PTYS/ASTR 206 – Section 3 – Homework 5 – Assigned 4/9/09

NAME:____(PRINT CLEARLY)

- Homework is due in class on Thursday April 16th.
- Late homeworks can be turned in class on Tuesday April 21st for 50% credit.
- Homeworks turned in later than this receive 0%.
- Students are encouraged to discuss approaches to solving homework problems with each other; however, all work submitted must be the student's own**. Do not turn in identical homeworks!** See the syllabus for more information.

Hint: Each of these questions should be quick to answer. If you find yourself engaged in a long chain of complicated reasoning or more than a few lines of math then something is probably wrong! Make sure to start this early and talk to the TA or myself with any questions.

Question 1: Jupiter's Interior

The rocky core of Jupiter has a radius of about 5,500km (close to the size of the Earth) and contains about 2.6% of Jupiter's mass. What is the density of Jupiter's core? [look up the mass of Jupiter to get started – this problem is pretty similar to Q1 of the last homework]

PTYS/ASTR 206 – Section 3 – Homework 5 – Assigned 4/9/09

The average density of the Earth is about 5500 kg m^{-3} and the average density of Earth's iron core is about 13000 kg m^{-3}. Compare these to the density of Jupiter's core you just derived, by what factor is the answer larger/smaller and why is it so different to the Earth?

Jupiter's rocky core is probably at a temperature of about 25,000K. Rocks on the Earth's surface melt close to 1,000K yet we think Jupiter's core is solid. How can this rocky material be solid at such high temperatures?

Question 2: Gas giant rotation

Jupiter and Saturn both rotate very quickly (in about 10 hours). Material at the equator rotates fastest, how fast do this material move? Fill in the following table to find out, you'll need to figure out the planets' circumference to know the physical distance the material moves in one rotational period. Express the speed this material moves at in km s^{-1}, be careful with units.

	Rotation period	**Radius**	**Circumference**	**Speed @ Equator**
Jupiter	9.925 hours	71,492 km		
Saturn	10.5 hours	60,268 km		
Earth	24 hours	6,378 km		

The circumference of a planet is $2\pi R$, where R is the planet's radius.

PTYS/ASTR 206 – Section 3 – Homework 5 – Assigned 4/9/09

How many times faster does the equator of Saturn move compared to the equator of the Earth? Why does this really fast rotation lead to flattening of the planet?

We describe flattening by figuring out the difference between the polar and equatorial radii as a fraction of the equatorial radius e.g. Mars bulges by 20km at the equator (R_e– R_p = 20km) and its equatorial radius is 3396km, so the flattening is 20km/3396km or 0.006. How flattened are the three planets we've been talking about? Use this table.

	Equatorial Radius	**Polar Radius**	**R_e – R_p**	**Flattening**
Jupiter	71,492 km	66,854 km		
Saturn	60,268 km	54,364 km		
Earth	6,378 km	6,357 km		

Earth and Mars are both made of the same stuff and both rotate at roughly the same speed yet they are flattened by different amounts. Why is the Earth more/less flattened than Mars?

PTYS/ASTR 206 – Section 3 – Homework 5 – Assigned 4/9/09

Question 3: Io's Volcanoes

Io spews out so much volcanic material that the surface is buried by a 1cm thick layer every year. If every square meter of Io's surface is covered like this then we can figure out the volume of this material. There are formulas in the previous homeworks for surface area and volume of a sphere.

What is the surface area of Io in square meters (the radius of the body is 1820km)?

If all these square meters are covered with 1cm of material every year, what is the volume of this material in cubic meters?

What is the volume of Io? How many years does it take Io's volcanoes to produce that volume of material? (we're ignoring the fact that the density of Io is higher in the interior here).

How many times over solar system history (4.5 billion years) has Io turned itself inside-out like this? What does that mean for the age of Io's surface?

PTYS/ASTR 206 – Section 3 – Homework 5 – Assigned 4/9/09

Question 4: Saturn's rings

The Huygens gap is a prominent gap in Saturn's ring system located about 117,000 km from Saturn's center. This gap is clear because any particles orbiting in it would be in a 2:1 resonance with one of Saturn's moons i.e. the particle would orbit twice for one orbit of the Moon. The repeated interactions with this Moon would eject these particles from the gap.

Kepler's 3rd law tells us that the period (P) and size (a) of an orbit are related by:

$$a^3 \propto P^2 \qquad or \qquad a \propto P^{2/3}$$

If we double the period of the orbit then by what factor does the size of the orbit increase?

If the period of the moon causing the Huygens gap has an orbital period twice that of particles in the gap then how large is the orbit of this moon?

The orbits of some of Saturn's moons are listed here:

Moon	Mimas	Enceladus	Tethys	Dione	Rhea
Orbital radius (in km)	185,000	238,000	295,000	377,000	527,000

Which of these moons is the likely candidate for clearing the Huygens gap?

PTYS/ASTR 206 – Section 3 – Homework 5 – Assigned 4/9/09

Question 5: The Roche limit

Moons of planets get pulled apart by tidal forces if they get too close. The smallest orbit that a moon can have before it can no longer hold itself together by self-gravity is called the Roche limit and given by:

$$2.4\left(\frac{\rho_P}{\rho_m}\right)^{1/3} R_P$$

where R_p is the radius of the planet and the ρ_p and ρ_m are the densities of the planet and moon respectively. If the density of Saturn is 687 kg m^{-3} and its moons are made of ice (density ~1000 kg m^{-3}) then how big is this orbit?

Compare this to the distance that Saturn's moons orbit at (see table in question 4). Are they inside or outside this limit?

The rings of Saturn are mostly inside this Roche limit, which is why the ring particles cannot clump back together to form a big moon. Why doesn't Saturn's tidal forces destroy the ring particles themselves though?

PTYS/ASTR 206 – Section 3 – Homework 5 – Assigned 4/9/09

The Earth has a density of about 5500 kg m^{-3} and its moon is made of rock (density ~3350 kg m^{-3}). What's the Roche limit in this case? Is the Moon inside or outside this limit?

Artificial Earth Satellites are within the Roche limit, why doesn't Earth's tidal forces rip them to pieces?