Titan, Mars and Earth : Entropy Production by Latitudinal Heat Transport

Ralph D Lorenz, Jonathan I Lunine, Paul G Withers

Lunar and Planetary Laboratory, University of Arizona, Tucson, AZ 85721-0092, USA

Christopher P. McKay

NASA Ames Research Center, Moffett Field, CA 94035, USA.

Abstract. Temperature contrasts between warm tropics and cool high latitudes depends on how efficiently heat is transported by the atmosphere (and oceans) from the tropics. This heat transport is generally assumed to be proportional to atmospheric pressure, but we show with a simple model that this prediction fails by orders of magnitude for Mars and Titan. However, a basic principle, previously postulated for the Earth, does successfully predict the heat flows and zonal temperatures of Mars and Titan. The circulation predicted by this Maximum Entropy Production (MEP) principle is consistent with Titan's observed zonal structure and the winds and CO_2 frost cycle on Mars. The principle makes powerful predictions where detailed information is lacking, such as on the early Earth and on possibly habitable extrasolar planets.

1. Introduction

The Earth's hydrosphere can be considered as a heat engine: motions are driven by the flow of heat from a hot reservoir (the tropics) to a cold one (midlatitudes and polar regions). The work that can be performed by this heat flow depends on the temperatures of the reservoirs : a large temperature drop for a given heat flow corresponds to a higher thermodynamic efficiency, work output, and entropy production. If the heat flow F (expressed as a power in W, or a flux in Wm^{-2}) flows from a warm reservoir at T_0 to a cool one at T_1 (figure 1) the entropy production dS/dt is simply defined as (F/T_1-F/T_0). If the heat flow is zero, the entropy production is zero too, and each region of the planet is in radiative equilibrium. If the heat flow is maximized, the planet is isothermal, T_0=T_1 and entropy production is again zero. In between, however, entropy production is positive, and has a single maximum value.

Using zonal energy-balance models, several workers have demonstrated [Paltridge, 1975], [Wyant et al., 1988] [Grassl, 1990] that the Earth's climate seems to be in this maximum entropy production state.

It is not clear why the system should 'choose' this state: however, ideas in nonlinear dynamics suggest that this state is natural for complex systems. While a fluid system with fixed boundary conditions will minimize its entropy production, one with adequate degrees of freedom (like a sufficiently thick planetary atmosphere) can instead adjust the boundary conditions and the internal flows to maximize this quantity.

Paper number 2000GL012336
0094-8276/01/2000GL012336$05.00

Further support for the idea is the recent observation that the vertical temperature structure of the Earth's atmosphere is consistent with an MEP profile of vertical heat transport by convection [Ohmura and Ozuma, 1997].

In this paper we develop a simple model for latitudinal heat flow on Earth, and apply it to Mars and Titan to determine the applicability of MEP for those bodies.

2. A Simple Model : Application to Earth

Conventional zonal energy balance climate models e.g. [North et al., 1989] parameterize the heat transport processes as proportional to the latitudinal temperature gradient, such that $F=2D(T_0-T_1)$ where F is a heat flow per unit area and D a coefficient of meridional heat diffusion and measures the 'ease' with which the atmosphere transports heat. Empirically, the Earth's climate can be reproduced with D$\sim$0.6-1.1 $Wm^{-2}K^{-1}$, corresponding to the observed heat flow F of 20-40Wm^{-2} or 4-6x10^{15} W per hemisphere - see figure 1. If heat transport were purely due to sensible heat by meridional winds, then we would expect $D\sim\rho C_p Hv/R$, with H the scale height $\sim$ 8km, C_p is specific heat capacity of $\sim$1kJ$kg^{-1}K^{-1}$, ρ is surface atmospheric density of 1.25 kgm^{-3} , average meridional wind v of 1 ms^{-1} and planetary radius R$\sim$6370km, we obtain 1.5 $Wm^{-2}K^{-1}$, the correct order of magnitude. In reality of course, winds are complex, and latent heat transport and the thermohaline circulation in the oceans play significant roles : all of these phenomena are buried in the parameter D. Where these models have been applied to Mars or paleoearths, D has either been a free parameter, or is scaled to the Earth's present value using physical arguments (e.g. from the expression above D is proportional to ρ. A more detailed recent parameterization [Williams and Kasting, 1997] suggests D is proportional to $(P/P_o)(C_p/C_{po})(m_o/m)^2(\Omega_o/\Omega)^2$, where P, C_p and m are respectively the surface pressure, specific heat capacity and relative molecular mass of the atmosphere, and Ω is the planetary rotation rate; subscript o indicates the Earth. Notionally, for other planets there are two additional factors : the effective value of D is inversely proportional to gravitational acceleration, and to the square of the planetary radius. With and without these additional factors the expected values for D would be D$\sim$220 to 10^4 $Wm^{-2}K^{-1}$ for Titan, and D$\sim$0.001 to 0.01 $Wm^{-2}K^{-1}$ for Mars, the former so high because of Titan's small size and slow rotation, the latter so small principally because of the thin atmosphere.

A simple two-box model (Figure 1), represents annually-averaged conditions on a planet. An estimate of D, the low and high-latitude insolations I_0,I_1, and the dependence of

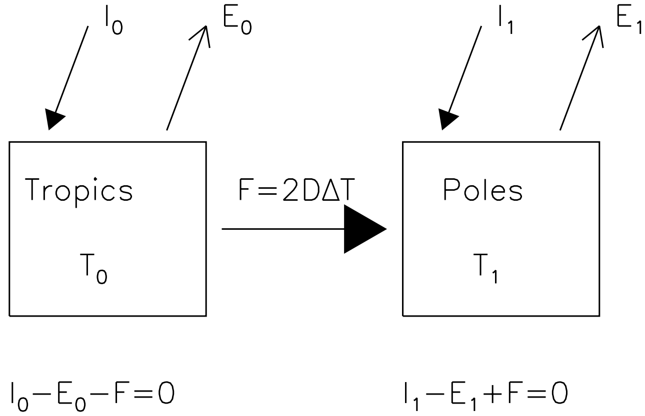

Figure 1. Two-box model - boxes are of equal surface area, the left corresponding to equatorial regions (bounded by 30 degrees latitude) at temperature T_0, the right 'polar' regions at temperature T_1. Each box has an absorbed solar flux I_0 and I_1. Between the boxes, there is a latitudinal heat flow F, assumed to be proportional to the temperature difference between the boxes, i.e. $F=2D(T_0- T_1)$. Each box also has an outgoing thermal flux E_0, E_1. E, I and F are all normalized by area and have units of Wm^{-2}. D has units $Wm^{-2}K^{-1}$: the heat flow F per unit area divided by twice the temperature difference. These terms are equivalent to the more conventional differential formulation for zonal models where for each latitude band $I=E+d/dx[D(1-x^2)dT/dx]$ where x is the sine of the latitude (here, the boxes are separated by $\Delta x=0.5$, thus $F=DdT/dx$, or $F=2D\Delta T$.)

outgoing thermal radiation on temperature $E_i=f(T_i)$ specify the system, predicting the temperatures T_0, T_1. If, for simplicity , $E_i=A+BT_i$, with $B\sim 4\sigma T^3/(1+0.75\tau)$, σ the Stefan-Boltzmann constant, andτ the infrared optical depth (typically of order one), T a typical planetary temperature in K we obtain the temperature contrast $\sigma T=(T_0-T_1)$ as $\Delta T=(I_0-I_1)/(4D+B)$ It can be shown that the maximum entropy production $(F/T_1-F/T_0)$ occurs for $D\sim B/4$. For Earth ($I_0=300$ Wm^{-2}, $I_1=170$ Wm^{-2}, $B=4\sigma 288^3/2\sim 1.35$-$5.4$ $Wm^{-2}K^{-1}$) this expression gives $D\sim 0.35$-1.3 $Wm^{-2}K^{-1}$.

While this simple exercise is hardly a rigorous proof of MEP's relevance to the Earth's climate (which has been demonstrated elsewhere), it shows agreement with observations that is remarkable given the model's few assumptions.

3. Titan

For Titan, taking $D=10^2\sim 10^4$ $Wm^{-2}K^{-1}$ from physical scaling, we would expect $\Delta T\sim 0.01$K, far smaller than the $\sim$4K indicated by infrared emissions at 530cm^{-1} ($T_0\sim 93$K, $T_1\sim 89$K [Samuelson et al., 1997]). To reproduce the observed temperatures requires a much weaker heat transport, namely $0.01<D<0.04$ $Wm^{-2}K^{-1}$. Figure 2 shows that the entropy production function peaks right in this range, at 0.02 $Wm^{-2}K^{-1}$, and agrees with the simple expression $D=B/4\sim 2\sigma 90^3/4=0.01$-$0.04$ $Wm^{-2}K^{-1}$. This result is far below what conventional scaling would lead us to expect: Titan's atmosphere is about 20 times *less* efficient at transporting heat even though it is 4 times denser than Earth's. Physically, it may be that the meridional heat transport is suppressed : Titan's dark polar hood [Smith et al, 1981] is suggestive of a polar vortex which might suppress mixing and on Earth a similar vortex allows Antarctic ozone levels to drop in winter. Another factor may be the pinning [Stevenson and Potter, 1986] of polar temperatures by condensation of methane and nitrogen, accounting not only for the magnitude of the equator-pole difference, but also the symmetry between poles.

Whatever processes are physically happening on Titan, the magnitudes of latitudinal heat transport and the resultant temperature contrasts (assuming that the 530cm^{-1} data reflects a real near-surface temperature contrast (see [Flasar, 1998]) appear to be successfully predicted from the MEP principle. Indeed, working from MEP alone, one can determine constraints on the meridional circulation that account for other observations such as Titan's seasonally-varying haze asymmetry. As before, if $D\sim\rho CpHv/R$, with $H\sim 30$km, $Cp\sim 1kJkg^{-1}K^{-1}$, and $\rho=5.4$ kgm^{-3}, then for $D=0.03$ $Wm^{-2}K^{-1}$, meridional velocity $v\sim 0.1$ mm/s. This is entirely consistent with the asymmetry which requires that the atmosphere turns over less than once per Titan year, i.e. $v << R/P$ where $R\sim 2575$km and P is period 29.5 years, or $v<$ 3mm/s.

4. Mars

We can also apply the principle to Mars, where $B\sim 2\sigma 200^3=0.45$-2 $Wm^{-2}K^{-1}$. Although the low D values predicted by pressure scaling also reproduce coarse annually-averaged temperatures (Figure 2) they in fact fail to reproduce Martian climate completely (extreme high latitudes get too cold during winter.) The models are 'fixed' by introducing an additional term to account for the advective flow due to the sublimating polar CO_2 cap [James and North, 1982], or by artificially pinning of the cap to a condensation temperature [Hoffert et al., 1981], [Wood and Page, 1992]. These model approaches effectively introduce additional heat transport such that D is actually higher and is in agreement with the MEP principle which suggest D of 0.1-0.5 $Wm^{-2}K^{-1}$. Again, this inferred D can predict the intensity of Martian winds and seasonal cycles, since with a temperature difference of $\sim$50K this implies a heat transport of $\sim$25 Wm^{-2} - consistent with typical windspeeds of a few ms^{-1} and $\sim 10^9$ J over half a Martian year, or the latent heat of about 1m thickness of CO_2 frost, as observed.

It is interesting to note that while the MEP principle suggests that a lower D is appropriate for Titan than physical parameterizations predict, for Mars MEP suggests a higher D. Yet in both cases the MEP solution successfully predicts the observed temperatures, despite the fact that it is nearly (except via radiative control of B) independent of the atmospheric pressure and composition. The agreement of Mars and Titan with MEP suggests the extremum format of the Earth's climate is not coincidental.

The principle probably also holds for the hot atmosphere of Venus which is essentially isothermal : however, Venus' atmosphere is so thick that pressure scaling and MEP give similar results. Although the MEP's disregard of parameters of conventional meteorological importance such as rotation rate is disconcerting, its predictive capability is impressive.

5. Conclusions and Applications

Applying the principle to climates in the early solar system, it follows from the proportionality of D to dE/dT (i.e. $\sim$B and thus to T itself), that the equator- to-pole temperature contrast ΔT remains fairly constant when the solar constant is lower. On Titan, ΔT would therefore be several

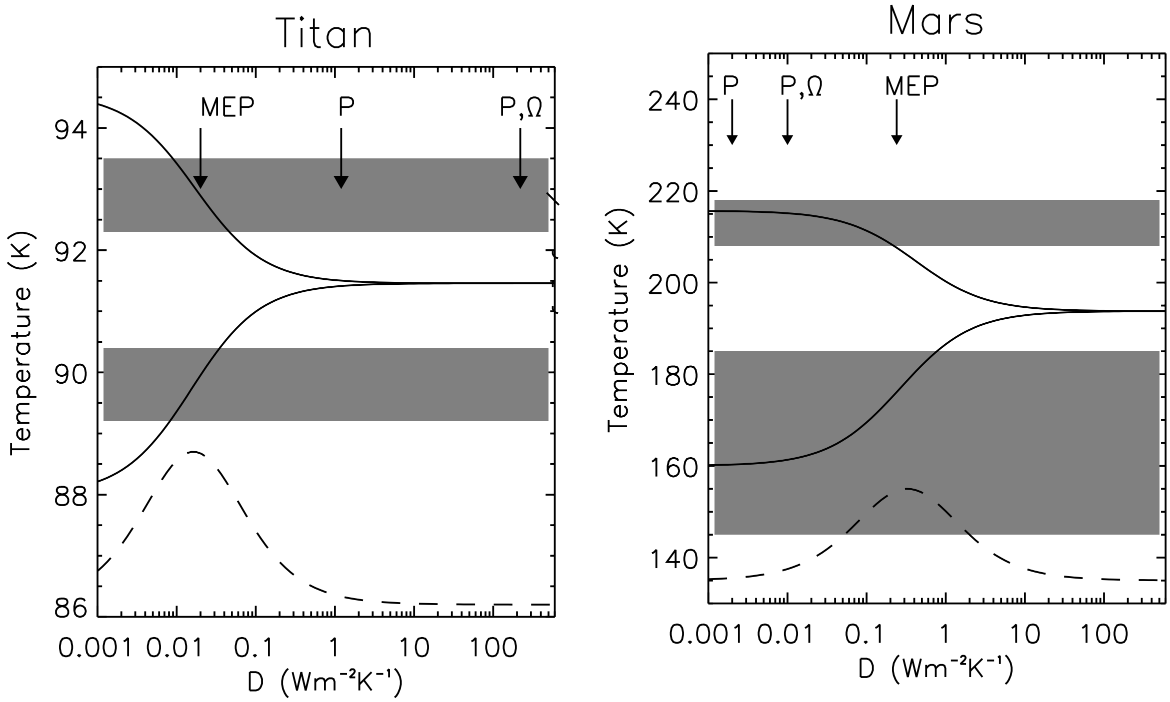

Figure 2. Observed temperature contrasts on Titan (a) and Mars (b) agree with predictions at maximum entropy production. Shading indicates observed annually-averaged temperature ranges for the regions 10- 20 deg and 40-60 deg latitude. Solid curves are model temperatures (upper curve tropics; lower curve polar regions) as a function of D : dashed curves at bottom are the entropy production dS/dt=(F/T_1-F/T_0) in arbitrary units. It is seen that the temperatures are successfully predicted where dS/dt has a maximum. Values of D predicted from scaling pressure (P), or pressure and rotation rate (P,Ω) are marked: the arrows marked MEP correspond to the maximum entropy value inferred from the simple expression D=$\sigma T^3/2$. High (>10Wm^{-2}K^{-1}) values for D predict near-isothermal conditions and do not agree with observations. Low (<0.01Wm^{-2}K^{-1}) D values appear compatible for Mars but not for Titan, but see text for discussion of the failure of small D for the Mars case. Note that the temperature slopes are steepest at MEP.

K, as at present, and the polar temperatures would fall below the condensation point of the atmosphere. A collapsed (Mars-like pressure, Triton-like composition) episode may have prevailed for a long period of Titan's history, and may be revealed by surface observations of Titan from Cassini [Lorenz et al.,1997].

The propensity for runaway glaciation of the Earth is somewhat dependent on D. For large D the climate is unstable since both equatorial and polar temperatures T_0 and T_1 are coupled to both insolations I_0,I_1, the latter being sensitive to the ice sheet position by ice-albedo feedback. For small D, however, the equatorial regions remain close to local radiative equilibrium, and since they receive much more than the planetary average insolation, remain unfrozen [Endal et al., 1982]. The MEP principle suggests that when insolation was lower, so was D and previous work has shown that that runaway glaciation therefore need never have occurred [Gerard et al, 1990].

For Mars, the heat transport and temperature contrasts mandated by the MEP principle have important implications. First, since even in a higher-pressure atmosphere temperature contrasts are significant, globally-averaged climate models will overpredict the greenhouse warming (i.e. τ) required to make the warmest part of Mars' surface habitable and allow ancient valley networks to form. E.g. $\tau \sim 5.3$ is required to bring global average surface temperatures up to 273K, the melting point of water. However, for low-latitude regions to have habitable annual- average conditions requires only $\tau \sim 3.7$. Since τ scales typically as a CO_2 pressure raised to a power less than 1, this corresponds to a factor of ∼2 reduction in required atmospheric pressure. Secondly, since at least up to a point the atmospheric pressure is controlled by the polar temperature [Gierasch and Toon, 1973] the latitudinal temperature contrasts must be taken into account in models of Mars' past and future atmospheric evolution.

The MEP principle's great shortcoming is that while it predicts the correct climate state, as illustrated by our Mars and Titan discussion, it sheds little light on what physical processes are occurring to cause it. Physically-based models such as those applied to Mars to date, and General Circulation Models in general, have importance as tools for understanding since individual mechanisms can be manipulated. GCMs can be usefully compared with models using MEP to identify unincluded or inadequately- parameterized processes : [Goody, 2000] recently highlighted entropy errors in GCMs. For situations where sufficiently detailed information to usefully constrain GCMs is lacking, such as extrasolar planets and the climates of the early solar system, simple MEP-based models appear both convenient and powerful.

Acknowledgments. RDL Thanks the Cassini project for support. JIL and CPM acknowledge the support of the NASA Planetary Atmospheres Program.

References

Endal, A. S. and Schatten, K. H. The Faint Young Sun-Climate Paradox: Continental Influences *J. Geophys. Res.*, **87**, 7295-7302 (1982)

Flasar, F. M. The Dynamic Meteorology of Titan *Planet. Space Sci.*, **46**, 1125-1147 (1998)

Gerard, J-C, Delcourt, D. and Francois, L. M. The Maximum Entropy Production Principle in Climate Models: Application to the Faint Young Sun Paradox, *Q. J. Roy. Met. Soc.* **116**, 1123-1132 (1990)

Gierasch, P. J. and Toon, O. B. Atmospheric Pressure Variation and the Climate of Mars *J. Atmos. Sciences*, **30**, 1502-1508 (1973)

Goody, R. Sources and Sinks of Climate Entropy *Q. J. Roy. Met Soc.* **126** 1953-1970 (2000)

Grassl, H. The Climate at Maximum Entropy Production by Meridional and Atmospheric Heat Fluxes *Q. J. Roy. Met. Soc.* **107**, 153-166 (1990)

Hoffert, M. I., A. J. Callegari, C. T. Hsieh and W. Ziegler Liquid Water on Mars: An Energy Balance Climate Model for CO_2/H_2O Atmospheres *Icarus*, **47**, 112-129 (1981)

James, P. B. and G. North, J. The Seasonal CO_2 Cycle on Mars : An Application of an Energy Balance Model *J. Geophys. Res.* **87**, 10271-10283 (1982)

Lorenz, R. D., McKay, C. P. and Lunine, J. I. Photochemically-Driven Collapse of Titan's Atmosphere *Science* **275**, 642-644 (1997)

North, G. R., Cahalan, R. F. and Coakley, J. R. Energy Balance Climate Models, *Reviews of Geophysics and Space Physics*, **19**, 91-121 (1989)

Ohmura H. and Ozuma A. Thermodynamics of a Global-Mean State of the Atmosphere : A State of Maximum Entropy Increase *J. Climate*, **10**, 441-445 (1997)

Paltridge, G. W. Global Dynamics and Climate Change: A System of Minimum Entropy Exchange, *Q. J. Roy. Met. Soc.*, **101**, 475-484 (1975).

Samuelson, R. E., Nath, N. R. and Borysow, A. Gaseous Abundances and Methane Supersaturation in Titan's Troposphere *Planet. and Space Sci.*, **45**, 959-980 (1997)

Smith B A. et al Encounter with Saturn : Voyager 1 Imaging Results, *Science*, **212**, 163-182 (1981)

Stevenson, D. J. and A. E. Potter Titan's Latitudinal Temperature Distribution and Seasonal Cycle, *Geophys. Res. Lett.*, **13**, 93-96 (1986)

Williams, D. and Kasting, J. F. Habitable Planets with High Obliquities, *Icarus*, **129**, 254-267 (1997)

Wood, S. E. and Paige, D. A. Modeling the Martian Seasonal CO_2 Cycle: 1. Fitting the Viking Lander Pressure Curves, *Icarus* **99**, 1-14, (1992)

Wyant, P. H., Mongroo, A. and Hameed, S. Determination of the Heat-Transport Coefficient in Energy-Balance Climate Models by Extremization of Entropy Production *J. Atmos. Sci.*, **45**, 189-193 (1988)

R. D. Lorenz, J. I. Lunine and P. G. Withers, Department of Planetary Sciences, Lunar and Planetary Laboratory, University of Arizona, Tucson, AZ 85721. USA.
(email rlorenz@lpl.arizona.edu)

C. P. McKay, NASA Ames Research Center, Moffett Field, CA 94035, USA. (email mckay@gal.arc.nasa.gov)

(Received September 13, 2000; revised November 14, 2000; accepted November 16, 2000.)