# The mass distribution of planets

Clues from orbital dynamics

Renu Malhotra
The University of Arizona

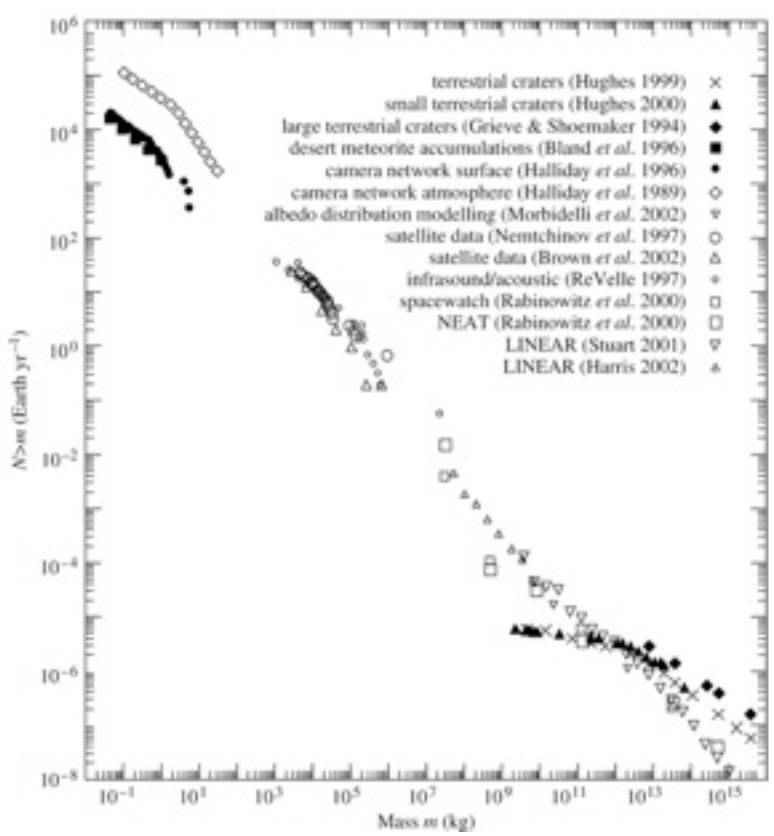
partially in Malhotra (2015), ApJ

See also: Tremaine 2015, Pu & Wu 2015

# Why should we care about the mass function?

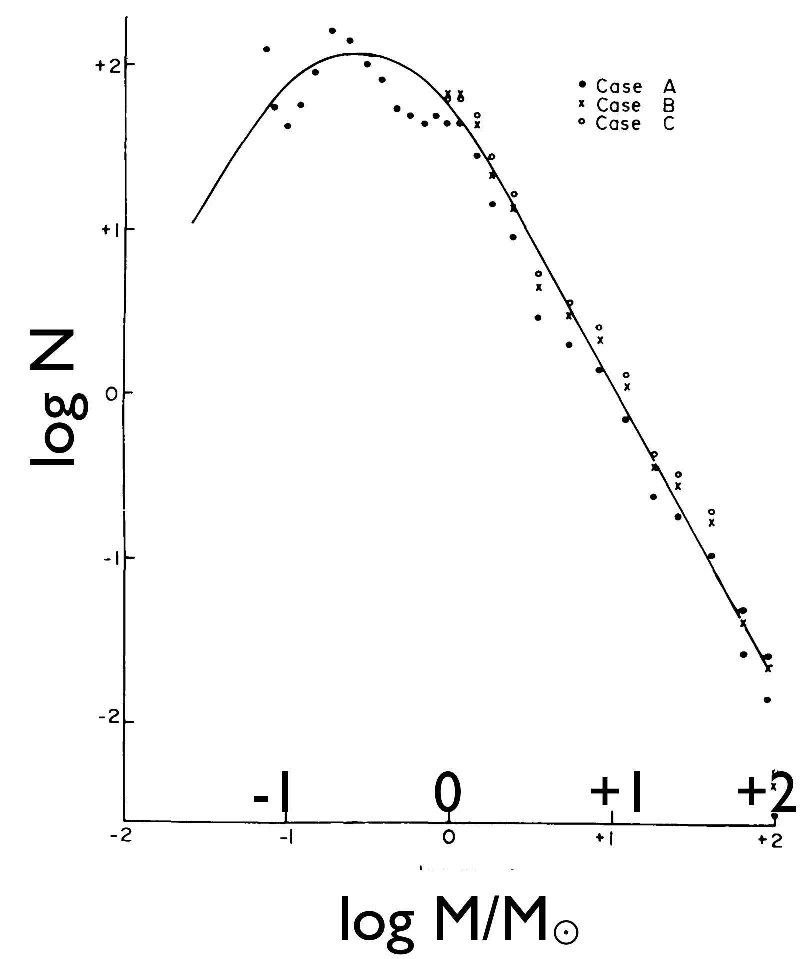
1. it would inform us about the abundance of planets like Earth
2. features in the mass function signal various physical processes

*Example: mass function of Earth impactors*



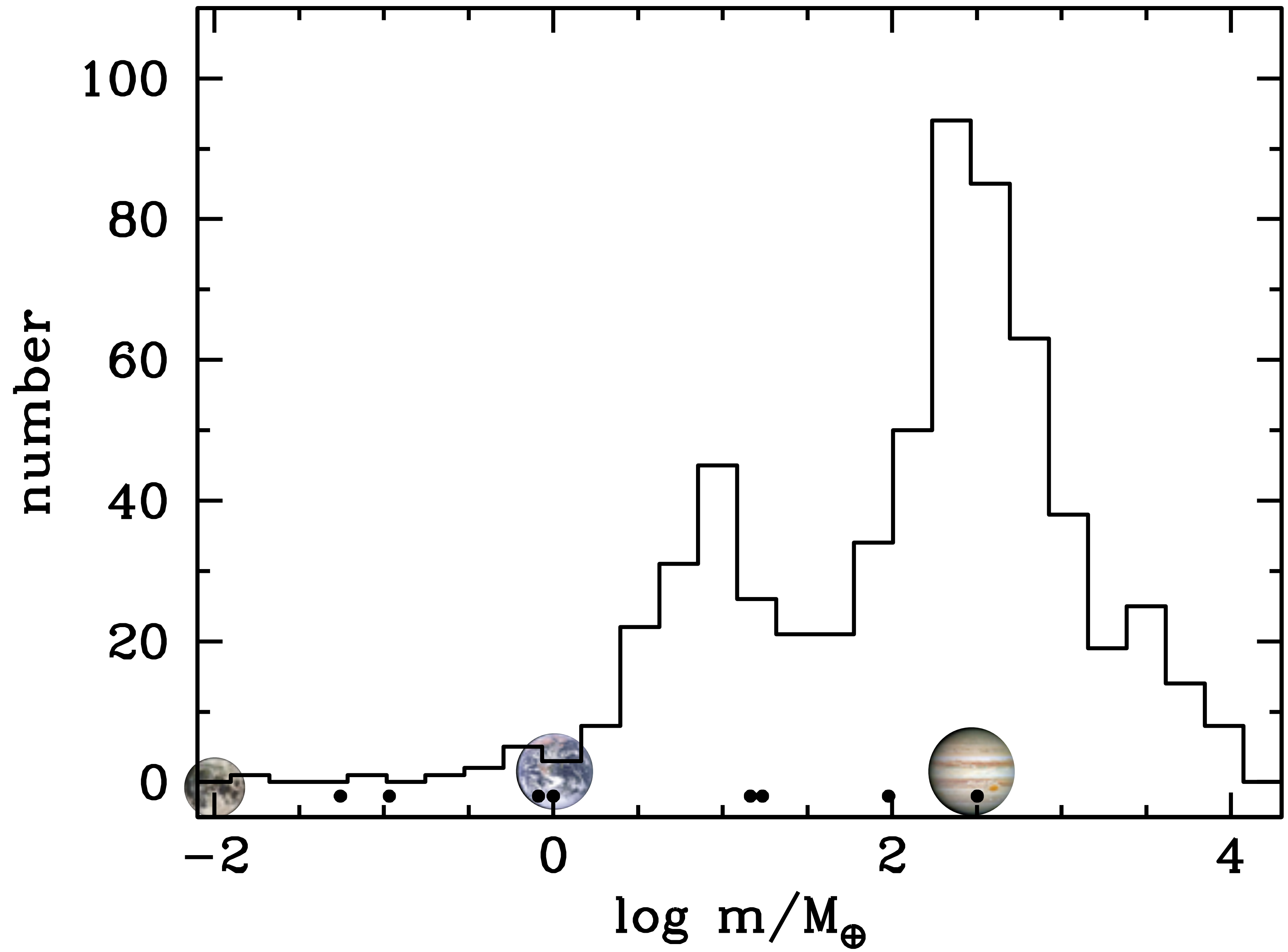

*Bland, 2005*

*Example: mass function of stars in the solar neighborhood*




*Rana, 1987*

# Observed exoplanet mass distribution




Confirmed exoplanets with **measured masses**
(total number: 617)
data from http://exoplanetarchive.ipac.caltech.edu/ (May 20, 2017)

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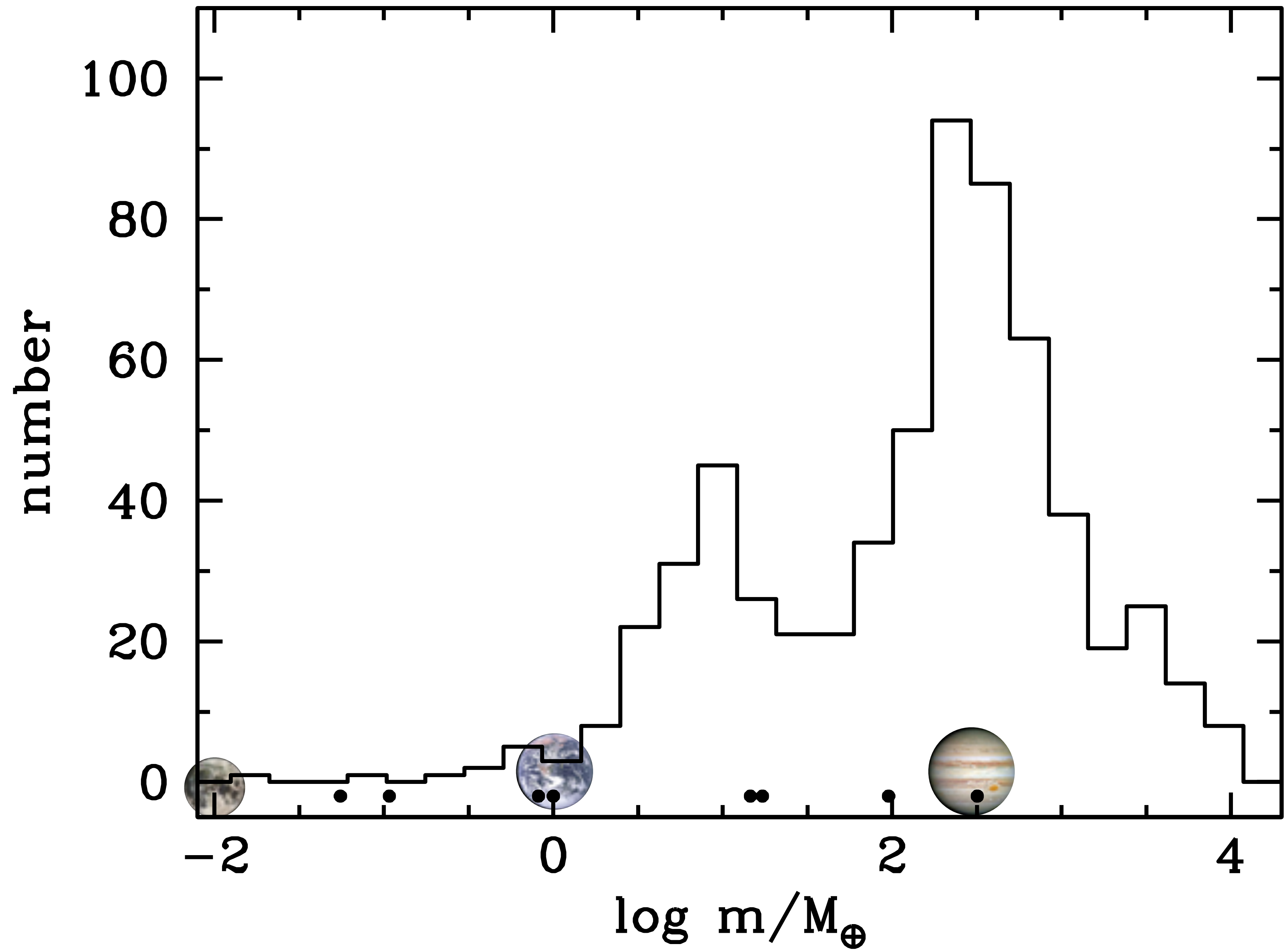


biased!

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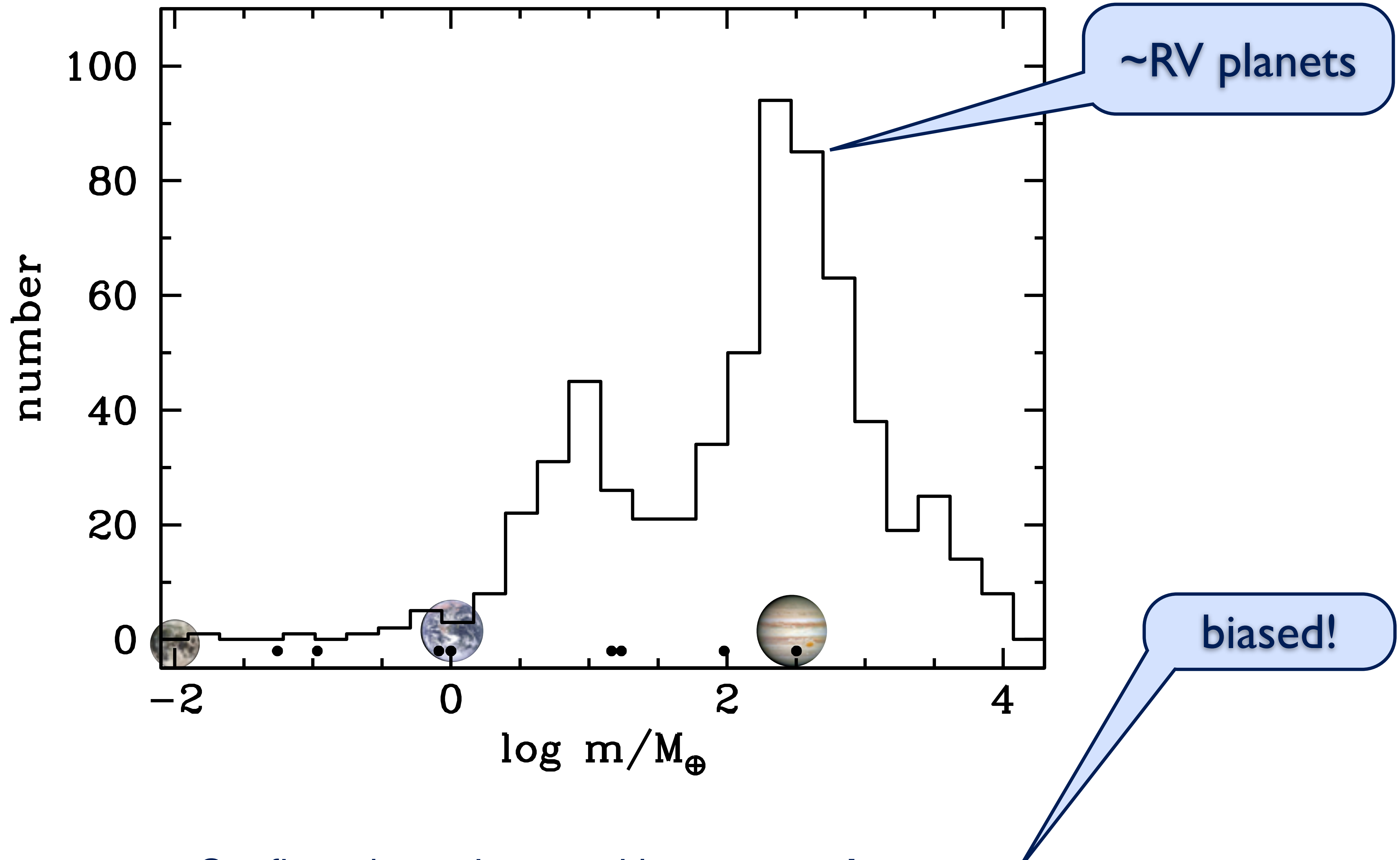


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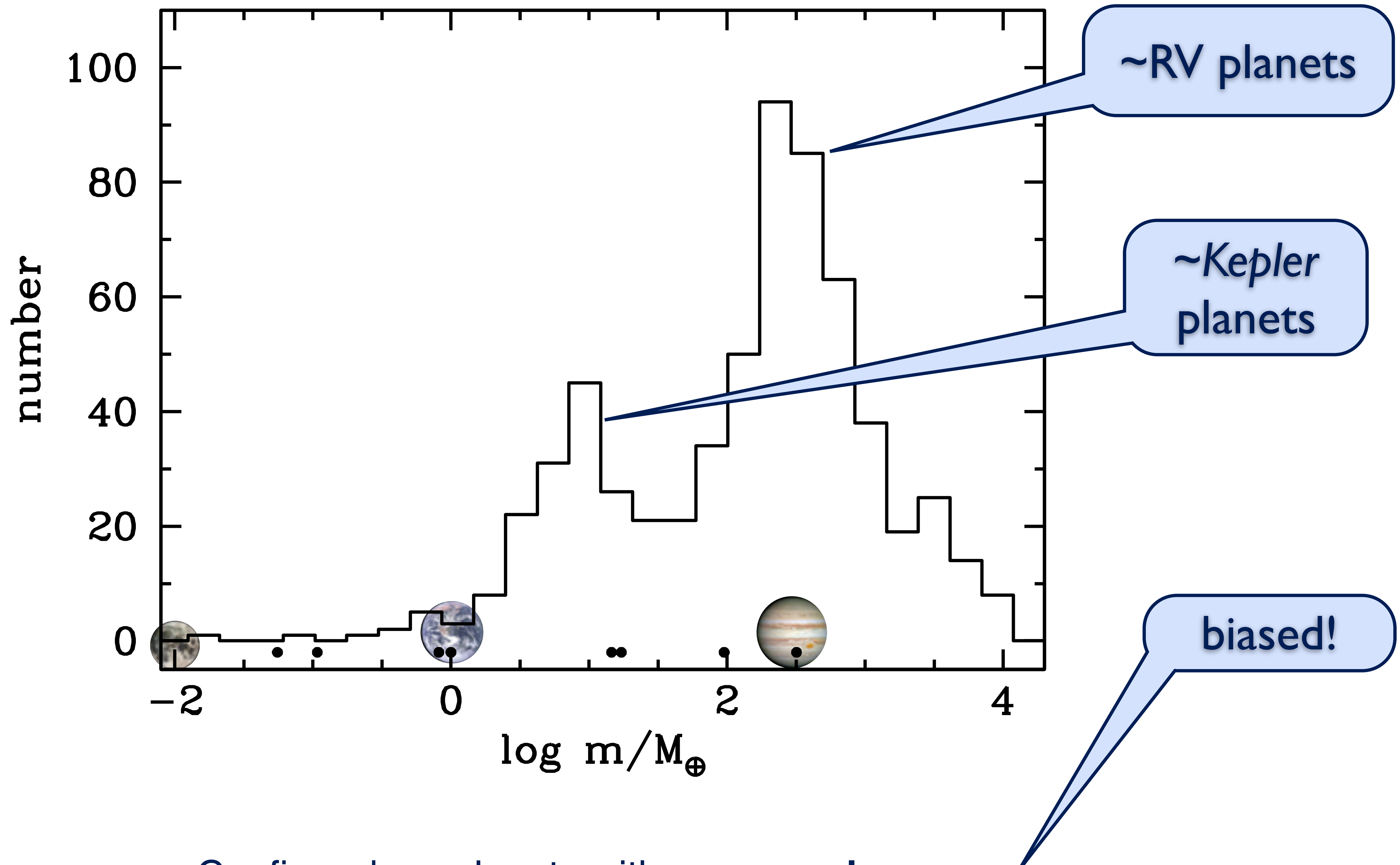


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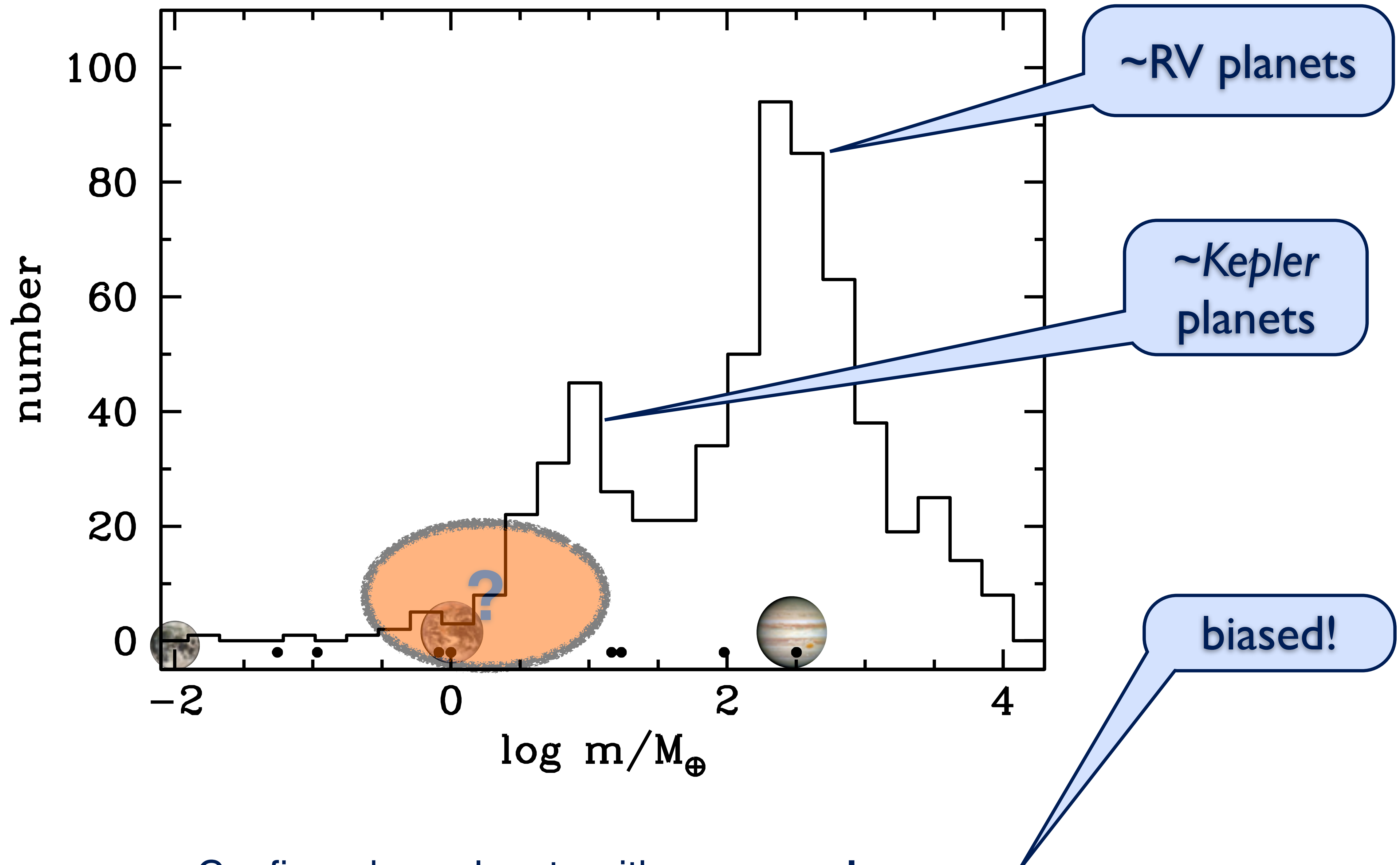


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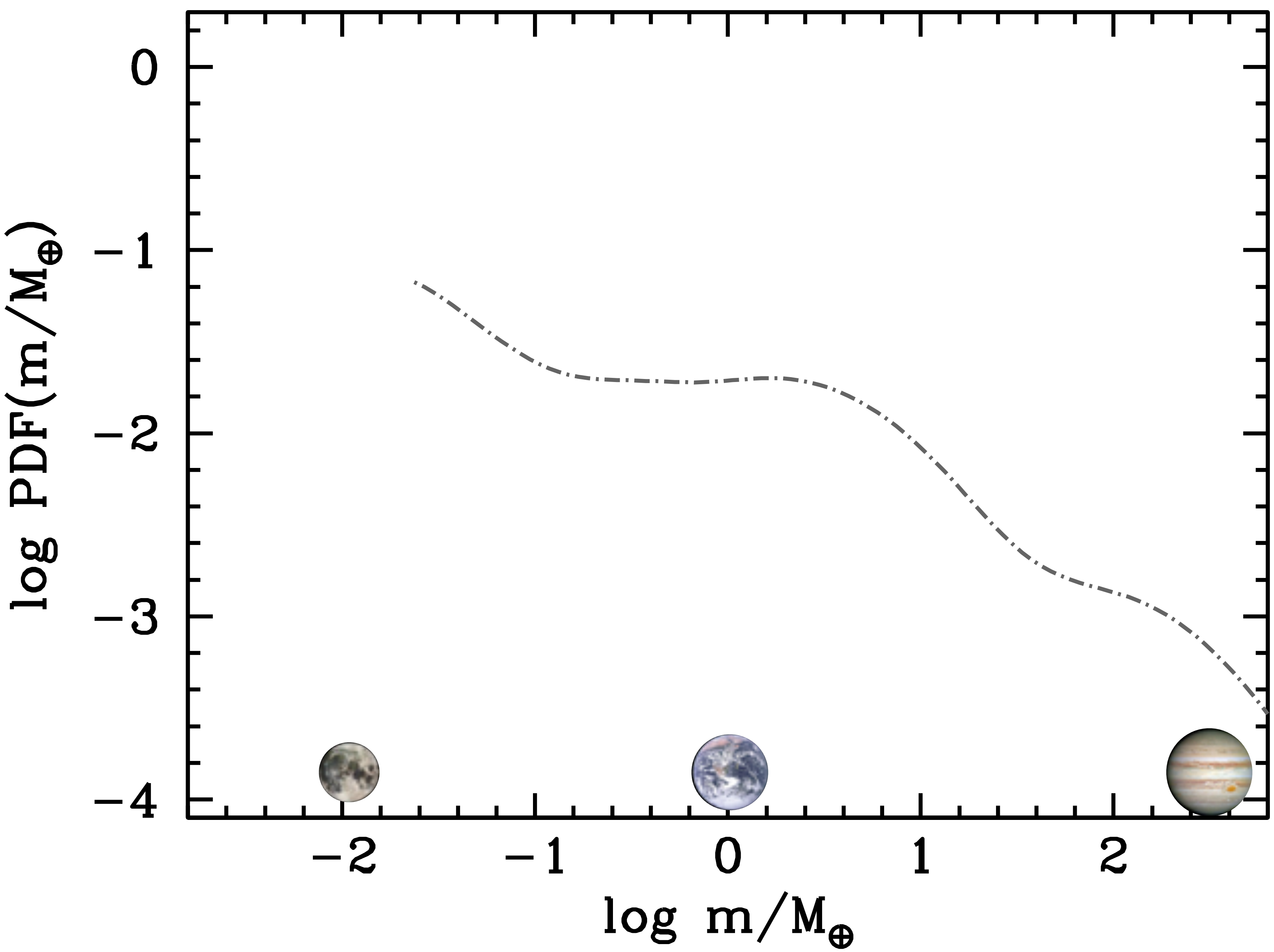


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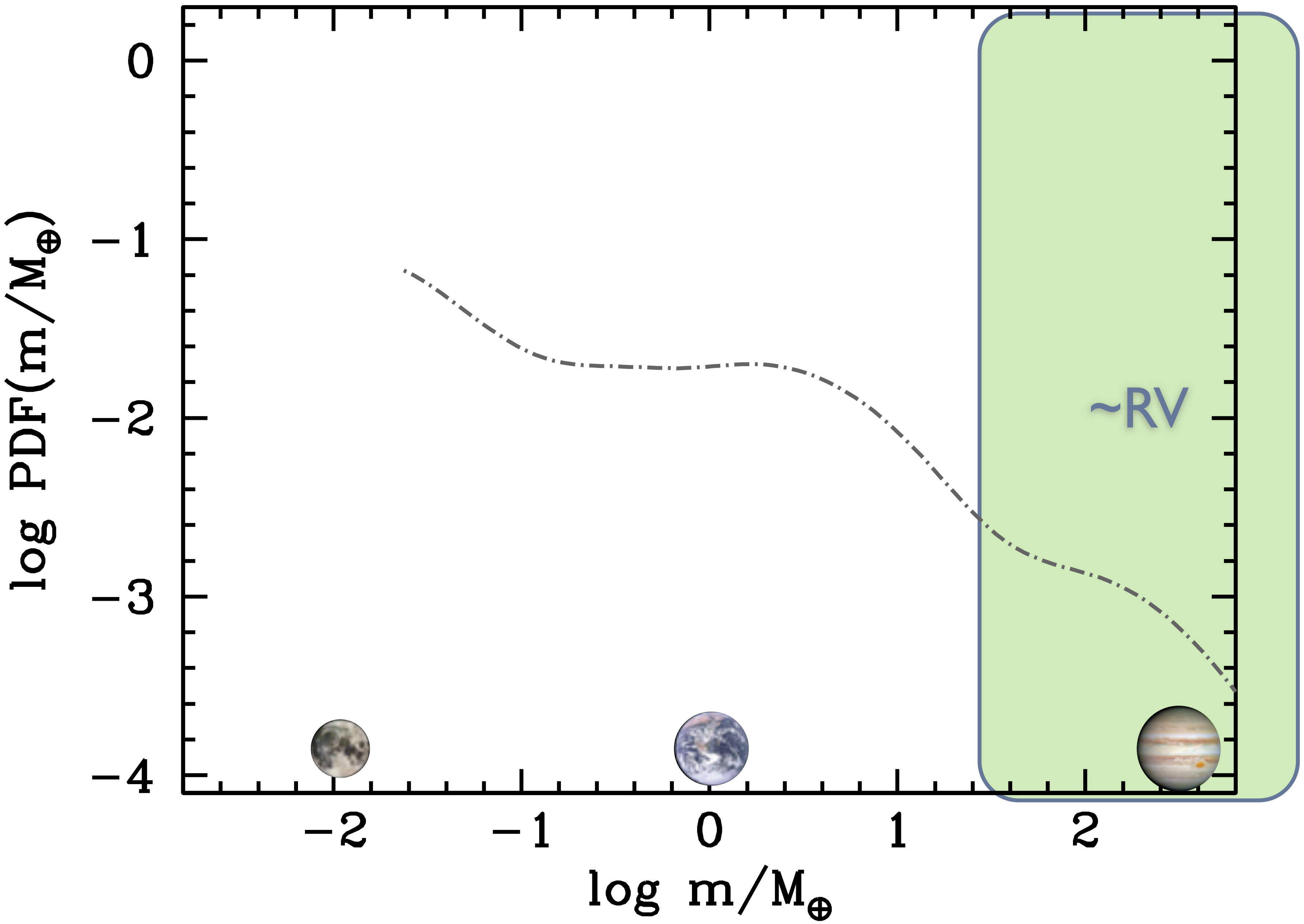


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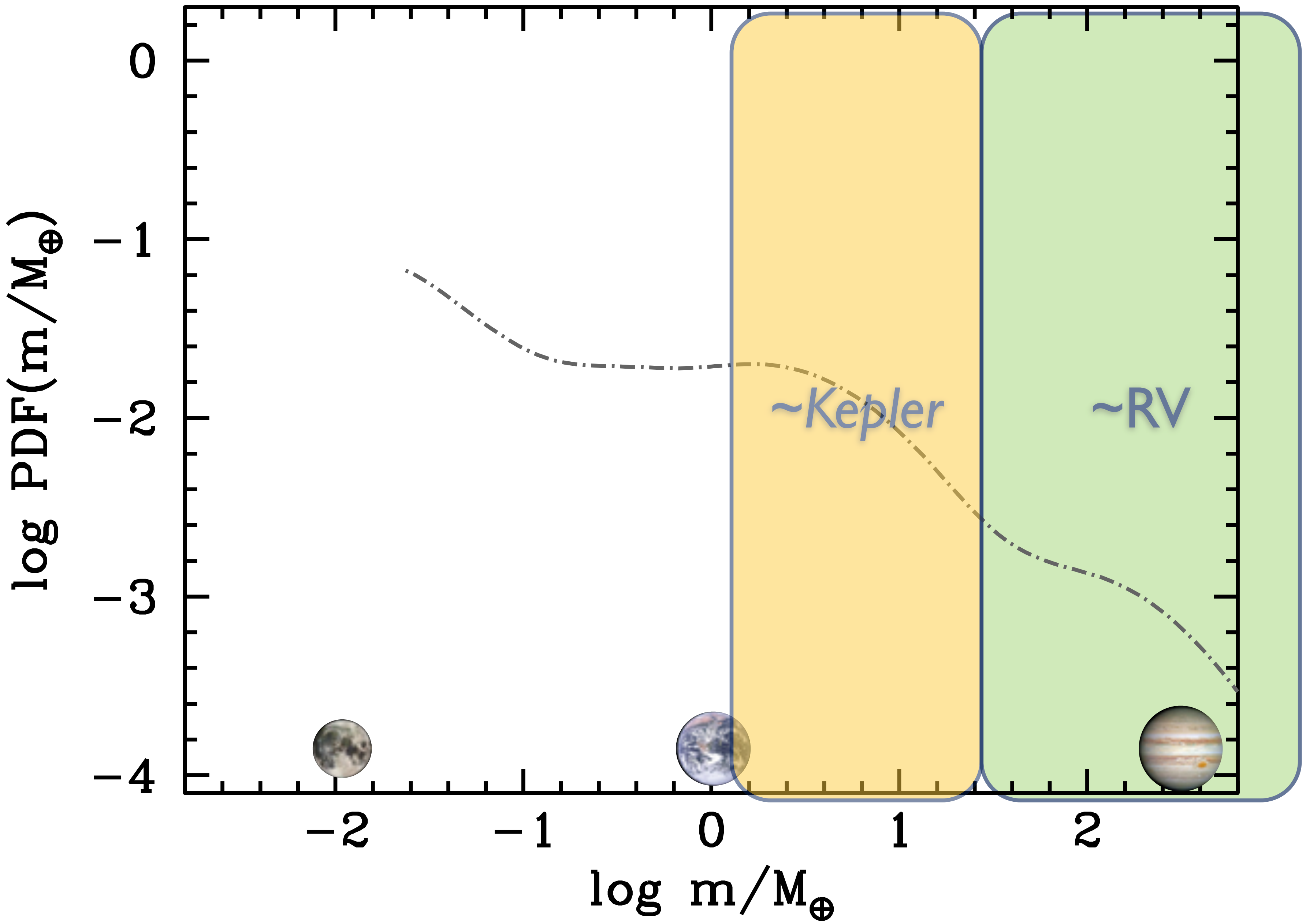


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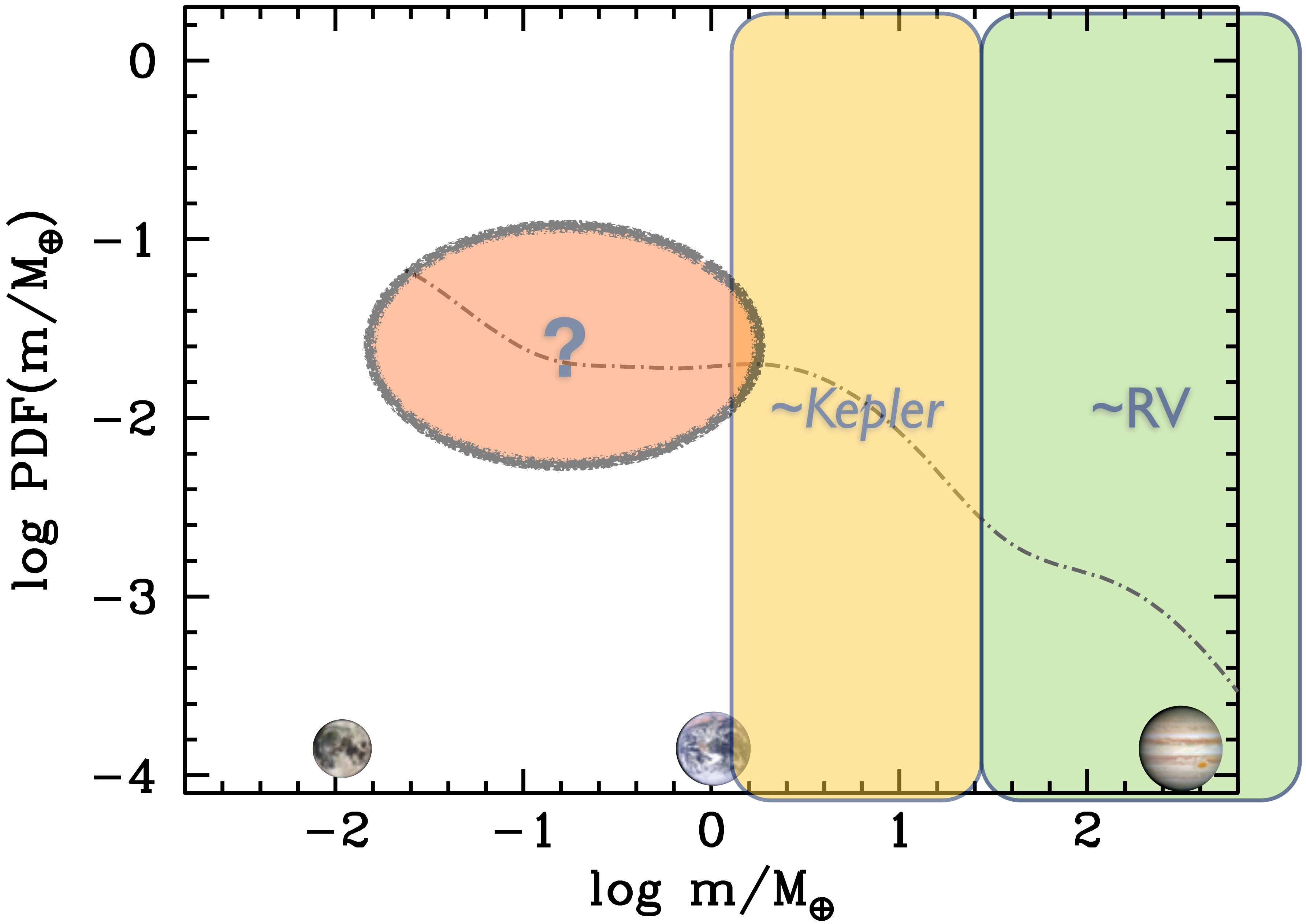


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# Planet masses from orbital periods?

- Orbital periods of nearly all exoplanets are well-determined
- Stellar host masses are fairly well determined

# Orbital spacing related to Period Ratio

Dimensionless orbital separation

$$D = \frac{a_{out} - a_{in}}{\frac{1}{2}(a_{out} + a_{in})}$$

by Kepler's 3rd Law

$P_j^2 \sim a_j^3$

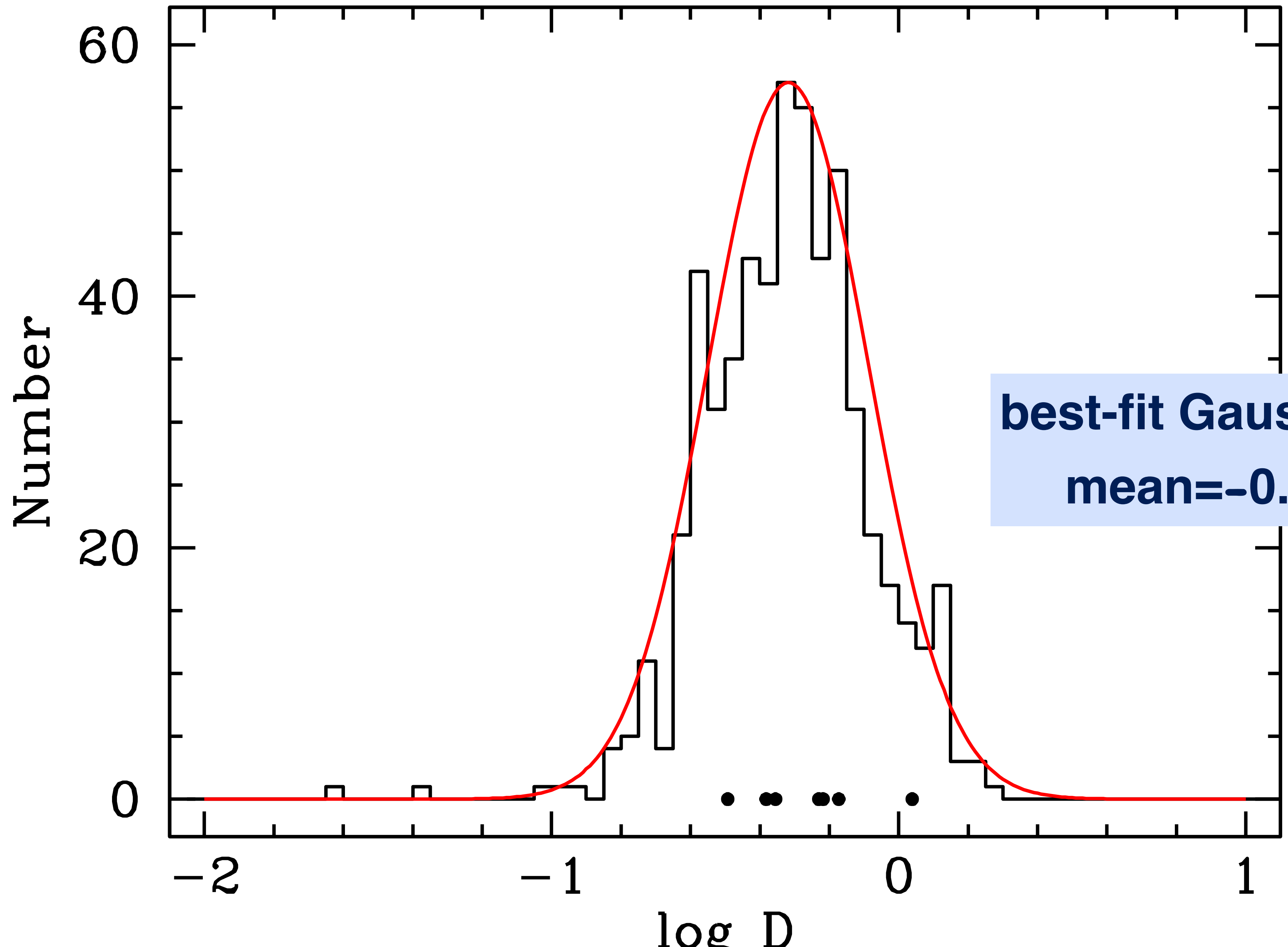
$$= 2(P^{2/3}-1)/(P^{2/3}+1)$$

$$P = P_{outer}/P_{inner}$$

# Distribution of dimensionless orbital separation

adjacent planets in *Kepler* multis

*D* is log-normal!




**best-fit Gaussian distribution:**
**mean=-0.318, s.d.=0.231**

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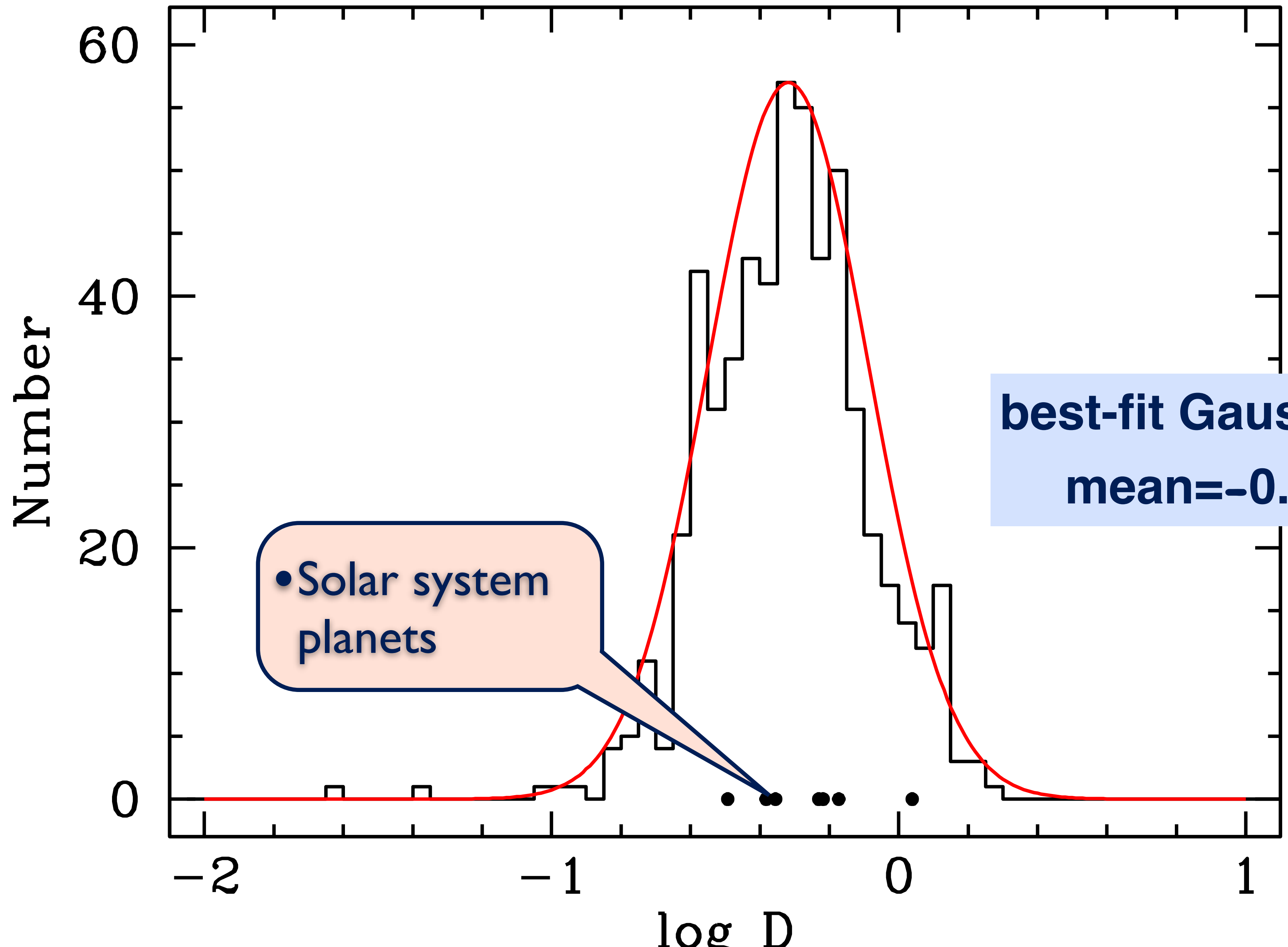


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# Dynamical Stability

**more massive planets tend to need larger spacings**

# Dynamical Stability

## Two planets

minimum orbital separation is ~3.46 times mutual Hill radius

*G.W. Hill, 1878*
*Gladman, 1993*

$$\mathcal{D} = 2\sqrt{3}\left(\frac{m_1 + m_2}{3m_*}\right)^{\frac{1}{3}}$$

# Dynamical Stability

**N>2 planets**

no analytical criterion

**empirical: generalize Hill's criterion**

$$\mathcal{D} = K \left(\frac{m_1 + m_2}{3m_*}\right)^{\frac{1}{3}}$$

$$\log\left(\frac{m_1 + m_2}{m_*}\right) = 3(\log \mathcal{D} - \log K) + \log 3$$

***K* > 3.46**... but by how much?

likely depends upon planet multiplicity (N),
eccentricities ('angular momentum deficit', AMD),
planet mass ratios ($m_1/m_2$),
age of the system ('dynamical age', $t/T_1$)

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eccentricities ('angular mome
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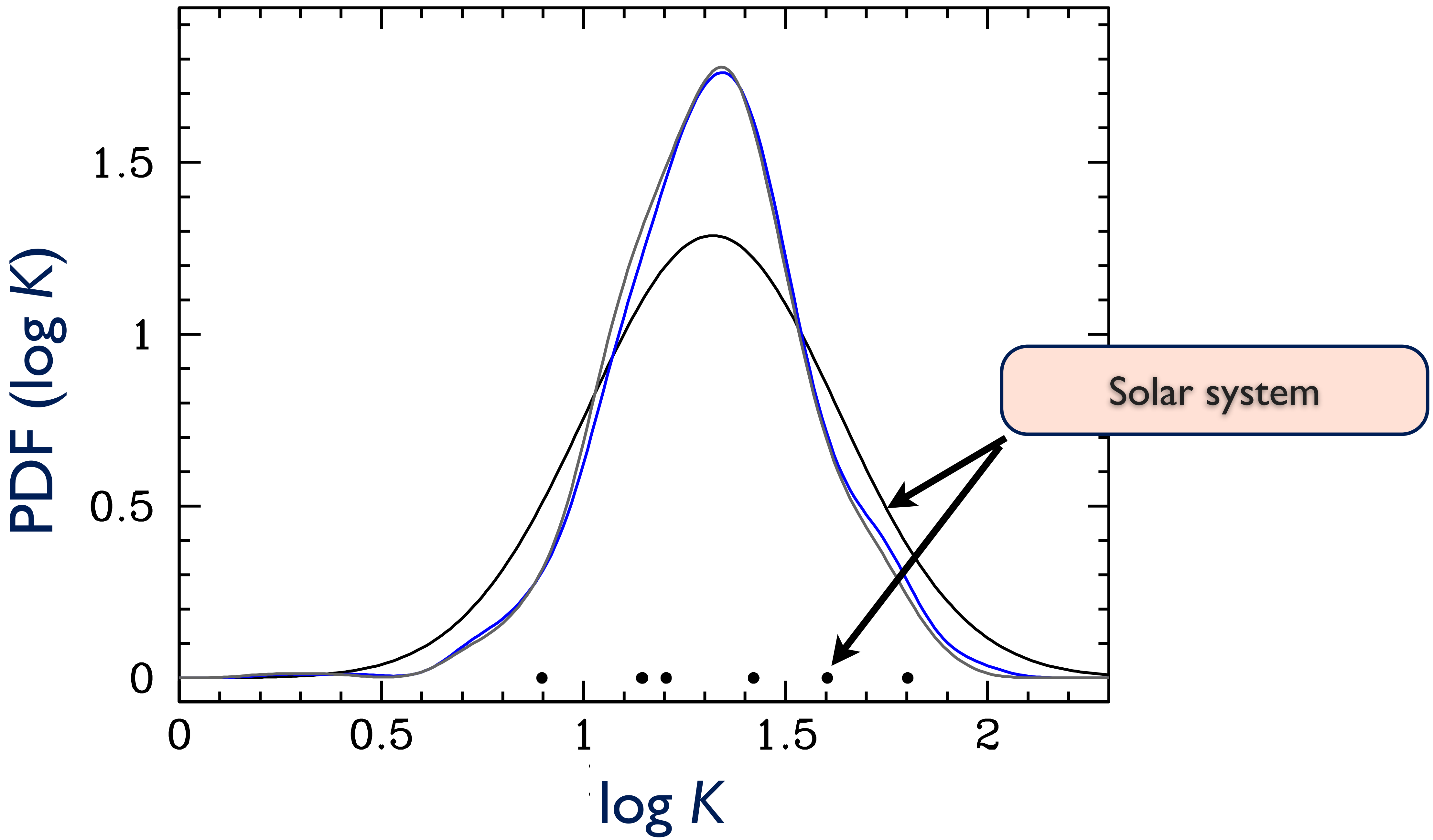
Ansatz: log *K* is Gaussian
mean = 1.32, s.d. = 0.31
(solar system mean & s.d.)

# Distribution of K

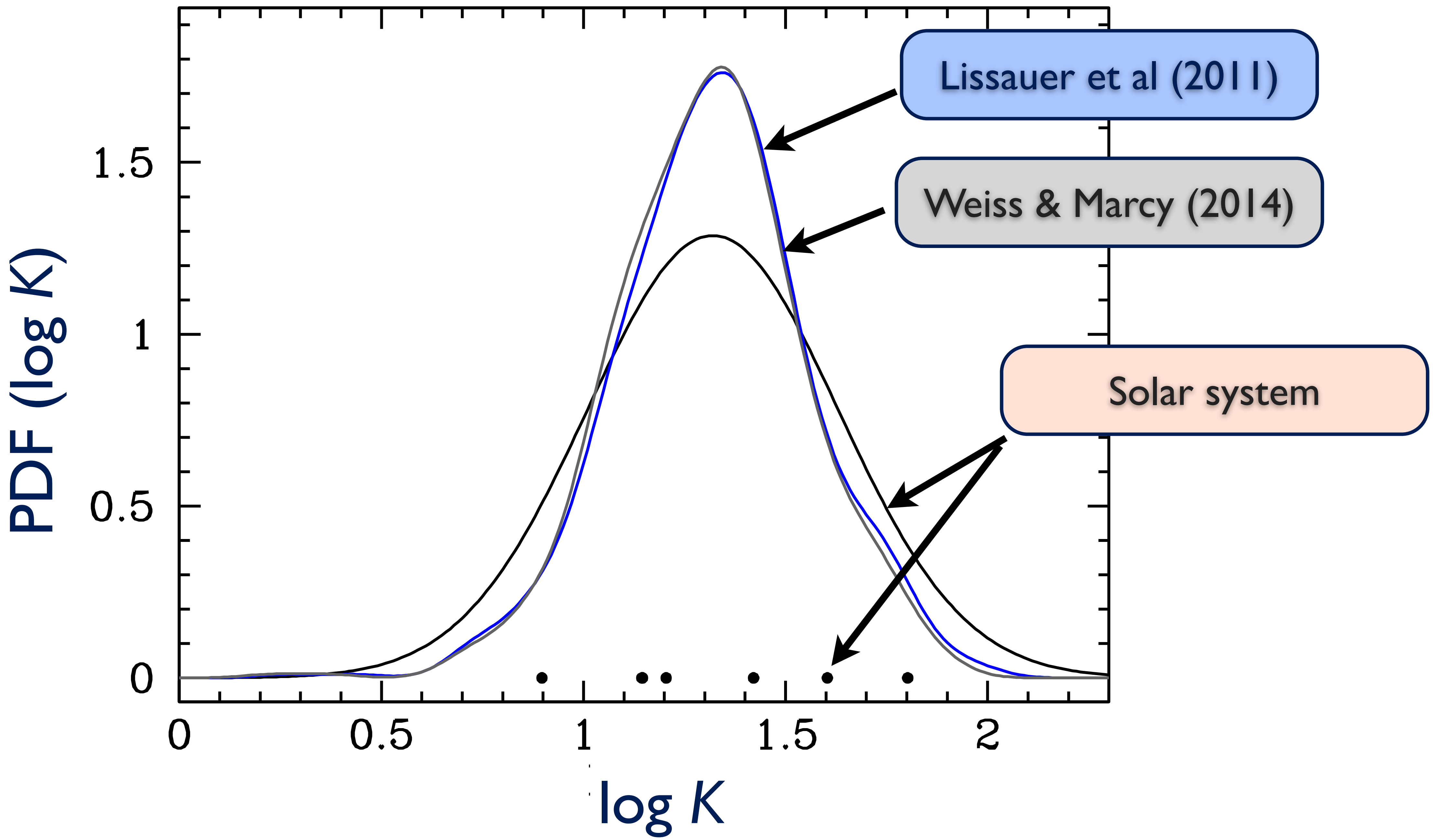
*Look to Solar System*



# Distribution of K

*Look to Solar System*

& *Kepler* multis with use of mass-radius relationship(s)



# planet pair masses from orbital separations

Hill's criterion for two planets
...generalized:

$$\mathcal{D} = K\left(\frac{m_1 + m_2}{3m_*}\right)^{\frac{1}{3}}$$

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Gaussian
(from observations)

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Gaussian (from observations)

Gaussian (by ansatz) mean = 1.32, s.d. = 0.31 (solar system mean & s.d.)

Gaussian mean=4.44, s.d.=1.16

# Individual planet masses

PDF of $(m_1+m_2)/M_*$

Stellar masses $M_*$ are fairly well determined (*Kepler*)

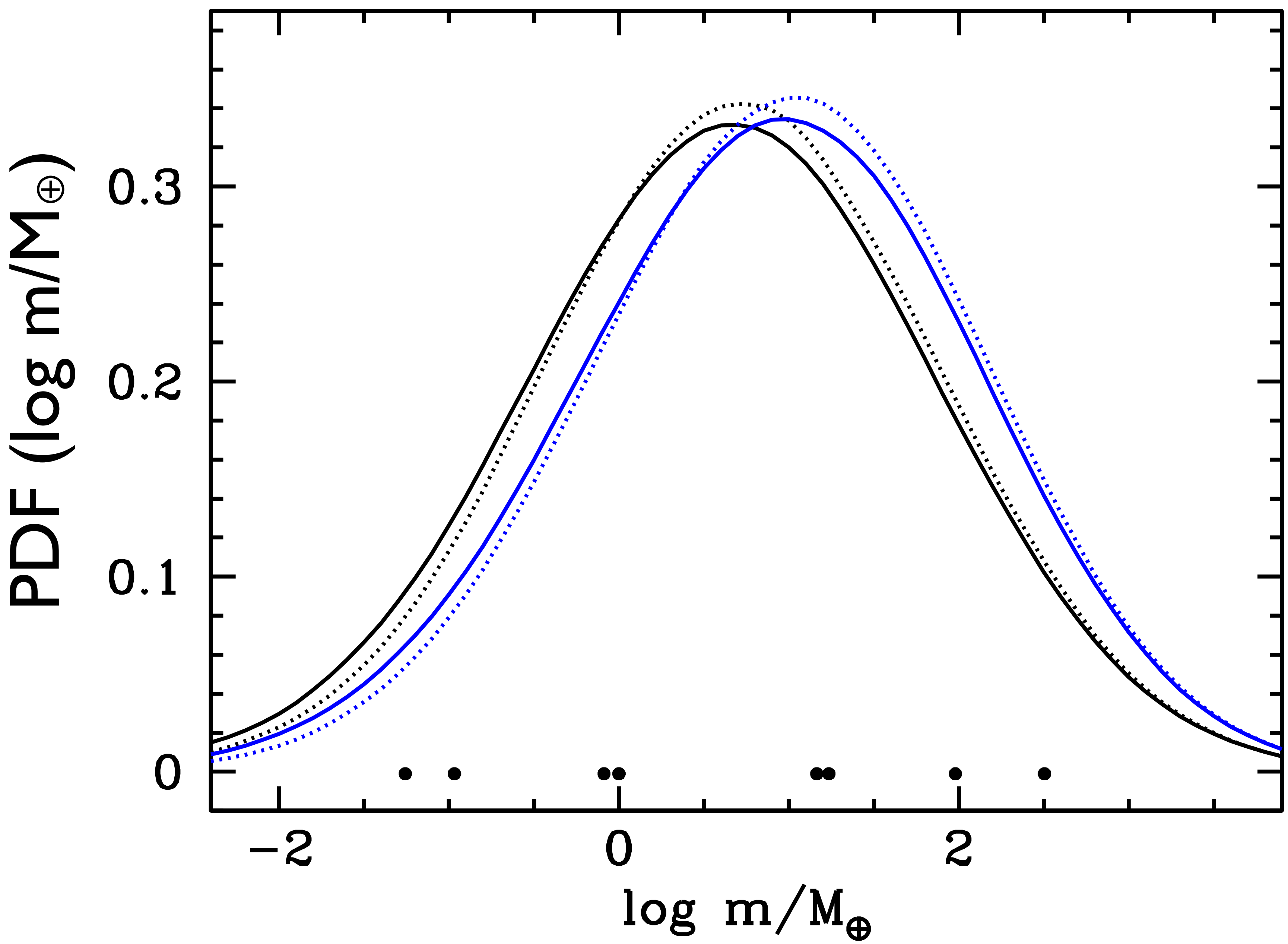
Assume ***$\min(m_1,m_2)/\max(m_1,m_2)$*** is random on (0,1)

or half-Gaussian on (0,1) if neighbor planets tend to be of similar mass

Distribution of individual planet masses

# Planet mass distribution: theoretical estimate

PDF of log(planet mass/earth-mass)



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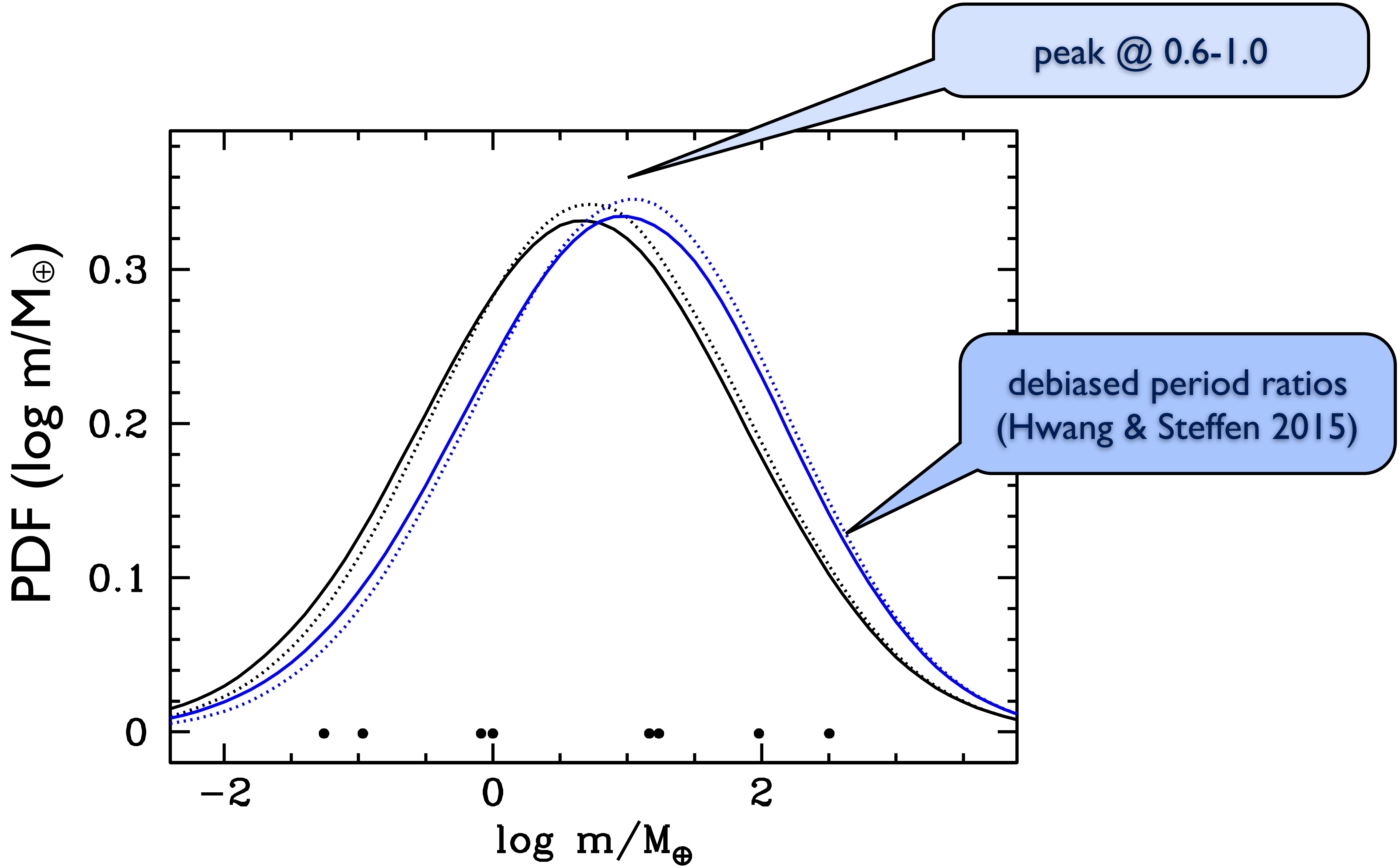
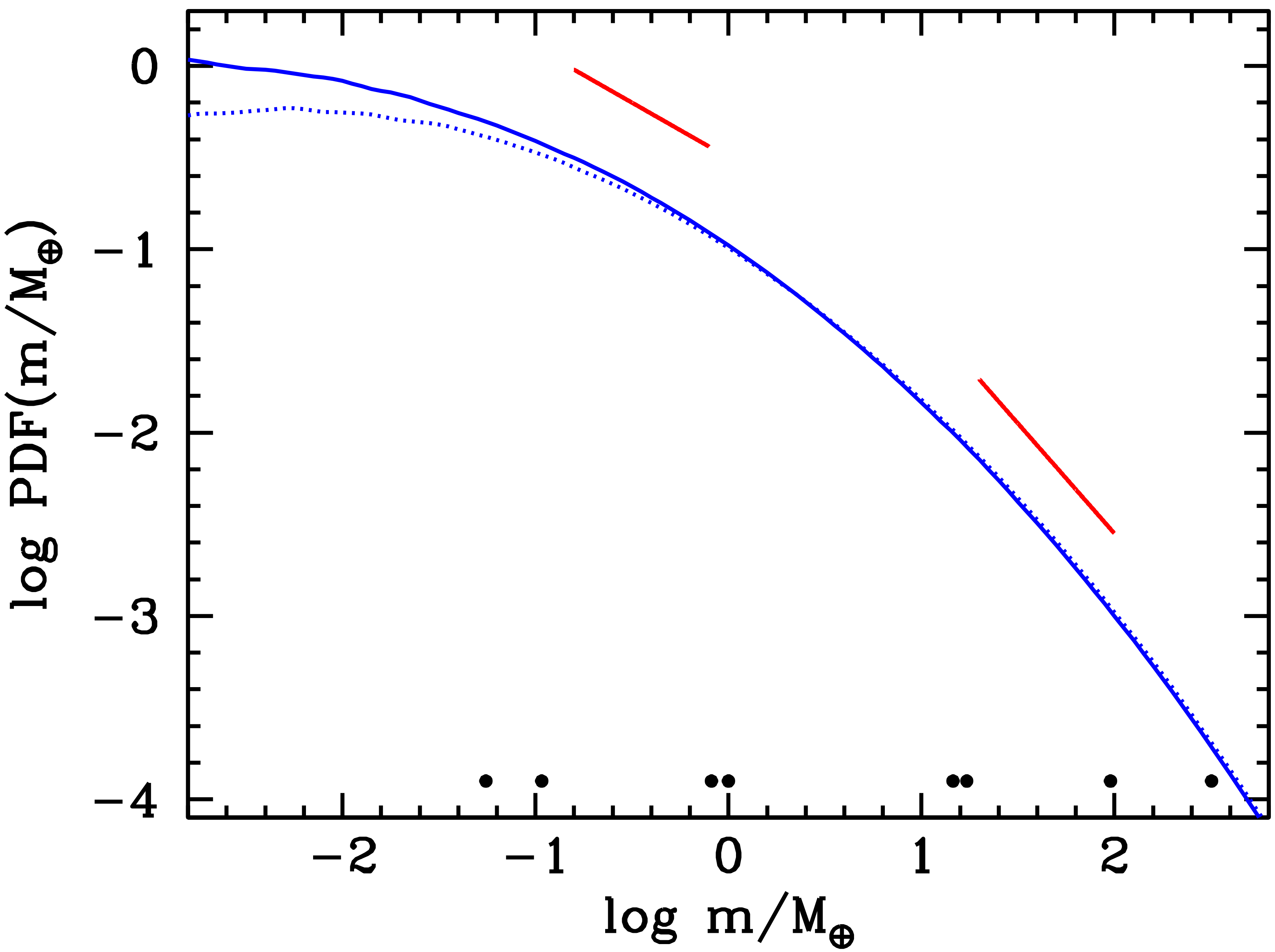


# Distribution of planet masses: theoretical estimate

log-log plot of PDF of (planet mass/earth-mass)



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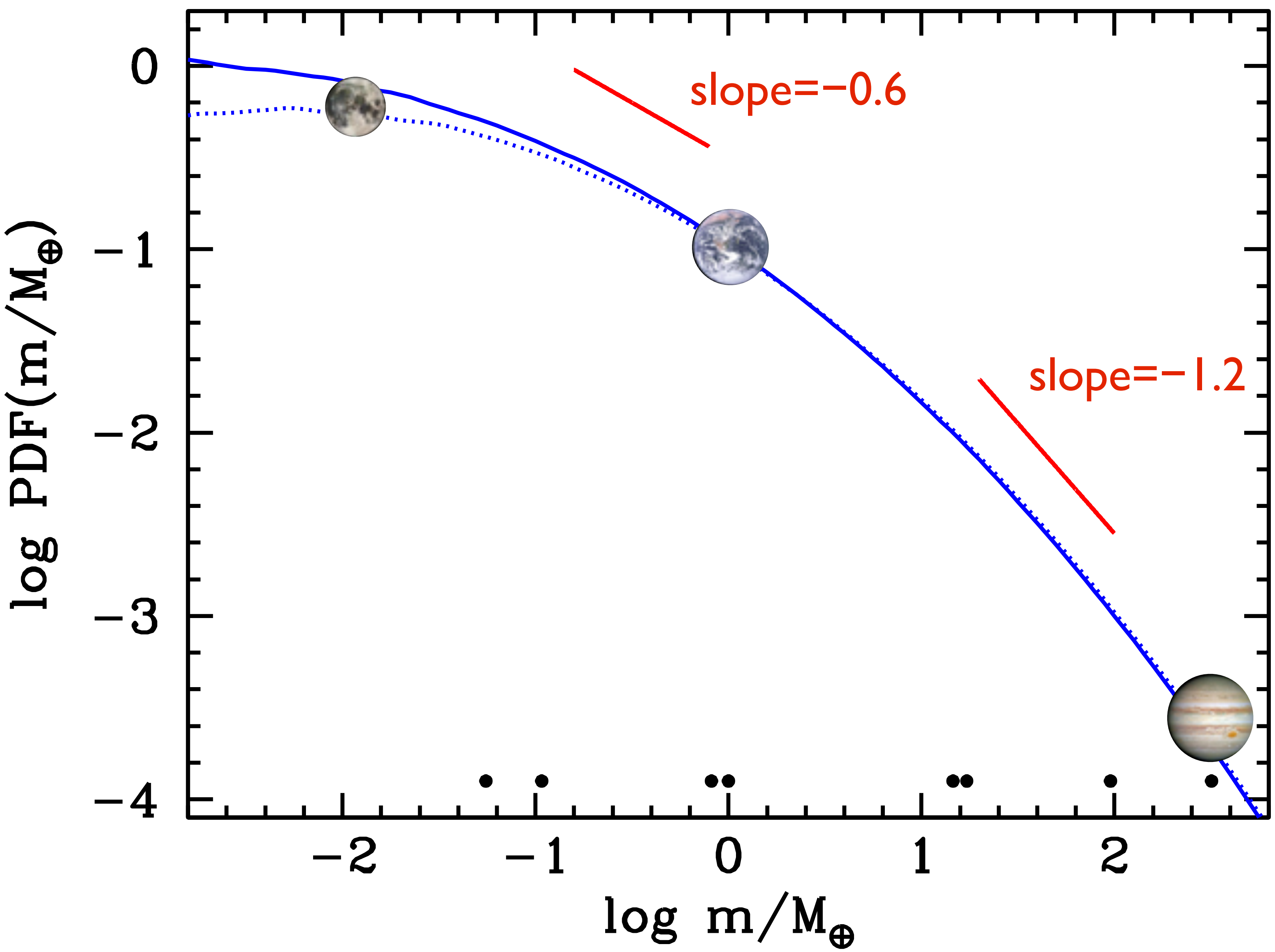

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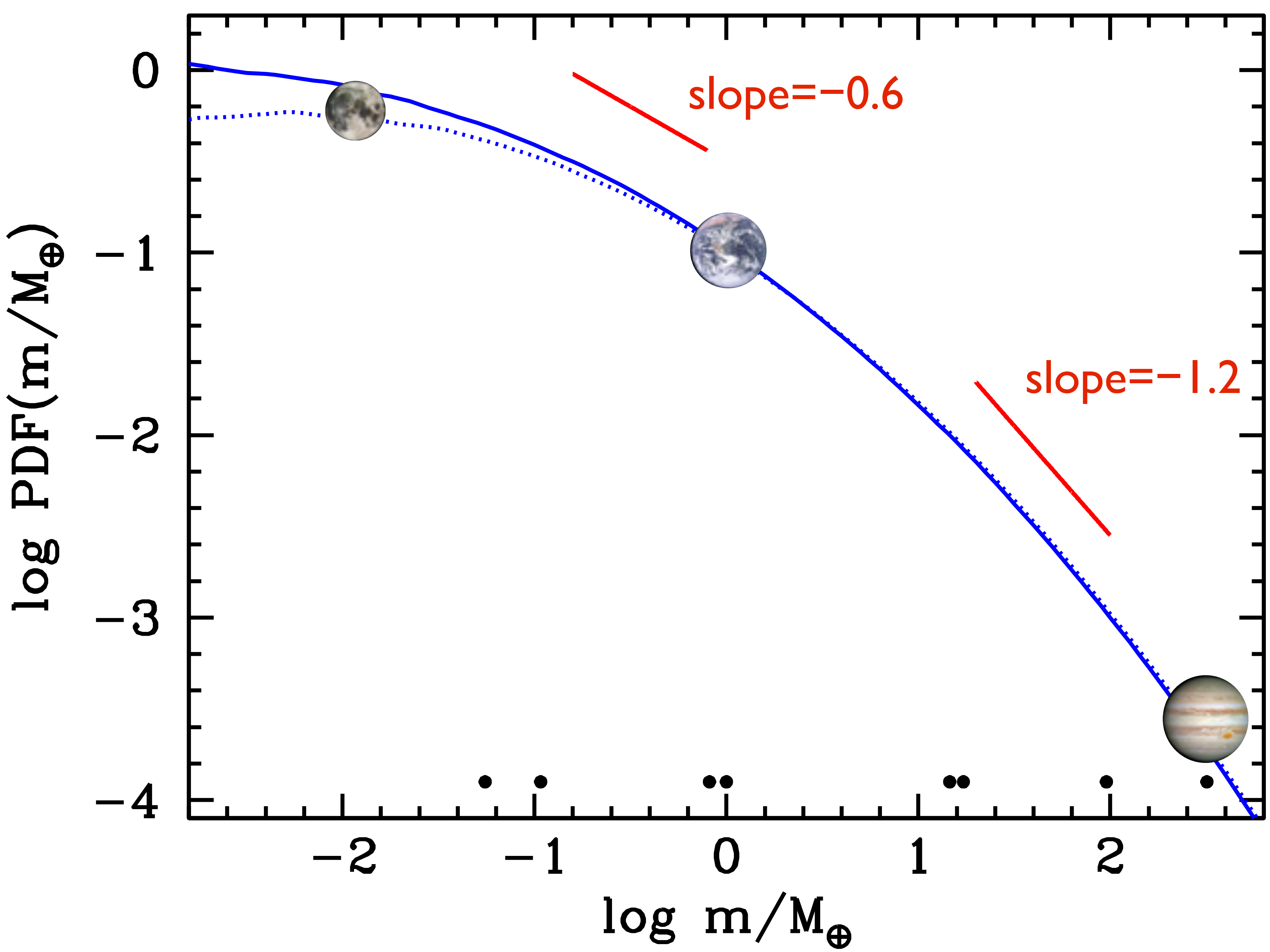
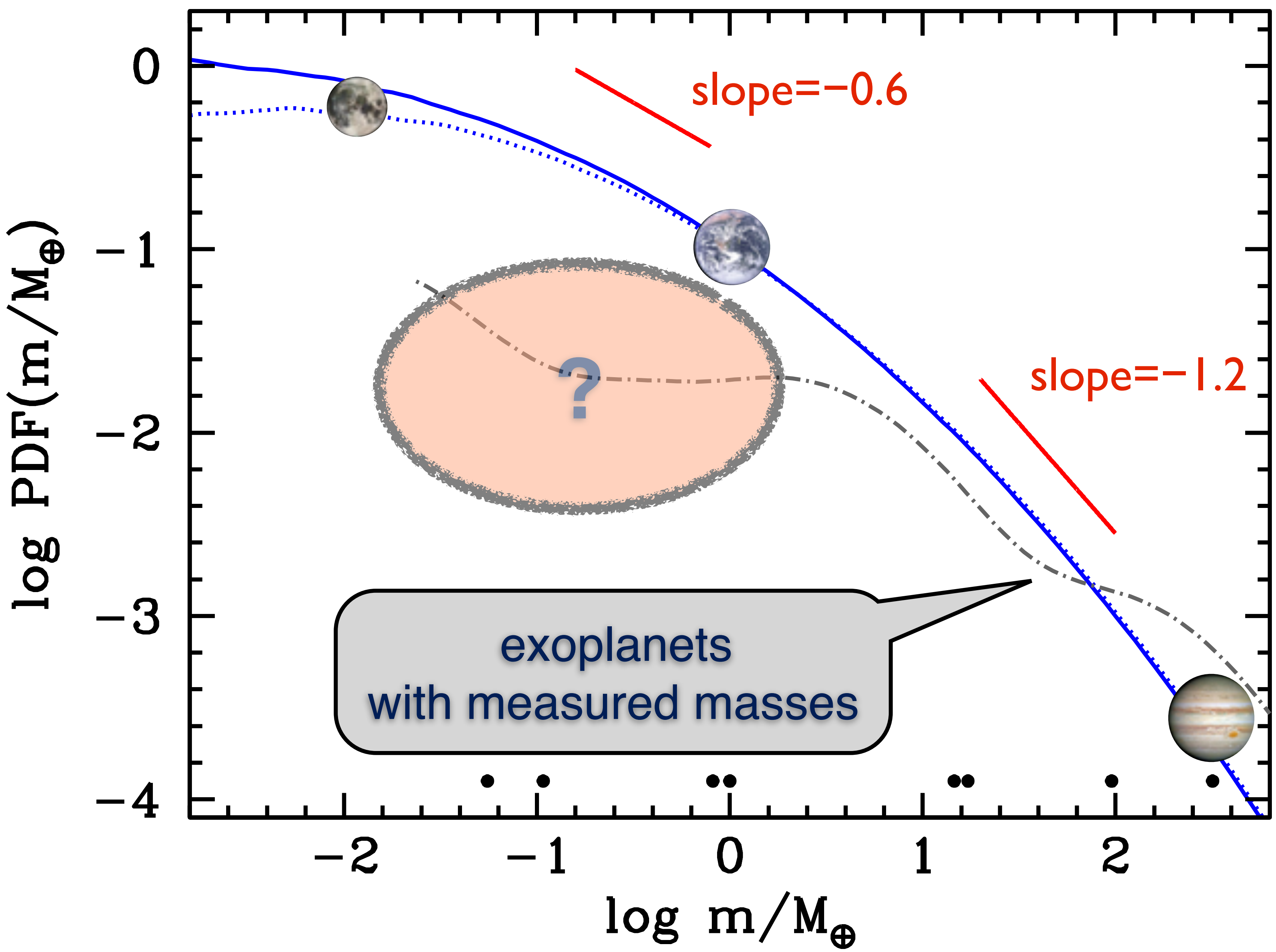



Earth-mass planets are ~$10^3$ more abundant than Jupiter-mass planets
The most common planets are of mass m < $M_\oplus$

# Distribution of planet masses: comparisons with other estimates



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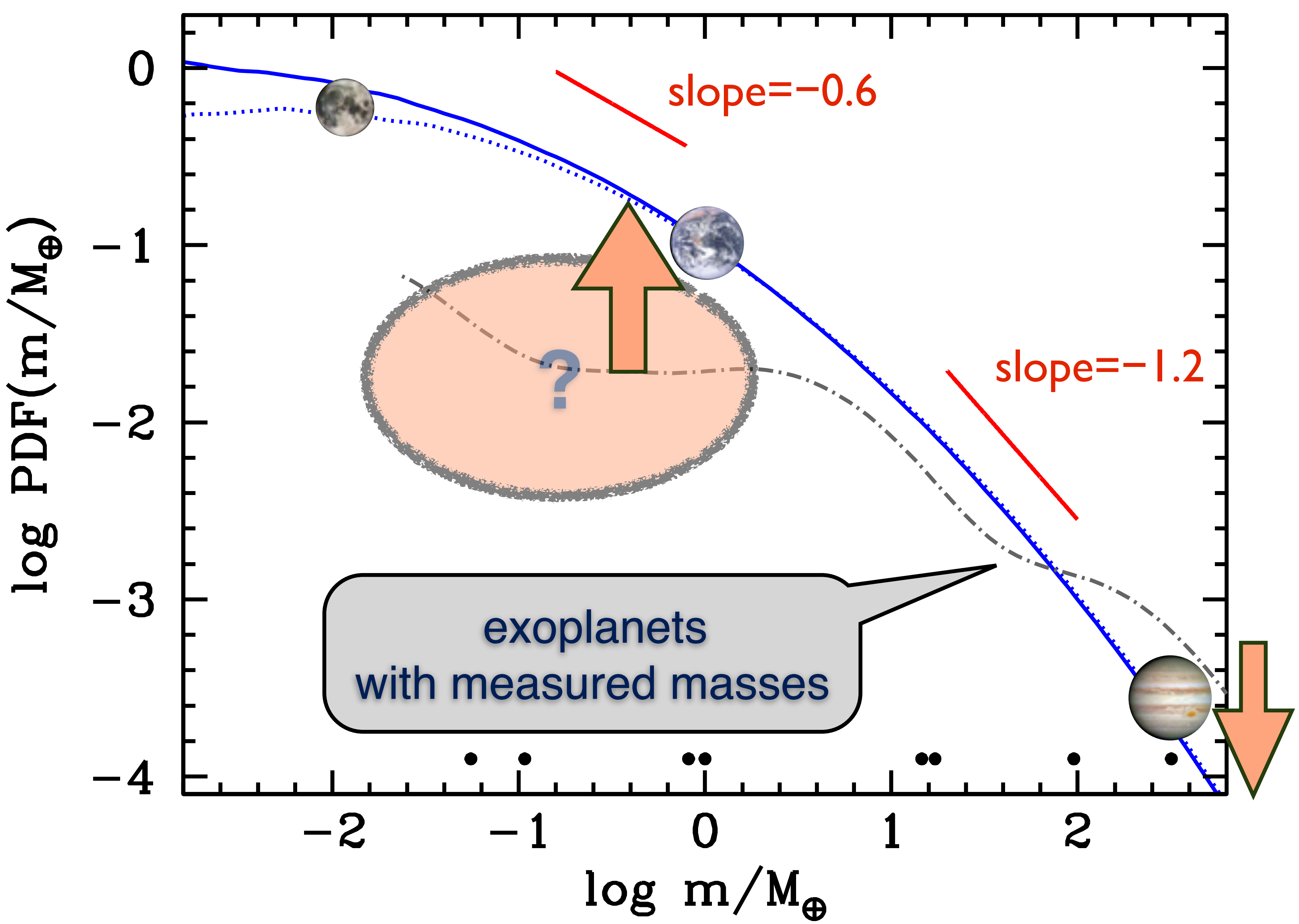

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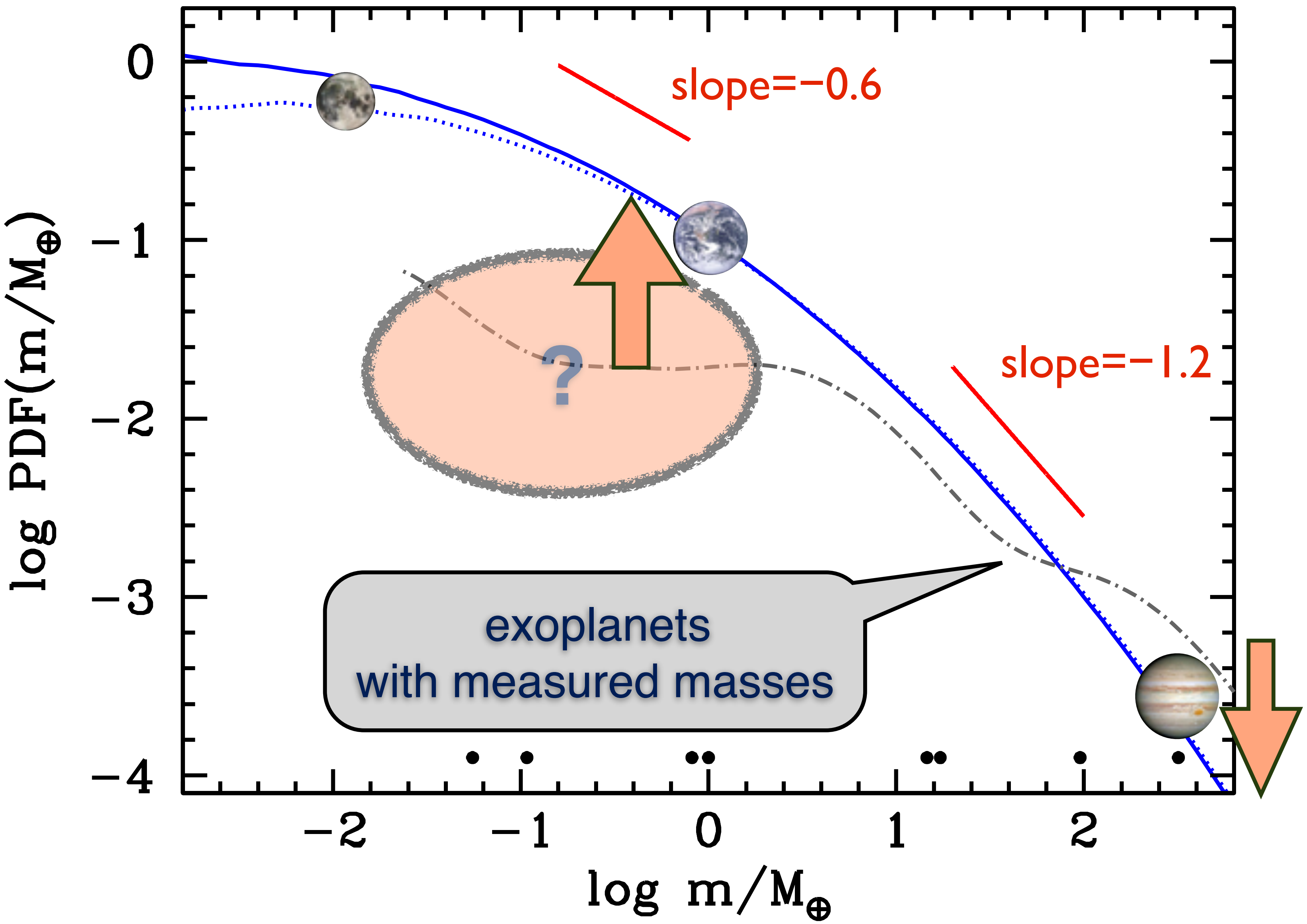


(microlensing) Sumi et al. (2010):
slope = −1.7±0.2 for m >10 M⊕

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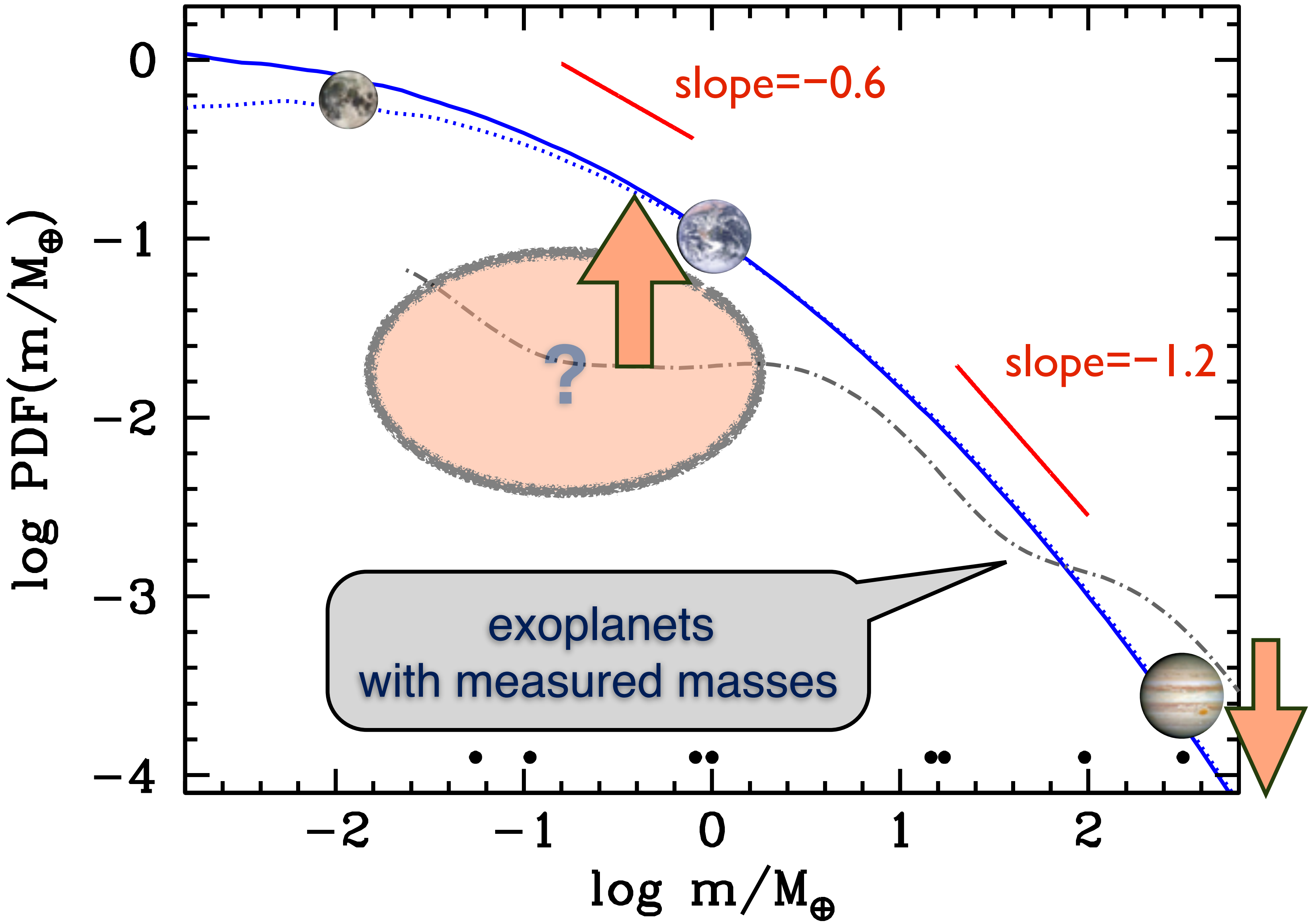


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slope = −1.7±0.2 for m >10 M⊕

(RV)Howard et al. (2012):
slope = $-1.48^{+0.12}_{-0.14}$ for 3-1000 M⊕

# Summary

- *Kepler* **data** of multiple-planet inner solar systems

  ➟ **orbital separations ~ log-normal**

- **Dynamical stability ➟ planet masses related to orbital separations**

- **With a simple ansatz, we derive that the planet mass function …**

  - **is a rolling power law, shallower at lower masses**

  - **Earth-mass planets are ~1000 x more common than Jupiter-mass planets**

  - **the most common planet mass, mode $m < M_\oplus$**