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GRAVITATIONAL TIDE IN THE ATMOSPHERE OF TITAN

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ABSTRACT

The presence of a tide in Titan's atmosphere as a result of the Saturnian gravitational field is suggested. Since Titan is believed to rotate synchronously or near-synchronously, this tide will be virtually stationary with respect to Titan's surface and may be a classic example of an 'equilibrium tide'. Titan atmosphere models to date are based on measurements made near points orthogonal to the sub-Saturn point. Consideration of the possible tidal effects suggests that surface pressure near the sub and anti-Saturnward points may be about 1% greater than at where these measurements were made. Other atmospheric tides on Titan and prospects for experimental observation of the tide are briefly considered.

Keywords : Titan, Atmosphere, tides, gravitation

1. INTRODUCTION

All fluids exposed to the gravitational fields of more than one body are subject to tidal phenomena. The tide of Earth's liquid seas is the most well-known, although solid body and atmospheric tides also occur.

The rise and fall of the sea, and their correlation with the sun and moon, have been known since ancient times. It was Newton, however, who first suggested (ref.1) that tides would also exist in the atmosphere, although he believed that they would be too small to be detected:

> *(of the sun and moon) "In the atmosphere, indeed, they will excite such a flux and reflux as they do in the sea, but with so small a motion that no sensible wind will be thence produced."*

In fact, the wind and pressure variations caused by the sun and moon are detectable, but it is difficult to separate tidal effects from the many other, larger, variations that also occur. Laplace (ref.2) was the first to attempt to detect an atmospheric tide, namely the diurnal pressure variation due to the moon's gravity. He analysed an 8-year series of pressure readings from the Paris Observatory and obtained an amplitude of 0.05 mmHg (6.7 Pa) - somewhat larger than the 'true' value, but of the same order of magnitude.

Gravitational tides are a result of the radial differential in gravitational field strength of a body - the effect exploited in gravity-gradient attitude stabilisation of satellites. Since the gravitational acceleration acting on an element of a body may differ from that acting on the body as a whole, forces are generated which tend to distort (or rotate) the body such that its long axis points radially.

In planetary atmospheres, this effect manifests itself as a 'bulge' towards and away from the primary. At a given altitude, the pressure will be highest near the 'peak' of the bulge (or conversely, the altitude for a given isobar will be higher near this point.) On the earth (ref.2), the situation is greatly complicated by the rotation of the atmosphere relative to the sources of tidal excitation, such that tides are dynamic time-variant phenomena. The tidal effect of the moon is slightly greater than that of the sun, but both are dominated by a 'thermal tide' caused by the expansion of the atmosphere as a result of solar heating on the dayside.

The proximity of Titan to the giant planet Saturn suggests that tidal effects there will be significant. Sagan and Dermott (ref.3) calculated the magnitude of the tide in the seas of Titan - assuming seas exist - and found it to be of the order of 100m. Since Titan has a thick atmosphere, tidal effects will manifest themselves there too, and because Titan is in approximately synchronous rotation, the atmospheric tide will be static and far simpler to calculate than those on Earth.

2. CALCULATIONS

Consider Titan at a distance D (1.22e9m) from Saturn, which has a mass M_s (5.69e26 kg). Any given location P on Titan has an angle ϕ, the angle between the Saturn vector and the position vector OP (see fig 1). The length r of OP is simply the sum of Titan's radius R_t and the altitude h.

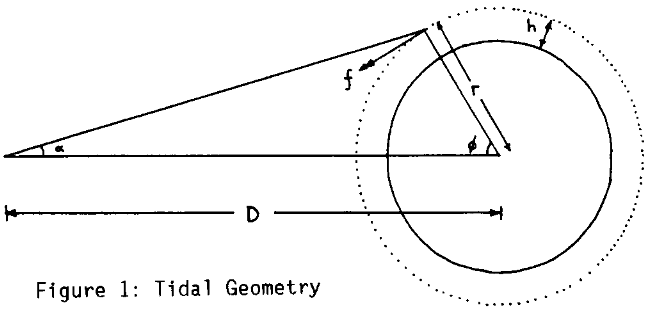

Figure 1: Tidal Geometry

The tidal acceleration due to the variation in gravitational acceleration has components (ref.3) parallel

$$\frac{2\cdot G\cdot M_s\cdot r\cdot\cos\phi}{D^3} \qquad (1)$$

and normal

$$\frac{G\cdot M_s\cdot\sin\alpha}{(D-r\cdot\cos\phi)^2} \qquad (2)$$

to the Saturn vector. G is the universal gravitational constant (6.673e-11 N/kgm²) and angle α is the angle subtended at Saturn by OP and is given by

$$\sin\alpha = \frac{r\cdot\sin\phi}{D-r\cdot\cos\phi} \qquad (3)$$

Thus the tidal acceleration f acting normal to OP (i.e. along the local horizontal) is given by

$$f = G\cdot M_s\cdot r\cdot\left[\frac{2}{D^3} + \frac{1}{(D-r\cdot\cos\phi)^3}\right]\cdot\sin\phi\cdot\cos\phi \qquad (4)$$

To calculate the pressure variation over the surface at a given altitude, the pressure distribution must be such as to balance the tidal force. Since for a given φ the tidal force is constant, the pressure is constant also. Hence we consider the equilibrium of a small element of atmosphere (fig.2)

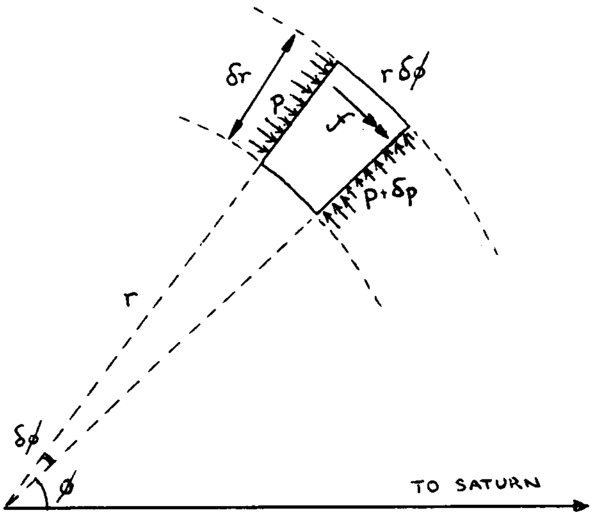

Figure 2: Equilibrium of an atmosphere element

thus

$$\delta p = r\cdot f\cdot\rho\cdot\delta\phi \qquad (5)$$

with p and p+δp being the pressures on either side of the element, ρ (rho) being the density of the element, calculated using the ideal gas law

$$\rho = \frac{P\cdot m}{R_o\cdot T} \qquad (6)$$

where R_o is the universal gas constant (8314 J/KgK), T the local temperature and m the relative molecular mass, assumed here at 28. (To a first approximation, the variation in density across the element is ignored.)

Combining (4),(5) and (6) yields the following

$$\frac{dp}{d\phi} = \frac{P\cdot m}{R_o\cdot T}\cdot G\cdot M_s\cdot r^2\left[\frac{2}{D^3} + \frac{1}{(D-r\cdot\cos\phi)^3}\right]\cdot\sin\phi\cdot\cos\phi \qquad (7)$$

Rewriting the right-hand side of the equation using the binomial expansion and rearranging gives

$$\frac{dp}{p} = \frac{3\cdot G\cdot M_s\cdot r^2\cdot m}{R_o\cdot T\cdot D^3}\cdot\Big[\cos\phi + (r/D)\cdot\cos\phi + (r/D)^2\cdot\cos^2\phi + .$$

$$\ldots\Big]\cdot\sin\phi\cdot\cos\phi\cdot d\phi \qquad (8)$$

Since the factor (r/D) is less than 0.01, we can ignore terms after the first. Thus, integrating gives the pressure variation with φ for a given altitude (we resubstitute R_t+h for r.)

$$\Big|\ln(p)\Big|_{\phi_2}^{\phi_1} = \frac{3\cdot G\cdot M_s\cdot m\cdot(R_t+h)^2}{2\cdot D^3\cdot R_o\cdot T}\Big|\cos^2\phi\Big|_{\phi_2}^{\phi_1} \qquad (9)$$

φ is a simple function of latitude Γ and longitude δ

$$\cos\phi = \cos\Gamma\cdot\cos\delta \qquad (10)$$

where on Titan the equatorial plane is assumed coincident with the orbital plane, and the prime meridian (0°E) points towards Saturn.

The only direct profile of Titan's lower atmosphere was obtained using the Voyager 1 radio-occultation measurements. Because of the flyby geometry (fig.3), these measurements give profiles near the equator at two locations near-orthogonal to the sub-saturn point (6°N 258°E and 8°S 76°E - ref.4) i.e. values of φ of 78° and 76° respectively. The standard atmosphere models are based on these data and the atmosphere is normally assumed to be spherically symmetric. It is suggested here that a set of correction factors be applied to take into account the geographical variation in atmospheric parameters due to the tide.

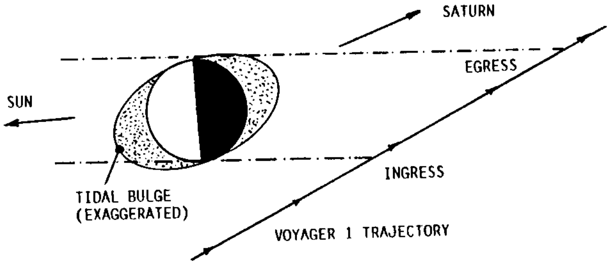

Figure 3: Voyager 1 Flyby Geometry

Table 1 lists the tidal correction factors (i.e. the antilogarithms of the expression in eqn.9) for a variety of altitudes and phi values. It is seen that the pressure at the surface varies from 0.9996 to 1.0069 of the nominal, the nominal being defined as the value for phi=77 degrees (the mean of the phi-values for the Voyager measurements on which the models are based.)

Phi h(km)	0	15	30	45	60	75	90
0	1.0069	1.0064	1.0051	1.0032	1.0014	1.0001	0.9996
5	1.0073	1.0068	1.0054	1.0035	1.0015	1.0001	0.9996
10	1.0079	1.0073	1.0058	1.0037	1.0016	1.0001	0.9996
20	1.0086	1.0080	1.0064	1.0041	1.0018	1.0001	0.9995
30	1.0092	1.0085	1.0068	1.0043	1.0019	1.0002	0.9995
40	1.0095	1.0088	1.0070	1.0045	1.0020	1.0002	0.9995
50	1.0095	1.0088	1.0070	1.0045	1.0020	1.0002	0.9995
60	1.0089	1.0082	1.0065	1.0042	1.0019	1.0002	0.9995
80	1.0056	1.0052	1.0041	1.0026	1.0012	1.0001	0.9997
100	1.0049	1.0046	1.0036	1.0023	1.0010	1.0001	0.9997
150	1.0045	1.0041	1.0033	1.0021	1.0009	1.0001	0.9998
200	1.0044	1.0041	1.0033	1.0021	1.0009	1.0001	0.9998
250	1.0045	1.0041	1.0033	1.0021	1.0009	1.0001	0.9998
300	1.0045	1.0042	1.0033	1.0021	1.0009	1.0001	0.9998
400	1.0048	1.0044	1.0035	1.0022	1.0010	1.0001	0.9997
600	1.0055	1.0051	1.0041	1.0026	1.0012	1.0001	0.9997
1000	1.0075	1.0070	1.0056	1.0036	1.0016	1.0001	0.9996

TABLE 1 : TIDAL CORRECTION FACTORS (UNITY FOR PHI=77 degrees)

Table 2 lists parameters for a variety of altitudes extracted from the Lellouch-Hunten model (ref.5): densities are quoted for the maximum, minimum and nominal cases, plus the nominal case with the tidal correction applied for $\phi=0°$ (i.e. maximum tidal perturbation.)

Altitude (km)	Density Min (kg/m3)	Density Nominal (kg/m3)	Density Max (kg/m3)	Tidal Factor	Density Nom+Tide (kg/m3)	Nominal Temp (K)
0	4.57E+00	5.29E+00	6.01E+00	1.0069	5.33E+00	97.2
5	3.95E+00	4.45E+00	4.95E+00	1.0073	4.48E+00	91.4
10	3.38E+00	3.67E+00	3.96E+00	1.0078	3.70E+00	85.8
20	2.08E-01	2.24E-01	2.40E-01	1.0086	2.26E-01	78.6
30	1.19E-01	1.28E-01	1.37E-01	1.0092	1.29E-01	74.4
40	6.45E-01	7.00E-01	7.55E-01	1.0095	7.07E-01	72.7
50	3.40E-01	3.71E-01	4.02E-01	1.0095	3.75E-01	73.3
60	1.73E-01	1.90E-01	2.07E-01	1.0089	1.92E-01	79.0
80	4.53E-02	4.98E-02	5.43E-02	1.0055	5.01E-02	127.9
100	2.05E-02	2.33E-02	2.61E-02	1.0049	2.34E-02	146.7
150	4.48E-03	5.34E-03	6.20E-03	1.0045	5.36E-03	167.6
200	1.28E-03	1.55E-03	1.82E-03	1.0044	1.56E-03	174.9
250	3.42E-04	4.92E-04	6.80E-04	1.0045	4.94E-04	180.2
300	9.55E-05	1.73E-04	2.63E-04	1.0045	1.74E-04	184.0
400	8.49E-06	2.55E-05	4.33E-05	1.0048	2.56E-05	187.3
500	8.48E-07	4.39E-06	8.01E-09	1.0052	4.41E-06	184.3
600	9.07E-08	8.39E-07	1.68E-06	1.0055	8.44E-07	184.3
700	1.11E-08	1.52E-07	4.06E-07	1.0094	1.53E-07	115.5
800	2.13E-09	1.32E-08	1.08E-07	1.0089	1.33E-08	129.5
900	6.29E-10	1.85E-09	1.27E-08	1.0077	1.86E-09	158.2
1000	1.99E-10	3.97E-10	1.71E-09	1.0075	4.00E-10	170.7

TABLE 2 : DENSITY AND TEMPERATURE MODEL (L-H ref.6) PLUS TIDAL EFFECTS

It is seen that the effect of the tide is small compared with the the uncertainty in the model.

surface to a change in pressure of .0069x1.53 bar, or 1055 Pascals, detectable with even a crude barometer.

The variation with altitude (assuming constant molecular weight) is indicated in figure 4. The initial rise is due to the drop in temperature with altitude until the 'cold trap' at about 40km altitude where the temperature falls to about 73K, or about three-quarters of its surface value. The subsequent behaviour is also dominated by the temperature profile (the peak at 700km due to low temperature at the mesopause), with a slight upward trend superimposed due to the increase in radius. However, the values at high altitude should be regarded with caution as many other unmodelled factors (zonal winds (ref.7), solar heating etc.) begin to be important. However, near the surface, in the planetary boundary layer, wind velocities will be low, and the tidal effects should attain a static equilibrium.

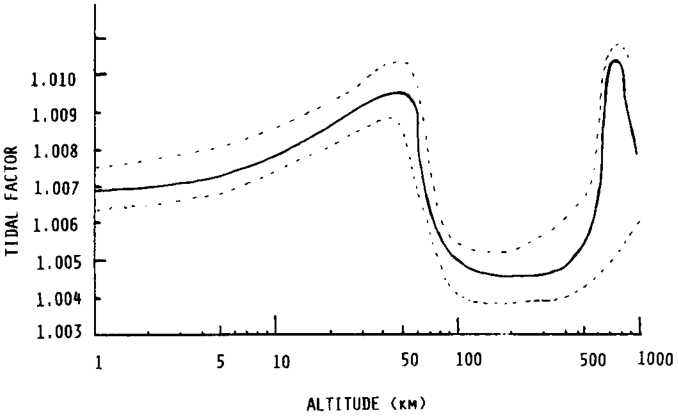

Figure 4 : Theoretical Tide as a function of Altitude

Suspicion that Titan's atmosphere may not be spherically symmetric was aroused when new atmospheric data on Titan was been obtained during the occultation of the star 28 Sagitarii by Titan in July 1989 (refs.8,9). Sicardy et al. (ref.9) found that the best-fit atmosphere profile was obtained by assuming an oblate atmosphere, the oblateness being 0.017 at the 250 microbar level (250km) and it was suggested that this oblateness is due to superrotating zonal winds. An oblateness of 0.017 at 250km, corresponds to a change in isobar height level of 0.017x(2575+250)=48km.

The stellar occultation data would also be consistent with a prolate atmosphere, with its long axis pointing towards Saturn. However, computing the pressure perturbation at 250km due to the conjectured tidal forces yields 0.0044x26=0.114Pa, corresponding to a height change of only about 200m. Hence oblateness due to super-rotation is the most likely explanation for the data.

Thus the effect of the tide is much smaller at high altitude than that due to the suggested oblateness. At the surface, however, superrotation should not affect pressure, so the tidal effect will predominate. Further, the tide will cause variation with both latitude and longitude, whereas an oblateness is a function of latitude only.

3. ENGINEERING IMPLICATIONS

The orbital arrival geometry, and the subsequent flyby tour requires that the Huygens probe arrives from approximately the anti-saturnward direction, probe, and the selected entry parameters (Flightpath angle Gamma=-65°, B-plane angle θ=-80°) place the entry point near 22°N, 200°E. This is close (ϕ=30°) to the centre of the tidal bulge (see figure 5.)

Since the atmosphere is distended by the tide at the descent point, the time for descent will be affected. An accurate assessment would require the previous correction factors to be applied to the atmosphere model, and a descent simulation performed. However, a reasonable estimate can be made quite simply. Since most of the descent is performed at terminal velocity, with the aerodynamic drag on the probe (or probe+parachute) in equilibrium with its weight, it is clear that at any given height

$$\beta g=\tfrac{1}{2}\rho V^2 \qquad (11)$$

with β the ballistic coefficient of the probe, g being local gravitational acceleration. Descent time τ is proportional to the reciprocal of descent speed V. Since the atmosphere is in hydrostatic equilibrium, the surface pressure P_0 is proportional to the integrated density of the atmosphere above it, then the total descent time varies as

$$\tau \propto (P_0)^{\frac{1}{2}} \qquad (12)$$

For ϕ=30° the surface pressure correction factor is 1.005. Thus for a probe descent time of approximately three hours (10800 seconds) the tide prolongs the descent by about 30 seconds, assuming the surface of Titan to be perfectly spherical and not itself distorted by tidal forces.

4. OTHER ATMOSPHERIC TIDES ON TITAN

In addition to the gravitational tide due to Saturn, We can also consider the effect of energy from Saturn, i.e. a Saturn-induced thermal tide. Saturn not only reflects an appreciable portion of the sunlight incident upon it, but also radiates away the energy released by the sinking of helium through less dense hydrogen.

The solar constant at the Saturnian system is about 15 W/m². Saturn has an seasonally-averaged cross-section (ref.10) of 1.04E16 m² and an albedo of 0.34, thus Saturn reflects about 5.3E16 W of power, with an approximately 'solar' spectrum. At Titan, this corresponds to a maximum flux of 5.7E-3 W/m², which can probably be neglected.

In addition to this, however, Saturn re-emits the absorbed solar radiation, plus the energy generated internally by helium differentiation, in the infra-red, with a peak emission of about 50 microns. This emission has a total power of 1.98E17 W, or 0.01W/m².

This value too is sufficiently small compared with the solar constant that the thermal emission of Saturn can probably be ignored, although note that in parts of infra-red region of the spectrum, the incident power spectral density from Saturn may exceed that from the sun.

There is a significant solar-driven thermal tide, but this is complicated by the fact that, relative to the Titan surface, the sun moves at an angular rate dictated by Titan's motion around Saturn. This thermal tide has already been considered by Leovy (ref.11), and may lead to high-altitude jets in Titan's zonal wind field.

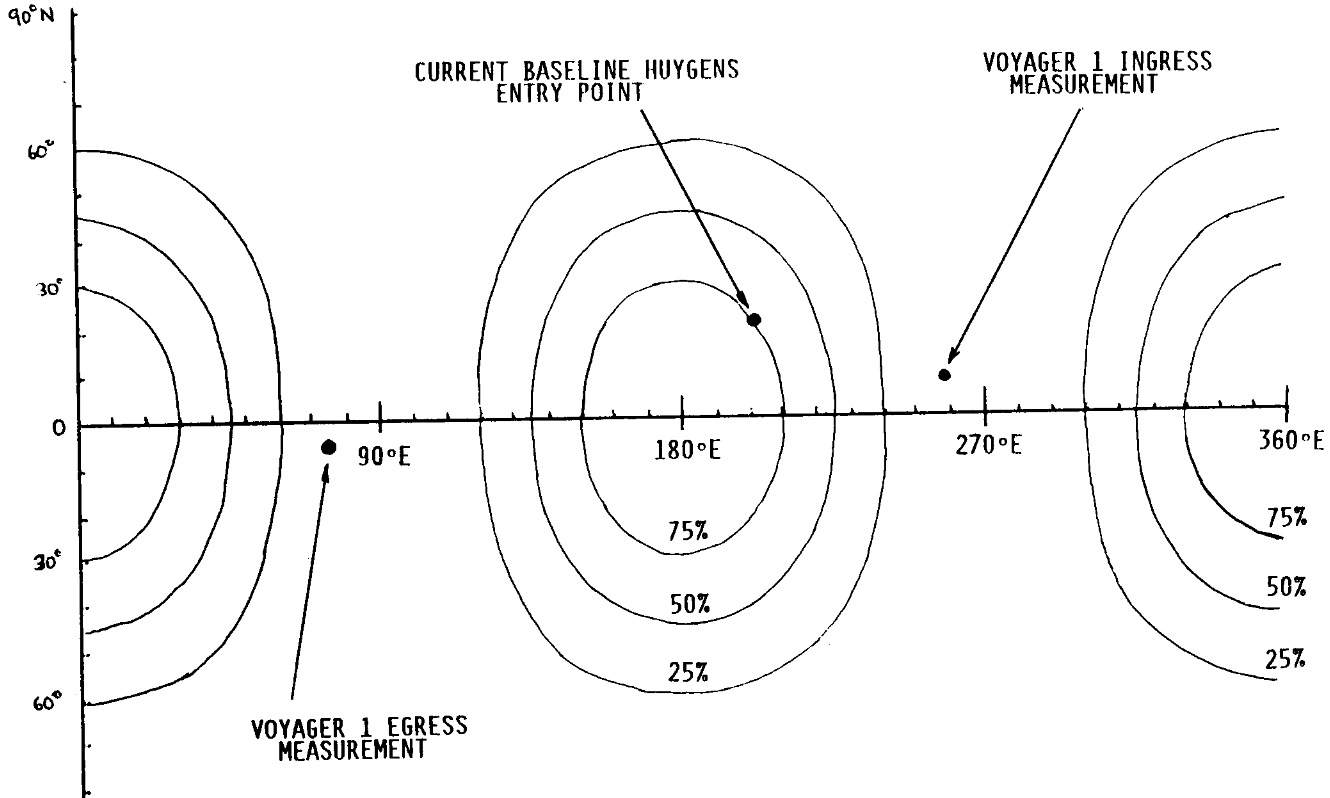

Figure 5 : Variation of Tide over the Surface

The solar-driven gravitational tidal forces also rotate once-per orbit with respect to Titan, but owing to the very large distance from the sun, are negligible in magnitude (2 millionths of the magnitude of the tide due to Saturn.) Tides on Titan due to the other Saturnian moons (ref.3) are similarly negligible.

5. FUTURE WORK AND OBSERVATIONS

The analysis presented here has not considered a number of complicating factors which may merit further study. Among these are the interaction of the tidal bulge with Titan's presumed zonal wind system and the motion of the sub-saturn point on Titan due to the eccentricity of Titan's orbit and possible non-synchronous rotation. The model presented here is essentially one-dimensional - extension into two or three dimensions would be necessary to examine these factors in detail.

As the Huygens probe only makes measurements along its descent path (i.e. over a very small range of geographical locations) and the tidal effects are small compared with the uncertainty in current models, it will probably be impossible to detect the tidal effect on pressure using sensors on the probe.

Instruments on board the Cassini Saturn Orbiter spacecraft may be able to confirm the presence of an equilibrium tide - multiple radio-occultation measurements, over a range of longitudes, should build up a tomographic picture of the atmosphere. However, the surface pressures inferred from the Voyager 1 radio occultations have an accuracy of about 20 mbar, or about twice the magnitude of the conjectured tide. Thus, even allowing for improvements in the radio-occultation technique, the tide will be at the threshold of observability. Other orbiter measurements may be of use, but it is possible that the tide may defy detection even by Cassini.

A potential difficulty is the detection of the tide in the presence of other variations in atmospheric pressure (oblateness and diurnal variations.) However, these problems are far less troublesome than on Earth.

The most obvious methods of verifying the tide are (1) direct measurement of pressures by a mobile platform or a network of fixed surface stations spread over Titan (see for example, comparable measurements planned for Mars (ref.12)), or (2) accurate monitoring of the evolution of the orbit of a satellite of Titan - a related technique was used to determine upper atmospheric density on Venus (refs. 13, 14) from the orbital decay of the Pioneer Venus orbiter. Unfortunately, neither of these possibilities is likely to be realised in the near future.

6. CONCLUSIONS

The gravitational effects of Saturn on Titan's atmosphere have been considered. It is conjectured that at certain locations on Titan, there will be an increase in atmospheric density and pressure of the order of 1% (relative to models based on Voyager 1 radio-occultation data.) It will, however, be a challenging task to confirm the presence of the tide experimentally.

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