PTYS/ASTR 206 – Section 3 – Homework 6 – Assigned 4/23/09

NAME:_____(PRINT CLEARLY)

- Homework is due in class on Thursday April 30th.
- Late homeworks can be turned in class on Tuesday May 5th for 50% credit.
- Homeworks turned in later than this receive 0%.
- Students are encouraged to discuss approaches to solving homework problems with each other; however, all work submitted must be the student's own. **Do not turn in identical homeworks!** See the syllabus for more information.

Hint: Each of these questions should be quick to answer. If you find yourself engaged in a long chain of complicated reasoning or more than a few lines of math then something is probably wrong! Make sure to start this early and talk to the TA or myself with any questions.

Question 1: Mass of Pluto

Kepler's third law applies to Charon orbiting Pluto, but not in as simple a way as it applied to planets orbiting the sun. In the latter case we could directly relate semi-major axis (in AU) to period (in years). This time we'll write Kepler's 3rd law out in full and use meters and seconds instead of AU and years. When we do this it reads:

$$a^3 = 1.7 \times 10^{-12} \quad M_{P+C} \quad P^2$$

Let's forget for the moment that Pluto is orbiting Charon as well as Charon orbiting Pluto. Look up Pluto and Charon's separation (that's the 'a' in the above formula) and period (that's the 'P' in the above formula). What is the combined mass of Pluto and Charon (M_{P+C})? [*Make sure the separation and period you use are in meters and seconds respectively*]

$a = 17\,536\ km = 1.7 \times 10^7\ m$

$P = 6.39\ days = 5.5 \times 10^5\ sec$

$$a^3 = 1.7 \times 10^{-12}\ M_{p+c}\ P^2$$

$$(1.7 \times 10^7)^3 = 1.7 \times 10^{-12}\ (5.5 \times 10^5)^2\ M_{p+c}$$

$$4.91 \times 10^{21} = 0.52\ M_{p+c}$$

$$9.45 \times 10^{21} = M_{p+c} \quad \text{(Answer is in kg)}$$

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Pluto's radius is twice that of Charon. How much higher is Pluto's volume than Charon's? What is Pluto's volume as a fraction of the total volume?

Volume $= \frac{4}{3}\pi\,(\text{Radius})^3$

charon: $V_c = \frac{4}{3}\pi (R_c)^3$

pluto: $V_p = \frac{4}{3}\pi (R_p)^3 = \frac{4}{3}\pi (2R_c)^3 = 8.\left[\frac{4}{3}\pi R_c^3\right] = 8.V_c$

The volume of pluto is eight times that of Charon

Plutos volume as a fraction of the total $= \frac{V_p}{V_p+V_c} = \frac{8V_c}{8V_c+V_c} = \frac{8V_c}{9V_c} = \frac{8}{9}$

If both objects have the same constant density then what is Pluto's Mass on its own?

Total Mass is 9.45×10^{21} kg.

Pluto has $\frac{8}{9}$ of the total volume.

if density is constant then: $\frac{8}{9}$ volume $= \frac{8}{9}$ mass

Pluto has $\frac{8}{9}$ of the total mass: $\frac{8}{9} \times (9.45 \times 10^{21}) = 8.4 \times 10^{21}$ kg

Question 2: Orbit of Pluto and Charon

Charon orbits Pluto every 6.4 days. Pluto and Charon also both rotate every 6.4 days. How would Charon appear to move to an observer on Pluto?

Charon would appear to stay in the exact same place as seen by an observer on Pluto.

Only observers on one side of Pluto can see Charon.

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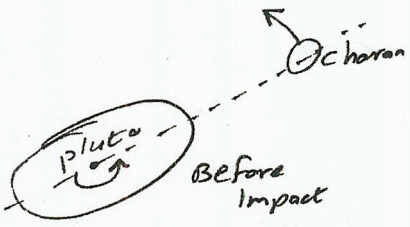
The Pluto/Charon system is thought to have formed in a collision (much like the Earth/Moon system). If something were to hit Pluto today and Pluto's rotation were to speed up then what would happen to the orbit of Charon? What would happen to Pluto's new rotation rate?

If pluto were to speed up then the tidal bulge would lead charon. ie.



This would cause Charon to speed up (attracted towards the bulge) and so drift outwards. It would also cause Pluto's rotation to slow down (bulge is attracted towards Charon

What is the angular size of Charon as seen from Pluto? What is the angular size of the sun as seen from Pluto? Could an observer on Pluto see a total solar eclipse?

$$\text{Angular size} = \frac{\text{physical size}}{\text{distance}}$$

$$\text{Charon}: \quad \frac{1207\ \text{km}}{17\,536\ \text{km}} = 0.069\ \text{radians}$$

$$\text{Sun}: \quad \frac{1.4 \times 10^6\ \text{km}}{30\ \text{AU}} = \frac{1.4 \times 10^6\ \text{km}}{30 \times (1.5 \times 10^8\ \text{km})} = 0.0003\ \text{radian}$$

Any value between 29 & 49 AU is acceptable

The sun is much smaller so an observer on pluto could see a total solar eclipse

Question 3: Triton

Why do we think that Triton is a captured object that didn't form alongside with Neptune?

It orbits in a retrograde direction and so cannot have formed in a disk of material surrounding Neptune.

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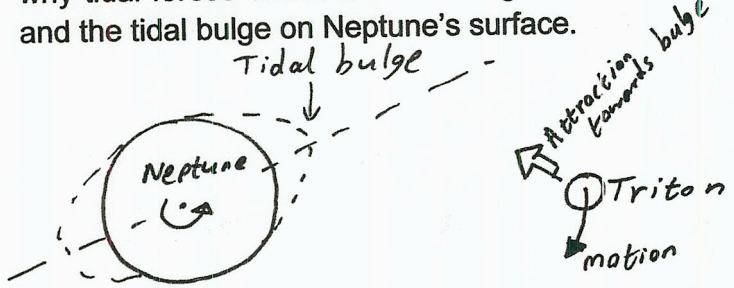
The size of Triton's orbit changes by 10cm/year. From a sketch of Triton's orbit explain why tidal forces cause the orbit to get smaller. Start by drawing Neptune, Triton's orbit and the tidal bulge on Neptune's surface.



Triton is attracted to Neptunes tidal bulge.

So Triton slows down in its orbit.

slowing down causes it to spiral inwards.

We saw in the last homework that the smallest orbit that a moon can have before it can no longer hold itself together by self-gravity is called the Roche limit and given by:

$$2.4\left(\frac{\rho_P}{\rho_m}\right)^{1/3} R_P$$

where R_p is the radius of the planet and the ρ_p and ρ_m are the densities of the planet and moon respectively. If the density of Neptune is 1640 kg m^{-3} and Triton is 2060 kg m^{-3} then how big is this distance?

$$2.4\left(\frac{1640}{2060}\right)^{1/3} R_{Neptune}$$

$$2.2 \quad R_{Neptune}$$

$$2.2 \quad (24\,764\ km) = 54\,480\ km$$

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Triton currently orbits at 355,000 km. If its orbit continues to shrink at 10cm/year then how long will it be before it passes inside Neptune's Roche limit? What will future space tourists flock to see after this happens?

Current Distance from Roche limit = 355,000 km − 54,480 km

= 300 520 km

Speed = 10 cm / year

$$\text{Time} = \frac{300\,520 \text{ km}}{10 \text{ cm/year}} = \frac{300\,520\,000 \text{ m}}{0.1 \text{ m/year}} = 3 \times 10^9 \text{ years}$$

In 3 billion years Triton will break up & form a spectacular ring system around Neptune.

Question 4: Kuiper Belt Objects

What are plutinos? How do they differ from the cubewanos?

Plutinos are kuiper belt objects in a special resonance with pluto. They orbit twice every time Neptune orbits three times.

Cubewanos have no special connection to Neptunes orbit. They orbit the sun between 30 & 50 AU.

A large Kuiper belt object named Ixion has a semimajor axis of 39.68 AU. What is its orbital period? Neptune's orbital period is ~165 days, is Ixion a plutino or a cubewano? [You can use the simple form of Keplers 3rd law here]

From Kepler's 3rd law $a^3 = P^2$ (When a is in AU & P is in years) so $P = a^{3/2}$

$P = (39.68)^{3/2}$ years = 250 years

2 x Ixion's orbit period = 2 x 250 yrs = 500 yrs

3 x Neptune's orbital period = 3 x 165 yrs = 495 yrs

This is a close match (only 1% difference) so Ixion is a plutino

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Why is Pluto no longer considered a planet? Changing Pluto's status was a little controversial; do you agree with the decision?

One of the criteria for planethood introduced in 2006 was that the object has to clear its neighborhood of other bodies. Pluto hasn't done this so it's no longer considered a planet.

Personally, I don't consider Pluto a planet, but also think the definition is unclear.

Question 5: Comets

One theory for where all Earth's oceanic water comes from is that it was delivered by comets early in solar system history. If a comet is a sphere 10km in diameter how many cubic meters of water does each one deliver (let's ignore the difference in density between ice and water and all the other junk contained in the comet)?

Volume is $\frac{4}{3}\pi R^3$ $\quad R = 5\text{ km} = 5000\text{ m}$

$$\text{Volume of comet} = \frac{4}{3}\pi (5000)^3 \text{ m}^3$$
$$= 5.2 \times 10^{11} \text{ m}^3$$

Earth's oceans cover 75% of the globe and are 4km deep how much water do they contain (this is **very** similar to question 3 on HW 5)? How many comets do you need to deliver this much water?

$$\text{Surface area of Earth} = 4\pi R_E^2$$
$$= 4\pi (6400\,000)^2 \text{ m}^2$$
$$= 5.1 \times 10^{14} \text{ m}^2$$

75% of this = ocean area = 3.9×10^{14} m^2

Oceans are 4 km deep (= 4000 m)

Volume of ocean is depth x area = $4000 \times 3.9 \times 10^{14}$ m^3

$$= 1.6 \times 10^{18} \text{ m}^3$$

$$\text{No. of comets needed} = \frac{\text{Volume of ocean}}{\text{Volume of comet}} = 3.1 \times 10^6$$

About 3 million comets

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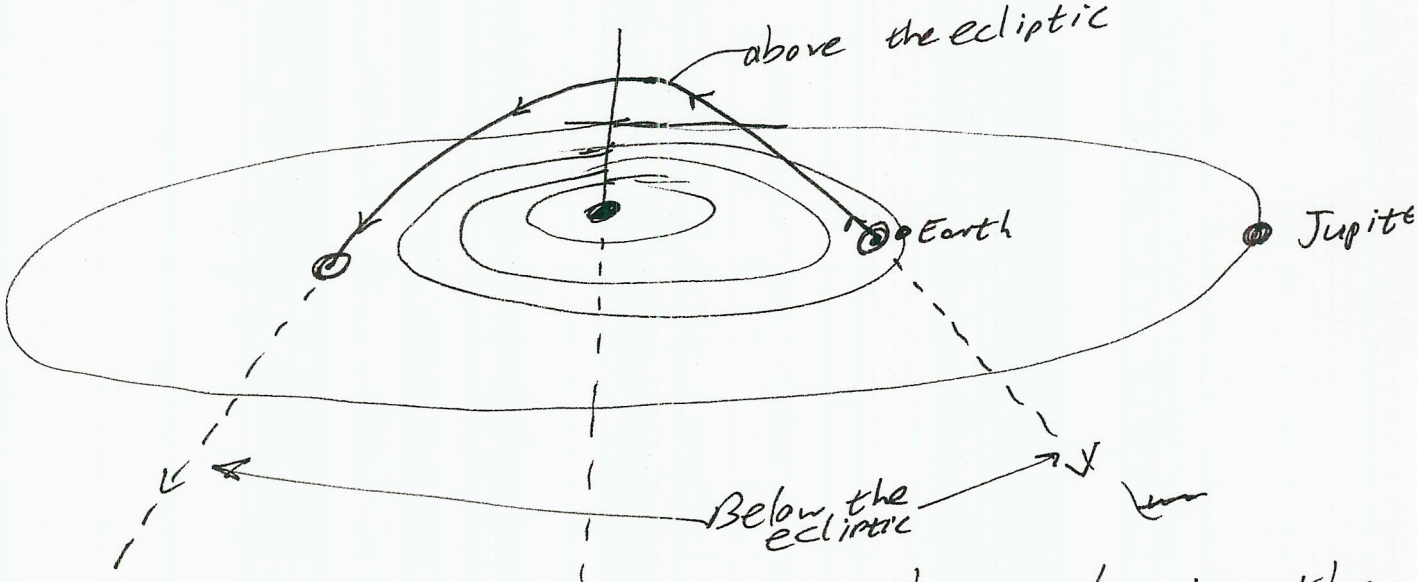
Comet Hale Bopp entered the inner solar system (perihelion was ~0.9 AU) with an inclination of almost 90° back in 1997 (it will return in 4530). Based on the inclination of its orbit, where did this comet likely originate?

The high inclination suggests it come from the Oort cloud.

Comets from the kuiper belt typically have low inclinations.

This comet is likely to survive quite a long time in this orbit. Why do you think that is? [You could try and sketch a picture of the solar system and Hale-Bopp's orbit]

Looking at the solar system from the side



Hale Bopp orbits almost at right angles to the plane of the solar system. It can only hit a planet at two points in its orbit so it's likely to last a long while