Work output of planetary atmospheric engines: dissipation in clouds and rain

R. D. Lorenz
Lunar and Planetary Laboratory, University of Arizona, Tucson, AZ 85721-0092, USA

N. O. Rennó
Dept of Atmospheric Sciences, University of Arizona, Tucson, AZ, USA

Received 12 July 2001; revised 5 October 2001; accepted 15 October 2001; published 25 January 2002.

[1] We compute the dissipation due to the drag of falling raindrops and cloud droplets, and find both to be significant sources of dissipation on Earth and possibly Titan. On other planets, they appear to be insignificant. We compare this dissipation with the latent heat transported by the drops and the thermodynamic efficiency with which it can be converted into work. This comparison suggests moist convection on Earth and Titan is similar, but on Jupiter latent heat alone does not provide enough work to lift the condensate against gravity. *INDEX TERMS:* 0343 Atmospheric Composition and Structure: Planetary atmospheres (5405, 5407, 5409, 5704, 5705, 5707), 5739 Planetology: Fluid Planets: Meteorology (3346), 1854 Hydrology: Precipitation (3354)

1. Introduction

[2] Despite the popular conception of clouds "floating" in air, each and every cloud droplet is pulled towards the ground by gravity. For clouds to persist requires that work be done to oppose their terminal velocity, or to replace them. Just as the original function of steam engines was to lift water, a principal output of the atmospheric heat engine is to raise water vertically.

[3] In this paper, we discuss the dissipation of the work output of the climate system, and in particular consider the dissipation due to friction around falling cloud drops. We find that the Earth's average cloudiness is a significant source of dissipation. We consider the same effect in other planetary atmospheres, and find that the Jupiter cloud deck appears to be quite consistent with the idea that the cloud is limited by the available work output.

2. Dissipation in the Atmosphere

[4] The original estimate of the dissipation in the atmosphere (equivalently, in steady-state, the work output of the atmospheric heat engine) appears to be due to [*Brunt*, 1926, 1952], who suggested on the basis of a typical wind speed and a turbulent boundary layer velocity profile that the dissipation was (probably overestimated as) $\sim$5 Wm^{-2}.

[5] Estimates of this order, usually quoted as $\sim$2 Wm^{-2}, appear to have been adopted widely — see e.g. [*Wulf and Davis*, 1952], [*Lorenz*, 1960], [*Peixoto and Oort*, 1992]. Some comfort is taken in modern texts that this dissipation is well under the Carnot (ideal) limit on the work output of the climate system.

[6] This Carnot limit is given by the product of the convective heat flux (typically assumed $\sim$80–100 Wm^{-2}) and the ideal efficiency $\eta = \Delta T/T$, where T is the hot-end temperature of the engine (the average surface temperature of 288K) and $\Delta T \sim$38K, the difference between T and the effective temperature (the temperature at which, on average, heat is rejected from the planet). These values — subject to some uncertainty, yield η = 13% and thus the work output of the climate system is limited to about 11–13 Wm^{-2}.

[7] It is rarely remarked what happens to the rest of this "available" work — a discrepancy comparable to the forcing by anthropogenic greenhouse gasses. Some workers, e.g. [*Michaud*, 1995] contend that the dissipation is severely underestimated, and that the system somewhere must be dissipating its full work output. One possibility is that small-scale variations in windspeed, unresolved by GCMs, may significantly augment dissipation.

[8] Additionally, [*Pauluis et al.*, 2000] have noted that aerodynamic friction by hydrometeors may be a significant ($\sim$2Wm^{-2}) source of dissipation in the climate system; see also [*Rennó*, 2001] for discussion. The aerodynamic forces on a drop are a combination of viscous and pressure (or inertial) forces. Small, slowly-falling drops at low Reynolds number R_e are dominated by viscous forces: the kinetic energy of a drop is quickly converted into heat by molecular viscosity (in effect the momentum deposited into the atmosphere is rapidily diffused over such a large mass of air that the net velocity of that air mass is negligible). For $R_e \gg 1$, the pressure forces predominate and the momentum continuously added by the weight of the drop creates downdrafts, with the fraction of work dissipated by immediate viscosity only $A/(B + R_e)$ — with A,B constants of order $\sim$10. This follows from Stokes' law and the definition of Reynolds number. The remainder will be dissipated somewhere — in the turbulent wakes of drops or perhaps on the ground.

3. Dissipation by Falling Droplets

[9] The terminal velocity v of a particle of radius r is computed as follows: at terminal velocity the weight of a particle is balanced by the drag, and thus

$$v^2 = (2mg/S\rho C_d) \tag{1}$$

where $m = 4/3\pi\rho_l r^3$ and $S = \pi r^2$, with ρ, ρ_l the densities of the air and particle. C_d is a drag coefficient, usually of order one (except for very small particles) and is parameterized as e.g. [*Lorenz*, 1993]

$$C_d = (24/R_e)\left(1 + 0.197R_e^{0.63} + 2.6 \times 10^{-4}R_e^{1.38}\right) \tag{2}$$

[10] Since $R_e = 2rv\rho/\mu$ is dependent on velocity, this system of equations needs to be iterated a handful of times to converge (we ignore the flattening of large drops). Let us assume there are enough droplets to create a cloud optical depth τ, where to a first order τ = N S, with N the column number density of particles. If all the drag is dissipated immediately, the work done per unit time per unit area is Nmgv.

0094-8276/02/2001GL013771$09.00

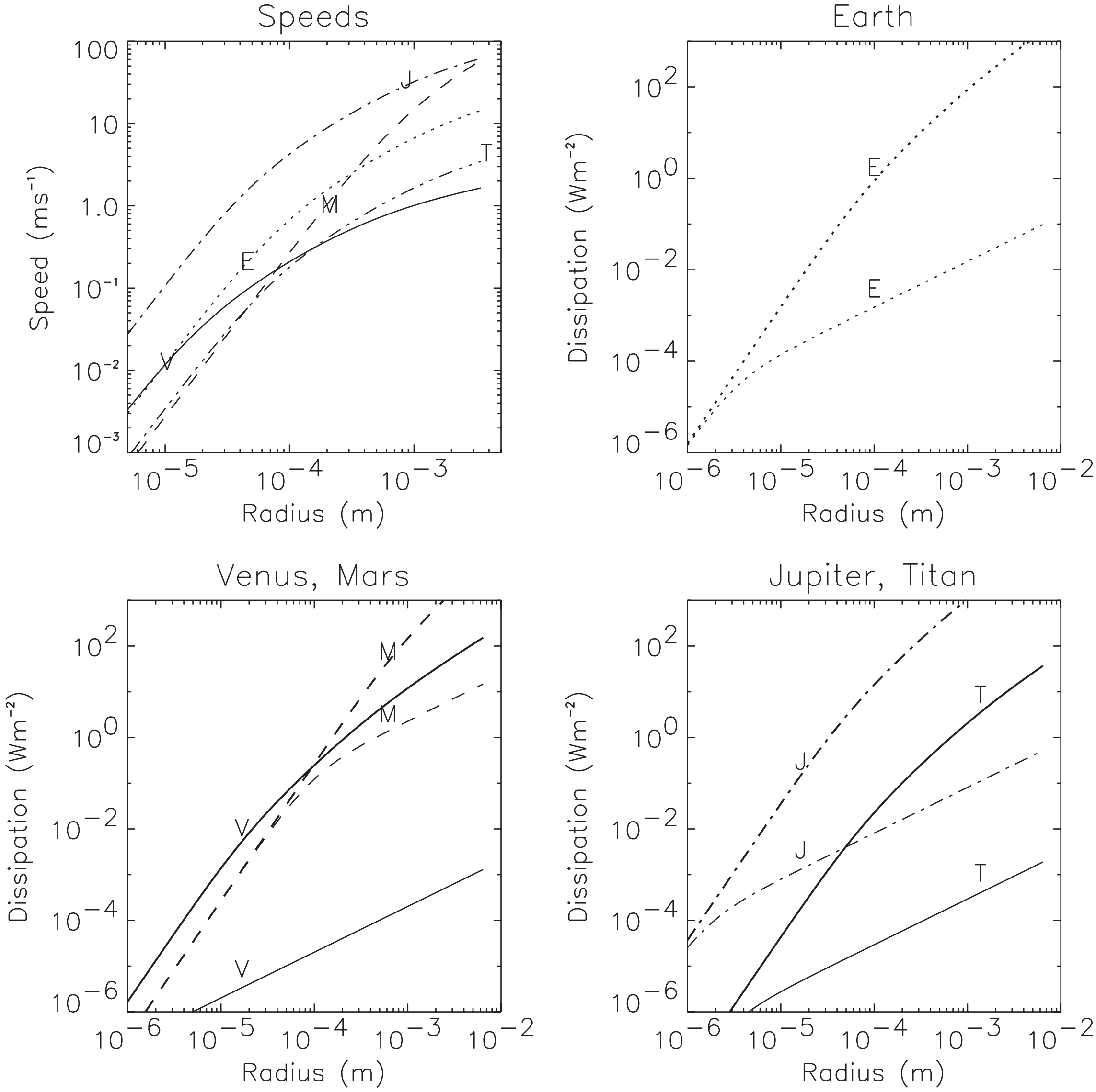

Figure 1. Terminal descent speeds as a function of drop size for Earth (dotted curve), Mars (dashed), Venus (solid), Jupiter (dot-dash) and Titan (triple-dot-dash). Subsequent panels show dissipation due to a unity optical depth of cloud particles for four planets, as a function of particle size. The upper curve is the dissipation assuming all the particle drag contributes to dissipation, the lower curve shows the viscous portion. Earth, Mars and Titan graphs are for surface conditions; Venus at 60km altitude, Jupiter at the 1 bar level.

[11] Fewer large particles are required for a given optical depth, since the larger particles individually occupy more area. However, since the total weight of material required is less for small particles to achieve that optical depth, and because those small particles fall comparatively slowly, it follows that far less work is required to support that optical depth with small particles. The work fluxes required to support a unity optical depth of particles as a function of particle size on various planets are shown in Figure 1.

4. Earth

[12] Let us first consider the Earth, and the work done by raindrops and cloud particles. The typical annual rainfall is around 1m depth, or 1000 kgm^{-2}. For raindrops of diameter 6mm, or mass 10^{-4} kg, this corresponds to 10^7 drops per m^2 per year. If drops fall around 1km, each drop dissipates 1J of work, and thus for 10^7 drops/yr, 0.3 Wm^{-2}, a not inconsiderable number.

[13] We can consider rain as a "cloud": each drop has an area of 3×10^{-5} m^2, thus 10^7 drops per m^2 form a layer with $\tau = 300$. However, falling at $10ms^{-1}$ from 1km takes only 100s, or 3×10^{-5}yr, and thus the *average* optical depth τ due to rain is $\sim$0.001. From Figure 1 we see that unity optical depth of such particles requires around 500 Wm^{-2} to remain aloft, or $\sim$0.5 Wm^{-2} for average $\tau = 0.001$. Drops falling over 2km altitude clearly would double this figure, and using a fall of 5km gives essentially the same result as [*Pauluis et al.*, 2000] of $\sim$2Wm^{-2}.

[14] But what of 'real' clouds themselves? The canonical cloud optical depth on Earth is usually assumed to be $\sim$4 [*Rossow and Schiffer*, 1999], with r $\sim$ 10^{-5}m. From Figure 1 we require dissipation of 2.5mWm^{-2} per unit optical depth, or only $\sim$0.01Wm^{-2} in total, and hence clouds are an insignificant dissipation source. Note that clouds are in general only weakly sensitive to the full drag assumption, since for small Re, $A/(B + R_e)$ is close to unity.

[15] A number of workers haveargued that the cloud optical depth figure, derived using 1-D radiative transfer models applied to satellite measurements, is in fact a severe underestimate. When the three-dimensionality of clouds is considered, an average optical

depth of over an order of magnitude higher may be appropriate. Such a figure is also compatible with a crude estimate [*Pruppacher and Jaenicke*, 1995] based on equating the average liquid water path of .34 kgm^{-2} to a number density of 10 μm particles, yielding τ = 58. Such an optical depth would require 0.15Wm^{-2} to remain aloft. This is a small, but not inconsiderable, number and increases sharply as the particle size increases.

5. Venus and Mars

[16] Venus partly owes its brilliance to a thick cloud deck. Measurements from descent probes (see, e.g. *Marov and Grinspoon*, 1998]) suggest that the main cloudopacity ($\tau \sim 30$) lies between about 50 and 60km, although in fact three distinct layers and particle modes are present.) Most particles have radii of the order of 2 μm, although the lower layers include some ~10μm particles. (Note that most of this cloud is recycled material, although a small photochemical H_2SO_4 component is produced photolytically above the cloud and would contribute a small steady-state opacity even in the absence of vertical convective motions.)

[17] Figure 1 shows dissipations of around 20μWm^{-2} per optical depth for 2μm particles, rising to about 2mWm^{-2} for the larger droplets. Thus the total required is less than 0.06Wm^{-2}. Balloon measurments reported in [*Crisp and Titov*, 1997] suggest heat transports of around 40 Wm^{-2} — consistent with the downward solar flux dropping from about 80 Wm^{-2} above the clouds to around 20 below them. The ΔT across the cloud layer is of the order of 90K, where ambient temperatures are around 360K, thus $\eta \sim 0.25$, although this quantity (like the others) is somewhat uncertain. Thus the work generation capacity of the Venusian atmosphere at these altitudes is around 10 Wm^{-2} and is far in excess of that required to maintain the cloud deck aloft.

[18] On Mars, upward convective heat transport near the surface as inferred from Viking lander measurements is of the order of 20 Wm^{-2} during the afternoon, with a probable conversion efficiency of $\Delta T/T \sim 10\%$, giving an available work of rather less than 1Wm^{-2}, averaged over the day. Dust and ice clouds have optical depths a little below unity and particles of the order of r = 1–10μ and hence the dissipation is negligible.

6. Jupiter

[19] The upwelling internal heat flux from Jupiter is about 10 Wm^{-2}, and since it arises from the deep, hot interior, is presumably efficiently converted into work. Inspecting Figure 1 suggests that an optical depth can be supported of particles 100μm or smaller, to around 100 for 1 μm particles. The 1-bar altitude shown is appropriate for NH_3 clouds — water clouds at around 6 bar (since particles at deeper levels have slower terminal velocities) can support somewhat higher optical depths. These appear to be reasonable values.

7. Titan

[20] We may now apply the work analysis in a predictive fashion to Titan, where the possibility of methane rainfall somewhere beneath the photochemical haze layers has long been recognized [*Toon et al.*, 1988], [*Lorenz*, 1993]. Recently, spectroscopic observations [*Griffith et al.*, 2000] have detected rapidly evolving, and therefore probably precipitating, clouds. Spectral fits determine their altitude to be ~27 km. The atmosphere at these altitudes is believed to be supersaturated, and thus cloud particles would rapidly grow into raindrops, up to 9 mm in diameter.

[21] The amount of sunlight ultimately expressed as convection is a somewhatuncertain number on Titan, determined from the flux imbalance in a radiative transfer model [*McKay et al.*, 1989] of around ~1% or 0.04 Wm^{-2}. ΔT/T between surface (94K) and effective (82K) temperatures gives η = 13%, as for Earth.

[22] Inspecting Figure 1, we then see that the 5×10^{-3} Wm^{-2} work production could only support a unity optical depth of particles smaller than about 50μm. Widespread clouds or fogs of large particles are therefore ruled out.

[23] As raindrops fall at around 2 ms^{-1} [*Lorenz*, 1993], the fall duration of a drop would be of the order of 10^4 seconds, for ~20 km depth of fall. Thus if one optical depth of raindrops (equivalent to about 1.5 cm annual rainfall depth) falls per year, we have an average optical depth of 3×10^{-4}. A unity optical depth layer of 9mm drops would require some 10 Wm^{-2} of work: dividing the actual work production by this amount yields an average optical depth of raindrops of 5×10^{-4}. Thus the available work limits annual rainfall on Titan to at most about 2 cm. This is a similar to a rainfall estimate of about [*Lorenz*, 2000] 1cm derived from equating the convective heat flux to the latent heat of vaporization of the fluid.

[24] Note that the substantial opacity ($\tau \sim 3-10$) due to submicron photochemical aerosols — as on Venus — implies a tiny dissipation that is not forced by convection.)

8. Wet and Dry Efficiencies

[25] The near-equivalence of the work and energy limits on precitipation for Titan prompt a more general consideration — how does the work done in lifting a drop relate to the work that can be done by the latent heat transported by it? The work done in lifting a drop through height h is mgh, while the work that can be done by its latent heat is $mL\eta$, or $mL\Gamma h/T$ where L is the specific latent heat of the condensing fluid and Γ the lapse rate. Let us define a ratio, χ, of the work done to the work available, and for simplicity use the dry adiabatic lapse rate ($\Gamma = g/c_p$ with c_p the specific heat of the gas) as a basis for comparison. $\chi = mgh/(mhLg/c_pT) = c_pT/L$. This quantity (which will be a factor of ~1.5 larger for typical, partly moist, lapse rates on Earth and Titan) then indicates how much extra work is required (from sensible heat) to raise the material. For Earth, χ = 0.1, indicating that only a part of the work available from a reversible engine driven by latent heat is required to raise the water substance. For Titan, similarly χ = 0.2. However, for Mars and Venus, χ = 0.4 and for Jupiter χ = 1.6. This latter value, in excess of unity, means that latent heat alone is not enough to lift the condensate — additional heat in the form of sensible heat must be supplied.

9. Conclusions and Applications

[26] We have discussed the work performed by vertical convection in the Earth's atmosphere and find that clouds may be a small but significant sink for the work generated by the atmospheric heat engine. Work in maintaining cloud drops aloft is negligible on other planets except possibly Titan.

[27] We derive an upper limit on annual methane rainfall on Titan of 2cm. We have derived a measure of the role latent heat plays in precipitation work, and find similar values for Titan and Earth, suggesting Titan may be a particularly fruitful Earth analog on which to study convection. Jupiter is qualitatively different from Earth and Titan in that latent heat alone cannot be driving moist convection.

[28] **Acknowledgments.** RDL Thanks the Cassini project for support. We thank Akos Horvath for useful comments on the terrestrial cloud thickness. We thank the referees for useful criticism that sharpened our understanding of this problem.

References

Brunt, D., Energy in the Earth's Atmosphere *Phil. Mag. S.7*, Vol. 1, 523–532, 1926.

Brunt, D., *Physical and Dynamical Meteorology*, Cambridge University Press, England (2nd ed), 1952.
Crisp, D., and D. Titov, The Thermal Balance of the Venus Atmospherein *Venus II.*, S. W. Bougher, D. M. Hunten and R. J. Phillips (eds), University of Arizona Press, Tucson, 353–384, 1997.
Griffith, C. A., J. L. Hall, and T. R. Geballe, Detection of Daily Clouds on Titan, *Science*, *290*, 509–513, 2000.
Lorenz, E. N., Generation of Available Potential Energy and the Intensity of the General Circulation, pp. 86–92 in *Dynamics of Climate*, R. L. Pfeffer (ed), Pergamon Press, 1960.
Lorenz, R. D., The Life, Death and Afterlife of a Raindrop on Titan Planetary and Space, *Science*, *41*, 647–659, 1993.
Lorenz, R. D., The Weather on Titan, *Science*, *290*, 467–468, 2000.
Marov, M. Ya., and D. H. Grinspoon, *The Planet Venus*, Yale University Press, New Haven, CT, 1998.
McKay, C. P., J. B. Pollack, and R. Courtin, The Thermal Structure of Titan's Atmosphere, *Icarus*, *80*, 23–53, 1989.
Michaud, L. M., Heat to Work Conversion during Upward Convection Part 1: Carnot Engine Method, *Atmospheric Research*, *39*, 157–178, 1995.
Pauluis, O., V. Balaji, and I. M. Held, Frictional Dissipation in a Precipitating Atmosphere, *J. Atmos. Sci.*, *57*, 989–994, 2000.
Peixoto, J. P., and A. H. Oort, *Physics of Climate*, American Institute of Physics, New York, 1992.
Pruppacher, H. R., and R. Jaenicke, The processing of water vapor and aerosols by atmospheric clouds, a global estimate, *Atmos. Res.*, *38*, 283–295, 1995.
Rennó, N. O., Comments on "Frictional dissipation in a precipitating atmosphere", *J. Atmos. Sci*, *58*, 1173–1177, 2001.
Rossow, W. B., and R. A. Schiffer, Advances in understanding clouds from ISCCP, *Bull. Amer. Meteor. Soc.*, *80*, 2261–2287, 1999.
Toon, O. B., C. P. McKay, R. Courtin, and T. P. Ackerman, Methane Rain on Titan, *Icarus*, *75*, 255–284, 1988.
Wulf, O. R., and L. Davis, Jr., On the Efficiency of the Engine Driving the Atmospheric Circulation, *J. Meteorology*, *9*, 79–82, 1952.

R. D. Lorenz, Department of Planetary Sciences, Lunar and Planetary Laboratory, University of Arizona, Tucson, AZ 85721, USA. (rlorenz@lpl.arizona.edu)

N. O. Rennó, Department of Atmospheric Sciences, University of Arizona, Tucson, AZ 85721, USA. (renno@atmo.arizona.edu)