PTYS 411 – Geology and Geophysics of the Solar System
Homework #3 – Assigned 2/27 due 3/13

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1. *Heating from Al^{26}.* The following table appeared in McCord and Sotin (2005). C_o is the mass fraction of that element in the silicate and H_0 is the heat produced by 1 kilogram of the element (not the rock) at time zero.

Table 5. Radiogenic Heating Parameters Used in the Models[a]

Element	C_0, ppb	H_0 (mW/kg of Element)	Half-Life, Myr
^{26}Al	450	138	0.716
^{60}Fe	0.8	74.7	1.5
^{40}K	430	0.0619	1250
^{232}Th	130	0.0204	14000
^{235}U	17.5	0.401	704
^{238}U	52.4	0.104	4470

The number of radioactive decays at any one time (t) is: $\frac{\partial N}{\partial t} = -\lambda N$
Where λ is the decay constant (equal to ln(2)/half-life) and N is the number of radioactive atoms remaining.

Show that the heat produced at any one moment in time is $\frac{\lambda H C_0}{\mu} e^{-\lambda t}$ per kilogram of rock. Where H is the energy per decay, μ is the atomic weight Show also that energy produced at t=0 is $H_o = \frac{\lambda H}{\mu}$ for 1kg of the element and is $C_o H_o$ for 1kg of the rock.

Aluminum 26 dominates at the earliest times, how much heat is each element producing per kilogram of rock at t=0? Use values in the above table.
How long before Fe^{60} is providing more heat them Al^{26}, how long before the long-lived isotopes e.g. U^{235} are more important than ^{26}Al?

Studying these isotopes allows us to figure out the chronology of what happened when in the early solar system. The earliest solids (CAIs in chondrites) had initial concentrations of Al^{26} of $6x10^{-5}$ times that of Al^{27}. That ratio is only $8x10^{-6}$ in the earliest chondrules, how much time elapsed before these chondrules formed?

2) *Bingham flows.* The driving stress in a Bingham fluid must exceed a finite yield value in order for flow to start. Equate the stresses at the base of the flow to this yield value to get an expression for the thickness of the flow. For a given slope and lava composition, are lava flows thicker on Earth or Moon?

The flow spreads out laterally until the driving stresses fall below this yield stress. Since the pressure driving this lateral spreading increases with depth show that the flow will take on a parabolic cross-section.

Show that the minimum width of the flow is $\frac{\sigma_y}{\rho g \sin^2(\alpha)}$ (where σ_y is the yield stress and α is the slope). For a given slope and lava composition, are lava flows wider on Earth or the Moon?

How is the flow width to thickness ratio related to the slope? Will this be the same on Earth and the Moon?

(Adapted from Melosh 2011)

Steep flow fronts are observed at the edges of broad, extensive lava flows on the lunar Mare. These lava flows are considerably thicker than terrestrial lava flows, often reaching 100 m in height. The average slope of one such flow is about 0.5°. Compute the Bingham yield stress of this lava flow. Typical terrestrial basaltic lava flows have yield stresses of several thousand Pa. Is lunar magma substantially stronger than terrestrial magma?

3) Regolith generation. Here we'll step through a simple model devised by Shoemaker in the late 1960s that we talked about in class.

Craters form a continuous surface cover at diameters below the equilibrium diameter (D_{eq}). These small craters overlap and produce the fine-grained, poorly sorted, impact debris known as regolith. The efficiency of these craters in forming regolith depends on the depth to which they disturb the underlying material; these depths are commonly about D/4 (D=crater diameter). Craters larger than D_{eq}, although they also break up material into finer particles, tend to form isolated impact structures and not a continuous debris blanket.

Here, we'll assume that the surface is saturated at 4% of the geometric saturation (where hexagonally packed craters occupy 90.5% of the terrain's area). Show that the number density of carters in this saturated population is:

$$N_{eq} = c_{eq} D^{-2}$$

Where c_{eq} is a constant independent of crater size or landscape age, what is the value of c_{eq} in this case? This is the number of craters that exist, but not the number produced (as new craters remove old ones in a saturated surface).
The actual production function of craters is: $N(>D) = c\, D^{-b}$.

Find an expression for D_{eq} in terms of c_{eq} and c (age coefficient in the non-equilibrium population). This Show that:

$$D_{eq} \propto time^{1/b-2}$$

Show that the fractional area covered by all craters between D and D_{eq} is given by:

$$f(D, D_{eq}) = \frac{\pi b c_{eq}}{4(b-2)}\left[\left(\frac{D_{eq}}{D}\right)^{b-2} - 1\right]$$

At some crater diameter (smaller than D_{eq}) the target area is completely covered (i.e. f=1) and the bottoms of these craters are interconnected so that the depth to which they disturbed the underlying material will be the average regolith thickness.

Recall that the regolith thickness will be a quarter of this diameter and rearrange the above expression to give this mean regolith thickness in terms of D_{eq}.

Observations of the craters on the lunar mare suggest that b is 3.8 and that this terrain is saturated in craters 250m in diameter (D_{eq}) and smaller. Armed with this info and the relationships you just derived, how thick is the regolith on the lunar Maria? (Estimates from Apollo 16 seismic data are 10-15m… so this model isn't bad!)

How does the mean regolith thickness change with time? If it took 3.8 Gyr to build up this regolith cover then how long did it take to build up a 1m thick covering?
How thick will the regolith cover be at the end of the solar system ~5Gyr from now?

4) By the time this homework is due, we'll be about half of the way through the course. If you haven't already, it's time to devote some thought as to what your class project is going to be.

   Write a short description of what you'd like to do for your class project. I'm only looking for about one short paragraph here, but you should lay out what you want to do and its relevance to the course material.

   I don't expect anyone to sign their name in blood (although you certainly could if you wanted to). Laying out an idea doesn't commit you to doing this for your project later, but this is a chance to get the slowest part of the process (i.e. deciding on what to do) out of the way before things get crazy at the end of the term. You should treat this like any other homework problem; talk about it amongst yourselves (and to me) if that will help. Points are given for how well thought out the idea is.

   The advice I'll give up front is to pick something that sounds interesting to you without consideration of how difficult/easy you think it will be. Once you identify the problem you can always break off a manageable chunk that will be enough (and not too much) for a class project.

5) Geotherms and satellite heating. If Io's surface heat flux is 3 W m^{-2} then how much heat is produced per kilogram in the interior? Compare this with what a typical piece of solar system rock (chondrite) produces via radioactive decay i.e. 4×10^{-12} W kg^{-1}. If Io's bulk composition is chondritic then what fraction of Io's heat comes from radioactivity rather than tides?

   If Europa has liquid water 4km below the surface and the average surface temperature is 110K then what is Europa's heat flux? How much radiogenic heat is produced in the rocky portion of Europa via radioactive decay? What fraction of Europa's heat comes from radioactivity rather than tides?

   If there were no tidal heating on Europa the how thick would the ice-shell be?

   How thick would the ice shell be on Ganymede if the rocky portion of that body produces radiogenic heat at the chondritic rate and the surface temperature is similar to Europa?

   (Europa's H_2O layer is ~150km thick and the radius of Ganymede's rocky interior is about 68% of the body.)