Introduction to Solar Radiative Transfer: I Basic Radiative Transfer

Han Uitenbroek
National Solar Observatory/Sacramento Peak
Sunspot NM, USA

Summerschool Sunspot, Jun 18 2008

Overview

- I Basic Radiative Transfer (Intensity, emission, absorption, source function, optical depth)

Overview

- I Basic Radiative Transfer (Intensity, emission, absorption, source function, optical depth)
- II Detailed Radiative Processes (Spectral lines, radiative transitions, collisions, continuum processes, Saha-Boltzmann, polarization)

Bibliography

- **Rutten:** Radiative Transfer in Stellear Atmospheres (http://www.astro.uu.nl/~rutten/Astronomy_lecture.html)

Bibliography

- **Rutten:** Radiative Transfer in Stellear Atmospheres (http://www.astro.uu.nl/~rutten/Astronomy_lecture.html)
- **Rybicki and Lightman:** Radiative Processes in Astrophysics

Bibliography

- **Rutten:** Radiative Transfer in Stellear Atmospheres (http://www.astro.uu.nl/~rutten/Astronomy_lecture.html)
- **Rybicki and Lightman:** Radiative Processes in Astrophysics
- **Gray:** Observation and Analysis of Stellar Photospheres

Bibliography

- **Rutten:** Radiative Transfer in Stellear Atmospheres (http://www.astro.uu.nl/~rutten/Astronomy_lecture.html)
- **Rybicki and Lightman:** Radiative Processes in Astrophysics
- **Gray:** Observation and Analysis of Stellar Photospheres
- **Mihalas:** Stellar Atmospheres

Bibliography

- **Rutten:** Radiative Transfer in Stellear Atmospheres (http://www.astro.uu.nl/~rutten/Astronomy_lecture.html)
- **Rybicki and Lightman:** Radiative Processes in Astrophysics
- **Gray:** Observation and Analysis of Stellar Photospheres
- **Mihalas:** Stellar Atmospheres
- **Shu:** The Physics of Astrophysics. I. Radiation

Bibliography

- **Rutten:** Radiative Transfer in Stellear Atmospheres (http://www.astro.uu.nl/~rutten/Astronomy_lecture.html)

- **Rybicki and Lightman:** Radiative Processes in Astrophysics

- **Gray:** Observation and Analysis of Stellar Photospheres

- **Mihalas:** Stellar Atmospheres

- **Shu:** The Physics of Astrophysics. I. Radiation

- **Allen:** Astrophysical Quantities

Short History

- **1802** Wollaston First to observe dark gaps in spectrum: spectral lines

Short History

- **1802** Wollaston First to observe dark gaps in spectrum: spectral lines
- **1814** Fraunhofer rediscovers lines. Assigns names e.g.,
  C (Hα), D (Na I), G (CH molecules), F (Hβ), and H (Ca II)

Short History

- **1802** Wollaston First to observe dark gaps in spectrum: spectral lines
- **1814** Fraunhofer rediscovers lines. Assigns names e.g., C (Hα), D (Na I), G (CH molecules), F (Hβ), and H (Ca II)
- **1823** Herschel realized spectra contain information on composition of source from flame spectra

Short History

- **1802** Wollaston First to observe dark gaps in spectrum: spectral lines
- **1814** Fraunhofer rediscovers lines. Assigns names e.g., C (Hα), D (Na I), G (CH molecules), F (Hβ), and H (Ca II)
- **1823** Herschel realized spectra contain information on composition of source from flame spectra
- **1842** Becquerel photographs spectra, discovers UV lines beyond the visible

Short History

- **1802** Wollaston First to observe dark gaps in spectrum: spectral lines
- **1814** Fraunhofer rediscovers lines. Assigns names e.g., C (Hα), D (Na I), G (CH molecules), F (Hβ), and H (Ca II)
- **1823** Herschel realized spectra contain information on composition of source from flame spectra
- **1842** Becquerel photographs spectra, discovers UV lines beyond the visible
- **1858** Bunsen and Kirchhoff discover wavelength correspondence between bright flame emission and dark solar absorption lines. Start of quantitative spectroscopy.

Why Radiative Transfer?

- Information on astronomical objects in general has to be recovered from the electromagnetic radiation the object emits and/or reflects.

Why Radiative Transfer?

- Information on astronomical objects in general has to be recovered from the electromagnetic radiation the object emits and/or reflects.
- Spectroscopic observation (remote sensing) and analysis is the only available method to determine the physical conditions of these astronomical objects.

Why Radiative Transfer?

- Information on astronomical objects in general has to be recovered from the electromagnetic radiation the object emits and/or reflects.
- Spectroscopic observation (remote sensing) and analysis is the only available method to determine the physical conditions of these astronomical objects.
- To analyze spectroscopic data meaningfully we need to understand how this physical information is encoded in the radiation **Radiative Transfer**.

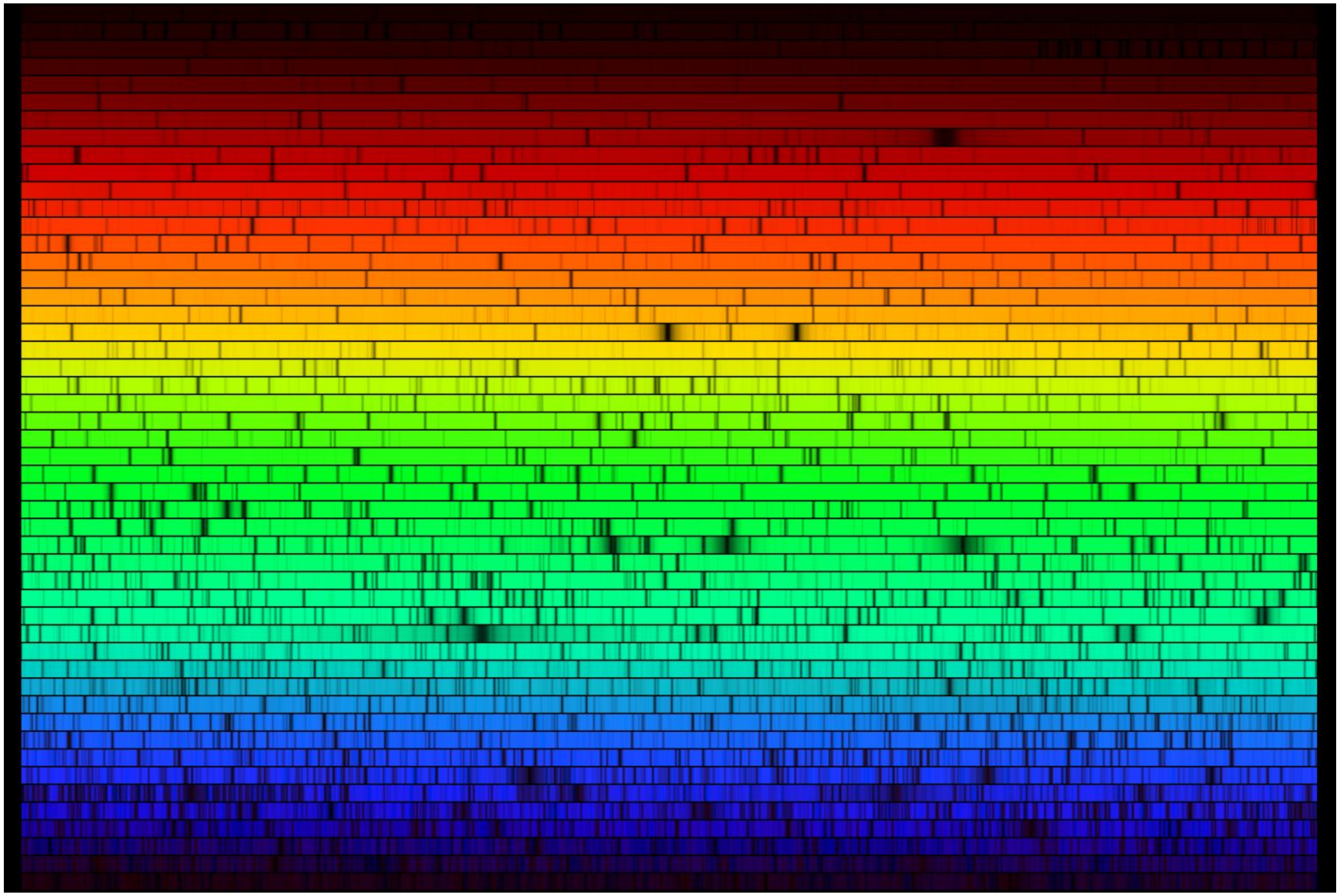
Why Radiative Transfer?

- Information on astronomical objects in general has to be recovered from the electromagnetic radiation the object emits and/or reflects.
- Spectroscopic observation (remote sensing) and analysis is the only available method to determine the physical conditions of these astronomical objects.
- To analyze spectroscopic data meaningfully we need to understand how this physical information is encoded in the radiation **Radiative Transfer**.
- Observational techniques need to be applied to obtain the relevant properties of the radiation signal.

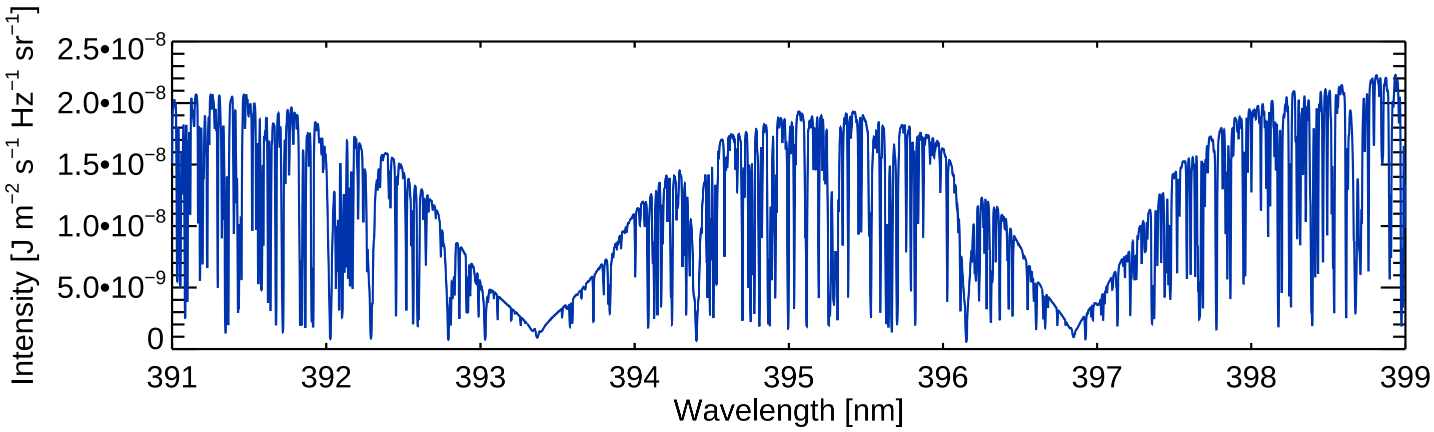
Why Radiative Transfer?

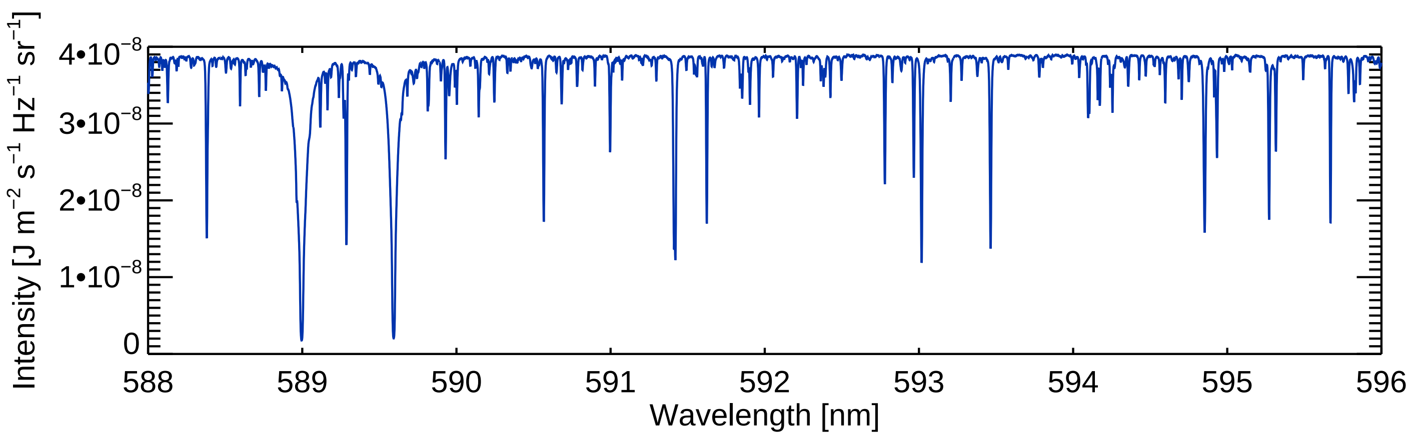
- Information on astronomical objects in general has to be recovered from the electromagnetic radiation the object emits and/or reflects.
- Spectroscopic observation (remote sensing) and analysis is the only available method to determine the physical conditions of these astronomical objects.
- To analyze spectroscopic data meaningfully we need to understand how this physical information is encoded in the radiation **Radiative Transfer**.
- Observational techniques need to be applied to obtain the relevant properties of the radiation signal.
- We need to understand how the radiative signal is modified as it travels from the object to our instruments.

Solar line spectrum

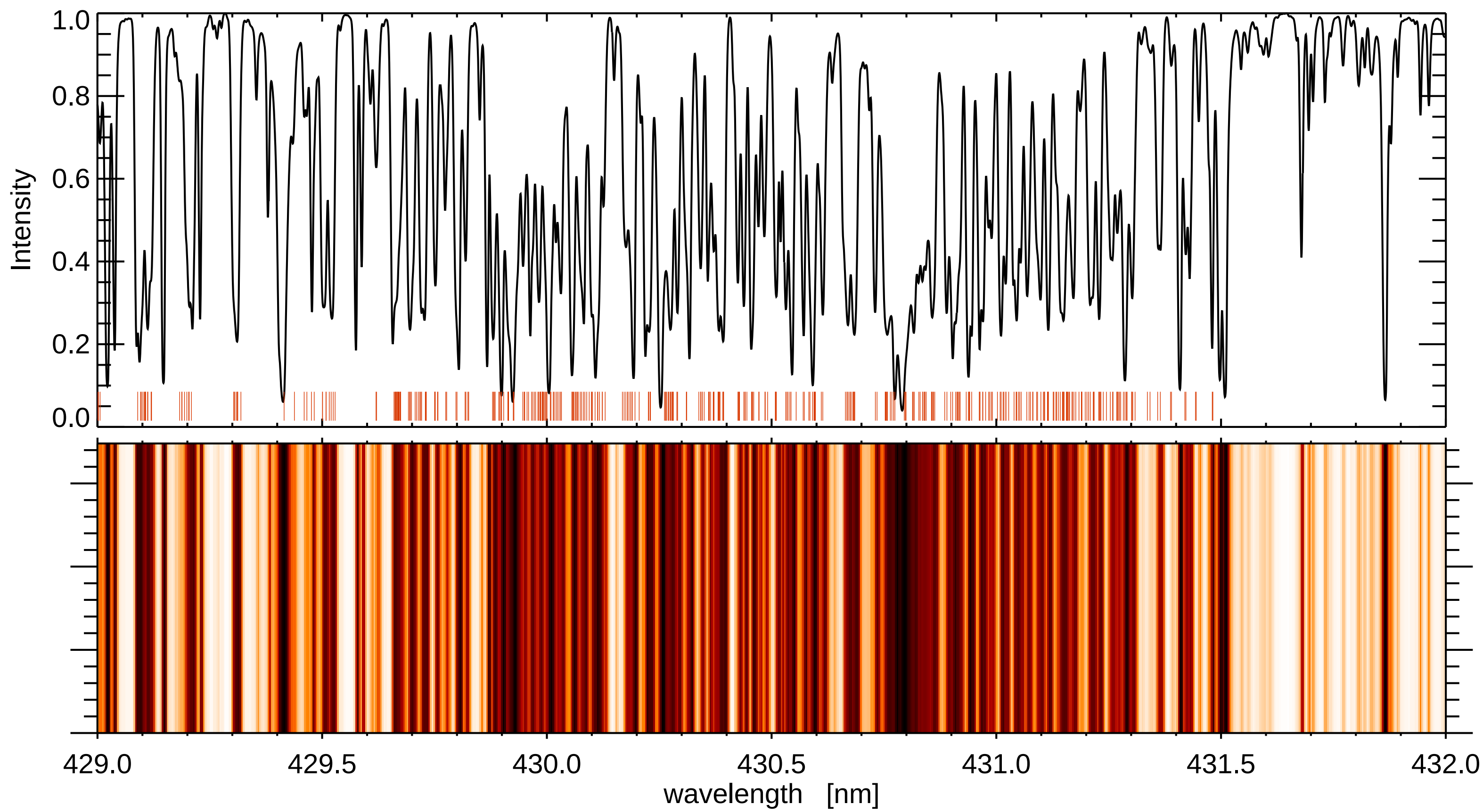


Differences in Spectral Lines

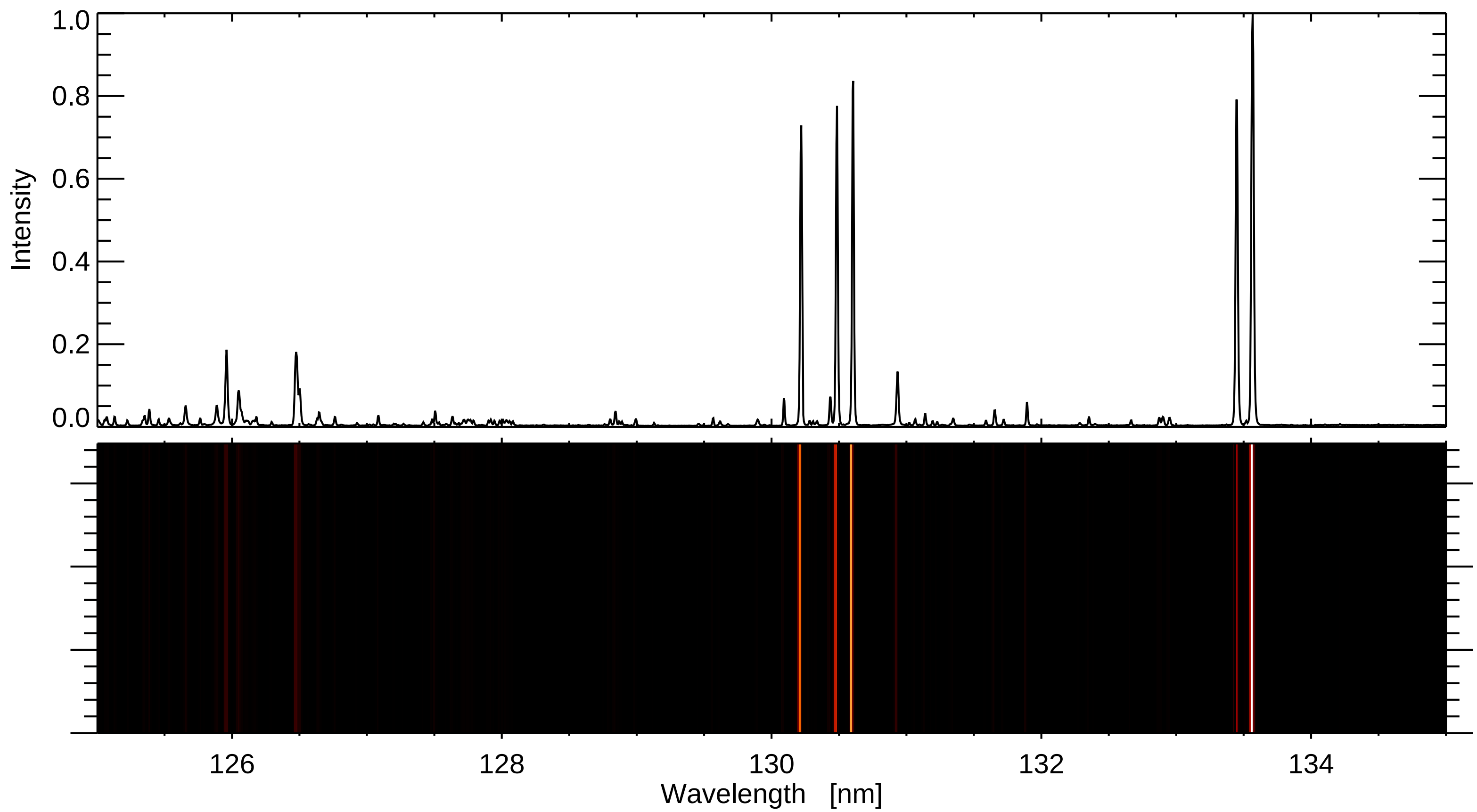




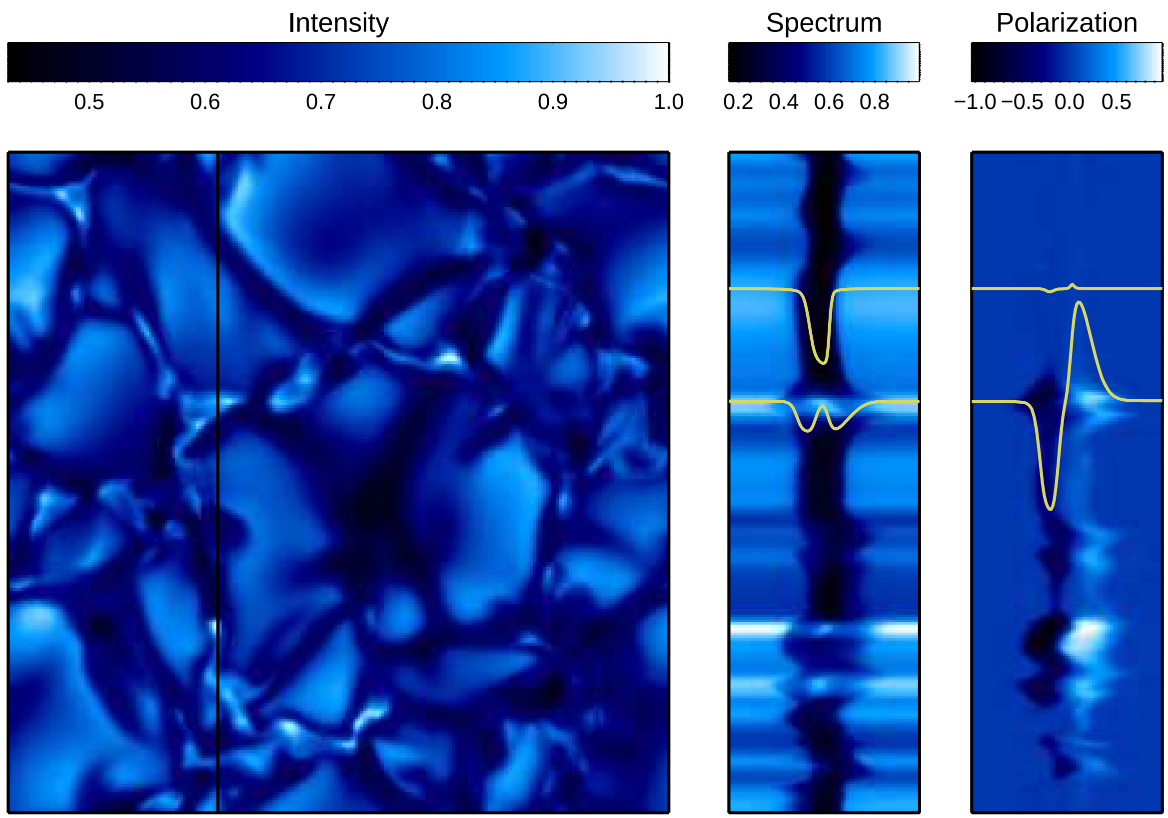
Molecular lines in the solar spectrum



In the UltraViolet the Spectral Lines are in Emission



Spatially Resolved Spectral Lines



Basic Radiative Transfer: Radiation Field

Specific Intensity:

$$\mathrm{d}E_\nu \equiv I_\nu(\vec{r}, \vec{l}, t)\, \mathrm{d}t\, \mathrm{d}A\, \mathrm{d}\nu\, \mathrm{d}\Omega \tag{1}$$
$$= I_\nu(x, y, z, \theta, \varphi, t) \cos\theta\, \mathrm{d}t\, \mathrm{d}A\, \mathrm{d}\nu\, \mathrm{d}\Omega$$

Units: J s^{-1} m^{-2} Hz^{-1} ster^{-1}

Mean intensity:

$$J_\nu(\vec{r}, t) \equiv \frac{1}{4\pi} \int I_\nu\, \mathrm{d}\Omega = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} I_\nu \sin\theta\, \mathrm{d}\theta\, \mathrm{d}\varphi \tag{2}$$

Units: J s^{-1} m^{-2} Hz^{-1} ster^{-1}

Basic Radiative Transfer: Radiation Field

Flux:

$$\mathcal{F}_\nu(\vec{r}, \vec{n}, t) \equiv \int I_\nu \cos\theta \, \mathrm{d}\Omega = \int_0^{2\pi} \int_0^{\pi} I_\nu \cos\theta \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\varphi \tag{3}$$

Units: J s^{-1} m^{-2} Hz^{-1}

Flux in radial direction:

$$\begin{aligned} \mathcal{F}_\nu(z) &= \int_0^{2\pi} \int_0^{\frac{\pi}{2}} I_\nu \cos\theta \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\varphi + \int_0^{2\pi} \int_{\frac{\pi}{2}}^{\pi} I_\nu \cos\theta \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\varphi \qquad (4) \\ &= \int_0^{2\pi} \int_0^{\frac{\pi}{2}} I_\nu \cos\theta \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\varphi - \int_0^{2\pi} \int_0^{\frac{\pi}{2}} I_\nu(\pi - \theta) \cos\theta \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\varphi \\ &\equiv \mathcal{F}_\nu^{+}(z) - \mathcal{F}_\nu^{-}(z) \end{aligned}$$

Basic Radiative Transfer: Local Changes

Emission:

$$dE_\nu \equiv j_\nu \, dV \, dt \, d\nu \, d\Omega \quad (5)$$

$$dI_\nu = j_\nu(s) ds$$

Units: J m^{-3} s^{-1} Hz^{-1} ster^{-1}

Extinction:

$$dI_\nu \equiv -\alpha_\nu I_\nu ds \quad (6)$$

Units: m^{-1}

Basic Radiative Transfer: Local Changes

Source function:

$$S_\nu = j_\nu / \alpha_\nu \tag{7}$$

Units: J s^{-1} m^{-2} Hz^{-1} ster^{-1}

$$S_\nu^{\text{tot}} = \sum j_\nu / \sum \alpha_\nu \tag{8}$$

$$S_\nu^{\text{tot}} = \frac{j_\nu^c + j_\nu^l}{\alpha_\nu^c + \alpha_\nu^l} = \frac{S_\nu^c + \eta_\nu S_\nu^l}{1 + \eta_\nu}, \quad \eta_\nu \equiv \alpha_\nu^l / \alpha_\nu^c \tag{9}$$

Basic Radiative Transfer: Transport Equation

Transport along a ray:

$$
\begin{aligned}
\mathrm{d}I_\nu(s) &= I_\nu(s+\mathrm{d}s) - I_\nu(s) = j_\nu(s)\mathrm{d}s - \alpha_\nu(s)I_\nu(s)\mathrm{d}s \\
\frac{\mathrm{d}I_\nu}{\mathrm{d}s} &= j_\nu - \alpha_\nu I_\nu \\
\frac{\mathrm{d}I_\nu}{\alpha_\nu \mathrm{d}s} &= S_\nu - I_\nu
\end{aligned} \tag{10}
$$

Optical length and thickness:

$$
\begin{aligned}
\mathrm{d}\tau_\nu &\equiv \alpha_\nu(s)\mathrm{d}s \\
\tau_\nu(D) &= \int_0^D \alpha_\nu(s)\mathrm{d}s
\end{aligned} \tag{11}
$$

Basic Radiative Transfer: Transport Equation

Transport along a ray:

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = S_\nu - I_\nu \tag{12}$$

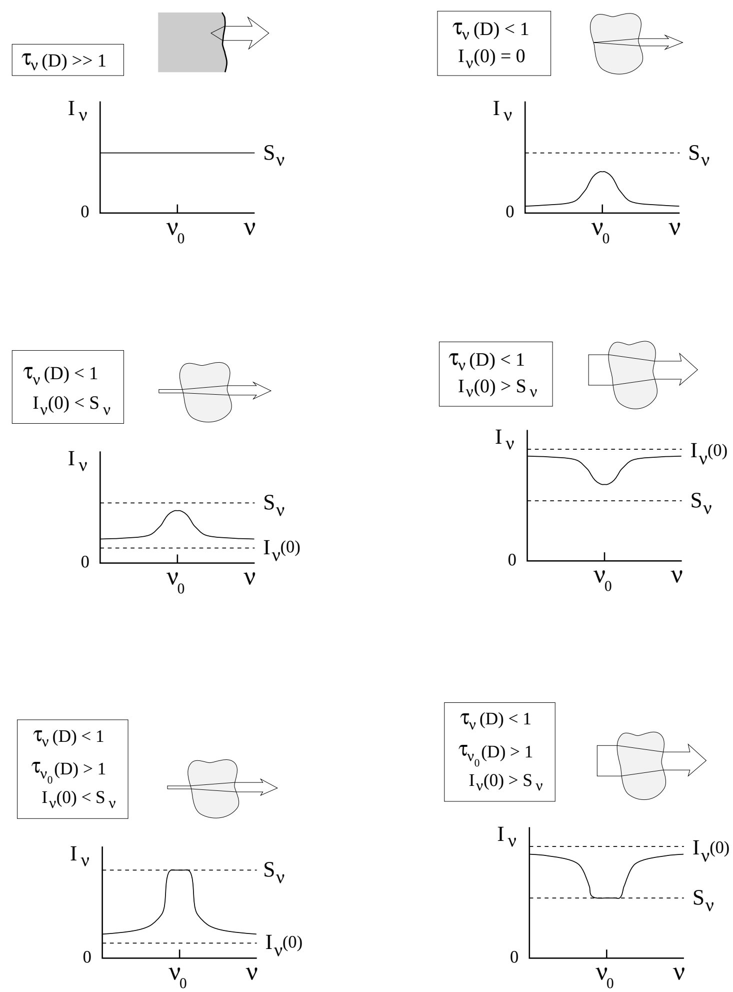
$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu(t)e^{-(\tau_\nu - t)}\mathrm{d}t$$

Homogeneous medium:

$$I_\nu(D) = I_\nu(0)e^{-\tau_\nu(D)} + S_\nu\left(1 - e^{-\tau_\nu(D)}\right) \tag{13}$$

Optically thick: $I_\nu(D) \approx S_\nu$
Optically thin: $I_\nu(D) \approx I_\nu(0) + [S_\nu - I_\nu(0)]\,\tau_\nu(D)$

Basic Radiative Transfer: Homogeneous Medium



Basic Radiative Transfer: Through an Atmosphere

Optical depth:

$$
\begin{aligned}
\mathrm{d}\tau_{\mu\nu} &= \alpha_\nu \mathrm{d}s \equiv -\alpha_\nu \frac{\mathrm{d}z}{|\mu|} && (14)\\
\tau_\nu(z_1) &= \int_{z_{\rm surf}}^{z_1} -\alpha_\nu \mathrm{d}z = \int_{z_1}^{z_{\rm surf}} \alpha_\nu \mathrm{d}z
\end{aligned}
$$

Standard plane parallel transport equation:

$$
\mu \frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = I_\nu - S_\nu \qquad (15)
$$

Basic Radiative Transfer: Through an Atmosphere

Formal solution in upward direction:

$$I_\nu^+(\tau_\nu, \mu) = \int_{\tau_\nu}^{\infty} S_\nu(t) e^{-(t-\tau_\nu)/\mu} \mathrm{d}t/\mu, \quad \mu > 0 \tag{16}$$

Formal solution in downward direction:

$$I_\nu^-(\tau_\nu, \mu) = -\int_{0}^{\tau_\nu} S_\nu(t) e^{-(t-\tau_\nu)/\mu} \mathrm{d}t/\mu, \quad \mu < 0 \tag{17}$$

Basic Radiative Transfer: Eddington–Barbier

Emergent intensity at the surface:

$$I_\nu^+(\tau_\nu = 0, \mu) = \int_0^\infty S_\nu(t) e^{-t/\mu} \mathrm{d}t/\mu \qquad (18)$$

Substitute power series:

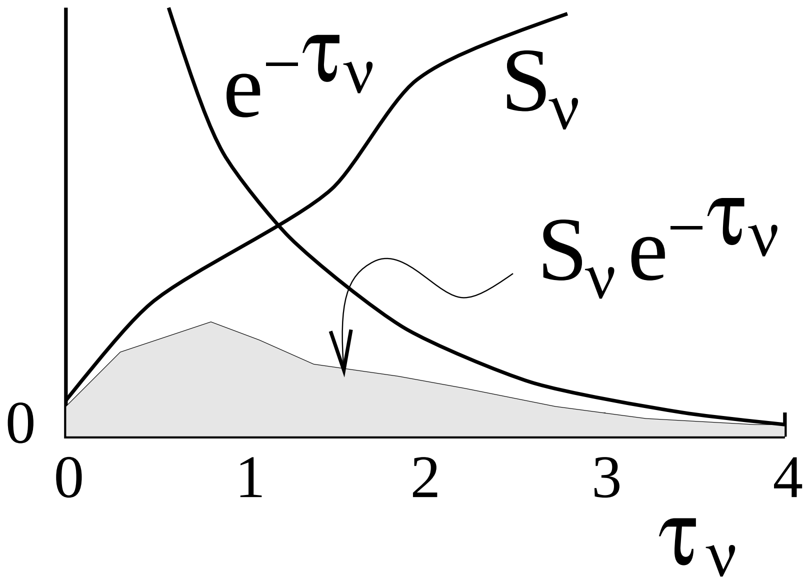
$$S_\nu(\tau_\nu) = \sum_{n=0}^{N} a_n \tau_\nu^n \qquad (19)$$

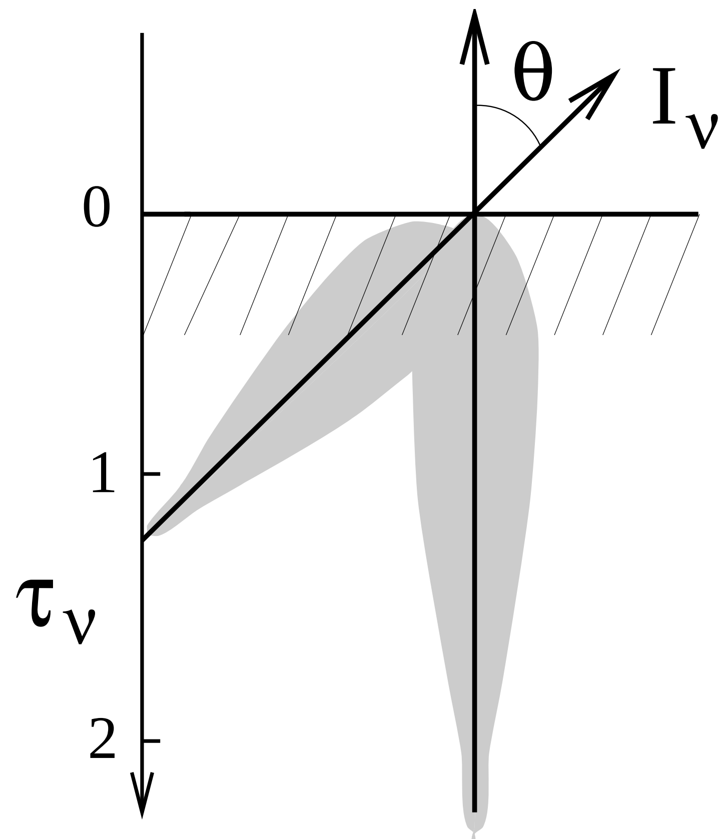
$$I_\nu^+(\tau_\nu = 0, \mu) = a_0 + a_1\mu + 2a_2\mu^2 + \ldots + n! a_N \mu^N$$

Eddington–Barbier relation:

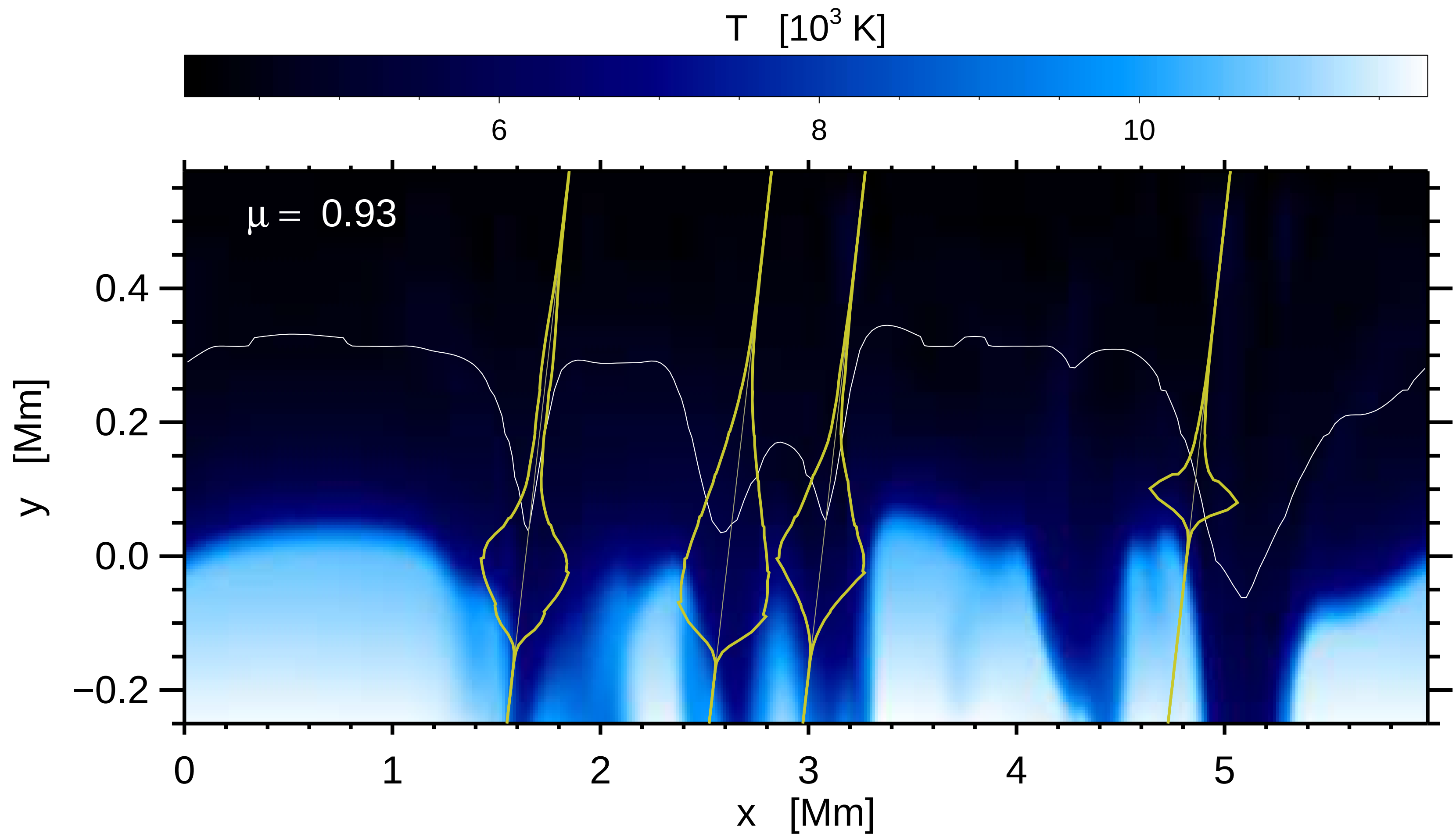
$$I_\nu^+(\tau_\nu = 0, \mu) \approx S_\nu(\tau_\nu = \mu) \qquad (20)$$

Basic Radiative Transfer: Eddington–Barbier

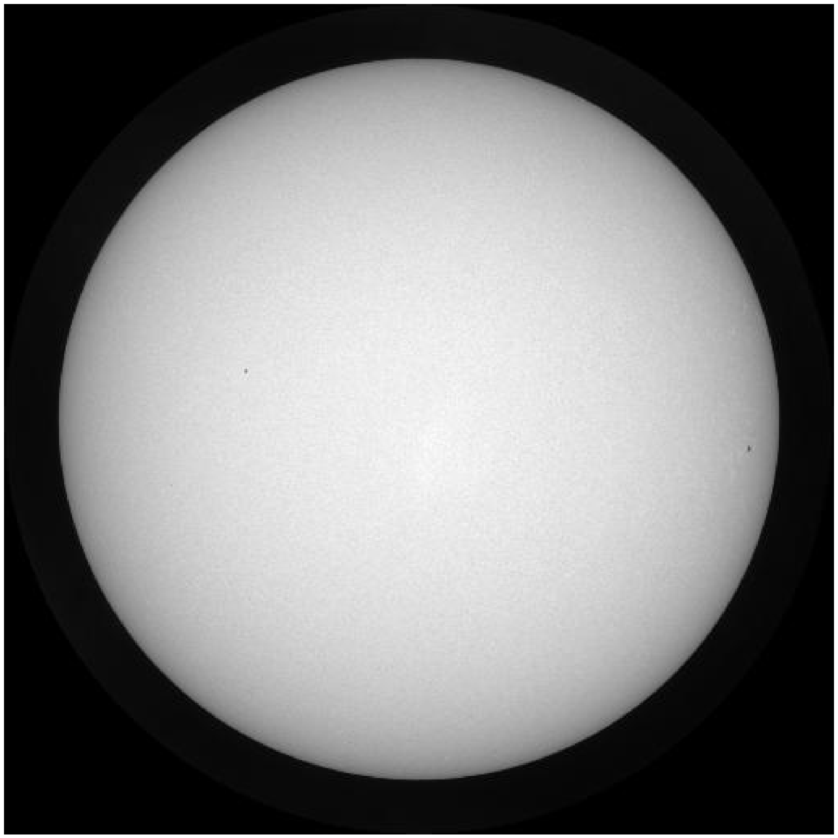


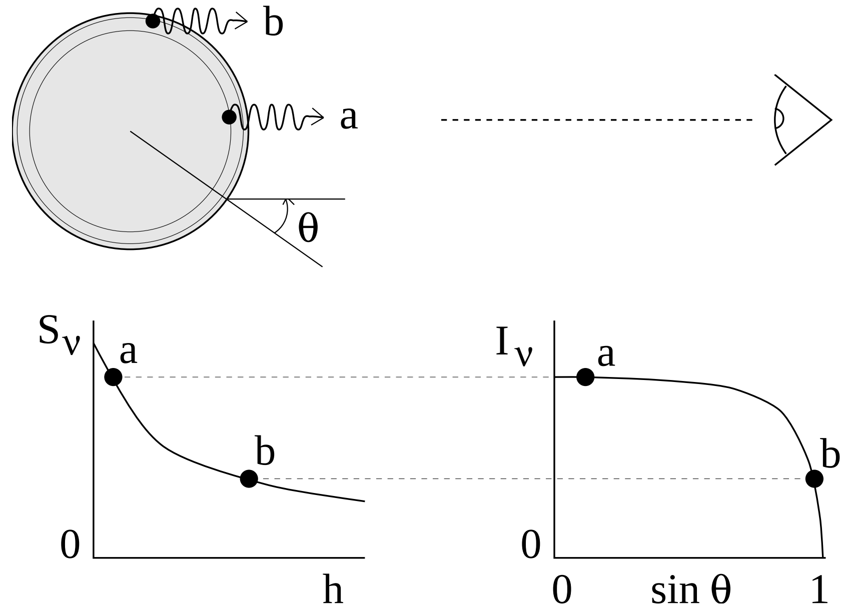


Basic Radiative Transfer: Eddington–Barbier



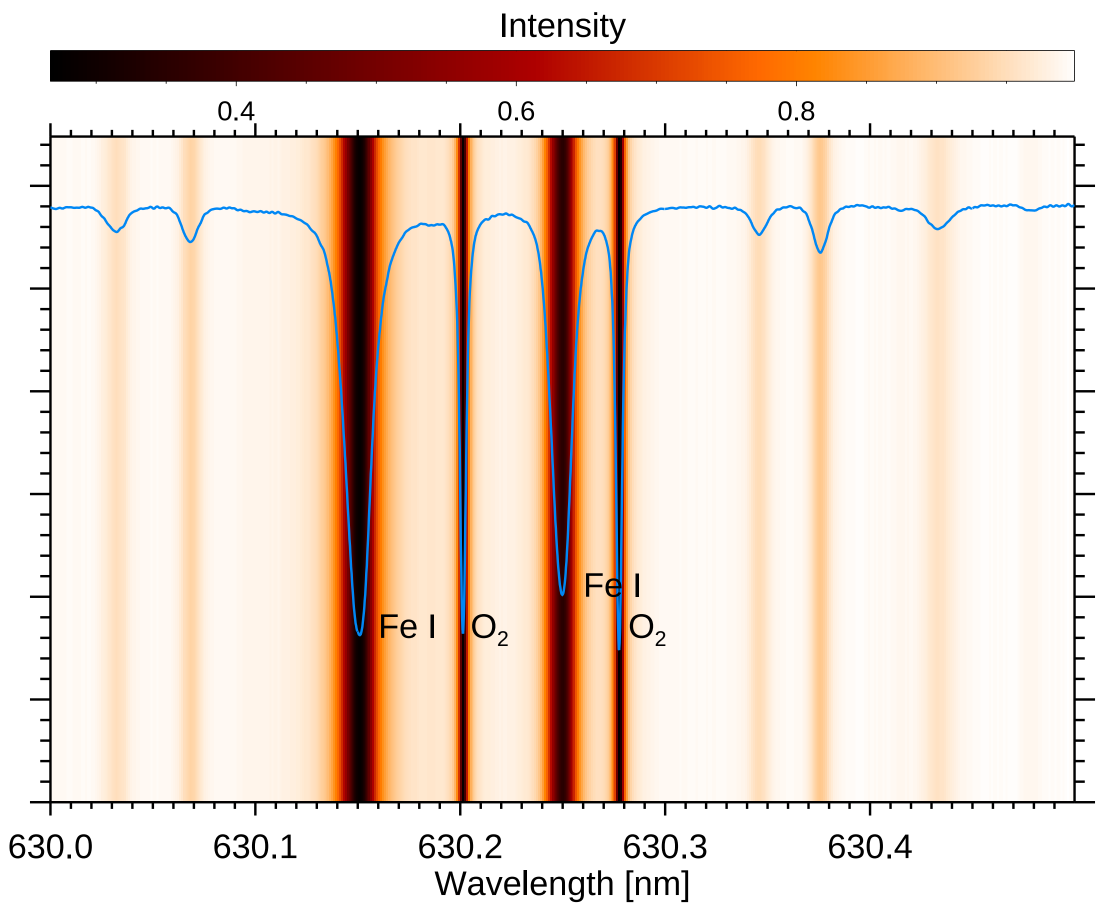
Basic Radiative Transfer: Limb Darkening





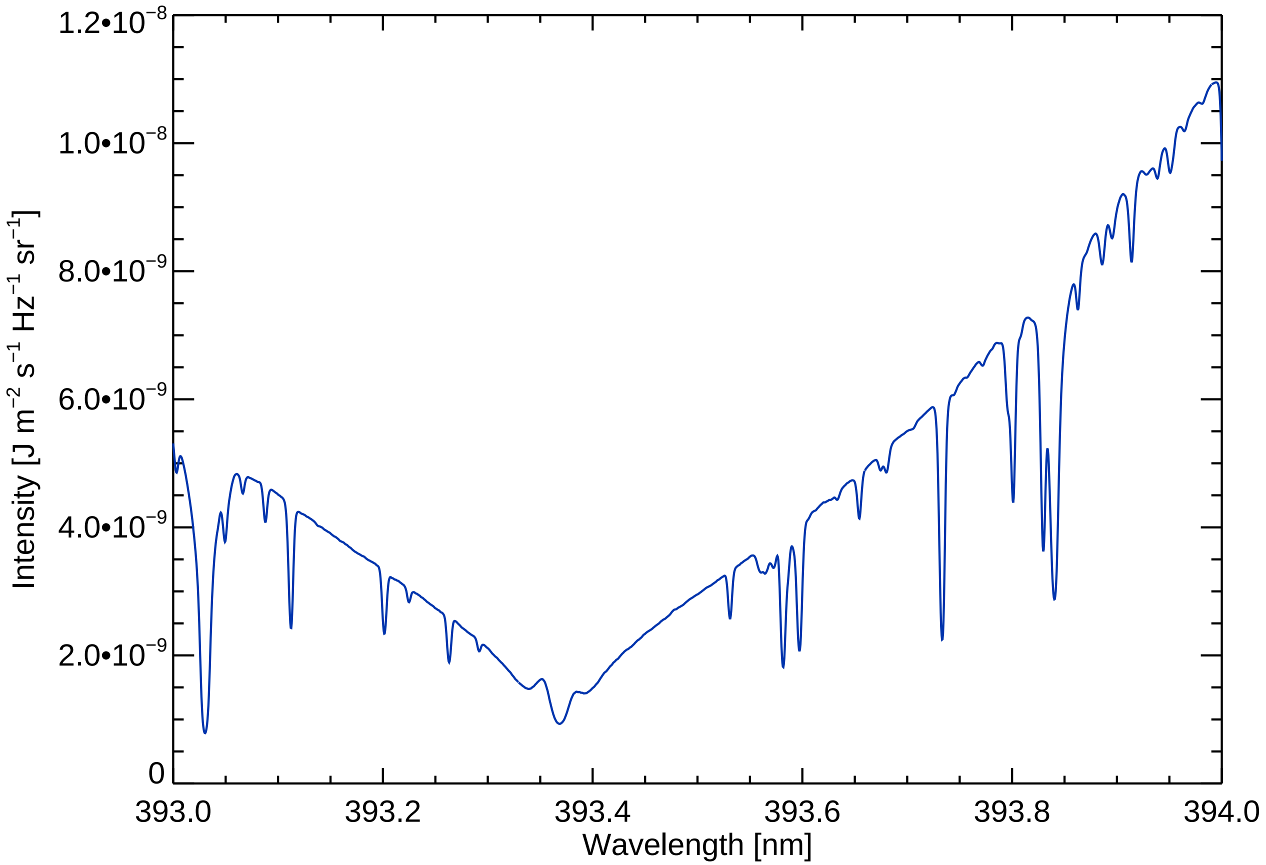
End Part I

Molecular Oxygen in the Earth Atmosphere



Back

Differences in spectral lines



Back

Invariance of Specific Intensity along Rays

Specific Intensity has been defined in such a way as to be independent of the source and the observer.

$$\mathrm{d}E_\nu = I_\nu \cos\theta \,\mathrm{d}t\,\mathrm{d}A\,\mathrm{d}\nu\,\mathrm{d}\Omega = I'_\nu \cos\theta' \,\mathrm{d}t\,\mathrm{d}A'\,\mathrm{d}\nu\,\mathrm{d}\Omega'$$

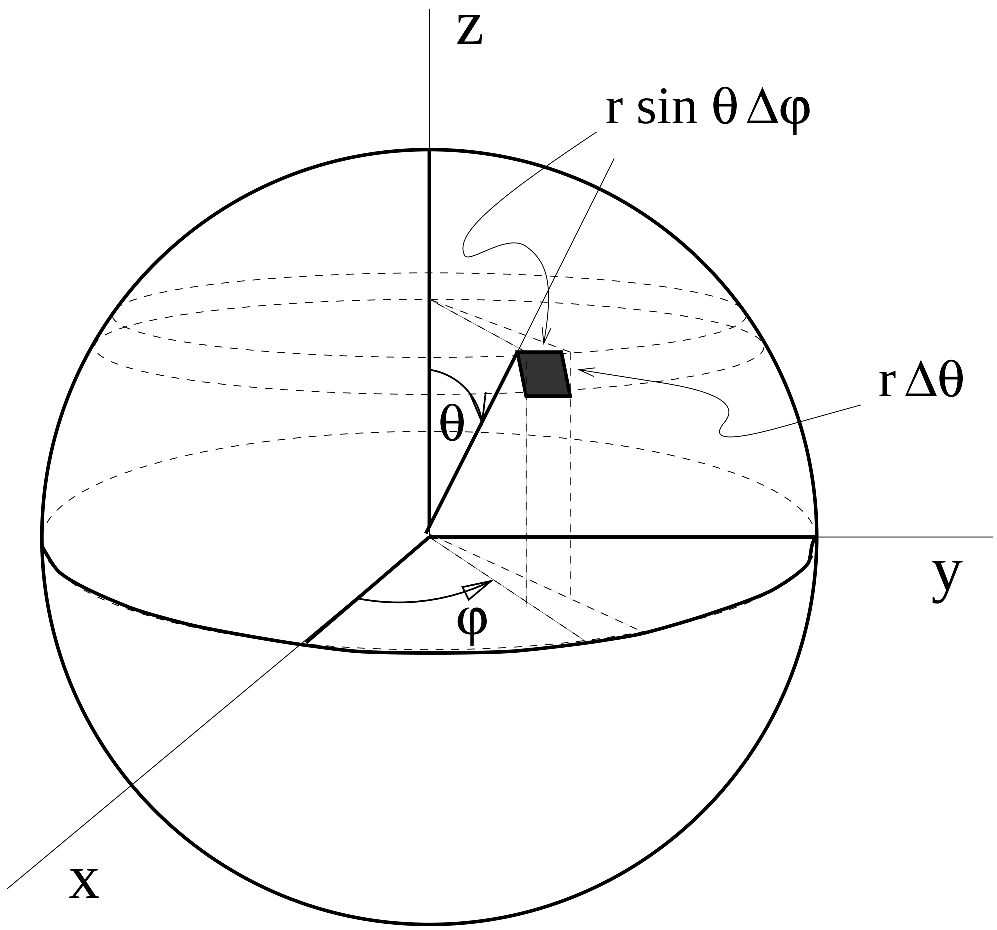
$$\mathrm{d}\Omega = \mathrm{d}A' \,\cos\theta'/R^2$$

$$\mathrm{d}\Omega' = \mathrm{d}A \,\cos\theta/R^2$$

$$I_\nu = I'_\nu$$

Back

Spherical Coordinates



Back