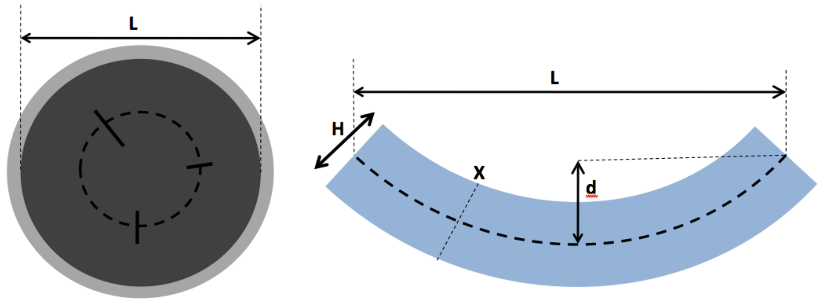
**PTYS 411 – Geology and Geophysics of the Solar System**
**Homework #2 – Assigned 2/13, due 2/27**

---

1) *Planetary thrust faults.* The subsidence of lunar basins under the weight of the mare basalts has generated thrust faults near their center. Imagine a circular plate, thickness H, has been downwarped a distance d. Its top surface is compressed and its lower surface is stretched while the material at a depth of H/2 feels zero strain (see cartoon below).




Show that the circumferential surface stresses (parallel to the rim) at point X (half way between center and rim) are given by:

$$\frac{4dHE}{L^2}$$ where E is Young's Modulus

Hints: Assume d << L in this problem. The dashed line in the cross-sectional view still has its original length, but the upper surface has been compressed. The warped surface is shaped like a section of a sphere. Stress is just Young's Modulus times strain so really this is asking you to show that strain (change of circumference divided by the original circumference) is $4dH/L^2$.

Try this for one of the mare where L is ~300km, H is ~50km (lithosphere at the time of loading) and d is ~2km. Is the resultant stress large enough to overcome typical rock strengths? (about 100 MPa)

If three thrust faults form with a typical dip then how much displacement will each fault experience?

2) *Moments of inertia*. A lot can be discovered from a planet from its moment of inertia. Moment of inertia depends on the geometry of the object: sphere vs empty shell vs point etc… but in general is given by k M $R^2$, where M is the mass, R is the radius and k is a constant e.g. for a point of mass M orbiting at distance R k=1, for a rotating thin hollow sphere M is the mass of the shell, R is its radius and k = 2/3.

   Use the moment of inertia of the thin hollow shell mentioned above to show that the moment of inertia of a homogeneous solid (and spherical) planet is $\frac{2}{5} M_P R_P^2$

   In a differentiated planet (radius $R_p$ with density $\rho_c$ in a core of radius $R_c$ and density $\rho_m$ in the mantle surrounding the core) the moment of inertia is:

$$I_{differentiated} = \frac{2}{5}\left[\frac{1+c\,x^5}{1+c\,x^3}\right] M_P R_P^2$$

   *where* $c = \frac{(\rho_c - \rho_m)}{\rho_m}$ *and* $x = \frac{R_c}{R_P}$. [Extra credit if you can prove this. The derivation isn't that bad. Break the previous integral into two parts.]

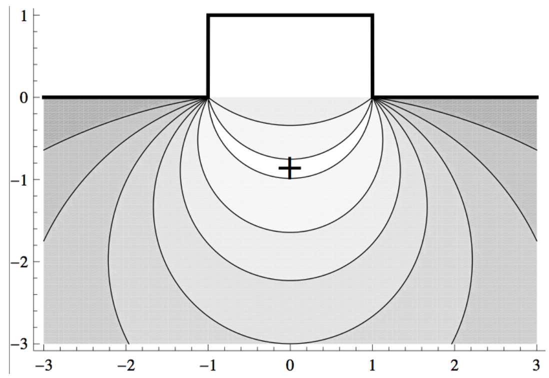
   Assume that the core is twice as dense as the mantle. Plot the value of the geometry-dependent constant $\frac{2}{5}\left[\frac{1+c\,x^5}{1+c\,x^3}\right]$ vs. x.

   Mars has a moment of inertia of 0.3662 $MR^2$. Use your plot to find x, and by extension the core size on Mars (the real value core radius has been estimated at ~0.48 $R_P$)?
   There are two solutions. Although it's clear which is the correct one, how would you distinguish between them if it wasn't so clear? Give a hand-waving explanation as to why are there are two solutions?
   The Moon has a moment of inertia of 0.3931 $MR^2$. Use your plot to find x, and by extension the lunar core size?
   If Mercury's core radius is 0.72 of its total radius and its moment of inertia is 0.33 $MR^2$ then what is the density ratio between its core and mantle?

   Titan is an exciting moon of Saturn that's very geologically active. Titan's mean density is 1880 kg $m^{-3}$, assume that it's made up only of different phases of water (~1000 kg $m^{-3}$) and rock (~3300 kg $m^{-3}$) and that it's fully differentiated. What moment of inertia factor do you expect Titan to have?
   Cassini tracking data published last year has shown this value to be 0.34. Compare this number to what you expected from the above calculation. What do we learn about Titan from this comparison?

3) *Io's mountains*

We discussed in class the maximum shear stress generated by a surface load is just a fraction (usually about a third to a half) of the peak load itself. For example, the rectangular block mountain and stress contours shown schematically here generates a peak shear stress in the subsurface of 0.352 $\rho gh$ at a depth of 0.865w (w=mountain width which we'll assume to be equal to h for now) (Melosh 2011). For the mountain to be supported then this stress must be less than the typical strength of rocks (~100 MPa).




Show that the maximum topography than can be supported like this is:

$$h \approx \left(\frac{0.7\ \ \sigma_y}{G\bar{\rho}\ \rho_c}\right) \Big/ R_{planet}$$

Where $\bar{\rho}\ \rho_c$ are the planet's mean density and crustal density respectively and $\sigma_y$ is the strength of rock.

In class we discussed how well (or not) this works for the terrestrial planets. Using the above relationship, how high are the highest mountains on Jupiter's moon Io predicted to be?
(crustal density is ~3000 Kg m$^{-3}$)

Io has prodigious amounts of volcanic activity, but also possesses non-volcanic mountains that appear to be tilted crustal blocks. In reality, these mountains top out at only ~17km. So something else is limiting their height.

Io's average heat flux is a whopping 2.5 W/m$^2$ (Earth's is a comparatively measly 0.08 W/m$^2$), but most of that come though local areas of volcanic activity. In general, only a few percent (let's say about 0.1 W/m$^2$) is conducted through the lithosphere. When rocks get to about half their melting temperature then they stop being able to support elastic stresses for long periods.

With this info, and the above diagram, in mind, how high can mountains on Io get?
(Thermal conductivity is about 3 Wm/K, rock melts at ~1200K and Io's surface temperature is ~100K.)

If Io's mantle has a density of 3300 Kg m$^{-3}$ then how deep of a crustal root would be required to support a 17km high mountain through Airy Isostasy? Knowing what you now know about Io's internal temperatures is this a reasonable way to support these mountains?

4) If roughly 10 major basins (>900 km in diameter) formed on the Moon during late heavy bombardment. How many craters greater than 1km in size formed during this period? [Hint: Use the slopes of the power laws shown in class and solve for the constants, don't forget the slope changes value at certain crater diameters.]

The gravitationally enhanced cross-section of the Moon is given by:

$$\pi r_{Moon}^{2}\left(1+\left({v_{Moon\_escape}}/{v}\right)^{2}\right)$$

If all these objects approach the Earth/Moon at v=15 km$^{-1}$, how many hit the Earth during the same late heavy bombardment period?

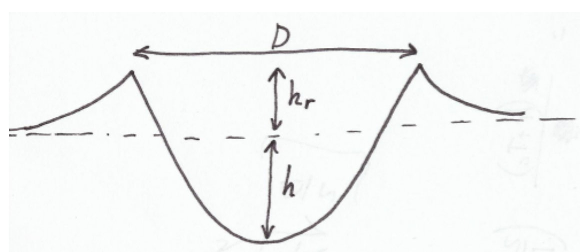
The actual impact speeds are given by: $v_i^2 = v^2 + v_{esc}^2$
What speeds do they hit each body at? How much extra impact energy did the Earth receive compared to the Moon?

When the velocity is size-independent like this, does most of the delivered impact energy come from the rarer large impacts or the more numerous small ones (and does the same hold true for craters less than 1km in size)?

Assume Lampson scaling for the connection between energy and crater size i.e. energy is proportional to crater diameter cubed.

**OPTIONAL EXTRA CREDIT QUESTION BELOW**

5) *Crater shapes.* Simple craters tend to be parabolas with h/D ~ 0.2. Ejecta blankets decrease in thickness according to the distance from the crater-center cubed. If volume is conserved in the crater creation process then derive the height of the rim ($h_r$) relative to the depth of the crater (h).




The answer is $h_r$ = 1/5 h

Hint: This is a challenge (but then that's why it's optional!). There are three volume integrals you need to do here. The interior bowl below ground and the above ground areas within and exterior to the rim. Volume is conserved so these should all sum to zero.