PTYS/ASTR 206 – Section 3 – Homework 5 – Assigned 4/9/09

NAME: Solution #5 (PRINT CLEARLY)

- Homework is due in class on Thursday April 16th.
- Late homeworks can be turned in class on Tuesday April 21st for 50% credit.
- Homeworks turned in later than this receive 0%.
- Students are encouraged to discuss approaches to solving homework problems with each other; however, all work submitted must be the student's own. **Do not turn in identical homeworks!** See the syllabus for more information.

Hint: Each of these questions should be quick to answer. If you find yourself engaged in a long chain of complicated reasoning or more than a few lines of math then something is probably wrong! Make sure to start this early and talk to the TA or myself with any questions.

Question 1: Jupiter's Interior

The rocky core of Jupiter has a radius of about 5,500km (close to the size of the Earth) and contains about 2.6% of Jupiter's mass. What is the density of Jupiter's core? [look up the mass of Jupiter to get started – this problem is pretty similar to Q1 of the last homework]

Jupiter's mass is 1.9×10^{27} kg

Jupiter's core is $0.026 \times 1.9 \times 10^{27}$ kg $= 4.94 \times 10^{25}$ kg

Volume of the core is: $\frac{4}{3}\pi R^3$

$\frac{4}{3}\pi (5500\,000\,m)^3$

$6.97 \times 10^{20}\ m^3$

Density $= \frac{mass}{volume} = 70\,884\ kg\,m^{-3}$

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The average density of the Earth is about 5500 kg m^{-3} and the average density of Earth's iron core is about 13000 kg m^{-3}. Compare these to the density of Jupiter's core you just derived, by what factor is the answer larger/smaller and why is it so different to the Earth?

Jupiter's core is $\frac{70\,884}{5\,500}$ = 12.9 times denser than Earth

Jupiter's core is ~~13000~~ $\frac{70\,884}{13\,000}$ = 5.5 times denser than Earth's core.

Jupiter's core is ~~very~~ dense because it is compressed by the weight of the over lying material.

Jupiter's rocky core is probably at a temperature of about 25,000K. Rocks on the Earth's surface melt close to 1,000K yet we think Jupiter's core is solid. How can this rocky material be solid at such high temperatures?

The high pressures at Jupiter's core allow the rock to stay solid.

Question 2: Gas giant rotation

Jupiter and Saturn both rotate very quickly (in about 10 hours). Material at the equator rotates fastest, how fast do this material move? Fill in the following table to find out, you'll need to figure out the planets' circumference to know the physical distance the material moves in one rotational period. Express the speed this material moves at in km s^{-1}, be careful with units.

	Rotation period	Radius	Circumference	Speed @ Equator
Jupiter	9.925 hours	71,492 km	449 197 km	12.57 km s^{-1}
Saturn	10.5 hours	60,268 km	378 675 km	10.02 km s^{-1}
Earth	24 hours	6,378 km	40 074 km	0.46 km s^{-1}

The circumference of a planet is 2πR, where R is the planet's radius.

$$\text{Speed} = \frac{2\pi . R}{\text{period in hours} * 3600}$$

↑ to convert to seconds.

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How many times faster does the equator of Saturn move compared to the equator of the Earth? Why does this really fast rotation lead to flattening of the planet?

Saturn's equator moves $\frac{10.02\ km\,s^{-1}}{0.46\ km\,s^{-1}} = 21.8$ times faster

Fast rotation increases the outward centrifugal forces. These forces are highest at the equator so the planet bulges outwards there.

We describe flattening by figuring out the difference between the polar and equatorial radii as a fraction of the equatorial radius e.g. Mars bulges by 20km at the equator ($R_e - R_p$ = 20km) and its equatorial radius is 3396km, so the flattening is 20km/3396km or 0.006. How flattened are the three planets we've been talking about? Use this table.

	Equatorial Radius	Polar Radius	$R_e - R_p$	Flattening
Jupiter	71,492 km	66,854 km	4638 km	0.0649
Saturn	60,268 km	54,364 km	5904 km	0.0980
Earth	6,378 km	6,357 km	21 km	0.0033

Earth and Mars are both made of the same stuff and both rotate at roughly the same speed yet they are flattened by different amounts. Why is the Earth more/less flattened than Mars?

Earth is less flattened.

Earth's gravity is higher and can overcome more of the outward centrifugal forces

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Question 3: Io's Volcanoes

Io spews out so much volcanic material that the surface is buried by a 1cm thick layer every year. If every square meter of Io's surface is covered like this then we can figure out the volume of this material. There are formulas in the previous homeworks for surface area and volume of a sphere.

What is the surface area of Io in square meters (the radius of the body is 1820km)?

If all these square meters are covered with 1cm of material every year, what is the volume of this material in cubic meters?

Surface area of Io $= 4\pi R^2 = 4\pi (1820\,000\,m)^2$
$= 4.16 \times 10^{13}\ m^2$

1 cm of material per square meter is $0.01\ m^3$
Volume of this material over the entire planet is this times the surface area. $= 4.16 \times 10^{11}\ m^3$ each year

What is the volume of Io? How many years does it take Io's volcanoes to produce that volume of material? (we're ignoring the fact that the density of Io is higher in the interior here).

Volume of Io $= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (1820000\,m)^3$
$= 2.5 \times 10^{19}\ m^3$

No. of years to produce this material is $\frac{2.5 \times 10^{19}\ m^3}{4.16 \times 10^{11}\ m^3 \text{ per year}}$
5.49×10^7 years
or ~ 55 million years

How many times over solar system history (4.5 billion years) has Io turned itself inside-out like this? What does that mean for the age of Io's surface?

How often has this happened in 4.5×10^9 years?
$$\frac{4.5 \times 10^9 \text{ years}}{5.5 \times 10^7 \text{ years}} = 82 \text{ times}$$

Io's surface is renewed many times over the age of the solar system so the surface is young

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Question 4: Saturn's rings

The Huygens gap is a prominent gap in Saturn's ring system located about 117,000 km from Saturn's center. This gap is clear because any particles orbiting in it would be in a 2:1 resonance with one of Saturn's moons i.e. the particle would orbit twice for one orbit of the Moon. The repeated interactions with this Moon would eject these particles from the gap.

Kepler's 3rd law tells us that the period (P) and size (a) of an orbit are related by:

$$a^3 \propto P^2 \quad or \quad a \propto P^{2/3}$$

If we double the period of the orbit then by what factor does the size of the orbit increase?

Orbit 1 : $a_1 \propto P_1^{2/3}$

Orbit 2 : $a_2 \propto P_2^{2/3}$ or $(2P_1)^{2/3}$ [If $P_2 = 2P_1$]

$$\frac{a_2}{a_1} = \frac{(2P_1)^{2/3}}{P_1^{2/3}} = 2^{2/3} = 1.59$$

The orbit increases in size by a factor of 1.59

If the period of the moon causing the Huygens gap has an orbital period twice that of particles in the gap then how large is the orbit of this moon?

The Huygens gap is at 117,000 km.
The moon with twice the orbital period of a particle in the Huygens gap is at

$1.59 \times 117{,}000$ km $= 185\,726$ km

The orbits of some of Saturn's moons are listed here:

Moon	Mimas	Enceladus	Tethys	Dione	Rhea
Orbital radius (in km)	185,000	238,000	295,000	377,000	527,000

Which of these moons is the likely candidate for clearing the Huygens gap?

Mimas matches the position of the moon that caused the Huygens gap.

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Question 5: The Roche limit

Moons of planets get pulled apart by tidal forces if they get too close. The smallest orbit that a moon can have before it can no longer hold itself together by self-gravity is called the Roche limit and given by:

$$2.4\left(\frac{\rho_P}{\rho_m}\right)^{1/3} R_p$$

where R_p is the radius of the planet and the ρ_p and ρ_m are the densities of the planet and moon respectively. If the density of Saturn is 687 kg m^{-3} and its moons are made of ice (density ~1000 kg m^{-3}) then how big is this orbit?

$$2.4\left(\frac{687\ \text{kg m}^{-3}}{1000\ \text{kg m}^{-3}}\right)^{1/3} 60{,}268\ \text{km}$$

127,629 km

Compare this to the distance that Saturn's moons orbit at (see table in question 4). Are they inside or outside this limit?

They are all outside this radius, as expected. If they were inside they would already have been destroyed.

The rings of Saturn are mostly inside this Roche limit, which is why the ring particles cannot clump back together to form a big moon. Why doesn't Saturn's tidal forces destroy the ring particles themselves though?

The ring particles themselves are not held together by self-gravity i.e. they are single particles, not collections of particles.
Single particles are held together by mechanical forces much stronger than self gravity.

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The Earth has a density of about 5500 kg m^{-3} and its moon is made of rock (density ~3350 kg m^{-3}). What's the Roche limit in this case? Is the Moon inside or outside this limit?

$$2.4 \left(\frac{5500 \text{ kg m}^{-3}}{3350 \text{ kg m}^{-3}}\right)^{1/3} 6378 \text{ km}$$

18,058 km

The moon orbits far outside this limit at a distance of 385,000 km. It is safe from Earth's tidal effects.

Artificial Earth Satellites are within the Roche limit, why doesn't Earth's tidal forces rip them to pieces?

Again, these satellites are held together by forces stronger than self gravity. This is basically the same reason that Saturn's ring particles survive except in this case we physically bolt the spacecraft together rather than fashioning it as a single piece.