Shock Viscosity and Rise Time of Explosion Waves in Geologic Media

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ABSTRACT

Near field velocity gauge data from many underground nuclear tests show that the rise time of the stress wave in geologic materials is extremely long—up to 300 ms at ranges of a kilometer or more in alluvium. Steady wave analysis of this data shows that stress waves traveling in salt, granite, alluvium, basalt and dry tuff also broaden as they propagate, reaching widths from 140m in salt to nearly 400m in alluvium. This broadening ceases when the stress in the wave drops below a threshold ranging from a few MPa for alluvium to ca. 60MPa for granite. The rise time and width of the stress wave is a unique function of particle velocity in each material studied: it does not depend on the yield of the explosion. Effective viscosities range from 5 x 10^4 Pa-sec in tuff to 3 x 10^5 Pa-sec in granite. These large viscosities permit realistic computer models of shock wave structure because they can stabilize numerical computations without artificial viscosity.

INTRODUCTION

A strong stress pulse or shock wave propagates into the ground following either a meteorite impact or the detonation of an explosive device. As it propagates the stress wave decreases in amplitude and changes shape because it engulfs more material and inelastic processes dissipate its energy. Much work has been devoted to this amplitude decrease[1-3] but relatively little study has been made of the shape changes. The most useful single-parameter specification of pulse shape is the rise time τ, here defined as the ratio between the maximum particle velocity change in the wave and the maximum acceleration during the rise $\tau = \Delta v_{max} / a_{max}$. The rise time is important for engineering applications such as the design of structures intended to resist damage from nearby explosions and the detection and analysis of remote impacts or explosions. The rise time also figures prominently in theories of rock fragmentation by impacts[4,5].

Any rate-dependent attenuation mechanism causes the rise time of a pulse to increase as its amplitude decreases e.g. [6,7]. Such an increase in rise time is commonly observed on particle velocity records from gauges placed close to underground nuclear tests[8], as well as in laboratory studies of small specimens[9]. This phenomenon is especially well shown in the records from the SALMON (5.3kT) and GNOME (3.1kT) nuclear tests in halite[10,11]. These tests were performed with the express purpose of evaluating ground motion.

Figure 1 illustrates the basic features of a stress pulse from the SALMON nuclear test. This is a velocity gauge record from a position 450 m from the “working point” (WP), the center of the nuclear explosion. It illustrates a small elastic precursor to the sharply rising principal wave, its somewhat less rapid fall, and later small oscillations. The rise time is measured as the interval between the points where the arriving wave rises from 10% to 90% of its final value, exclusive of the elastic precursor. Figure 2 is a reproduction of the velocity gauge records from the GNOME nuclear test. It clearly shows the decrease in rise time with increasing range from the working point.

This paper evaluates the data from SALMON, GNOME and nuclear tests in other geologic media to show that the rise time is a unique function of the peak particle velocity when the steady wave approximation is valid. The rise time is thus a material property and does not scale with device yield, as others have argued[12]. The rate dependence of the attenuation law is parameterized in terms of a nonlinear viscosity, but the physical processes that lead to this law are not fully understood.

THE STEADY WAVE APPROXIMATION

Steady wave analysis has been used to study the structure of shock fronts in metals[13,14], PMMA[15] and a number of oxides such as MgO and SiO_2[9]. In this work a steady, planar shock front is produced in a small sample by a gun-driven flyer plate. The pulse, which initially has a very short rise time, gradually relaxes to a constant (longer) rise time and propagates with constant strength. The measured rise time is attributed to details of dislocation motion or a viscous failure law. Note that the rise times for shock waves in gases are much shorter, with a pulse breadth comparable to the mean free path of a gas molecule[16]. Solids offer many more avenues for energy dissipation than gases and so the shock fronts are correspondingly larger.

In these experiments it is important to allow the pulse to propagate far enough to reach a steady state shape. A minimum requirement is that the pulse width w = □U, where U is the pulse velocity, is smaller than the sample size. Although this can be satisfied in metals, where pulse widths are fractions of a millimeter, it is not possible to perform laboratory experiments on rock where pulse widths reach 100's of meters. The underground test data is the best available for this purpose. Unfortunately, the stress wave from an explosion is not planar. Because the stress wave diverges spherically, the particle velocity (hence shock strength) declines with increasing range from the source. The wave thus cannot be steady. However, for large explosions the ratio between pulse width w and range R from the source is small enough that a quasi-steady situation may exist in which the stress pulse nearly attains its steady state rise time at each successive radius. The success of this approximation depends on the ratio $w/R << 1$. Small explosions may not satisfy this criterion, but w/R decreases with increasing explosive energy. Data from explosions larger than about 1kT yield appear to satisfy this condition. Figure 7 shows that the rise times from EVANS, a 0.03kT shot in tuff, fall well below the steady wave curve defined by larger yield shots. Numerical analysis must be employed when the steady wave approximation fails.

A steady wave moving with velocity U is described by a single variable $\xi = r - Ut$ in a moving coordinate system. Far in front of the wave ($\xi \to \infty$), the density $\rho(\xi \to \infty) \to \rho_0$, pressure $P(\xi \to \infty) \to P_0$ and particle velocity $v(\xi \to \infty) \to 0$. The first and second equations of one-dimensional hydrodynamics become[17]:

$$\rho(U - v) = \rho_0 U \tag{1a}$$

$$\rho_0 v U = P - \tau \tag{1b}$$

For solids the third (energy) equation can be neglected for all but the strongest shocks. The longitudinal stress τ contains the constitutive law of the material. If τ is an algebraic function of v or P there is no restriction on the sharpness of the stress pulse and the wave attenuates without broadening. However, if τ involves any derivatives (i.e., a rate dependent law) the situation is entirely different. For example, for a fluid of viscosity η,

$$\tau = 2\eta\dot{\varepsilon}_r = 2\eta\frac{\partial v}{\partial \xi} \tag{2}$$

In this case $\frac{\partial v}{\partial \xi}$ is infinite for a discontinuous shock and equation (1b) cannot be satisfied. The pulse must spread out so that τ remains finite. Because $\frac{\partial v}{\partial \xi}$ vanishes both far in front of and behind the shock, the Hugoniot jump conditions are still satisfied across the broadened stress wave.

A more general expression for τ that will be used to fit the data is a power law:

$$\tau = A\,\dot{\varepsilon}_r\left|\dot{\varepsilon}_r\right|^{n-1} = A\frac{\partial v}{\partial \xi}\left|\frac{\partial v}{\partial \xi}\right|^{n-1} \tag{3}$$

where A and n are constants. When $n = 1$ the linear law (2) is recovered, whereas for $n \neq 1$ equation (3) gives a more general "viscosity" law that behaves properly in the $\dot{\varepsilon} \to \pm\infty$ limits.

The equations of motion (1) may be solved with the generalized viscosity law (3) only after an equation of state $P(\rho)$ is specified. For cold rock under relatively small compression it is adequate to use

$$P = c^2 (\rho - \rho_0) \tag{4}$$

c is the bulk sound speed. Equations (1), (3) and (4) can be solved for the width of the stress pulse which, in the limit $v_{max} << c$, becomes

$$w = \tau c = \left(\frac{4A}{\rho_0}\right)^{1/n} \frac{1}{v_{\max}^{(2-n)/n}} \tag{5}$$

when $n = 1$ Newtonian viscosity), $w \sim 1/v_{max}$. The rise time is $\tau = w/c$. Note that w and τ do not depend upon initial pulse width: they are functions only of the maximum particle velocity v_{max} and material properties.

It must be emphasized that equation (5) is approximate: not only is the wave front assumed to be quasi-static, the equation of state (4) is unrealistically simplified and the rate dependent flow law (3) has no yield point. In reality, (3) should be valid until the stress drops to some strength limit, below which η decreases to near zero. The above treatment, however, is adequate for a first approximation: further work must be performed numerically.

DATA ANALYSIS

Equation (5) suggests that pulse widths (and thus rise time $\tau = w/c$) are a material property of the geologic medium in the steady wave limit. This suggestion was tested by plotting the peak particle velocity v_{max} observed in a number of underground nuclear tests vs. rise time τ.

Figures 3 and 4 present data obtained from nuclear tests in halite (GNOME and SALMON) and in granite (PILE DRIVER). These data were plotted in conjunction with information on gauge integrity and position–they constitute a "clean" data set. Note that the GNOME (3.1kT) and SALMON (5.3kT) data are indistinguishable on this plot supporting the hypothesis that the rise times are a material property. When rise time in halite is plotted versus either range R or yield-scaled range $R/W^{1/3}$, where W is the energy yield of the explosion, the two data sets form distinct arrays, as shown in Figure 5, where the yield-scaled rise time is plotted against the yield-scaled range. This observation directly contradicts the widespread assumption[12] that rise time scales as $W^{1/3}$.

The data in Figures 3 and 5 are fit by

salt: $\tau(\text{sec}) = 0.0380\, v_{max}\, (\text{m/sec})^{-.595}$ (6a)

granite: $\tau(\text{sec}) = 0.0874\, v_{max}\, (\text{m/sec})^{-0.81}$ (6b)

The corresponding values of n and A are tabulated in Table I. These rise time relations hold until v_{max} drops below a minimum particle velocity v_b:

salt: $v_b = 1.4$ m/sec (7a)

granite: $v_b = 3.8$ m/sec (7b)

The stresses associated with these minimum particle velocities are estimated from $P_b = \rho_0 c v_b$ (see Table I).

The fits to the data (6a) and (6b) should not be taken outside the range of the measurements shown in Figures 3 and 4. Comparing these fits to rise times for the oxides MgO and SiO_2, measured at much higher particle velocities[9], Figure 6 shows that the rise time may decrease much faster than (6a) and (6b) predict. Unfortunately, it is not possible to directly compare rise times for the same materials over this large range in particle velocity, but the crude comparison of rocks to oxides suggests that the rise time may be a strongly decreasing function of particle velocity for stronger shocks.

Although the date in Figures 3 and 4 are the cleanest available, the rise time vs. particle velocity for shots in tuff and alluvium (Figs. 7 and 8) can be roughly estimated. These data were obtained directly from the Perret and Bass[8] tabulation. "Bad" records due to noise or broken gages, gauges sited in different geologic materials, and refractions due to layering have thus not been removed. These plots must be treated as a "first-cut" -- they could be improved by examination of the actual gauge records and site geology. The salt and granite data showed that marked improvements may result. Nevertheless, the tuff data (Fig. 7) show the same general trend as the data in salt and granite. With the exception of the 0.030 kT EVANS event, the data show a steady increase of rise time with decreasing particle velocity down to v_b □ 1.5 m/sec, where the rise time stabilizes at approximately 0.08 sec (80 ms). EVANS may not fit the general trend because the steady wave approximation fails (i.e., w/R is too large: in this case w/R would be 0.6 in the steady wave approximation).

The data for alluvium (Fig. 8) show the same general trend: on a v_{max} versus □ plot, data from tests with a variety of yields nearly follow a single curve, although there is some tendency for large yield events to have longer rise times at large particle velocities. The strangest feature of the alluvium data is the positive slope below v □ 1 m/sec: the rise time actually <u>decreases</u> as the particle velocity declines. This fact was most clearly demonstrated in MERLIN (10kT)[18], but all the other alluvium data is consistent with this trend. The reason why the stress waves in alluvium appear to "shock up" is not known.

Table II is a summary of the rise time data for a variety of materials, where I have "force fit" a Newtonian viscosity, $n = 1$, decay law (2) to the data. This data is far less reliable than the corrected data for salt and granite (Table I). To give a feeling for the accuracy of the method, Table II also includes the salt and granite data. Note that the breakover velocity v_b is higher in Table II than in Table I -- this is due to the "force-fit", $n = 1$, steeper velocity dependence. Table II shows that, while variations occur, most geologic media have the same effective viscosity (i.e., rate dependence) within an order of magnitude. This high viscosity results in broad plastic wave fronts, which should allow effective numerical modeling of actual pulse shapes once a flow law analogous to (3) is established.

DISCUSSION

The data presented in this paper indicate that the rise times of stress pulses from large enough explosions depend only upon particle velocity and the properties of the media through which they propagate. This observation contradicts the widely held notion that rise times scale as $(\text{yield})^{1/3}$.

The large breadth of the quasi-static wave may make realistic numerical computation of pulse shapes in rock possible. Numerical codes that deal with stress waves are inherently unstable unless the shock front is spread over several computational cells[19, Chapter 12]. This smearing is generally accomplished by the introduction of an "artificial viscosity" whose function is to simulate the actual dissipation that occurs in a shock front. Without such dissipation no entropy jump can occur at the shock and the Hugoniot equations and equation of state cannot be simultaneously satisfied. For gases or metals this viscosity is much larger than the physical viscosity and the computed pulse rise times are numerical artifacts. However, the apparent viscosities for geologic media (Table II) are comparable to the required artificial viscosity. Thus with fine enough grid spacing the actual viscosity can stabilize the computation and the computed pulse shapes may be believed.

The fundamental reason for the large viscosities observed in geologic media is not presently known. It seems likely however, that the rate dependent dissipation is related to crushing of the rock by the stress wave. Whether the energy is lost by friction of one fragment against another, by the opening of cracks, or by the scattering of elastic energy from heterogeneities that are either inherent in the rock or created by fracture is unclear at the present time. The data presented here may help to constrain theories of rock fracture by strong stress waves.

Table I

Steady Wave Parameters for Salt and Granite

Material	Initial Density*	Sound Speed*	Power law	Coefficient of Eq. (3)	Breakover Stress
	ρ_0,kg/m^3	c, m/sec	n	A (mks)	P_b, MPa
Salt	2230	4669	1.25	3.6 x 10^5	15
Granite	2670	5122	1.11	8.5 x 10^6	58

*Data from [8]

Table II

Force Fit n=1 Rise Time vs. Particle Velocity*

Material	Rise time at breakover	Peak particle velocity at breakover	Effective† viscosity	Width of plastic wave at breakover
	τ_b, sec	v_b m/sec	η, Pa-sec	W_b , m
Granite	0.01±.01	10 - 20	2.9 x 10^5	57
Salt	0.02±.01	3-5	1.1 x 10^5	94
Alluvium	0.2±.1	1-2	1.2 x 10^5	360
Dry Tuff	0.06±.04	1-2	5 x 10^4	150
Basalt	0.015±.005	9-10	2 x 10^5	71

*Data on density ρ_0 and sound speed c are from [8]

†Viscosity is estimated from $A = 2\eta$, $\eta = \rho_0 \, c \, v_b \, \tau_b / 8$.

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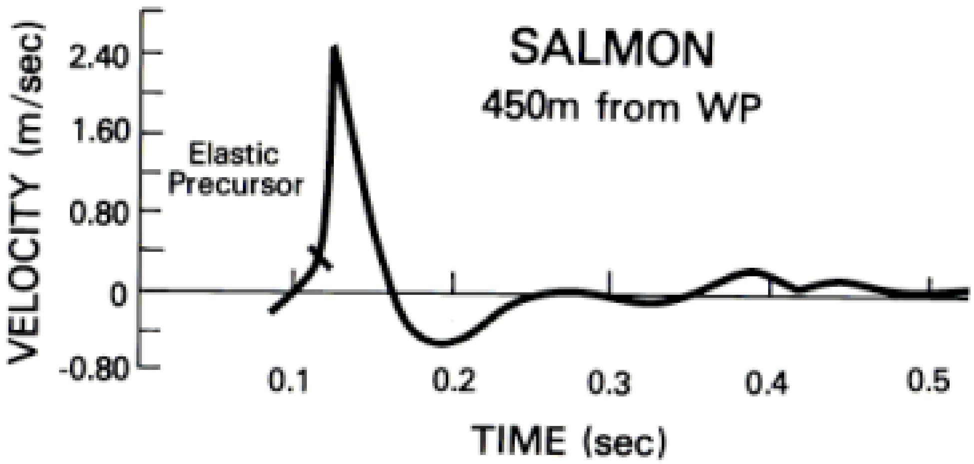
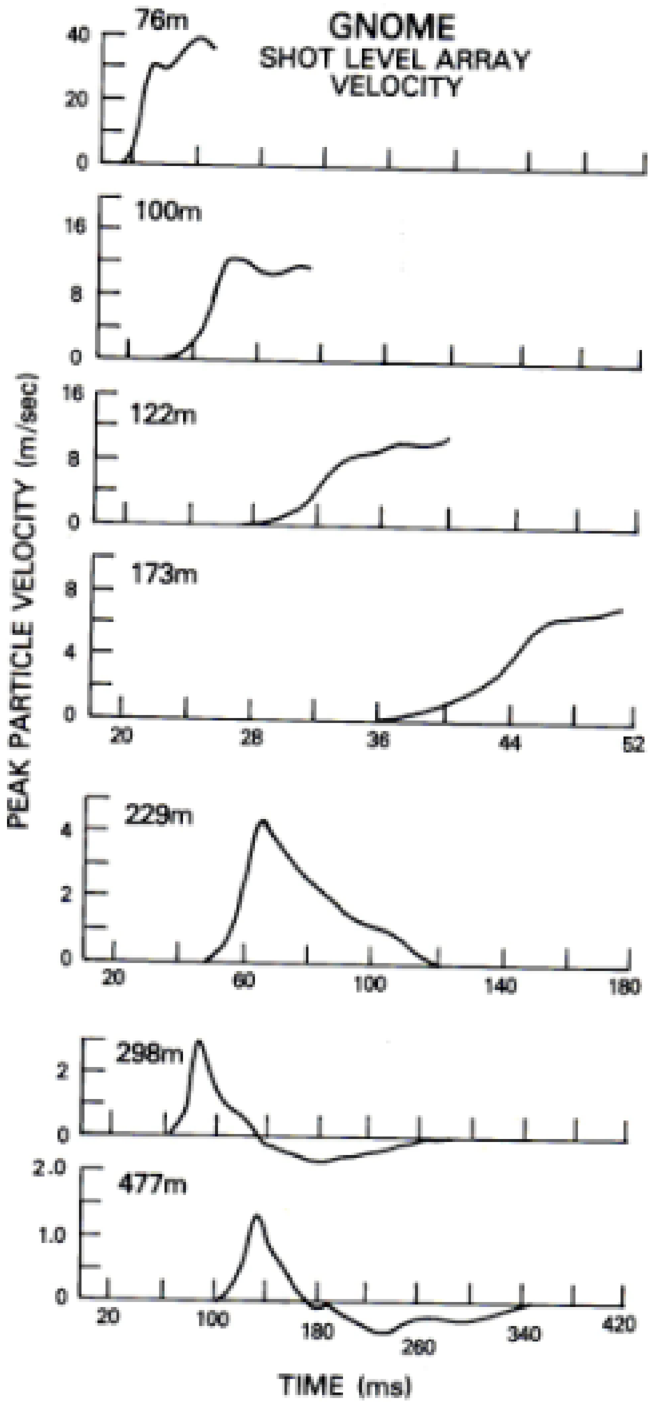
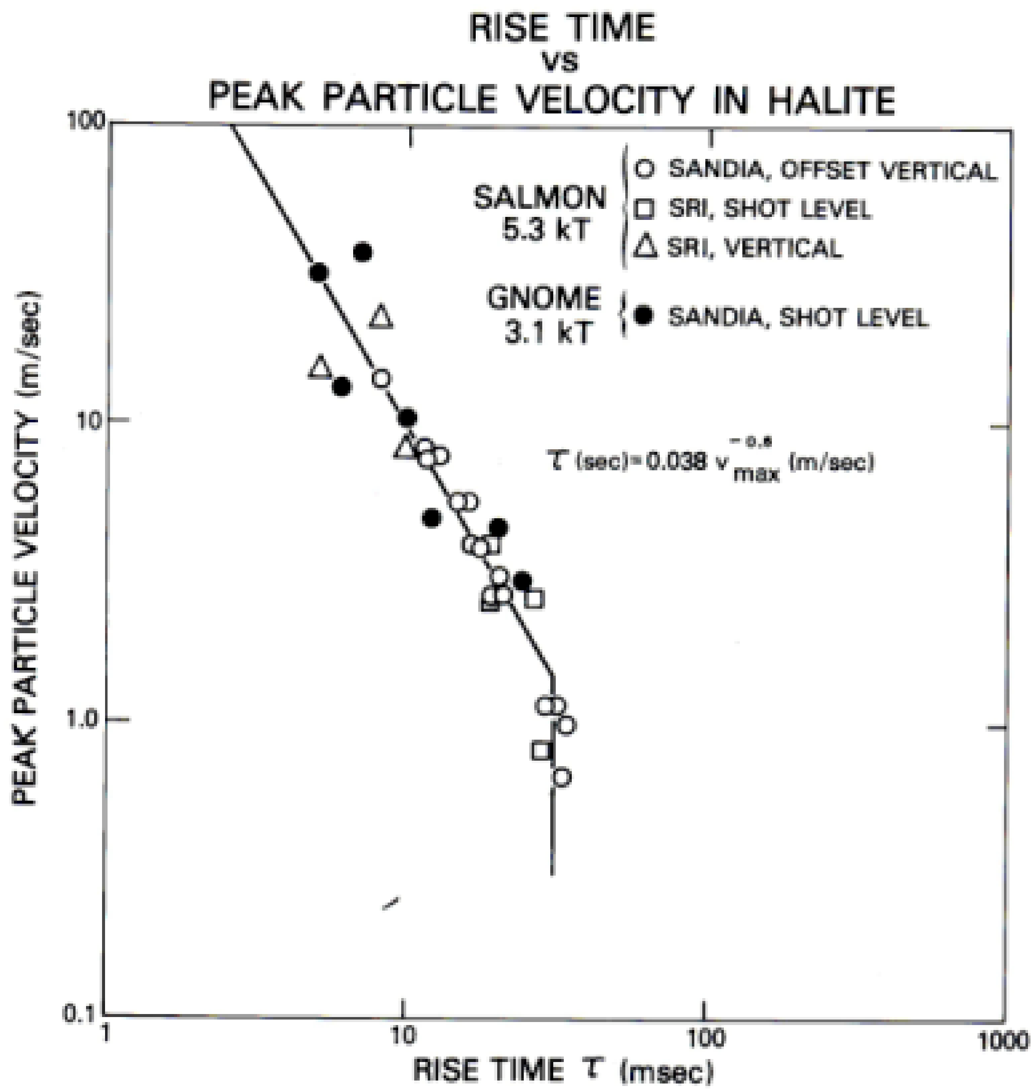


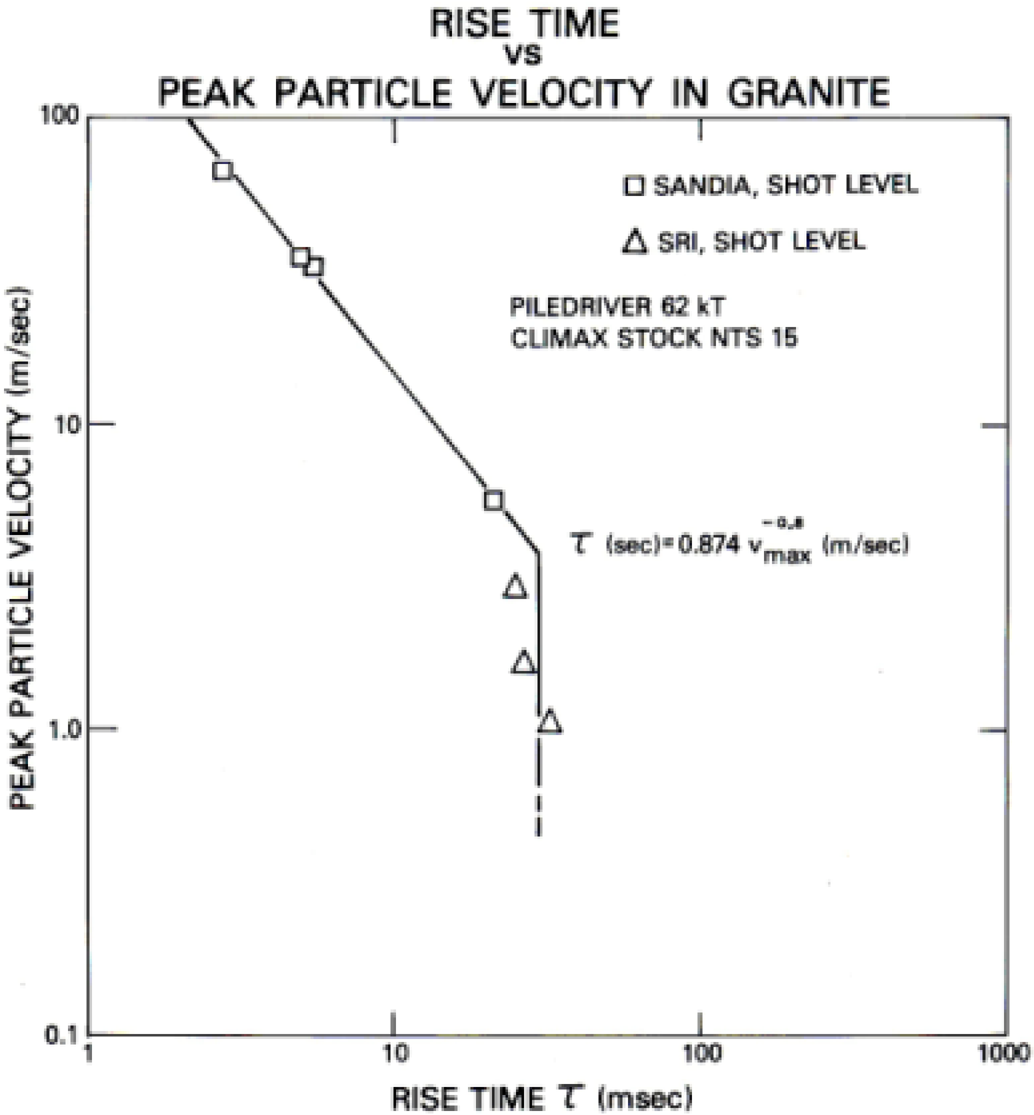
(1) Typical velocity gauge record from the 5.3 kT nuclear test SALMON. The record shows the arrival of a small elastic precursor, the main stress wave and a number of smaller oscillations after the main wave passes.



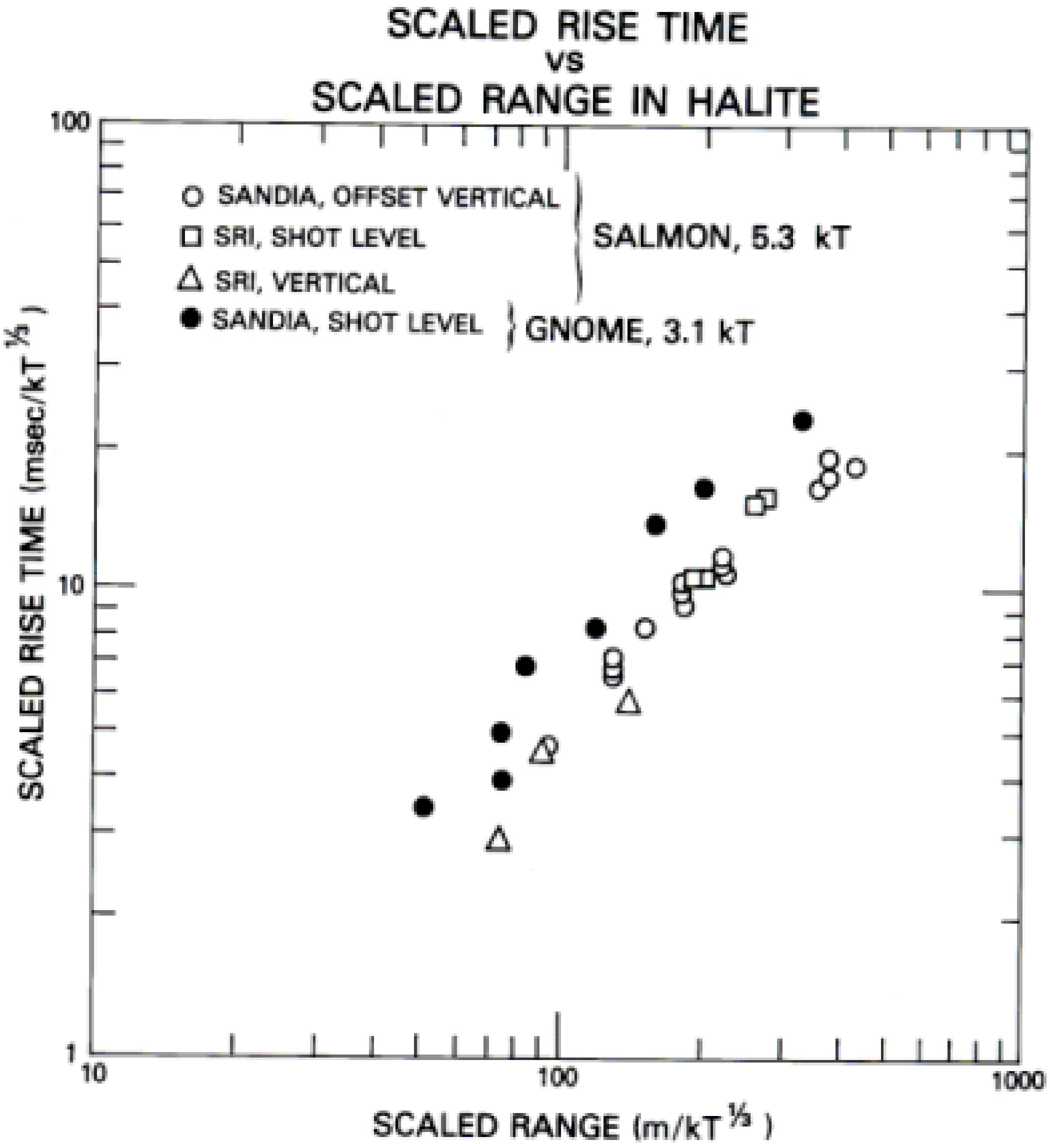
(2) Velocity gauge records from the 3.1 kT nuclear test GNOME. The increase of rise time with distance from the shot point is clearly shown in this series of records. Note that the time scale changes in the more distant records. The records close to the shot point are terminated by the destruction of the gauge.



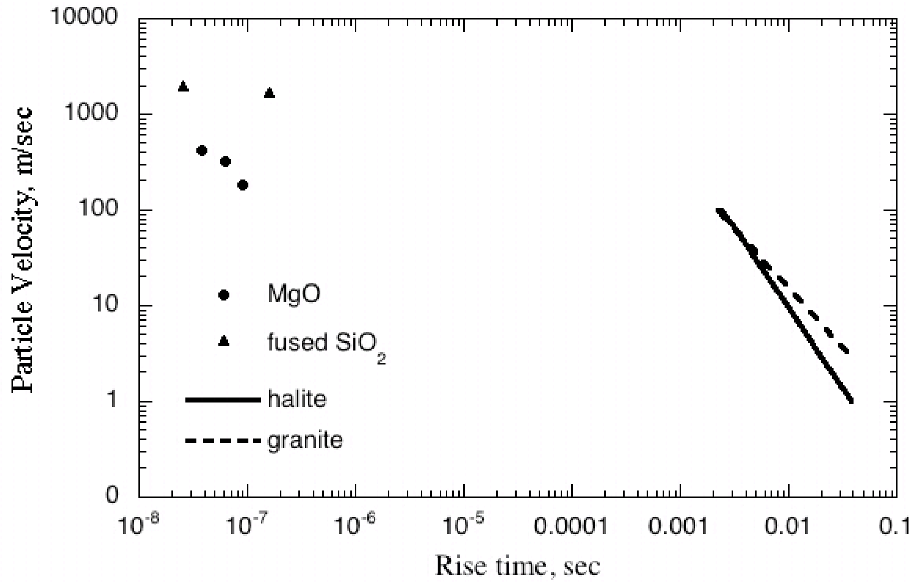
(3) Rise time vs. peak particle velocity for the SALMON and GNOME events in halite. The date sets overlap, showing that rise time is a direct function of peak particle velocity and supporting the idea that the rise time is a material property not directly related to device yield.



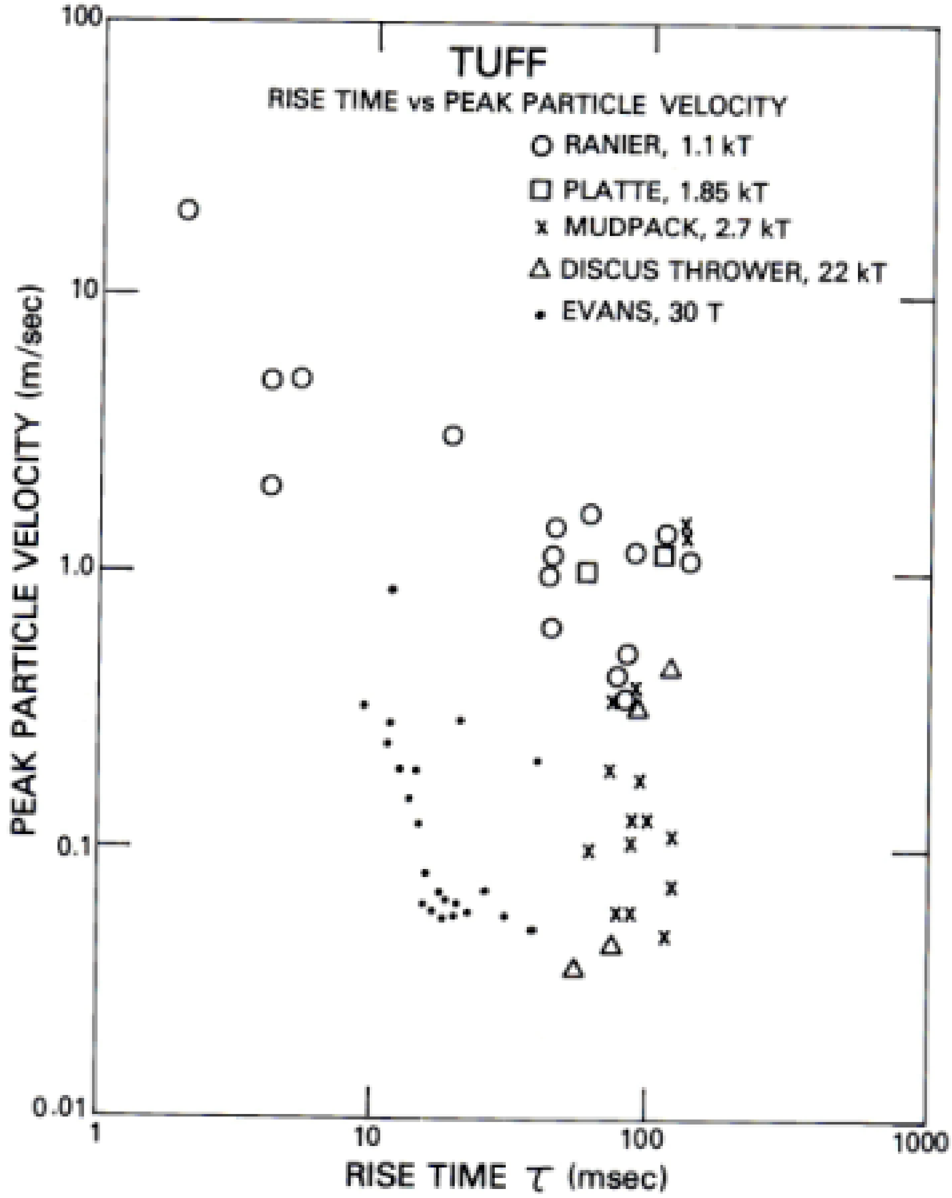
(4) Rise time vs. peak particle velocity in granite from the 56kT PILE DRIVER event[8]. The data form an array with structure similar to Figure 1, but with different parameters due to the different material.



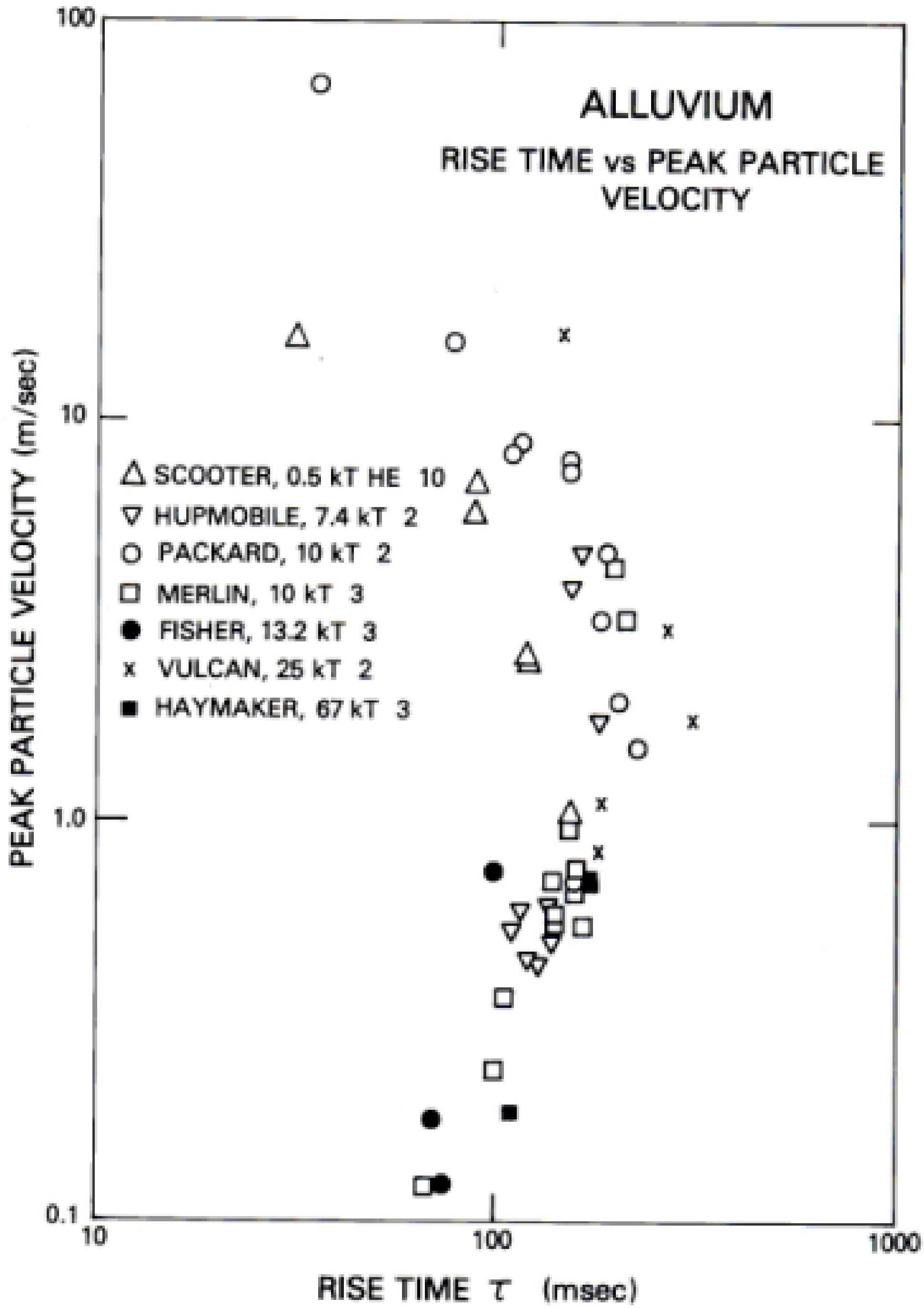
(5) Yield-scaled rise time vs. Yield-scaled range for the SALMON and GNOME explosions in halite. The data align in two distinct arrays, showing that yield scaling does not adequately describe the rise time behavior.



(6) Comparison of the fits (6a) and (6b) for rise time vs. particle velocity in nuclear explosions in halite and granite to laboratory data on the oxides MgO and fused SiO_2. The laboratory rise times were measured from Figs. 13 and 14 of reference 9.



(7) Rise time vs. peak particle velocity in dry tuff for a variety of yields. The data here are scattered but show the same general structure as Figures 3 and 4. The departure of the data for the low-yield EVANS test is probably due to the failure of the steady-wave approximation.



(8) Rise time vs. peak particle velocity in alluvium. The nearly unique curve for a variety of yields, ranging from 0.5kT to 46kT is evidence that rise time is a material property. The 0.5kT SCOOTER data have somewhat shorter rise times than the other data because the yield is too small for the steady wave approximation to be strictly valid.