PTYS 411 – Geology and Geophysics of the Solar System
Homework #1 – Assigned 1/30, due 2/13

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1) Time for clouds to collapse. We've seen that planet formation is pretty fast (planetary disks dissipate in 10 Myr) and cleaning up the debris through impacts takes a little longer (~100 Myr). But how long does it take to make a disk to begin with? Show that the freefall timescale for a giant molecular cloud to collapse is given by:

$$t_{ff} = \sqrt{\frac{3\pi}{32G\rho}}$$

where ρ is density and G is the gravitational constant.
The place to start here is to think of a particle in free-fall as being in a very large orbit. Orbit sizes and periods are related by Kepler's 3^{rd} law. You can see by looking at it that this is an approximate expression so be ready to make some approximations.

For typical cloud densities (a few 1000 H_2 molecules per cubic centimeter), how does this freefall timescale compare to the age of the solar system?

2) *Magma Oceans.* The heating for planetary bodies comes from three sources: 1. Heat of accretion from impacting planetesimals, 2. Short-lived radioisotopes (such as Al^{26}) and 3. Long-lived radioisotopes (such as decay of uranium).
The Earth accreted too late for short-lived isotopes to contribute much heat, but too quickly for long-lived isotopes to be important. So most of the heat in the initially formed Earth was delivered by the impacting planetesimals that it is comprised of.

Show that the total energy delivered by this mechanism is: $\frac{3GM_E^2}{5R_E}$

[Assume a constant density for the growing Earth and that impacting bodies have zero energy at infinity before free-falling onto the planet.]

Assume all this energy goes into heat, write an expression for the temperature rise of the planet [Use a heat capacity of 800 J kg^{-1} k^{-1}]. Is this enough to melt the body (Rock melts at ~1300K)? This model is very simplistic; give a reason why this temperature rise might be an overestimate and a reason that it might be an underestimate.

What is the minimum size a planet must grow to in order to be completely melted by this process? Compare this to the size of the terrestrial planets, the Moon and some of the largest asteroids. Should we expect magma oceans on newly formed planets?

3) Isostasy. On Venus plate tectonics is absent. Down-welling flows in Earth's mantle are usually associated with subduction whereas on Venus it's thought to cause shortening of the crust.
   Think of a linear strip of the crust that has a width w_o. It gets compressed and reduced in width to w. i.e. the compression factor is $C_f = w_o/w$. This compression builds mountains that are supported by Airy isostasy i.e. they float like icebergs in the Venusian mantle. Show that the mountain height is given by

$$h = T_L \frac{\rho_m - \rho_c}{\rho_m}\left(C_f - 1\right)$$

   where the mantle and crust densities are ρ_m (~3300 kg m^{-3}) ρ_c (~2750 kg m^{-3}) respectively and T_L is the thickness of the crust. How tall do these mountains get when crustal rocks get compressed by a factor of two?

   Assume a representative Venusian crustal thickness of 70km

   What free-air gravity anomaly would you expect over this feature (in milligals)?

   What Bouguer gravity anomaly would you expect (in milligals)?

4) More on Isostasy. The very large lunar south pole Aitken basin is currently about 8km deep and has no major gravity anomaly associated with it. If this impact originally excavated all the way through the crustal material (density 2800 kg m^{-3}) to the mantle (3300 kg m^{-3}) then how thick is the lunar crust?

5) Define the difference between a planet's lithosphere and crust. What sets the thickness of a planets lithosphere? Is the lithosphere of the Earth thicker or thinner than its crust (let's just assume we're talking about oceanic crust here)?

   Is the lithosphere on small planets like the Moon and Mars thicker or thinner than larger planets like Earth and Venus? Why?

   How does the behavior of surface and deeply buried rocks differ when they're put under stress?

   What's the pressure at the bottom on the Marineris trench on the Earth (~12km below sea level)? If I were to put a rock down there what differential stress would it experience? Would you expect typical rocks to fracture in this environment?

The next page has an extra credit problem…

6) *A Molten lunar core?* In this problem we'll assume that heat production continues inside the Moon from long-lived radioisotopes (a good assumption) and that the Moon is a homogeneous body (a bad assumption).

Heat inside the Moon is conducted toward the surface, since the Moon has had plenty of time to reach a steady state all the radiogenic heat is conducted out at the same rate as it's produced and the temperatures within the body are stable. Show that the energy production from radioactive decay per unit mass (H) is:

$$H = {}^{3q}\!/\!_{R_P \rho}$$

where q is the surface heat flux, ρ is density and R_p the radius of the planet. q was measured by the Apollo astronauts at about 16 mW m^{-2}, what values do you get for H?

The heat flux (F) inside the Moon from conduction is: $F = -k\frac{\delta T}{\delta R}$ where k is the thermal conductivity. Show that the temperature (T) inside the Moon is:

$$T = T_o + \frac{\rho H}{6k}\left(R_P^2 - R^2\right)$$

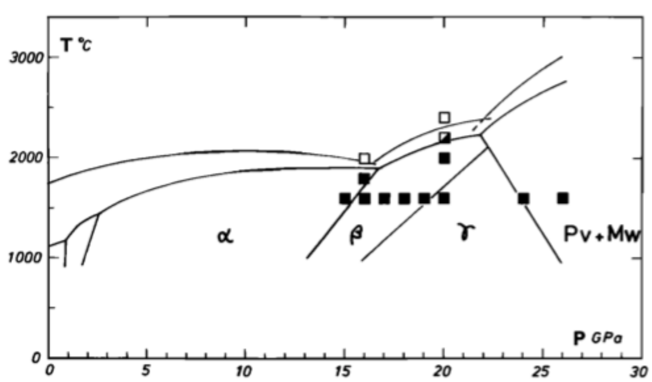
where T_o is the surface temperature (~300K).

What is the temperature at the center of the Moon? What's the main thing wrong with this model?

Melting has as much to do with pressure as temperature. Show that the pressure within the (again homogeneous) Moon is given by:

$$P = \frac{2\pi}{3}G\rho^2\left(R_P^2 - R^2\right)$$

[Hint – as with most of these integrals, start with an expression for a thin shell and integrate]. What's the pressure at the center of the Moon?



Look at the peridotite (similar to Earth's mantle and the lunar interior) phase diagram to the left (Takahashi, J. Geophys. Res., 1986); at this pressure what temperature do you need to have a melt?

Recent discoveries (Weber et al. Science 2011) indicate the Moon's core is molten.