No. 74 SCALE TRANSFER FOR LUNAR PHOTOGRAPHS

by D. W. G. Arthur

February 23, 1966

ABSTRACT

A method is described for transferring scale from one lunar photograph to another. Selenodetic controls are used, and the effects of systematic and random errors in the positions of the selenodetic points are largely eliminated by choosing a suitable pattern of points. The transfer errors are of the same order as those generally met in selenodetic measures.

1. *Statement of the Problem*

Communications LPL No. 73 describes a device and the associated technique for determining the focal length of a large refractor. The technique demands relatively little telescope time and can be alternated with normal lunar photography, so that precise focal lengths can be associated with lunar photographs taken on the same evening. Other methods for determination of focal length can be used, of course; here we consider generally the problem of transferring scale or focal length from photographs with known values to photographs with unknown focal lengths.

The method proposed here for transferring focal length depends on the use of the selenodetic standard points. It is thus relevant to examine briefly the limitations of these and to determine how their errors affect the transfer.

2. *Error Characteristics of Selenodetic Points*

The absolute errors of the selenodetic points are unknown, of course, but we can arrive at some idea of their nature and magnitude by comparing the various selenodetic triangulations. The two that compare most closely and that I assess as most reliable are the Breslau triangulation (Schrutka-Rechtenstamm 1958) and the ACIC triangulation (Meyer and Ruffin 1965). If the solid rectangular coordinates are denoted by (E, F, G), with E and F in the plane of the mean limb, the differences in E and F are quite small and show very little system. Certainly there is little systematic scale difference, as the average value of the differences of the quantity $\sqrt{(E^2 + F^2)}$ between the two triangulations is statistically insignificant.

Furthermore, the Breslau points, used at LPL, give limb radii that agree with the values on the photographs to 0.0002 of the radius. In short, the Breslau and ACIC points do not appear to have any scale error in the plane of the limb much exceeding 0.0001 of the moon's radius.

Comparison between the two triangulations for the third coordinate G does reveal slight systematic differences, but I believe these are unimportant in comparison with a much larger systematic error in G that affects both triangulations in common. Theoretical considerations indicate that systematic errors of this type are functions of G itself and tend to be constant for a constant value of G. Thus, these errors can be largely suppressed by using selenodetic points that all lie at much the same distance from the center of face. This configuration is also favorable for the elimination of errors arising from the computed values of the librations.

3. *The Transfer of Focal Length*

Assume that we have two plates, one of known focal length, the other unknown, indicated by the subscripts 1 and 2 respectively; let the known focal length for 1 be L_1. The focal length L_2 for the second plate is to be found.

The same configuration of selenodetic points is to be used on both plates; as noted above, this consists of points well removed from the center of face, all by approximately the same distance.

The computations are of the same type as those in *Comm. LPL* Nos. 60 and 61. Let (x, y) represent the refraction-free photographic coordinates and (E, F, G) the rectangular selenodetic coordinates taken from the Breslau list. Then the instantaneous rectangular coordinates are (X, Y, Z) where

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} a, & b, & c \\ e, & f, & g \\ i, & j, & k \end{pmatrix} \cdot \begin{pmatrix} E \\ F \\ G \end{pmatrix}. \qquad (1)$$

The projected coordinates in the plane of the limb are

$$\left. \begin{aligned} X' &= X/(1 - Z \sin s') \\ Y' &= Y/(1 - Z \sin s') \end{aligned} \right\}, \qquad (2)$$

where s' is the augmented semidiameter. If J is the focal length, then the coordinates (X', Y') are converted to the units of (x, y) by the factor $J \sin s'$. Hence we can write

$$\begin{aligned} x &= J \sin s'(X' \cos \theta - Y' \sin \theta) + H, \\ y &= J \sin s'(Y' \cos \theta + X' \sin \theta) + K. \end{aligned}$$

Introducing

$$\begin{aligned} p &= J \sin s' \cos \theta, \\ q &= J \sin s' \sin \theta, \end{aligned}$$

the last become

$$\left. \begin{aligned} x &= pX' - qY' + H \\ y &= pY' + qX' + K \end{aligned} \right\}. \qquad (3)$$

These are the observation equations for determining the parameters p and q by the method of least squares. To determine J we have

$$p^2 + q^2 = J^2 \sin^2 s',$$

or

$$J = \frac{\sqrt{(p^2 + q^2)}}{\sin s'}. \qquad (4)$$

Plates 1 and 2 each yield a value for J which we denote by J_1 and J_2. These may be called the selenodetic values of the local lengths. They will be in error because of errors in (E, F, G), errors in the librations, and errors in the measures (x, y) on each plate. The libration errors are minimized and probably rendered negligible by the chosen configuration for the points, while the errors in x and y can be minimized only by including sufficient points. The dominant errors are those of E, F, and G. It will be noted, however — if the plates have roughly the same libration — that these errors produce highly correlated errors in J_1 and J_2. Hence, with some precision, we can write for the case of similar librations

$$J_1/J_2 = L_1/L_2,$$

or

$$L_2 = L_1 J_2 / J_1. \qquad (5)$$

4. *The Precision of the Transfer*

The error-propagation laws for the focal length in the method sketched above are not simple. All we have to do, however, is to establish that the error of the computed focal length introduced by the method itself is not larger than 0.00005 of the focal length itself, since even with the best photography, the errors of the measures tend to be larger than this fraction of the lunar radius.

This check can be made by taking samples of fictitious controls and imposing errors of the expected magnitudes on their coordinates. Simulated transfers were made with errors of up to $0.001r$ in E and F and $0.003r$ in G, with libration differences of two degrees. These did not produce errors in L_2 larger than 0.00001 in units of the focal length and usually they were smaller. Ten to fifteen standard points were used in the transfers. The stated precision does not allow for errors in the measures (x, y), and it appears that if too few points are used in the transfer, the errors in x and y may be found more important than the errors in E, F, and G.

Acknowledgments. This research was supported by the Air Force Cambridge Research Laboratories, U.S. Air Force, under Contract AF19(628)–4332.

REFERENCES

Meyer, D. L., and Ruffin, B. W. 1965, *Coordinates of Lunar Features, Group I and II Solutions,* ACIC Tech. Paper No. 15, Aeronautical Chart and Information Center, U.S. Air Force, St. Louis, Mo.

Schrutka-Rechtenstamm, G. 1958, "Neureduktion der 150 Mondpunkte der Breslauer Messungen von J. Franz," *Sitz. Österr. Akad. Wiss. Math.-Naturw. Kl.,* Abt. II, 167 Bd.