THE RICHTMYER-MESHKOV INSTABILITY

Martin Brouillette
Département de Génie Mécanique, Université de Sherbrooke, Sherbrooke (Québec), Canada J1K 2RI; e-mail: martin.brouillette@gme.usherbro.ca

Key Words  shock-induced turbulence, shock-wave refraction

Abstract  The Richtmyer-Meshkov instability arises when a shock wave interacts with an interface separating two different fluids. It combines compressible phenomena, such as shock interaction and refraction, with hydrodynamic instability, including nonlinear growth and subsequent transition to turbulence, across a wide range of Mach numbers. This review focuses on the basic physical processes underlying the onset and development of the Richtmyer-Meshkov instability in simple geometries. It examines the principal theoretical results along with their experimental and numerical validation. It also discusses the different experimental approaches and techniques and how they can be used to resolve outstanding issues in this field.

1. INTRODUCTION

A wide variety of fluid motions can be generated, following the interaction of a shock wave with an interface separating two fluids of different properties. Any perturbation initially present on the interface will, in most cases, be amplified following the refraction of the shock. This class of problems is generally referred to as the Richtmyer-Meshkov instability (RMI). The basic mechanism for the amplification of perturbations at the interface is baroclinic vorticity generation resulting from the misalignment of the pressure gradient of the shock and the local density gradient across the interface. As the interface between the two fluids becomes more distorted, secondary instabilities, such as the Kelvin-Helmholtz shearing instability, develop and a region of turbulence and mixing ultimately results.

The RMI arises in the context of impulsively generated flows occurring in a wealth of man-made applications and natural phenomena. In inertial confinement fusion, the RMI causes mixing between the capsule material and the fuel within, limiting final compression and thus the ability to achieve energy break-even or production (Lindl et al. 1992). The RMI has also been used to explain the lack of stratification of the products of supernova 1987A and is now a required ingredient in stellar evolution models (Arnett 2000). The interaction of a shock wave with a flame takes place in many combustion systems, and the resulting instability plays an important role in deflagration-to-detonation transition (Khokhlov et al.
The RMI can also be used to promote mixing between fuel and oxidizer in supersonic and hypersonic air-breathing engines (Yang et al. 1993). Finally, this instability also has been invoked as a possible mechanism for explaining driver gas contamination in reflected shock tunnels when shock bifurcation due to the wall boundary layer is absent (Stalker & Crane 1978, Brouillette & Bonazza 1999).

The case of an interface under impulsive acceleration produced by shock interaction was first considered by Markstein (1957), although the first rigorous treatment of the shock-excited instability was the theoretical and numerical analysis of Richtmyer (1960). Shock-tube work by Meshkov (1969) then confirmed, at least qualitatively, the predictions of Richtmyer. This class of problems is thus known as the Richtmyer-Meshkov instability. It differs sensibly from the instability that develops when a finite sustained acceleration takes place between two fluids of different densities, as was first examined by Strutt (1900). Taylor (1950) developed the linear theory for the case of an interface between two incompressible fluids of different densities under gravitational acceleration. It was found that the instability takes place only when the light fluid accelerates into the heavy one and that the initial growth of the perturbations is exponential in time. The class of problems is since known as the Rayleigh-Taylor instability (RTI).

The emergence in the 1970s of inertial fusion as a potential power source has been a major impetus for the study of accelerated and shock-processed interfaces. The combination of compressible phenomena, such as shock interaction and refraction, with interface instability, including nonlinear growth and subsequent transition to turbulence across a wide range of Mach numbers, has been a challenge to theorists, experimentalists, and computer modelers alike.

Rupert (1992) summarized the state of the field at the start of the past decade, when wide discrepancies were observed between theory, experiments, and numerical simulations. In the past 10 years, however, dramatic progress has been accomplished in these arenas. Zabusky (1999) reviewed the application of the vortex paradigm, mostly through computer simulation, to accelerated interface problems such as the RMI; the material pertaining to vortex models and numerical simulation algorithms (e.g., Holmes et al. 1999) will therefore not be repeated here. This review focuses on the basic physical processes underlying the onset and development of the Richtmyer-Meshkov instability in simple configurations. It examines the principal theoretical results along with their experimental and numerical validation. It also discusses the various experimental techniques and how they can be used to resolve outstanding issues regarding the RMI.

2. BASIC CONSIDERATIONS

2.1. Configurations and Terminology

Figure 1 shows the basic configuration for the RMI. In the basic RMI problem, the shock is initially flat and traveling in a direction normal to the interface. The region over which the properties change across the interface can be infinitely thin, in which
Richtmyer-Meshkov Instability

Figure 1  Basic configuration for the Richtmyer-Meshkov instability in the rectangular geometry. (a) Discontinuous multimode interface. (b) Discontinuous single-mode interface: initial perturbation given by $\eta(y, t = 0) = \eta_0 \cos(2\pi y/\lambda)$. Two fluids, 1 and 2, initially at rest and having different properties (such as density $\rho$ and ratio of specific heats $\gamma$, for example), are separated by an interface that has an initial perturbation; a normal shock wave, traveling from top to bottom from Fluid 1 into Fluid 2 is about to interact with the interface (Figure 1a).

The geometry can either be two-dimensional, for which the initial interface corrugation varies only in the lateral direction for a rectangular geometry (Figure 1a,b) or in the circumferential direction for a cylindrical geometry. For the three-dimensional problem, the initial interface perturbation is prescribed in two lateral directions for either the rectangular or the spherical geometry. Finally, the initial perturbation can be described either by a single sine function of known wavelength and amplitude, in which case the interface is said to have a single-mode perturbation (Figure 1b), or by a superposition of two or more of these perturbations, in which case the interface is said to be multimode (Figure 1a).

Although there has been recent interest in the RMI in cylindrical (Zhang & Graham 1998) and spherical configurations (Meshkov et al. 1997), most of the work to date on the RMI has been performed in the rectangular geometry with single- or multimode initial perturbations. This review therefore focuses on these results.

2.2. Evolution of the Instability

Following the refraction of the incident shock wave, a distorted shock is transmitted into Fluid 2; then either a distorted shock or a rarefaction wave, depending on fluid properties, is reflected back into Fluid 1. As a result of this process, the interface has
Figure 2  Wave patterns resulting from the refraction of a shock wave at an interface (I)—case of a reflected shock. (a) A shock wave (T) is transmitted through the interface, and a shock wave (R) is reflected from the interface. The latter has been accelerated to a constant velocity by the refraction process. (b) Transverse waves develop behind the curved transmitted and reflected shocks. Transverse compression waves behind the reflected shock near the crest region cause a temporary slowdown of the perturbation growth rate. (c) Reverberation of these transverse waves between the shocks and the interface cause oscillations in the perturbation growth rate (after Holmes et al. 1995).

been impulsively accelerated to a constant velocity and travels in the same direction as the transmitted shock (Figure 2a). Two physically acceptable viewpoints can then be used to explain the subsequent distortion of the interface.

The first viewpoint examines the baroclinic vorticity generation resulting from the misalignment of the pressure gradient of the shock with the density gradient across the interface. For the case of a sinusoidal interface separating a light and a
Figure 3  Vorticity deposition at a light/heavy interface. (a) Initial configuration. (b) Circulation deposition and intensity of vortex sheet. (c) Subsequent deformation of the interface.

heavy fluid ($\rho_1 < \rho_2$) (Figure 3a), this generates counterclockwise vorticity on the right side of the perturbation and clockwise vorticity on the left (Figure 3b). An unstable vortex sheet of varying strength is thus created at the interface which subsequently leads to its deformation (Figure 3c). Indeed, much can be learned about the RMI in various configurations by using the vortex paradigm, and Zabusky (1999) has recently reviewed these issues. The present article will therefore not repeat this material.

The second viewpoint invokes the pressure perturbations caused by the distorted transmitted and reflected waves in the vicinity of the interface. Again, we examine the case of a single-mode light/heavy interface, for which both transmitted and reflected waves are usually shocks. Behind these waves, it is found in the crest region that the transmitted shock is slightly converging and that the transmitted shock is slightly diverging, whereas the reverse situation holds in the vicinity of the troughs (Figure 2a). Because of this, there is a slight, positive pressure perturbation near the crests and a negative pressure perturbation near the troughs in the heavy fluid, and opposite pressure perturbations in the light fluid. The effect of these localized pressure variations across the interface is to increase the penetration of the heavy fluid into the light one near the crests and the penetration of the lighter fluid into the heavy one near the troughs. The overall result is that the initial amplitude of the perturbation increases continuously after the passage of the incident shock. Similar arguments can also be made for the heavy/light interface (Sturtevant 1988).

These two viewpoints can be formally reconciled by noting that the pressure perturbations discussed in the previous paragraph lead to a difference in the tangential component of the fluid velocity across the interface, which is responsible for circulation. The pressure perturbations across the interface can thus be directly related to the vorticity initially deposited by the shock interaction (e.g., Fraley 1986).
Figure 4  Evolution of a single-mode perturbation. (a) Initial configuration. (b) Linear growth—crests and troughs are symmetric. (c) Start of nonlinear evolution—asymmetric spike and bubble development. (d) Roll-up of spike. (e) Emergence of small-scales and turbulent mixing.

For low initial amplitudes and early times, the growth of the perturbation on the interface is linear in time (Figure 4a,b). However, as the amplitude increases, the crests and troughs of the perturbations evolve asymmetrically and lead to the appearance of spikes of heavy fluid penetrating into the light one and bubbles of light fluid “rising” into the heavy one (Figure 4c). At some point the Kelvin-Helmholtz instability also becomes important, causing the roll-up (“mushrooming”) of the spike (Figure 4d) and the appearance of smaller scales. Eventually a turbulent mixing zone (TMZ) develops between the two fluids (Figure 4e).

3. THEORY

The interaction of a shock wave with an interface separating two different fluids leads to complex wave refraction phenomena and their study constitutes a research field in itself (Henderson 1989). In this review, however, we concentrate on the evolution of the interface following this interaction. Specifically, this section examines the theories describing the interface from the start of the interaction with the incident shock wave through the late nonlinear stages of the perturbation growth. Validation with experiments and numerical simulations is also presented.

3.1. Impulsive Model

The case of an interface under shock acceleration was considered by Markstein (1957), although the linearized response of the RMI on a single-scale discontinuous interface was first rigorously studied by Richtmyer (1960) (Figure 1b). Richtmyer proposed that, by the time the transmitted and reflected waves have traveled a long distance from the interface compared to the wavelength of the perturbation, the subsequent motion at the interface can be considered incompressible. In that case, Richtmyer used the linear theory of Taylor (1950) for the growth of the
amplitude $\eta$ of small single-mode perturbations on a discontinuous interface between incompressible fluids under gravitational acceleration $g$:

$$\frac{d^2\eta(t)}{dt^2} = kgA\eta(t),$$

where $k = 2\pi/\lambda$ is the wavenumber of the perturbation and $A \equiv (\rho_2 - \rho_1)/(\rho_1 + \rho_2)$ is the preshock Atwood number. By replacing the constant acceleration $g$ by an impulsive one $g = [u]\delta(t)$, where $\delta(t)$ is the Dirac delta function and $[u]$ the change in interface velocity induced by the refraction of the incident shock, and integrating Equation 1 once with respect to time, Richtmyer obtained the impulsive growth rate relation:

$$\dot{\eta}_{\text{imp}} = k[u]A\eta_0.$$  

The growth rate $\dot{\eta}_{\text{imp}}$ describes the development of interface instability long after shock refraction phenomena have taken place at the interface while the perturbation amplitude is still small enough to remain in the linear regime.

From the impulsive formula (Equation 2) we see that, because $\dot{\eta}_{\text{imp}}$ is constant, the growth of perturbations on a shock-processed interface is initially linear in time and that both light/heavy and heavy/light configurations are unstable. Indeed, for the light/heavy configuration ($A > 0$) the perturbation increases in amplitude from the start, whereas for the heavy/light configuration ($A < 0$) the perturbation initially decreases in amplitude before reversing its phase and growing linearly. The latter case is known as the indirect inversion case; another type of phase inversion is discussed below in the context of the compressible linear theory.

Richtmyer solved the linearized RMI problem numerically for light/heavy interfaces between perfect gases and strong incident shocks. He found agreement within 5%–10% of the impulsive incompressible formula (Equation 2), provided $\eta_0$ and $A'$, the postshock perturbation amplitude and Atwood number, respectively, (the primes denote the use of postshock properties), were used instead of the preshock values, i.e., $\dot{\eta}_{\text{imp}} = k[u]A'\eta_0$. Although shock refraction at an interface between two fluids of different properties is an inherently compressible process, the above equation and Equation 2 are usually considered incompressible relations because the perturbation velocities are assumed small with respect to the local speed of sound.

Meyer & Blewett (1972), using a two-dimensional Lagrangian inviscid hydrocode, performed a numerical simulation (NS) of the RMI for both light/heavy and heavy/light interfaces between perfect gases with moderately weak incident shocks. They found good agreement with Richtmyer’s modified formula for light/heavy interfaces, whereas for heavy/light interfaces they found that they could obtain better agreement by using the average of pre- and postshock perturbation amplitudes $(\eta_0 + \eta'_0)/2$ along with the postshock Atwood number $A'$.

To bridge this discrepancy between the light/heavy and heavy/light cases, Vandeboomgaard et al. (1998) proposed a single formula, valid for both cases,
Figure 5 Comparison between the impulsive models of Richtmyer (1960) (dashed lines), Vandenboomgaerde et al. (1998) (dotted lines), and the compressible theory of Fraley (1986) (solid lines). Case of a reflected shock: (a) $\gamma_1 = \gamma_2 = 5/3$, (b) $\gamma_1 = 5/3$, $\gamma_2 = 11/10$. Data taken from Mikaelian (1994) and Vandenboomgaerde et al. (1998). using the average of the pre- and postshock properties as $\dot{\eta}_{\text{imp}} = k[u](A\eta_0 + A'\eta'_0)/2$.
Because it is based on a linear theory, the impulsive formulation is valid only as long as the perturbation remains small, $\eta(t) \ll \lambda$. The amplitude at which the impulsive model begins to break down has been examined by Aleshin et al. (1988) who propose that its validity extends up to a time estimated as $t_{\text{max}} \approx 1/(k^2 A'[u] \eta_0')$. A similar relation is proposed by Grove et al. (1993) as $t_{\text{max}} \approx 1/(k^2 M_0 c_0 \eta_0')$. The extent of validity of the linear theory has also been examined in the context of the nonlinear theory, as discussed below, and the amplitude at which the linear theory is no longer valid is found to be around $\eta_{\text{max}} \approx 0.1 \lambda$. Because of the difficulty of performing accurate measurements at the required small amplitudes, early experimental results (e.g., Meshkov 1969, Benjamin 1992) were obtained in the nonlinear regime and, characteristically, did not agree well with the impulsive theory. However, recent low (Jones & Jacobs 1997) and high (Holmes et al. 1999) Mach-number experiments yield good agreement with the impulsive formulation.

For the linear problem, including the impulsive formulation, three dimensional initial perturbations in the rectangular geometry can be treated by using the global wavenumber of the perturbation $k = \sqrt{k_y^2 + k_z^2}$, where $k_y$ and $k_z$ are the wavenumbers of the perturbation in the two lateral directions. Also, arbitrary two- or three-dimensional perturbations can be Fourier decomposed into a series of harmonic perturbations, and the global growth rate is obtained by superposition of individual growth rates computed using either the impulsive formulation, when valid, or the linearized problem, as discussed in the next section.

When many successive waves interact with the interface one after the other, the impulsive model assumes that the overall growth rate is obtained by the sum of the growth rates induced by each individual wave. After $N + 1$ wave interactions, the growth rate is given by $\dot{\eta}_N = \sum_{i=0}^{N} \dot{\eta}_i$, where $\dot{\eta}_i$ is the impulsive growth rate induced by wave $i$, the first wave interaction corresponding to $i = 0$. This has been applied to shock waves as well as to incident rarefaction waves.

The RMI on a continuous interface was first examined theoretically by Mikaelian (1985) who treated the interface as a series of $M$ discrete discontinuous interfaces in the context of the impulsive theory. The growth rate is obtained from the eigenvalues of a system of $M \times M$ equations. Although $M = 5$ accurately models a smooth continuous interface, this treatment is not amenable to a compact closed-form expression. Instead, Brouillette & Sturtevant (1994) proposed to combine the model of Duff et al. (1962) for the RTI of a continuous interface with the impulsive model of Richtmyer to obtain the growth rate for the RMI at a continuous interface as $\dot{\eta}_{\text{imp}} = k[u] A' \eta_0' / \psi$. The parameter $\psi (\geq 1)$ is the so-called growth reduction factor, which depends on the density gradient at the interface, i.e., on interface thickness and Atwood number. It is obtained as the eigenvalue of the equation for the incompressible velocity perturbation. This approach has also been validated experimentally by Collins & Jacobs (2000).

In summary, impulsive formulations owe their attractiveness mostly to ease of computation. They give good results for small amplitudes and weak shock strengths, when compressibility effects following the initial shock interaction are
not important, in which case the difference between the various impulsive models is minimal. Another limitation is that they do not describe interface phenomena in the instants just following the refraction of the incident shock wave at the interface. For strong shocks and/or early times, one must therefore use the full compressible linear theory.

3.2. Compressible Linear Theory

As in most hydrodynamic problems, the linear stability of the RMI starts with the small-amplitude perturbation of the equations of motion. However, because of the inherent compressibility of the RMI, the unperturbed state is not steady and thus the time dependence cannot be separated from the linearized perturbation equations. Because of this, “decomposition of a perturbation into the normal eigenmodes is far from being straightforward” (Velikovich 1996).

The linearized perturbations equations, for the case of a reflected shock wave, were first derived for general fluids by Richtmyer (1960), who obtained their numerical solution for perfect gases. Using Laplace transforms in time, Fraley (1986) was the first to obtain an analytical solution for the asymptotic linear growth rate \( \dot{\eta}_\infty \) for the same problem as Richtmyer. For weak shocks, he found that [see Mikaelian (1994) for the correct expressions]

\[
\dot{\eta}_\infty = k[u] \eta_0 \left( A + \epsilon \frac{F}{\gamma_1} \right),
\]

where

\[
F = F(z, A) = \frac{1}{2} \left\{ (z - 1)^2 - 2 \frac{1 + A}{1 - A} - 2z \right. \\
+ \left. \frac{2}{z} \left( \frac{(1 + A)^2}{1 - A} + (1 - A)y^2 \right) \right\} \left[ \frac{1 - A}{z + 1} \right],
\]

with \( z \equiv [(1 + A)\gamma_2/(1 - A)\gamma_1]^{1/2} \). For \( \epsilon \to 0 \), this reduces to the impulsive Richtmyer formula (Equation 2). An interesting limit to Equation 4 is presented in the case of two fluids of equal densities. For \( A = 0 \), \( F(z, 0) = (z - 1)(z - 2)/2z \) with \( z = (\gamma_2/\gamma_1)^{1/2} \). The asymptotic growth rate is then \( \dot{\eta}_\infty|_{A=0} = k[u] \epsilon (z - 1)(z - 2)/2z \gamma_1 \). For \( y = 1 \) this reduces to the case of an “interface” between two identical fluids and, expectedly, the perturbation growth rate is zero. However, for \( z \neq 1 \), the growth rate on an interface separating two fluids of the same densities but having different \( \gamma \)s is not zero. This underscores the fact that the RMI is not just the shock-accelerated RTI, but that the inherently compressible wave refraction phenomenon is the driving force behind this instability. Indeed, this result illustrates that baroclinic vorticity can be deposited by a shock at an interface initially without a density gradient because shock refraction at the interface subsequently modifies the initial density gradient. For the special case \( A = 0 \) and \( z = 2 \), one also gets \( \dot{\eta}_\infty = 0 \), and this is known as the so-called “freeze-out” case of the RMI (Mikaelian 1994) for which no perturbation growth is observed even if
RICHTMYER-MESHKOV INSTABILITY

Fluid properties are different across the interface. Similar arguments can be made for arbitrary shock strength, although no compact closed-form analytical formula is available.

Yang et al. (1994) solved the linearized equations numerically for both the reflected shock and reflected rarefaction cases, whereas Velikovich (1996) obtained an analytic solution for the reflected rarefaction case using power series expansions. Figure 6 shows the linear solution for the RMI at a light/heavy interface. Three important features are characteristic of the linear evolution of the RMI: (a) The first kink in the linear growth rate (just after 50 $\mu$s in Figure 6b) is caused by the first convergence, near the crest of the perturbation, of the pair of waves behind the transmitted shock, which temporarily reduces the growth rate. (b) The subsequent oscillations in the linear growth rate are the result of perturbations in the pressure field resulting from transverse wave interaction behind both transmitted and reflected shocks, as shown in Figures 2b and c. (c) After sufficient time, the solution approaches an asymptotic value $\dot{\eta}_{\infty}$, which, for weak shocks, is well described by the impulsive formulation $\dot{\eta}_{imp}$.

In the parameter space for the linear RMI in perfect gases, two types of phase inversion of the initial perturbation are observed: (a) direct inversion, which takes place before or at the conclusion of the shock-interface interaction, such that $\eta_0'/\eta_0 < 0$ and (b) indirect inversion, with $\eta_0'/\eta_0 > 0$ and the perturbation ultimately reversing phase as the result of a negative asymptotic growth rate; impulsive models cannot correctly account for direct inversion.

Also, Yang et al. (1994) found that, although the acceleration $\ddot{\eta}$ is insensitive to random perturbations to the initial conditions, the asymptotic growth rate $\dot{\eta}_{\infty}$ does

Figure 6  Comparison between the impulsive model of Richtmyer (1960) (- - -), the compressible linear computation of Yang et al. (1994) (...), the front tracking numerical simulation of Holmes et al. (1995) (--), the nonlinear theory of Zhang & Sohn (1997) (--) and the experimental results of Benjamin (1992) (- + -). (left) Amplitude of perturbation vs time. Air/SF$_6$ interface, $M_s = 1.20$, $\eta_0 = 2.4$ mm, $\lambda = 37.5$ mm.
indeed depend on the initial conditions. This has important consequences on the late time nonlinear development of the RMI.

In the strong shock limit in gases, chemical dissociation becomes important, and this effect has been examined both theoretically and numerically by Samtaney & Meiron (1997) for the case of a reflected shock. Nonlinearity manifests itself quite rapidly and agreement between Richtmyer’s impulsive model for frozen chemistry and the equilibrium linear theory worsens as the Atwood number is reduced.

3.3. Nonlinear Theory—Single Mode Perturbation

The late time, nonlinear development of the perturbation has been studied in terms of the evolution of the spikes and bubbles forming on the interface (cf. Figure 4c). To extend the range of the RMI theory for the single-scale discontinuous interface beyond the linear regime, one technique is to perform a formal expansion of the perturbations beyond the first order.

The second-order solution yields a bubble velocity $u_b(t) = k[u]A'\eta_0'(1 - k[u]A'\eta_0')t$ and a spike velocity $u_s(t) = k[u]A'\eta_0'(1 + k[u]A'\eta_0')t$; the overall growth rate of the interface perturbation $\dot{\eta}(t)$ is just the average of bubble and spike velocity $\dot{\eta}(t) = [u_b(t) + u_s(t)]/2$. It is thus observed that the interface develops asymmetrically from early on, with spikes growing faster than the bubbles, except for $A' = 0$ then the interface development is symmetrical.

However, to ascertain the convergence and the temporal range of validity of the expansion, one must include a large number of terms. Velikovich & Dimonte (1996) performed an expansion to arbitrary order for incompressible flow in the limit that $A' = 1$. They also extended the range of validity of the expansion beyond its natural radius of convergence using the method of Padé approximants. They found reasonable agreement between their theory and results from laser-driven high Mach-number experiments.

By supposing that initial interface growth is well described by the linear compressible theory and that late time growth is governed by an incompressible nonlinear solution, an overall description, valid from early to late time, can therefore be obtained by appropriately matching the two solutions.

Zhang & Sohn (1996, 1997) thus treat the nonlinear problem with higher-order expansions and extend the range of validity of the series using Padé approximants. Using this matching technique, akin to asymptotic methods used in boundary-layer theory, they have treated two- and three-dimensional initial perturbations both for the cases of a reflected shock and for a reflected rarefaction. They obtain distinct bubble and spike velocities, and the overall growth rate at the interface, valid for both early and late times, is given by (Li & Zhang 1997)

$$\dot{\eta}(t) = \frac{\dot{\eta}_{lin}(t)}{1 + \xi \eta_0'^2 \dot{\eta}_{lin}(t) \xi_1 t + \max(0, \eta_0'^{12/5} - \xi_2) k^2 \dot{\eta}_{lin}(t) t^2},$$

where $\dot{\eta}_{lin}(t)$ is the time varying growth rate computed from the compressible linear theory, $\xi = 1$ or $\xi = -1$ in the absence or presence of phase inversion, respectively,
and $\xi_1$ and $\xi_2$ are geometrical parameters whose values differ for two- or three-dimensional configurations; in two dimensions, $\xi_1 = 1$ and $\xi_2 = A^2 - (1/2).$ For early times $t \to 0$, one can see that $\dot{\eta}(t) \to \dot{\eta}_{lin}(t)$, as expected. For very late times and relatively large $A'$, $\dot{\eta} \sim 1/t$, which appears correct, whereas for small $A'$, then $\dot{\eta} \sim 1/t^2$, which disagrees with vortex models (Jacobs & Sheeley 1996), as discussed below.

Figure 6 shows a comparison between this nonlinear theory, a front-tracking Euler computation and experiments for a light/heavy interface. Agreement between the nonlinear theory and the computation is very good, and the two correctly predict the average experimental growth rate measurement (Figure 6b). This figure also shows that the experiment was performed in the nonlinear regime, which explains why the measured growth rate was significantly lower than the results from the impulsive model and the linear theory. Although the computed growth rates agree with the experimental average, there is a constant discrepancy between the experimental and computed amplitudes (Figure 6a), which has been attributed to the effect of the plastic membrane used to initially form the interface in the experiment.

An alternate approach to the single-mode nonlinear problem is to use Layzer’s (1955) single-mode potential incompressible flow model to compute bubble and spike velocities. Hecht et al. (1994) considered an array of identical bubbles in two dimensions with $A' = 1$ and assumed that the flow is governed by the behavior near the bubble tips, supposed parabolic in shape. The asymptotic bubble velocity is obtained as $u_b(t) = 2\pi C/kt = C\lambda/t$, with $C = 1/3\pi$, which does not depend on the initial growth rate; this result has been confirmed by Mikaelian (1998). Zhang & Sohn (1997) compare their series approach with the potential flow model for the nonlinear bubble velocity and also find good agreement between the two for $A' = 1$. Zhang (1998) used a similar approach to compute the asymptotic spike velocity for $A' = 1$ and obtains a constant value that depends on the initial linear growth rate and on tip curvature; reasonable agreement with numerical simulations is obtained. Atwood number dependence has been examined by Alon et al. (1995) using two-dimensional NS, and the result $C = 1/3\pi$ is valid for bubbles down to $A' \approx 0.5$, whereas for $A' \to 0$, $C = 1/2\pi$, in accordance with vortex model calculations (Jacobs & Sheeley 1996). For $A' = 1$, they confirm that $u_s(t)$ is constant, whereas for $A' < 1$, they find that the spike growth is described by $u_s(t) \approx [(1 + A')/(1 - A')]C\lambda/t$.

Sadot et al. (1998) propose that the evolution of a single-scale perturbation from early to late times can be obtained by combining the linear impulsive result with the asymptotic bubble/spike potential flow model as $u_{b/s}(t) = u_0[1 + Bt]/[1 + Dt + E\lambda^2]$ with $u_0 = k\lambda'[u]\lambda_0$, $B = ku_0$, $D_{b/s} = (1 \pm A'k\lambda_0$, and $E_{b/s} = [(1 + A')/(1 - A')][1/2\pi C]k^2\lambda_0^2$, the plus sign being used for bubbles and the minus sign for spikes. For very late times, this yields $\dot{\eta}(t)$ for all $A'$. Good agreement is obtained between this formula and shock-tube experiments at low (1.3) and moderately high (3.5) incident shock Mach numbers.

Using the same formulation, Sadot et al. (2000) obtained good predictions for the evolution of spikes and bubbles in the case of two successive shock refractions at
a single-mode interface, originating from the incident shock wave and its reflection from the downstream end of the shock tube, as compared with low Mach-number experimental results and NS.

3.4. Nonlinear Theory—Multimode Perturbations

In most applications the initial perturbation is multimodal, with wavenumbers spanning many orders of magnitude. Also, as the RMI develops, the Kelvin-Helmholtz instability causes vortex roll-up, which further increases the range of physical scales. Ultimately, a three-dimensional turbulent mixing zone (TMZ) develops on the interface, even for a two-dimensional initial perturbation. Because of the wide range of spatial and temporal scales that need to be resolved, the direct numerical simulation of TMZ evolution due to the RMI is therefore not practical, and a number of physical models have been developed to address this problem.

In the late nonlinear stage, it is generally agreed that the time evolution of the overall thickness $h$ (i.e., peak-to-valley amplitude) of the TMZ evolves according to a power law $h \sim t^\theta$ where values ranging from $\theta = 0.25$ to $\theta = 1$ have been proposed. However, because the single-mode asymptotic bubble velocities appear to vary as $1/t$ (see above), then the TMZ time evolution could even be logarithmic. Most theoretical and experimental evidence also demonstrates that the TMZ evolution is not independent of initial conditions. This is because the kinetic energy available for the turbulent motions is deposited only during the interaction of the shock with the interface, and this depends strongly on the amplitude and wavenumber of the initial perturbation (Saffman & Meiron 1989, Brouillette & Sturtevant 1993).

Assuming isotropic turbulence in a thin fluid layer containing a fixed kinetic energy, Barenblatt (1983) found that $\theta = 2/3$ in the absence of dissipation; with dissipation $\theta < 2/3$ and depends on the turbulence model constants, which are usually adjusted from experimental data. The $\theta = 2/3$ value can also be obtained from simple dimensional arguments. Huang & Leonard (1994) have proposed a late-time similarity that also yields a power law decay. Using Saffman’s (1967) hypothesis, which bounds the integral moments of the vorticity distribution in the large scales, one obtains $\theta = 1/4$.

Another approach considers the relative growth rates of the many modes forming a multimode configuration. If this broad spectrum of modes is assumed to be dominated by the just-saturated mode, then the overall growth exponent is found to be $\theta = 1/2$ (Dimonte et al. 1995). On the other hand, by applying an impulsive acceleration to the RTI mixing result of Read (1984), Mikaelian (1989) obtained $\theta = 1$.

The late time evolution of the interface can also be viewed as governed by a competition between bubbles and spikes of various sizes and velocities (Sharp 1984). In this case, the bubbles and spikes are treated separately and the global interface thickness is obtained as the sum of bubble and spike sizes $h = h_b + h_s$, with the bubble width $h_b \sim t^{\theta_b}$ and the spike width $h_s \sim t^{\theta_s}$. Using a bubble merger
rate computed from the two-bubble potential flow model of Hecht et al. (1994). Alon et al. (1995) computed the late time evolution of a two-dimensional interface in terms of the competition process and found that $\theta_b \approx 0.4$ over the entire range of Atwood ratios. By analogy, spike penetration is well fitted by $\theta_s / \theta_b \approx 1 + A'$, as $\theta_s = \theta_b$ for $A' = 0$ (symmetrical spikes and bubbles) and increases to $\theta_s = 1$ for $A' = 1$ (linear spike growth rate). Similar results have been obtained by Rikanati et al. (1998) in the case of vortex, rather than bubble, merger. Sadot et al. (1998) obtained good agreement between the potential flow bubble competition model, low Mach-number shock-tube experiments and NS for a two-dimensional bimodal initial configuration. Oron et al. (1999) applied the same methodology to three-dimensional configurations and found that the growth exponent for bubbles is lowered to $\theta_b \approx 0.25$ because the narrower three-dimensional bubble-size distribution reduces the merging rate in three dimensions. Sadot et al. (2000) also validate the potential bubble competition model on a bimodal interface subjected to two successive shock interactions, in comparison with low Mach-number experiments and NS.

Another bubble dynamics approach uses a simplified two-phase flow model that assumes that, after the shock wave has imparted an impulse to bubbles and spikes at the interface, their momentum is reduced by the effects of drag. The solution of the resulting equations of motion also yields a power law dependence, where the growth exponent depends on the drag coefficient (Dimonte & Schneider 1997). Similar approaches have also been proposed by others (e.g., Cheng et al. 2000). The required coefficients are usually obtained from experimental data, and these models have a wide range of applicability (Dimonte & Schneider 2000).

A model attributed to Ramshaw (1998) uses a Lagrangian energy formulation to derive a differential equation describing the time evolution of a perturbed interface under variable acceleration. It contains two adjustable parameters, one related to viscous dissipation and the other to the growth exponent $\theta$. For an impulsive acceleration in the absence of dissipation, it recovers the linear impulsive formula for early time, whereas for late times it yields $\theta = 2/3$, in agreement with simple scaling arguments. As above, dissipation reduces $\theta$, and the required model coefficients are usually obtained from experimental data. Somewhat surprisingly, Ramshaw’s equation, although based on a totally different approach, has the same form as phenomenological models based on bubble-drag dynamics.

The most recent experimental results in a variety of initial configurations also exhibit wide variation in the growth factor, which may indicate that this problem is indeed quite sensitive to initial conditions. Laser-driven experiments reported by Dimonte & Schneider (1997) (three-dimensional initial perturbation, $A' \sim -0.6, M_s > 10$) yield $\theta = 0.5 \pm 0.1$, although the bubble competition model of Alon et al. (1995), with $\theta_b \sim 0.40$ and $\theta_s \sim 0.57$, produces a reasonable fit to that data. Another laser-driven experiment by Farley & Logory (2000) (two dimensions, $A = -0.85, M_s \sim 30$) obtains $\theta = 0.8$, but that experiment included an unstable sustained deceleration that might have increased the growth rate over the purely impulsive case. Dimonte & Schneider (2000) performed linear electric motor (LEM)
460  BROUILLETTE

experiments (three dimensions, $0.15 \leq A \leq 0.96$) and obtained $	heta_b = 0.25 \pm 0.05$
and $	heta_t = \theta_b[(1 + A)/(1 - A)]^{(0.21 \pm 0.05)}$. A shock-tube experiment by Prasad et al. (2000) (two-dimensions, $A \sim 0.7$, $M_s = 1.55$) measured very late time growth to
get $	heta = 0.26 - 0.33$, and this was found to model correctly the data of Sadot et al. (1998) because the latter was taken at relatively earlier times.

All in all, the current experimental evidence is not sufficient to narrow the
uncertainty on the growth exponents or to quantify the influence of density ratio or
shock strength. This is because the data are not sufficiently precise nor of adequate
duration to reveal the differences between the various models; bubbles and spikes
also need to be resolved separately to ascertain $	heta_b$ and $	heta_t$. These limitations are
further discussed in the next section, in the context of experimentation.

4. EXPERIMENTS

As in other fluid mechanics problems, the experimental investigation of the RMI
aims at learning new physics, validating theories and providing data on which mod-
els can be calibrated. Because real-world configurations are usually too compi-
lcated, experiments are usually performed in simplified geometries. The experi-
mental study of the RMI requires three essential ingredients: (a) means for producing a
shock wave or other type of impulsive acceleration, (b) an interface separating two
fluids having a well-characterized initial perturbation, and (c) diagnostic methods
allowing for the measurement of relevant quantities.

4.1. Shock-Wave Generation

In the context of the RMI, incident shock-wave Mach numbers in the range
$1 < M_s < 5$ have been produced in shock tubes. To allow for the use of most flow
diagnostics, a test section having a square or low (~1) aspect ratio rectangular cross
section is preferred. The optimum shock tube must also be sufficiently wide so that
wall effects can be minimized, and the driver and test sections must be long enough
to allow for sufficient test duration before the arrival of reflected waves from both
eads. On the other hand, strategically locating the end of the test section can allow
for the study of the interface as it is repeatedly processed by reflections of the pri-
mary shock wave reverberating between the end wall and the interface. High Mach
numbers can be produced either by increasing the driver pressure or by reducing the
test pressure. The latter option is not desirable, beyond a certain point, as boundary-
layer effects increase as the test pressure is decreased (Brouillette & Bonazza 1999).
On the other hand, large square test sections able to sustain high pressures pose
design and cost challenges. One solution, implemented by Anderson et al. (2000),
is to use thin metal plates to form a large square section that is strengthened by em-
bedding it in a concrete matrix contained in a larger circular pipe. A short optical
access section is built separately using thicker plates and no reinforcement. The
facility, having a 25 cm square section, is able to withstand a 20 MPa pressure load.
Gaseous (Dudin et al. 1997) or condensed-phase (Benjamin & Fritz 1987) explosives have also been used to drive strong shock waves in RMI experiments with gas/gas, liquid/liquid and liquid/solid interfaces, but because of the presence of the expansion (Taylor) wave propagating just behind a detonation front, the initial shock acceleration is immediately followed by an extended deceleration. These experiments therefore study the RMI immediately followed by the RTI.

Powerful laser radiation can also be used to drive shock waves in solids. Because beam inhomogeneities also excite instabilities, the preferred method is the indirect drive in which the laser irradiates the inside of a cylindrical enclosure (hohlraum) to generate an X-ray pulse (Figure 7). These drive X rays rapidly heat and ablate the target surface, launching a shock wave into an attached shock tube (Peyser et al. 1995). Shock waves in the range \( M_s \sim 10^{-30} \) have been produced in solids with square laser pulses (\( \sim 30 \text{ kJ at } \sim 5 \mu\text{m for } \sim 1-3 \text{ ns} \)). As in shock tubes, the duration of an experiment is limited by wave reflection phenomena from the ends, although test time could be extended by lengthening the laser pulse, which requires more drive energy.

Almost impulsive accelerations of an interface can be achieved by mechanically subjecting a free-moving test section to a large force of short duration. Dimonte et al. (1996) use a linear electric motor (LEM) to produce an acceleration pulse typically around 1500 m/s\(^2\) lasting around 10–20 ms (accelerations up to 7000 m/s\(^2\) have also been produced). Higher accelerations require a greater driving force but impose the strengthening of the test section, which increases its mass. Jacobs & Sheeley (1996) produced a quasi-impulsive acceleration of the test section by having it recoil from a spring after a period of free fall. The period of interaction with the spring corresponds to the duration of the quasi-impulsive acceleration.

![Figure 7 Laser-driven radiation experiments (from Farley & Logory 2000).](image)
4.2. Interface Formation

Ever since the first RMI experiments were performed by Meshkov (1969), interface formation has posed a challenge to experimentalists.

In shock tubes, a common method to form an interface between two gases is the use of a thin polymeric film of micron thickness. By supporting the membrane with thin wires and/or using a small pressure difference to create a bulge, well-characterized two- or three-dimensional discontinuous configurations can be produced. For single-mode experiments, good agreement is obtained with the appropriate theory for the perturbation growth rate, but perturbation amplitude is usually found to be smaller than predicted (Figure 6). For a multimode experiment on a nominally flat interface, perturbations are introduced by membrane rupture, and this is found to have an important effect on the subsequent TMZ evolution. The wide discrepancy between results obtained in different facilities (cf. Brouillette & Sturtevant 1993) has partly been attributed to the different compounds used to produce the films, film-curing time, film thickness, and thickness distribution, which all have an effect on the mechanical properties of the film and thus its rupture characteristics (Abakumov et al. 1996, Houas & Chemouni 1996). Zaytsev et al. (1985) showed that the membrane could be pyrolized by strong shocks but that its gaseous decomposition creates a continuous interface between the two test gases. The inertia of the membrane can also have an effect on the shock refraction at the interface (Brouillette & Bonazza 1999), and this gets worse with light test gases at reduced initial pressures. Recent results (Erez et al. 2000) with two different membrane thicknesses seem to show that the effect of the film on the late time, nonlinear growth of the single-mode RMI is negligible; for nominally flat interfaces, however, the TMZ development after the incident shock depends on film thickness, whereas the growth after the interaction with the shock reflected from the end of the test section is about the same for both films.

To avoid the membrane in shock tube experiments, Brouillette & Sturtevant (1989) and others have used a thin sliding plate that initially separates both gases; obviously, this requires a vertical shock tube. The plate is retracted prior to launching the shock, leaving behind a continuous interface whose thickness is controlled by the time delay between plate retraction and shock firing. The “pulling” action of the retracting plate can be used to introduce perturbations on the interface (Brouillette & Sturtevant 1994). Another membrane-free scheme has been implemented by Jones & Jacobs (1997) who used a downward flow of light gas and an upward flow of heavy gas that meet at the desired interface location and exit via lateral slots in the test section walls (Figure 8). This forms a relatively thin (≈5 mm) continuous interface whose initial shape can be controlled by oscillating the entire shock tube in the lateral direction. Typically, these low Mach-number experiments, free from membrane effects, exhibit good agreement with the impulsive theory (Brouillette & Sturtevant 1994, Jacobs & Collins 2000). The slab configuration, i.e., a certain fluid sandwiching a layer of another fluid, can also be created by using a contoured jet or gas curtain entering from one wall of the test section and exiting from the other side (e.g., Jacobs et al. 1993). Issues pertaining
to shock-tube experimentation of the RMI, specifically regarding wall effects and wave phenomena, have been discussed by Brouillette & Bonazza (1999).

Explosive and laser-driven experiments with solid/liquid or solid/solid interfaces enjoy well-defined initial perturbations that are machined on the interface, although this is a fabrication challenge for the miniature shock tubes used with laser drives. Gels can also be shaped at will and, under strong shock loading, lose most of their mechanical strength (Volchenko et al. 1989). With liquid/gas or liquid/liquid interfaces, a method for introducing well-characterized initial perturbations is to produce standing waves by laterally shaking the test section at the right frequency and amplitude (Jones & Jacobs 1997). Another method is to release bubbles of light fluid from the bottom of the test section and to initiate the experiment just as they pierce the interface (e.g., Dimonte & Schneider 2000). A problem of the liquid/gas interface, however, is the formation of a meniscus where the liquid meets the side walls of the test section that introduces a mostly undesirable initial perturbation.

4.3. Diagnostics

The experimental observation of the RMI requires performing measurements in a rapidly moving and changing environment. Ideally, two needs have to be fulfilled: *(a)* What does the interface and wave phenomena look like? *(b)* What are their kinematic, dynamic, and thermodynamic properties?

The usual flow-visualization techniques can be applied to the various RMI configurations. However, depending on the facility and experimental conditions, wall
effects can be important (Brouillette & Bonazza 1999), and integrating methods such as the shadowgraph/schlieren, interferometry, and X-ray radiography may not differentiate well between interface and wall phenomena. To date, most measurements on the RMI have been obtained by integrating methods, such as these flow visualization techniques, and also using laser absorption techniques.

However, the state of the art now resides with planar imaging techniques, which allow the observation of a two-dimensional slice of the flow, away from boundary phenomena on the windows. As long as the flow does not possess strong three-dimensional features, these techniques can then provide a representative view of the interface. Planar laser induced fluorescence (PLIF) uses seeding of one of the fluids across the interface with a species that fluoresces upon excitation at the right frequency. Because the fluorescence is proportional to the seed concentration, the two-dimensional density field in the illuminated region can then be obtained; PLIF can be used with liquids and gases. Planar Rayleigh scattering (PRS) uses the difference in the photon-scattering cross section between various gaseous species. Upon laser sheet illumination, the scattered signal at a given point is simply proportional to the concentrations of the two gases initially across the interface, and a two-dimensional density map of the flow field can thereafter be recovered. Recently, Prestridge et al. (2000) have used particle image velocimetry (PIV) to measure the circulation for the RMI resulting from the interaction of a weak shock with a gas curtain. These techniques are not suitable for experiments performed with plastic membranes, however, as the fragments impair the proper illumination and visualization of the interface region. Also, high-speed imaging is necessary to gather data over a certain period of a single experimental run, otherwise many runs are required to cover a given experimental condition, with the uncertainty associated with repeatability.

Lately, pointwise probing of interface properties has been attempted, mostly intended to obtain velocity or concentration measurements, using laser doppler or hot-wire anemometry. As in other applications, these techniques are useful to measure fluctuations within the turbulent interface and could provide valuable data for the tuning of simulations based on turbulence models. A useful source of reference material regarding diagnostic methods for the experimental study of the RMI can be found in Houas et al. (1999).

4.4. Summary

All in all, a variety of experiments are required to probe the RMI from low to high shock strength over a wide range of Atwood numbers. LEM experiments are useful to study the incompressible RMI for a variety of density ratios. The latest generation of large-size, high-strength, vertical shock tubes can perform membrane-free experiments in gases over a wide range of Mach numbers and could ultimately produce conditions that overlap with radiation-driven experiments, which cover the high Mach-number region. The increased use of planar imaging techniques allows a clear view of interface phenomena, away from deleterious wall effects.
In all cases experimental duration must be extended, and observation of the interface must be achieved from the start of the interaction through the late nonlinear, turbulent stages.

5. CONCLUSIONS

The interaction of a shock wave with an interface separating different fluids produces a rich variety of flow phenomena even in the simplest configurations. Over the past 10 years, much has been accomplished to understand the development of the instability at the interface, from the onset of shock refraction to the nonlinear evolution of the initial perturbation. Characteristically, the turbulent regime has posed the biggest challenge, and new experimental data will be required to generalize the time evolution of the interface over a wide range of initial conditions.

ACKNOWLEDGMENTS

This review is dedicated to the late Brad Sturtevant, who remains an inspiration for many of us. Special thanks to Riccardo Bonazza for his insightful comments and for his help with some of the reference materials.

Visit the Annual Reviews home page at www.AnnualReviews.org

LITERATURE CITED


Brouillette M, Sturtevant B. 1989. Growth
induced by multiple shock waves normally incident on plane gaseous interfaces. *Physica D* 37:248–63


BROUILLETTE

CONTENTS

FRONTISPICE xii
MILTON VAN DYKE, THE MAN AND HIS WORK, Leonard W. Schwartz 1
G.K. BATECHELOR AND THE HOMOGENIZATION OF TURBULENCE, H.K. Moffatt 19
DAVID CRIGHTON, 1942–2000: A COMMENTARY ON HIS CAREER AND HIS INFLUENCE ON AEROACOUSTIC THEORY, John E. Ffowcs Williams 37
SOUND PROPAGATION CLOSE TO THE GROUND, Keith Attenborough 51
ELLIPITIC INSTABILITY, Richard R. Kerswell 83
LAGRANGIAN INVESTIGATIONS OF TURBULENCE, P.K. Yeung 115
CAVITATION IN VORTICAL FLOWS, Roger E.A. Arndt 143
MICROSTRUCTURAL EVOLUTION IN POLYMER BLENDS, Charles L. Tucker III and Paula Moldenaers 177
CELLULAR FLUID MECHANICS, Roger D. Kamm 211
DYNAMICAL PHENOMENA IN LIQUID-CRYSTALLINE MATERIALS, Alejandro D. Rey and Morton M. Denn 233
NONCOALESCEACE AND NONWETTING BEHAVIOR OF LIQUIDS, G. Paul Neitzel and Pasquale Dell’Aversana 267
BOUNDARY-LAYER RECEPTIVITY TO FREESTREAM DISTURBANCES, William S. Saric, Helen L. Reed, and Edward J. Kerschen 291
ONE-POINT CLOSURE MODELS FOR BUOYANCY-DRIVEN TURBULENT FLOWS, K. Hanjalić 321
WALL-LAYER MODELS FOR LARGE-EDDY SIMULATIONS, Ugo Piomelli and Elias Balaras 349
FILAMENT-STRETCHING RHEOMETRY OF COMPLEX FLUIDS, Gareth H. McKinley and Tamarapu Sridhar 375
MOLECULAR ORIENTATION EFFECTS IN VISCOELASTICITY, Jason K.C. Suen, Yong Lak Joo, and Robert C. Armstrong 417
THE RICHTMYER-MESHKOV INSTABILITY, Martin Brouillette 445
SHIP WAKES AND THEIR RADAR IMAGES, Arthur M. Reed and Jerome H. Milgram 469
SYNTHETIC JETS, Ari Glezer and Michael Amitay
FLUID DYNAMICS OF EL NIÑO VARIABILITY, Henk A. Dijkstra and Gerrit Burgers
INTERNAL GRAVITY WAVES: FROM INSTABILITIES TO TURBULENCE, C. Staquet and J. Sommeria

INDEXES
Subject Index
Cumulative Index of Contributing Authors, Volumes 1–34
Cumulative Index of Chapter Titles, Volumes 1–34

ERRATA
An online log of corrections to the Annual Review of Fluid Mechanics chapters may be found at http://fluid.annualreviews.org/errata.shtml