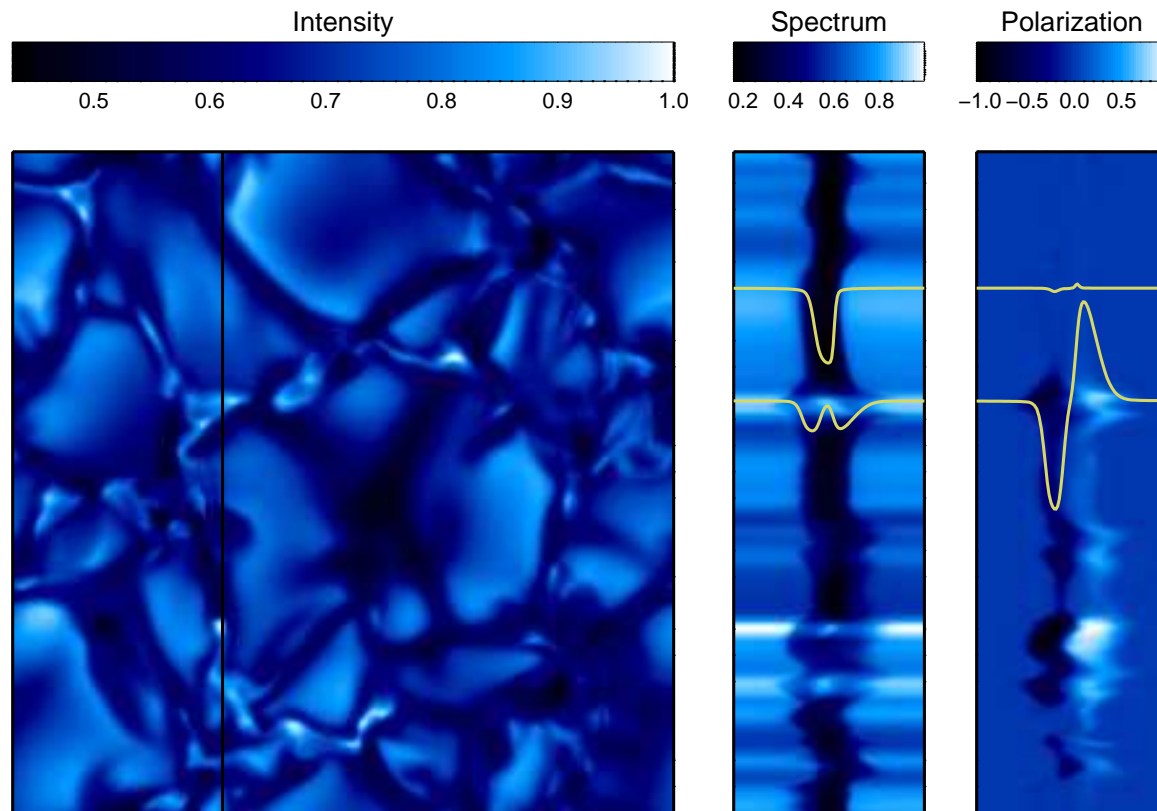
$$\Delta\lambda = \frac{v}{c}\lambda_0$$

Shift of wavelength due to motion: Doppler shift



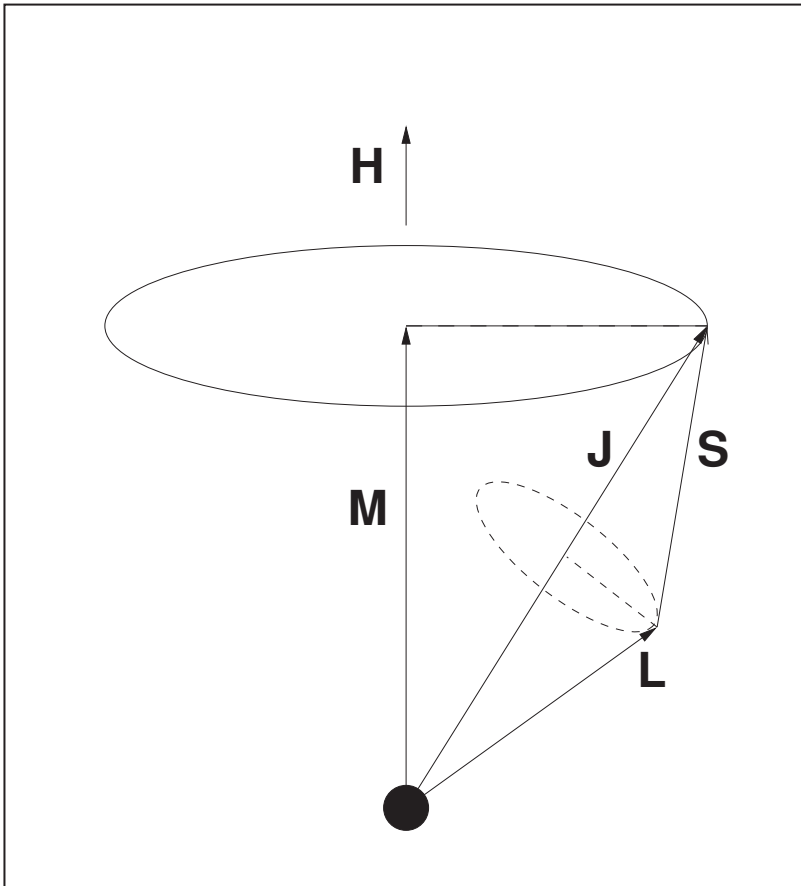
$$\Delta\lambda = \frac{v}{c}\lambda_0$$

Spatially Resolved Spectral Lines

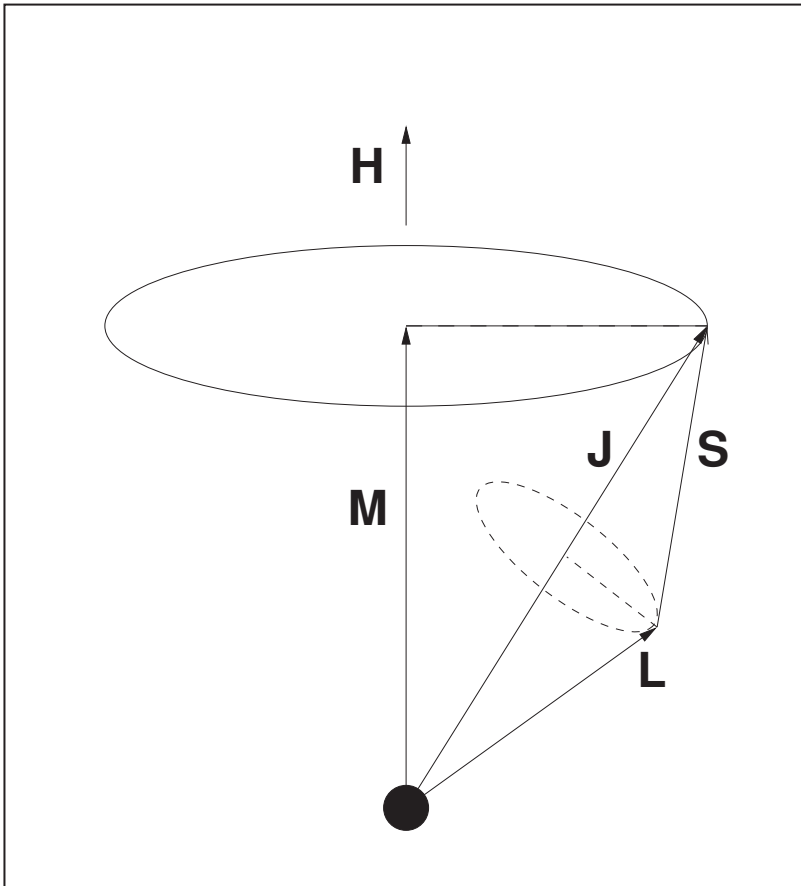


Polarized light

The Zeeman effect in atoms



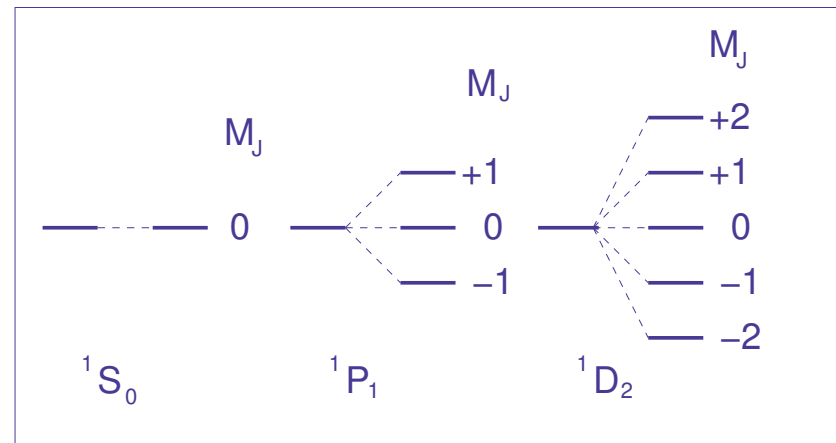
The Zeeman effect in atoms



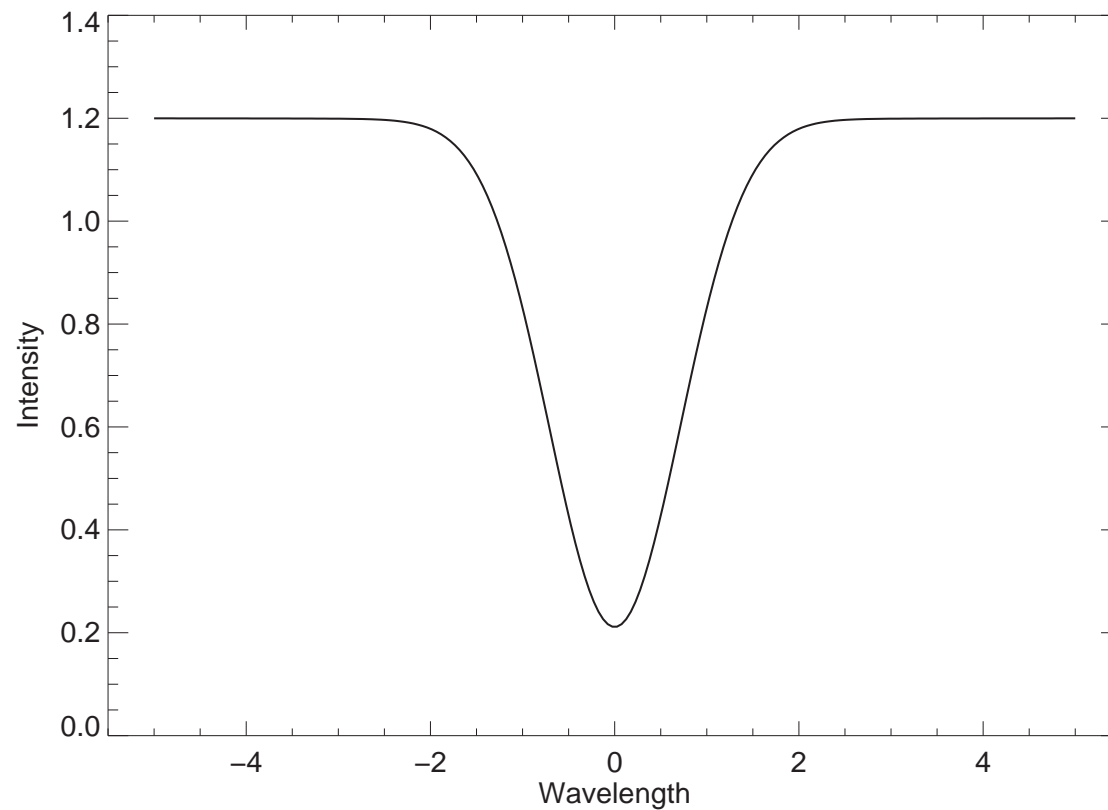
$$M_J = -J, -J + 1, \dots, J - 1, J$$

$$J_z = M_J \hbar$$

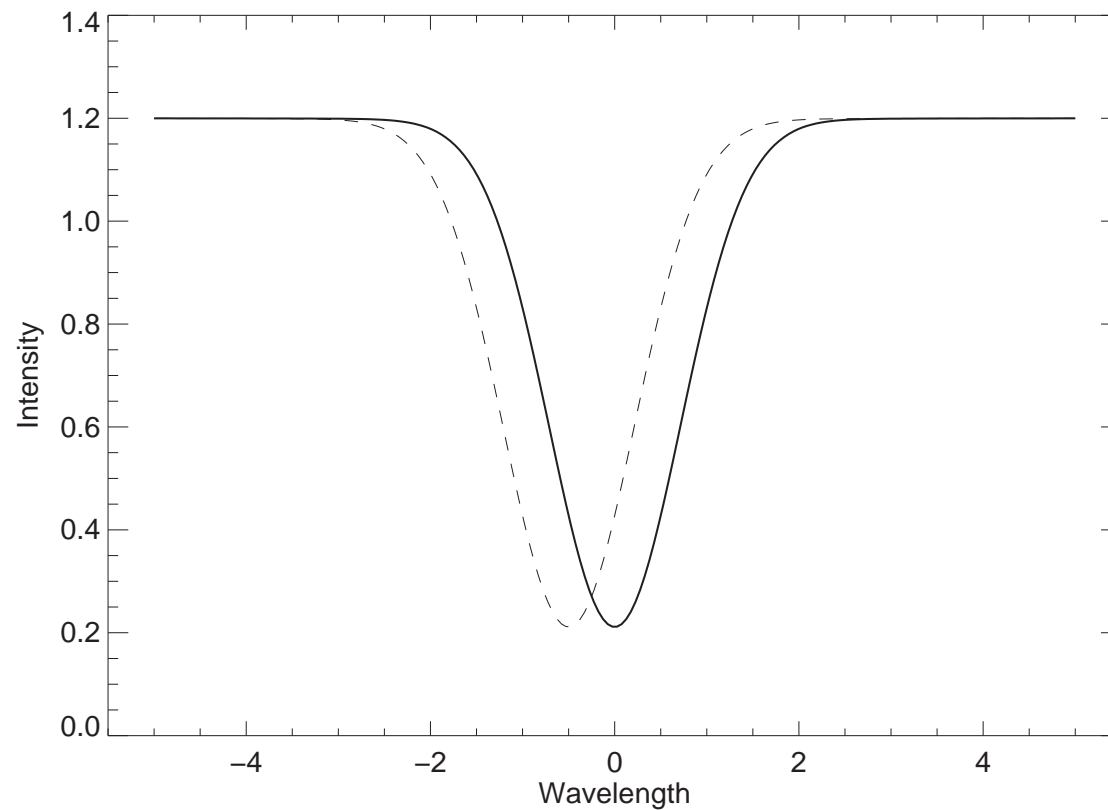
$$E = E_0 + g_L M_J \mu_H H$$



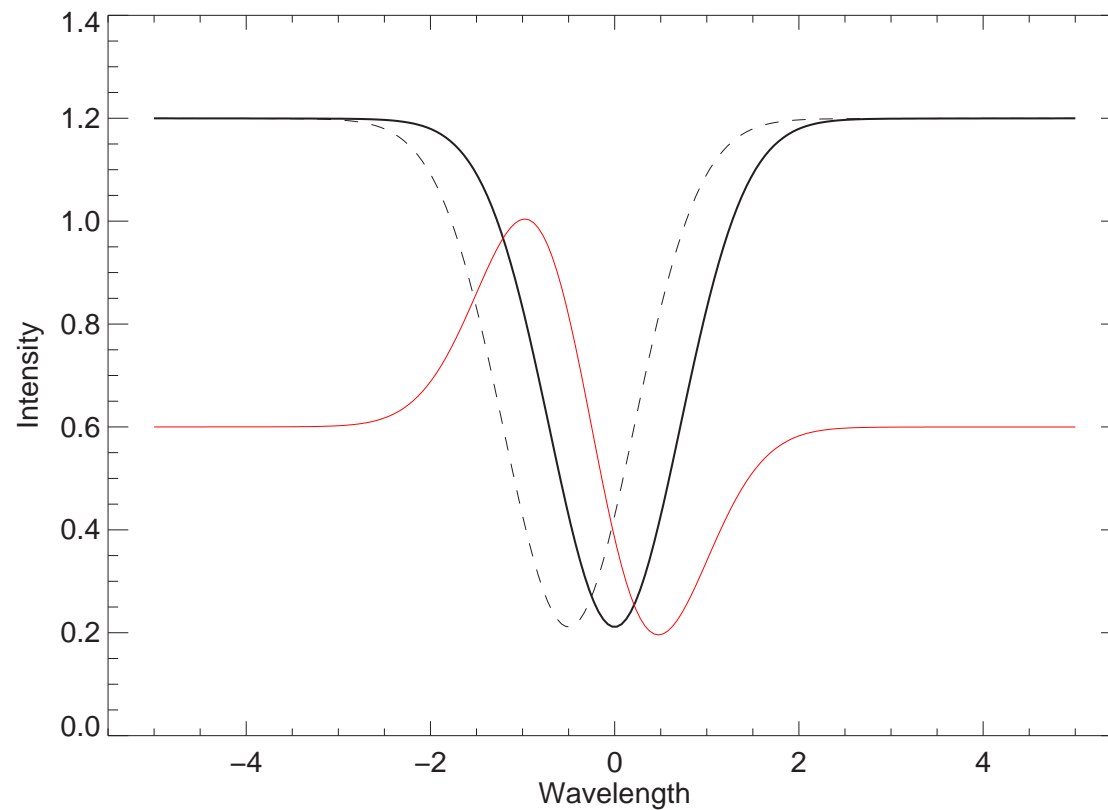
Measuring the magnetic field strength with circular polarization



Measuring the magnetic field strength with circular polarization

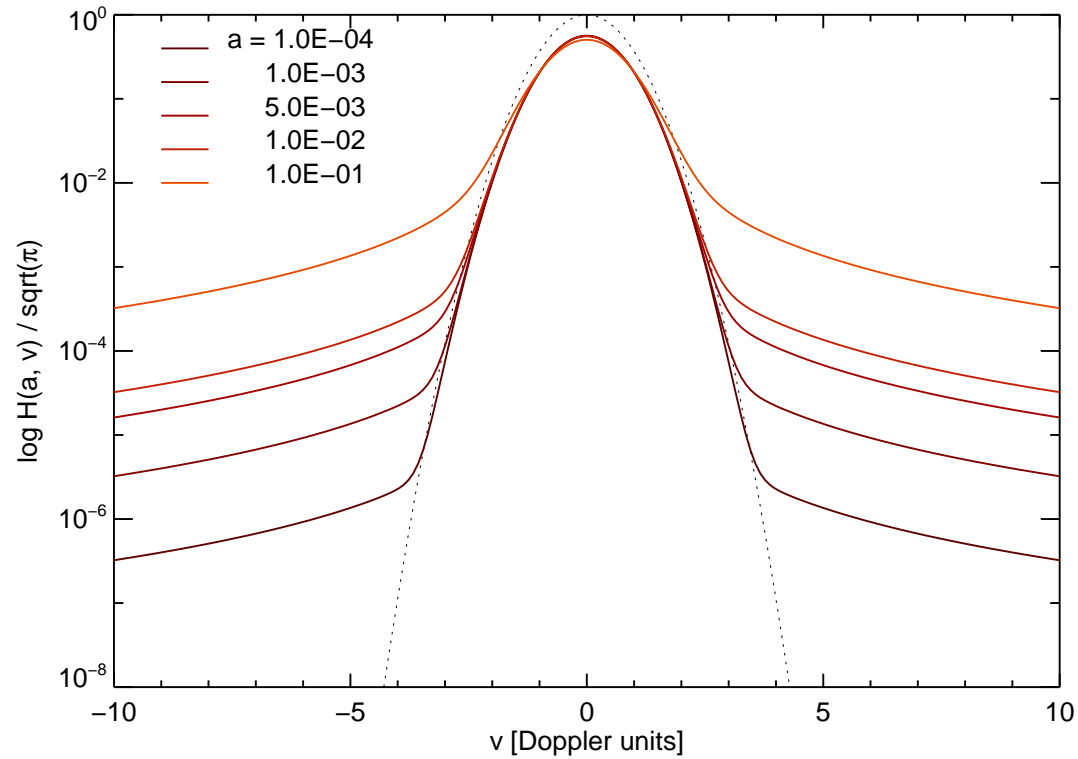


Measuring the magnetic field strength with circular polarization



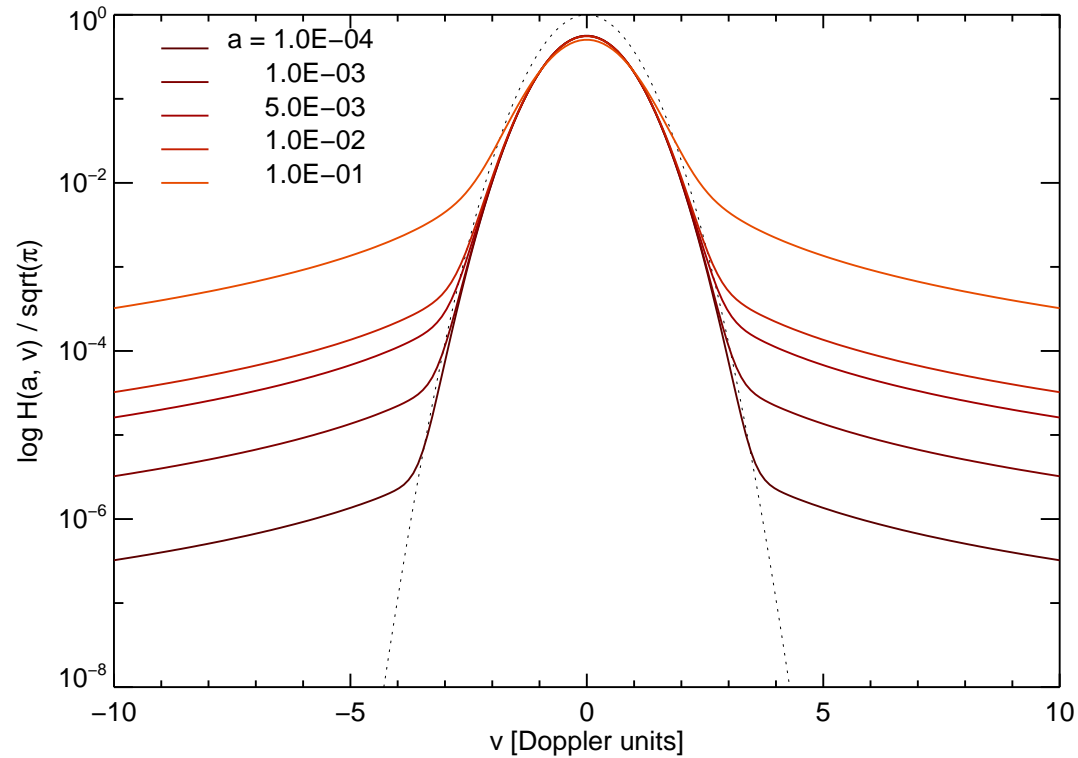
End Part II

Voigt Functions



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Voigt Functions



$$\psi(\nu - \nu_0) = \frac{H(a, v)}{\sqrt{\pi} \Delta\nu_D}$$

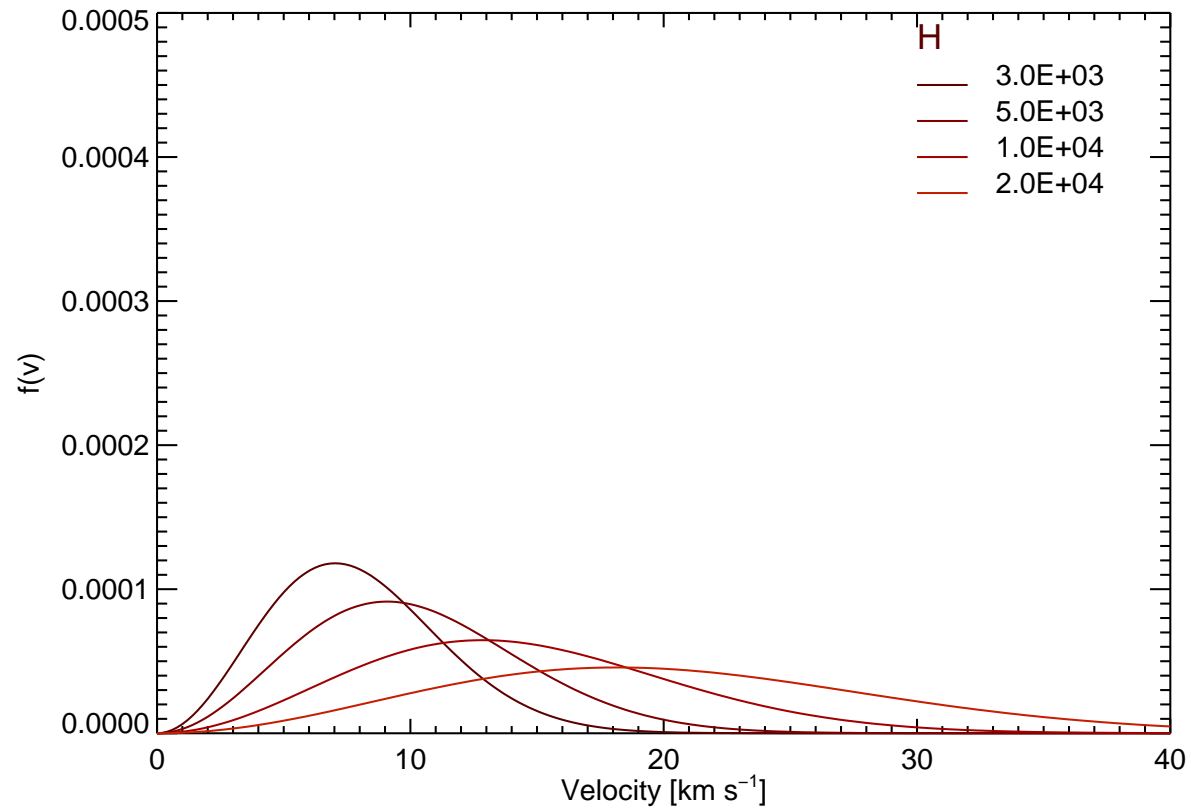
$$\Delta\nu_D \equiv \frac{\nu_0}{c} \sqrt{\frac{2kT}{m}}$$

$$v = \frac{(\nu - \nu_0)}{\Delta\nu_D}$$

$$a = \frac{\Gamma}{4\pi \Delta\nu_D}$$

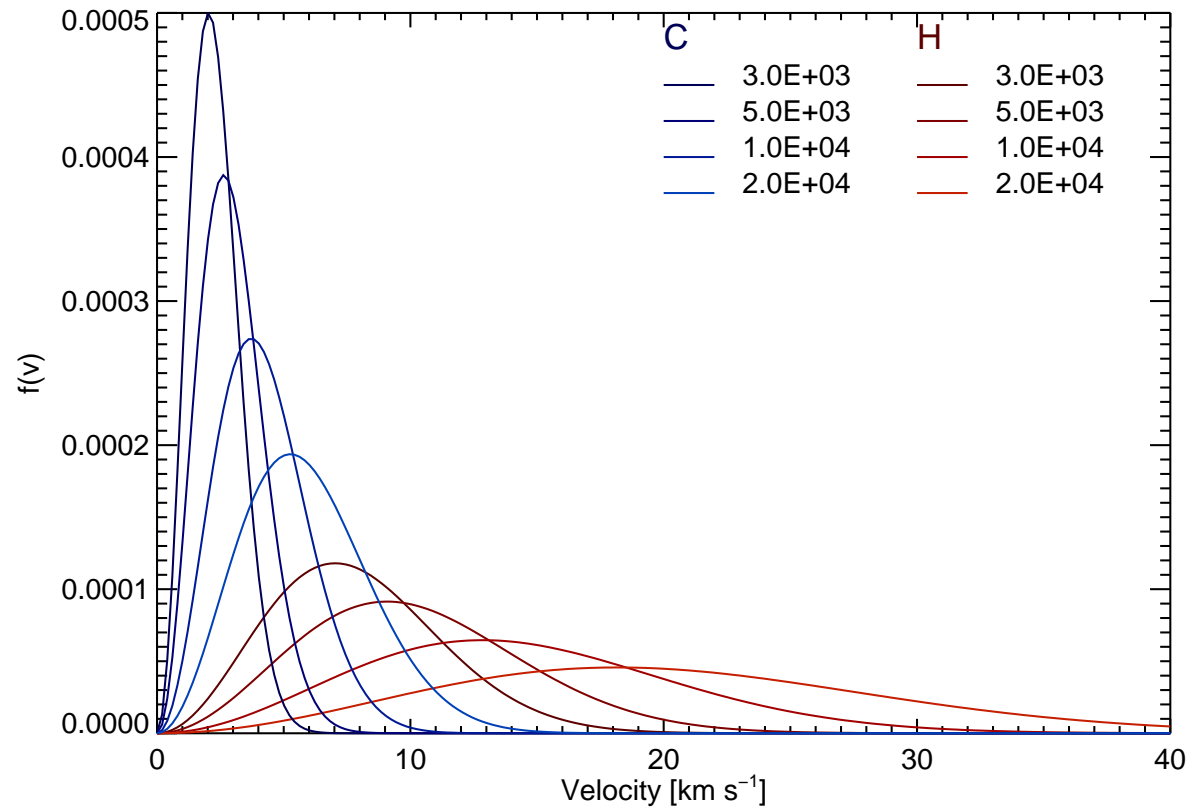
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Maxwellian Velocity Distribution



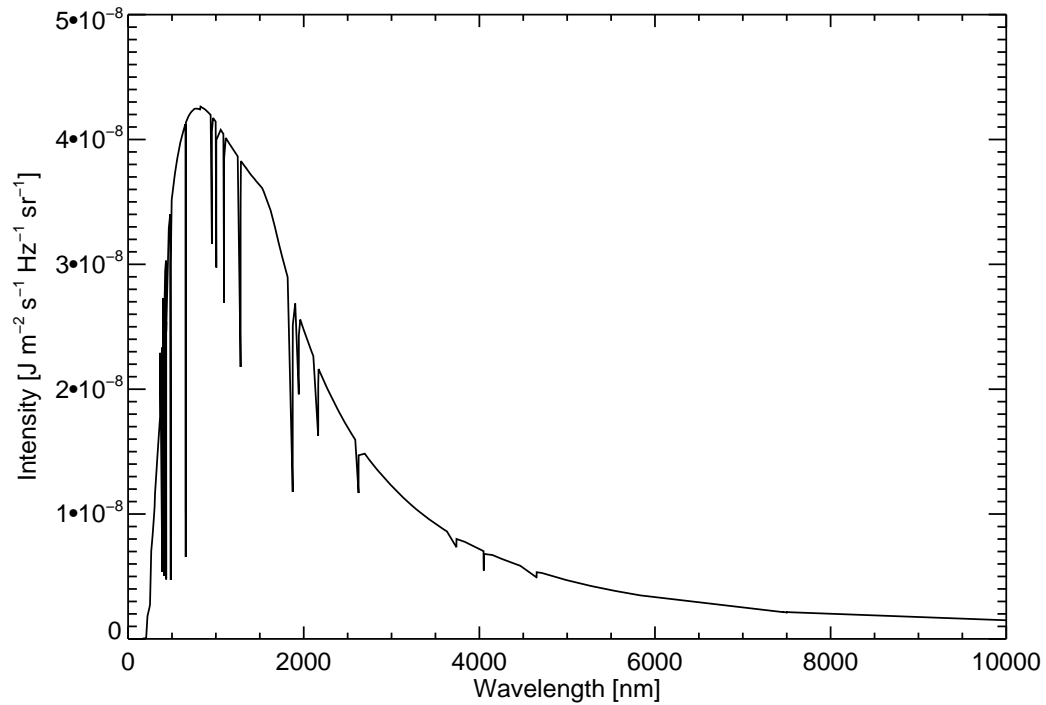
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Maxwellian Velocity Distribution



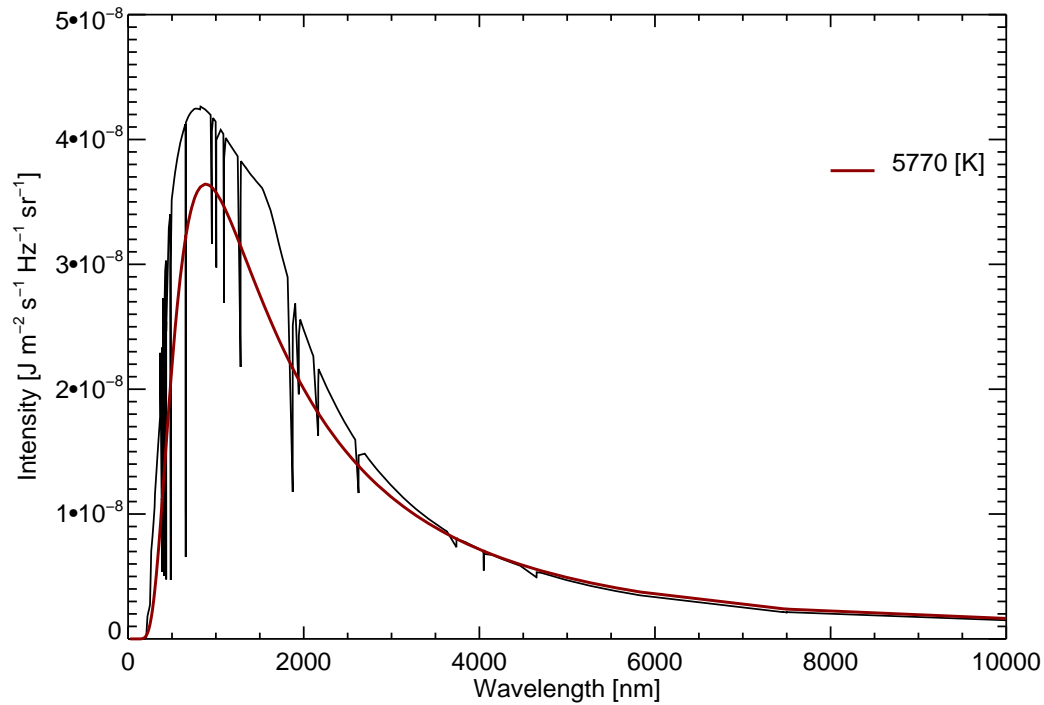
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The Solar Spectrum and Surface Temperature



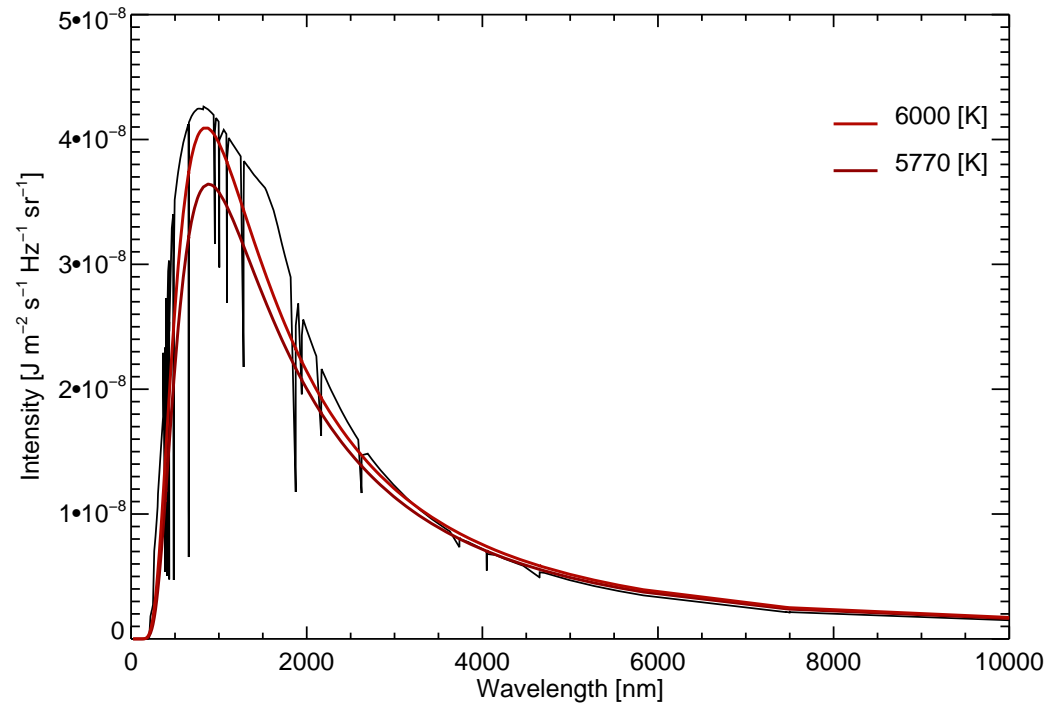
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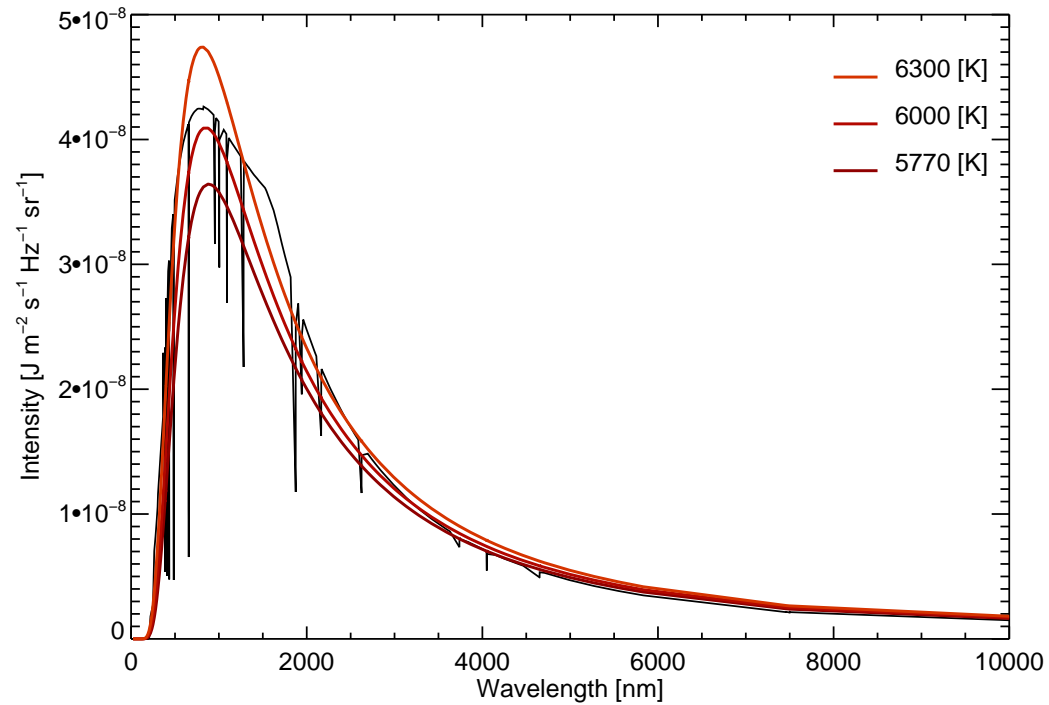
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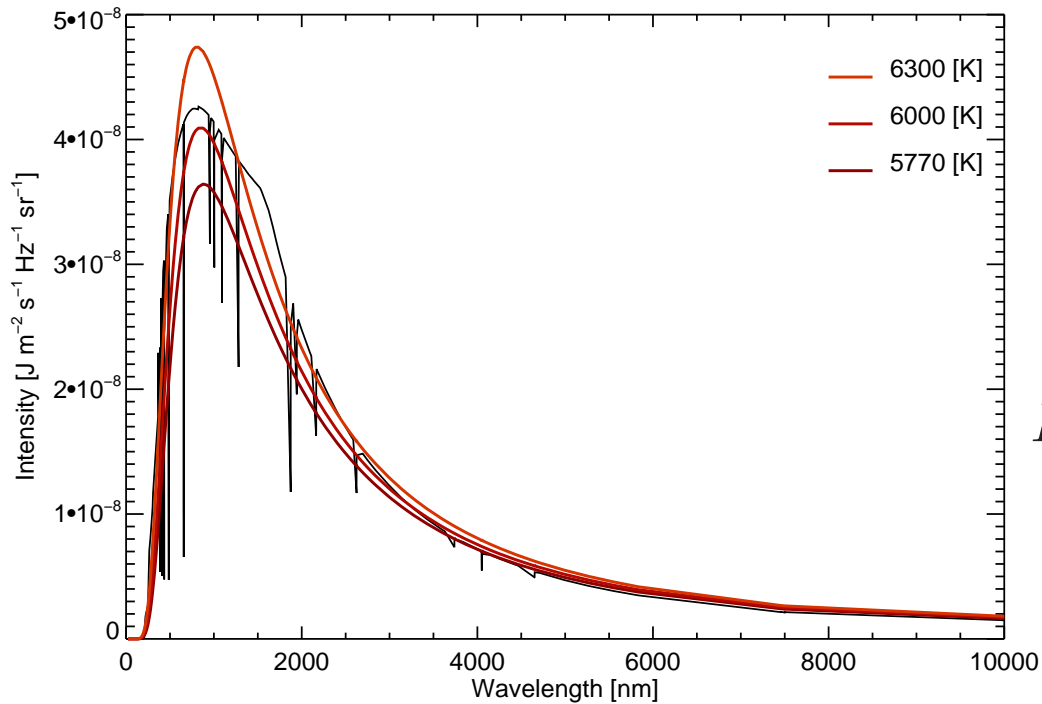
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The Solar Spectrum and Surface Temperature



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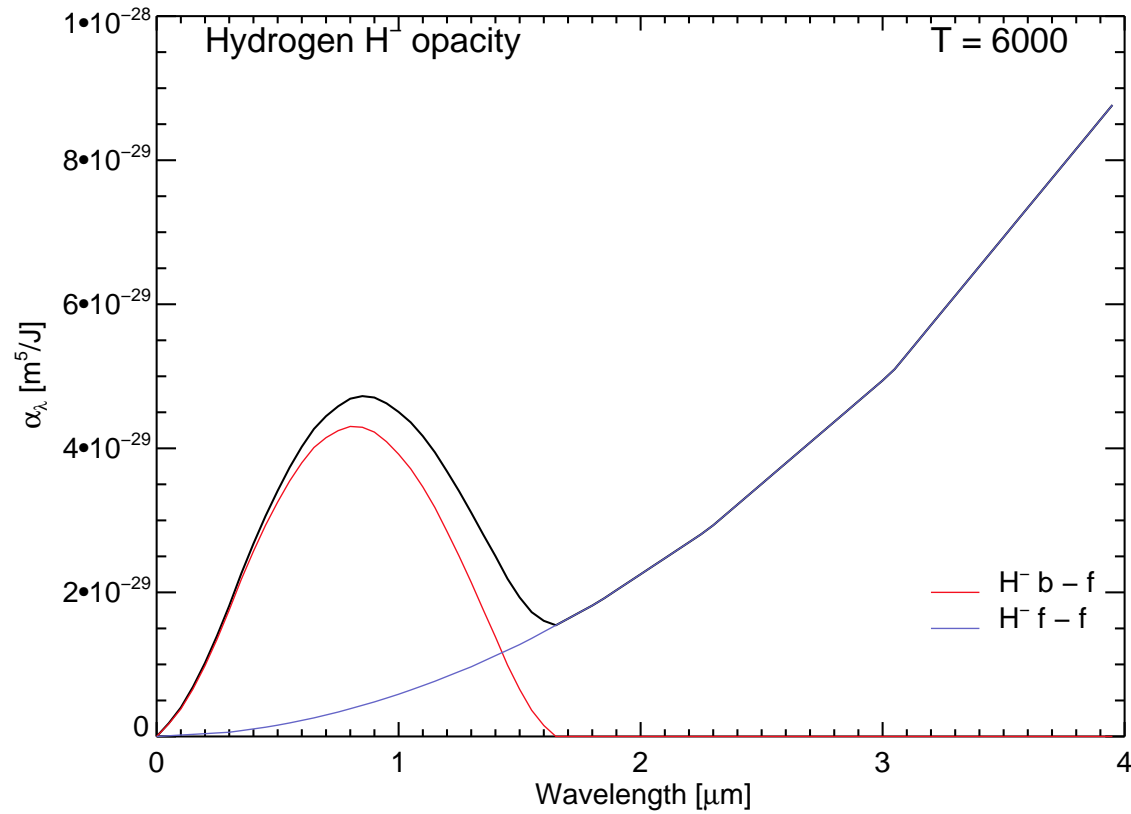
The Solar Spectrum and Surface Temperature



$$B_\nu(T) \equiv \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

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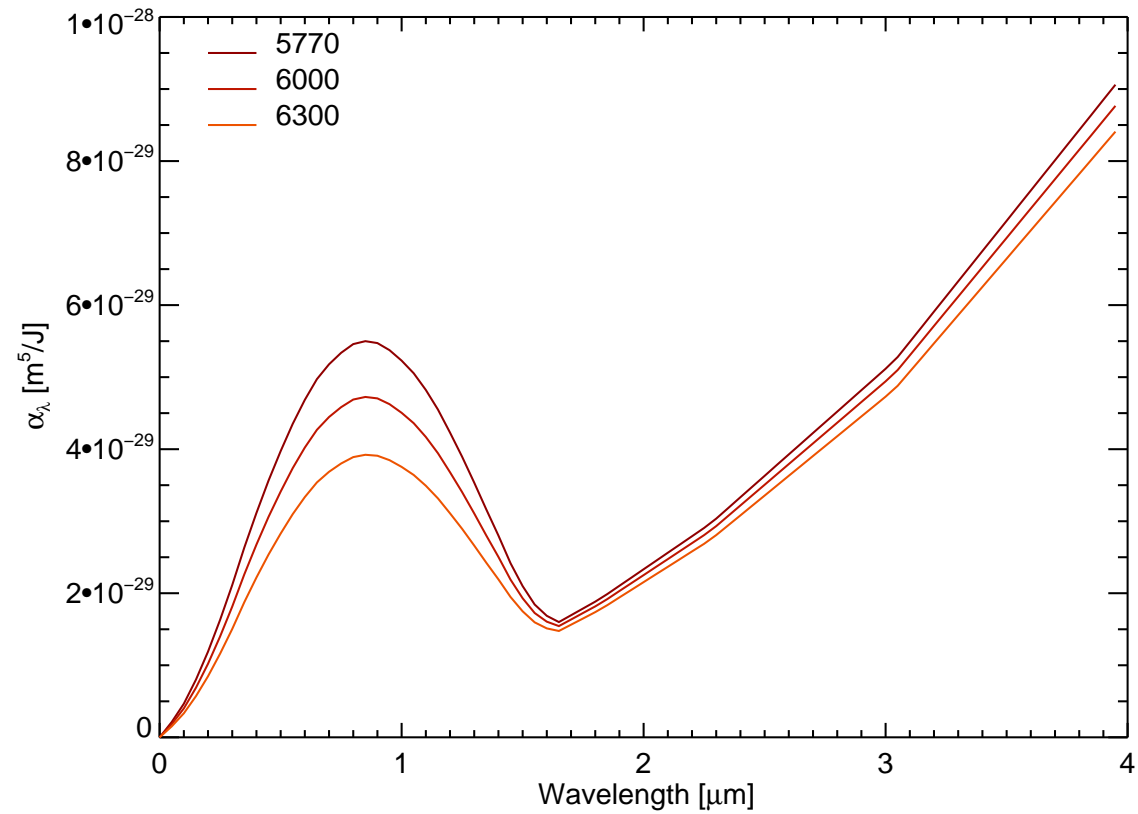
H⁻ Opacity



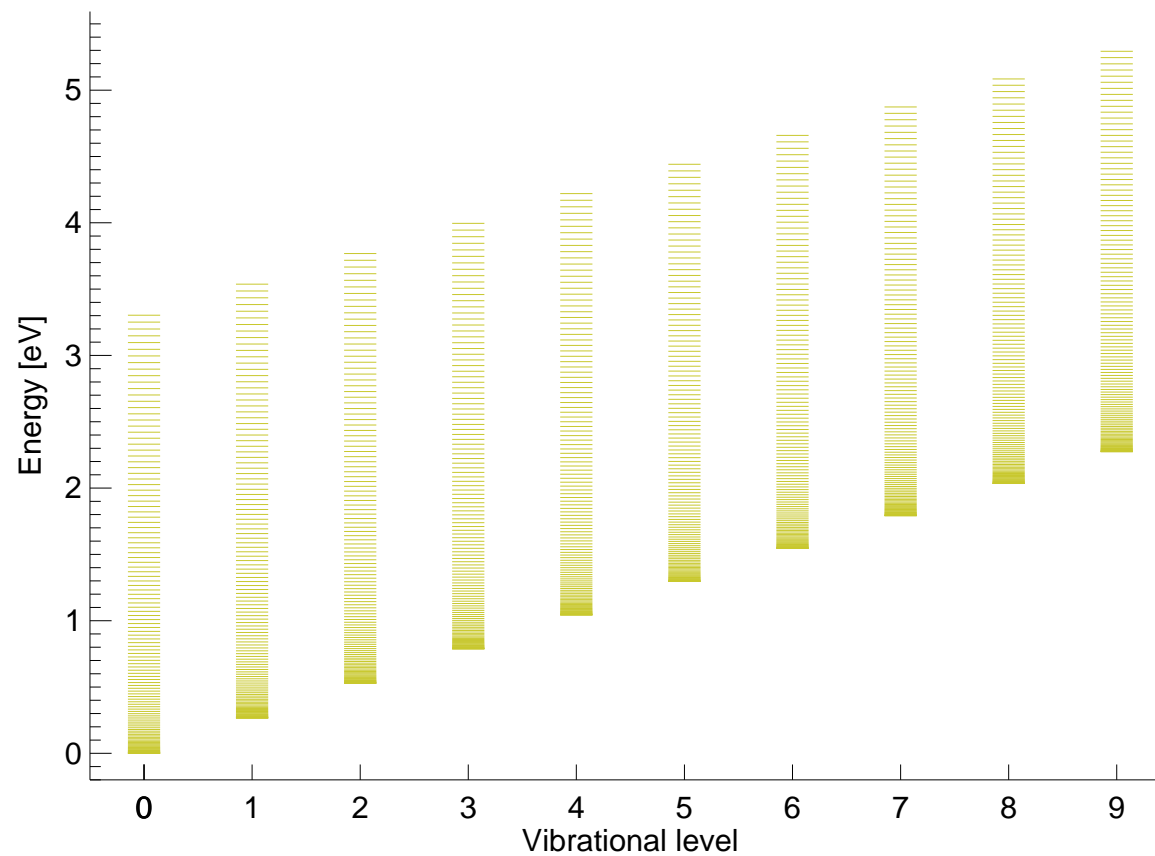
$$\Delta E^{\text{bf}} = 0.754 \text{ eV}$$

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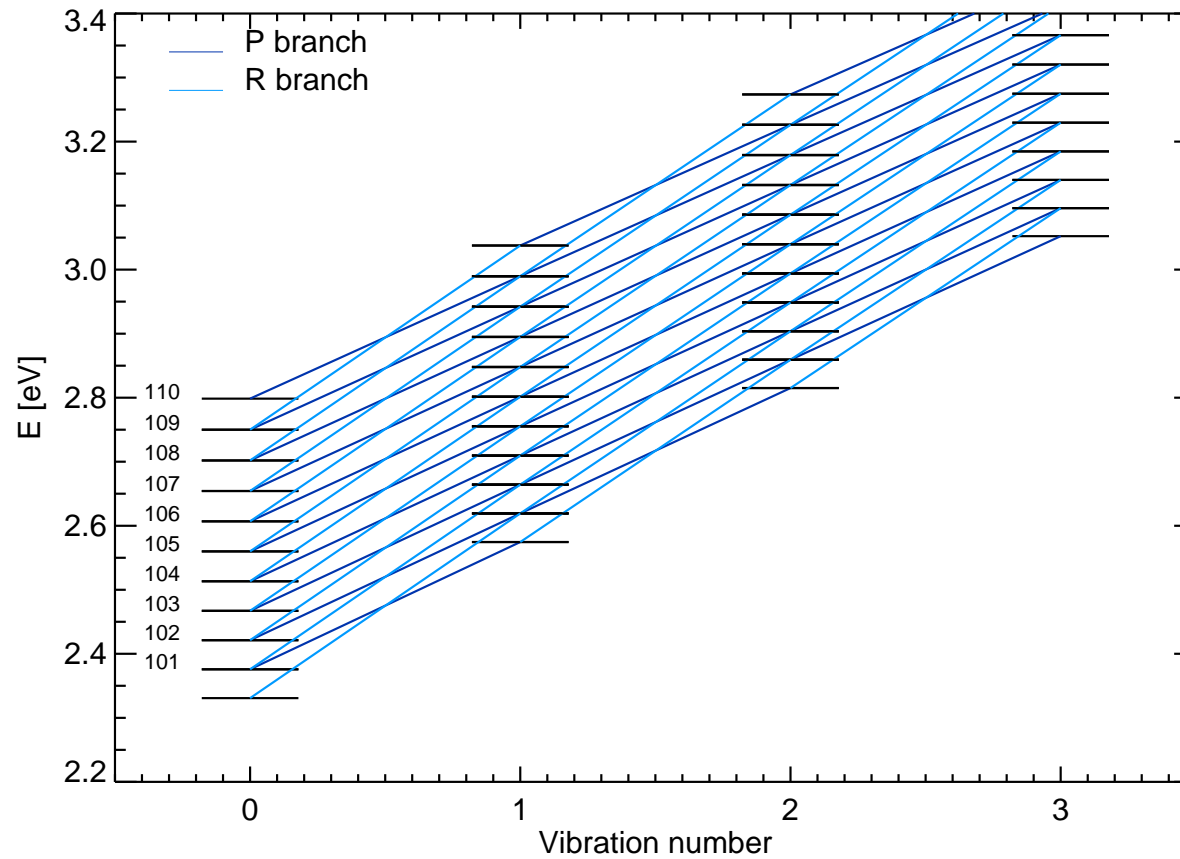
H⁻ Opacity Changes with Temperature



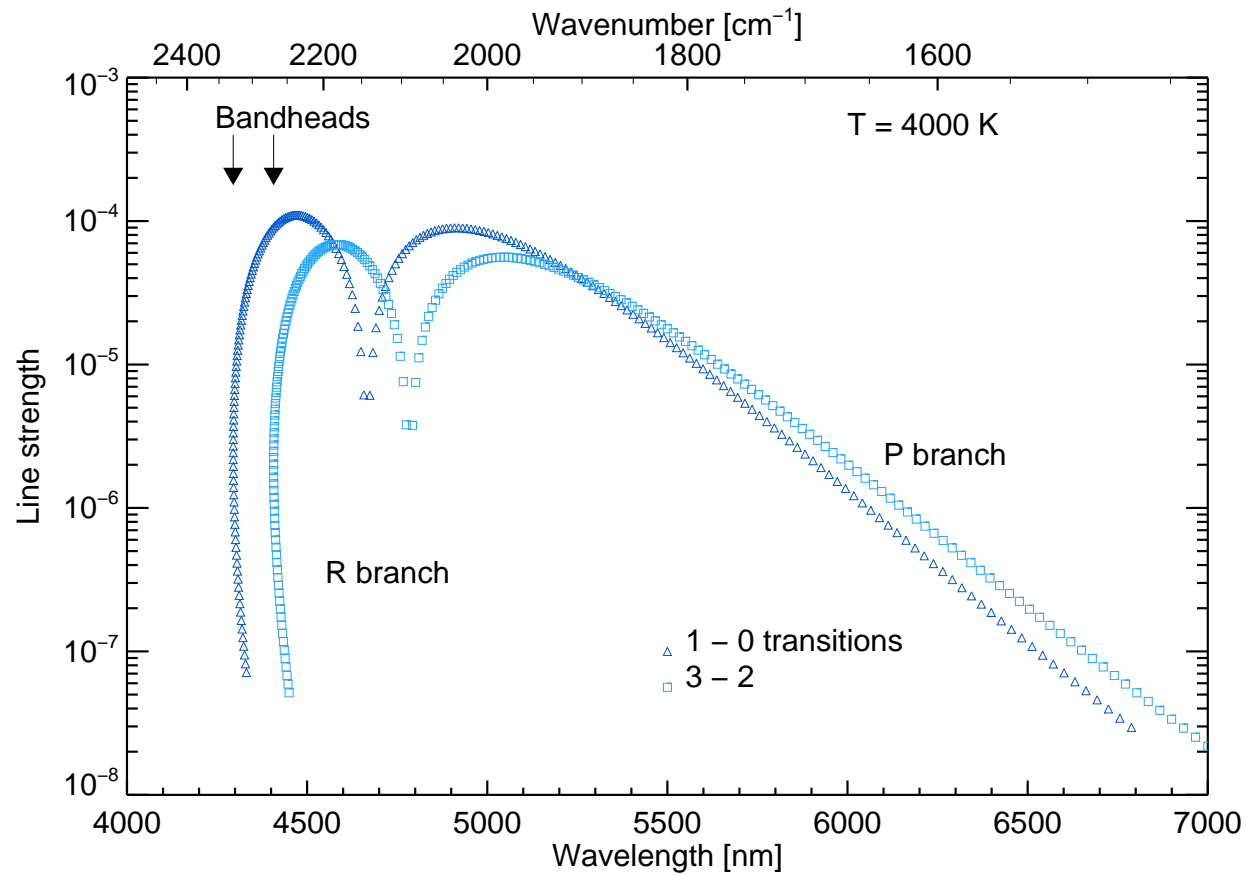
Energy levels of the CO Molecule



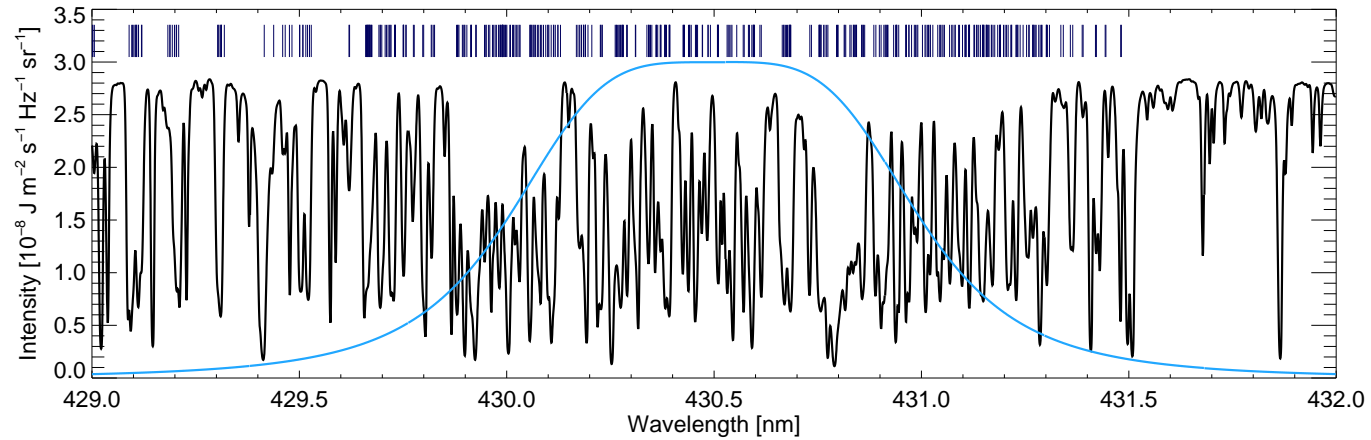
Vibration-Rotation Transitions in CO



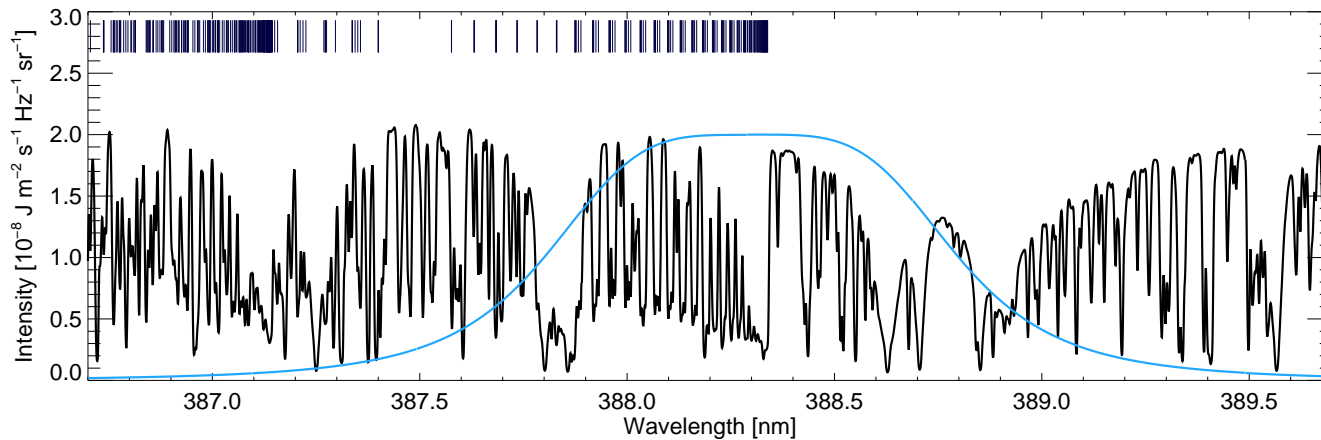
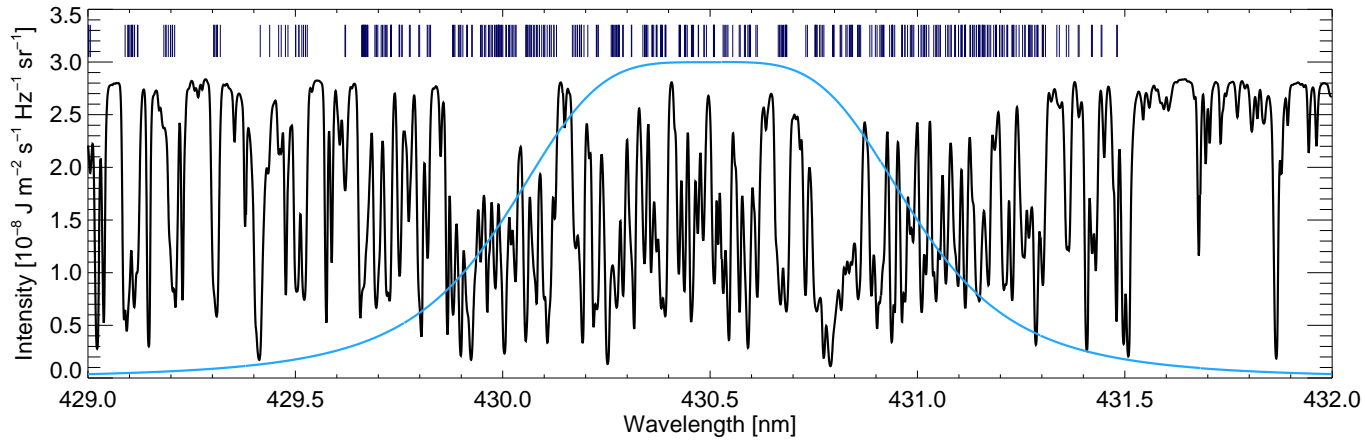
Molecular Lines are grouped in Bands



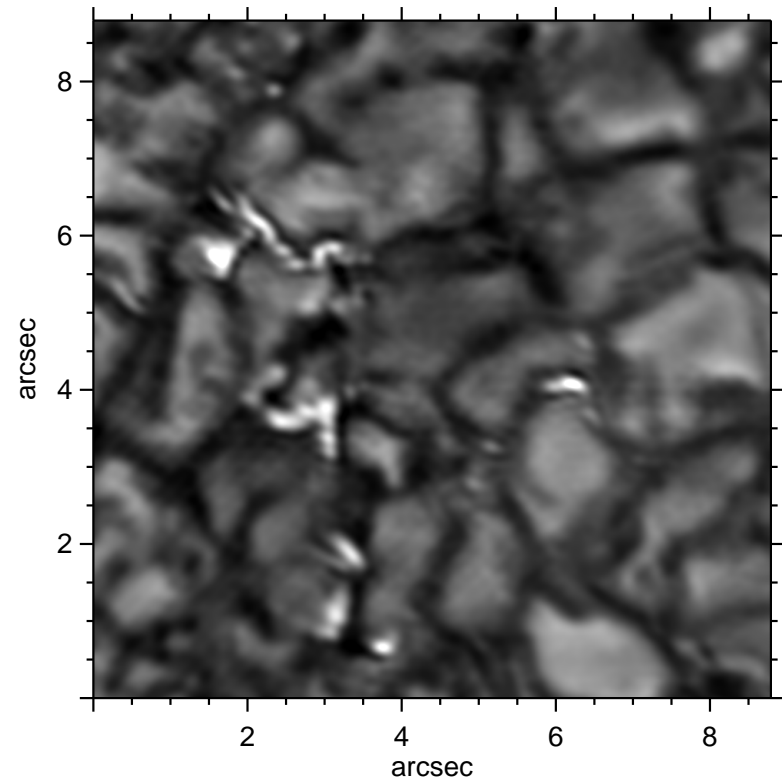
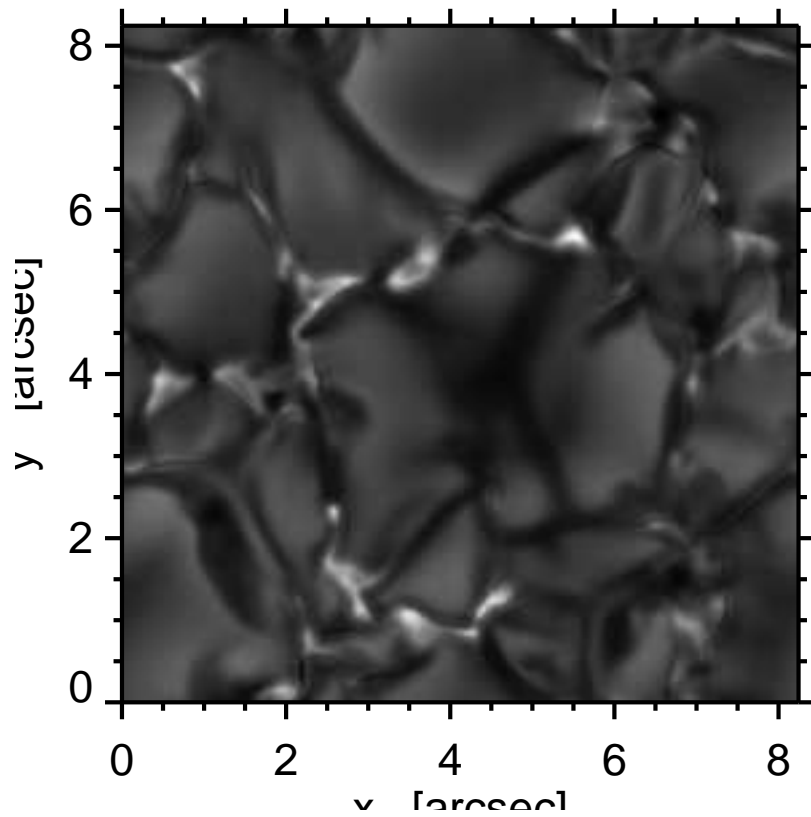
Molecular Bands in the Blue



Molecular Bands in the Blue

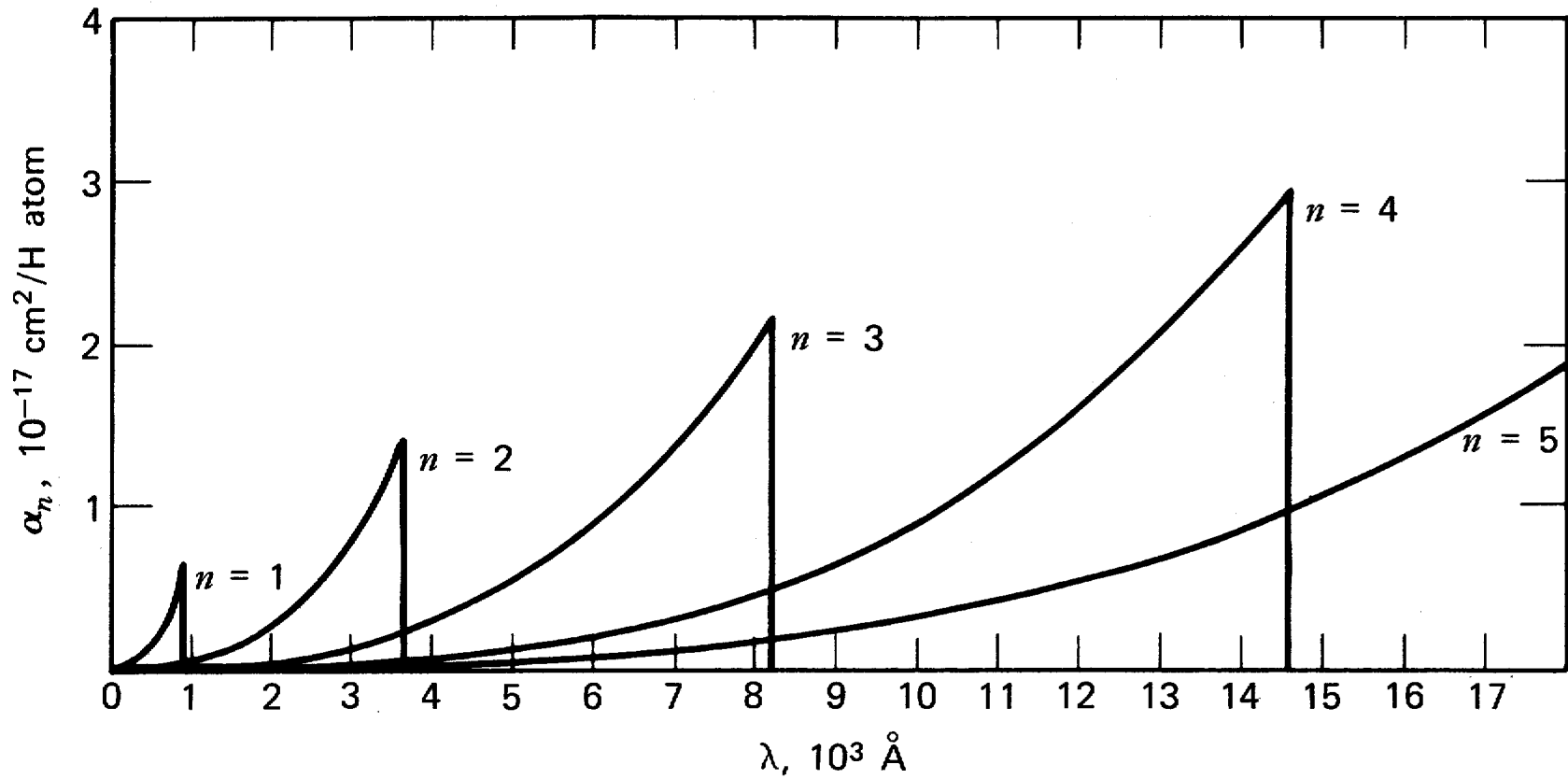


High resolution imaging in the G-band



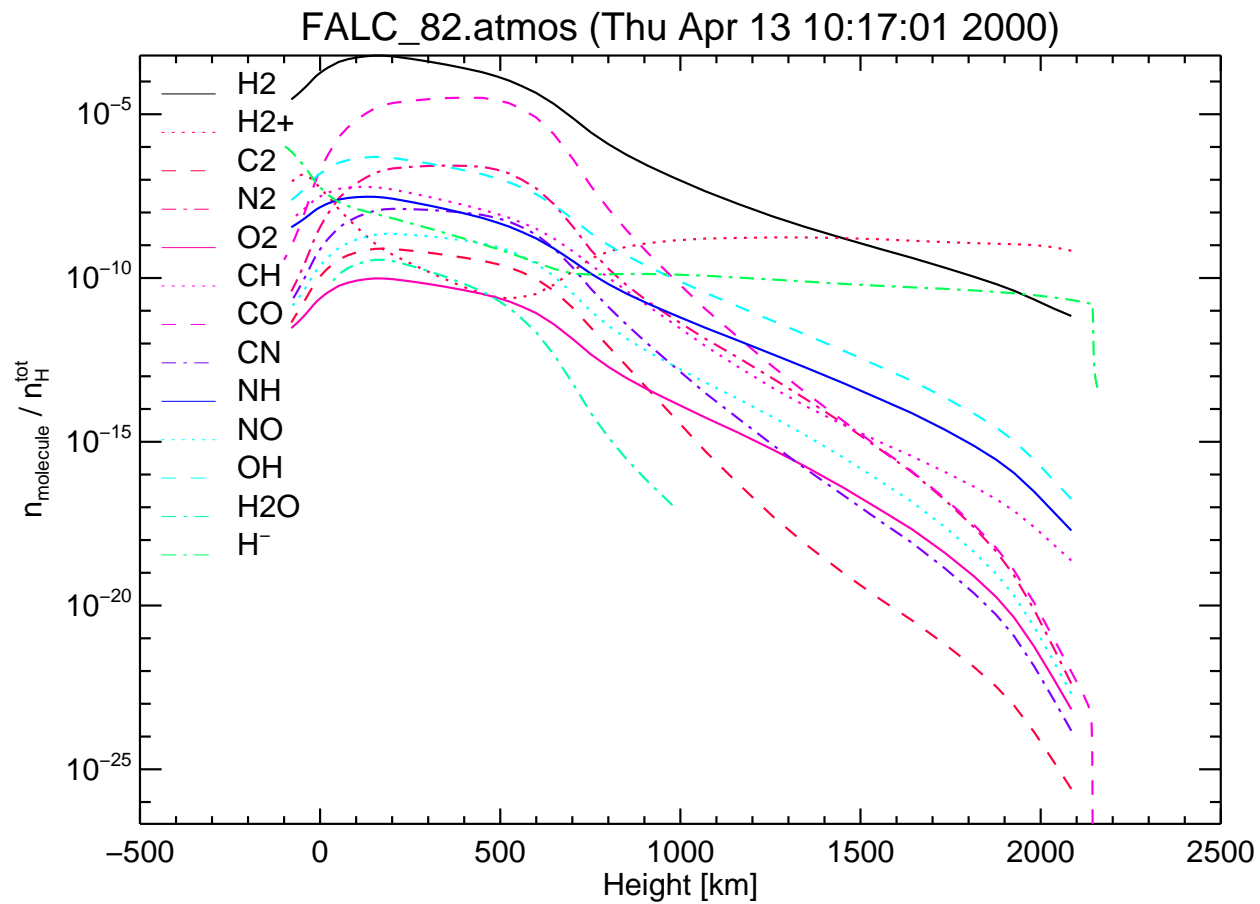
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Bound-Free Cross Sections of Hydrogen

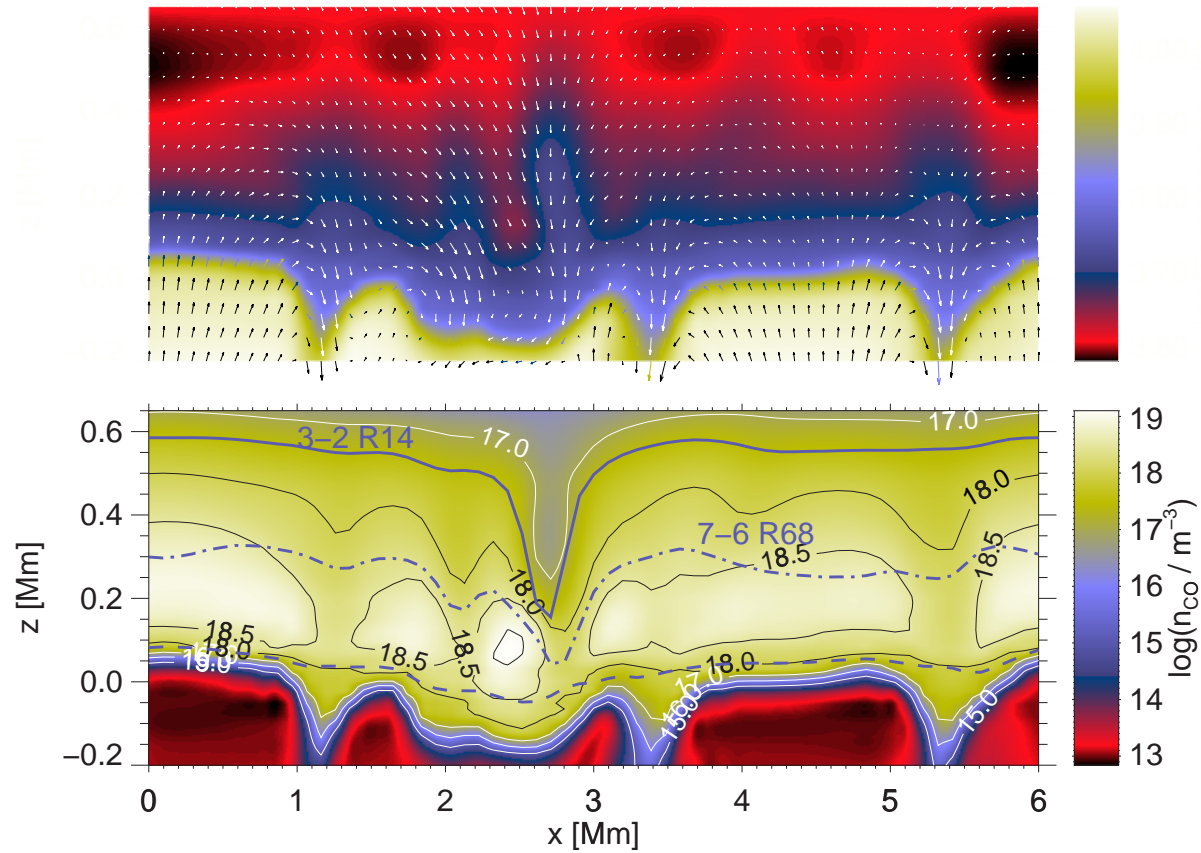


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Molecules in the Sun



Molecules in the Sun



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