PTYS 411: Geology and Geophysics of the Solar System
Homework #5 – Assigned 4/10, due 4/24

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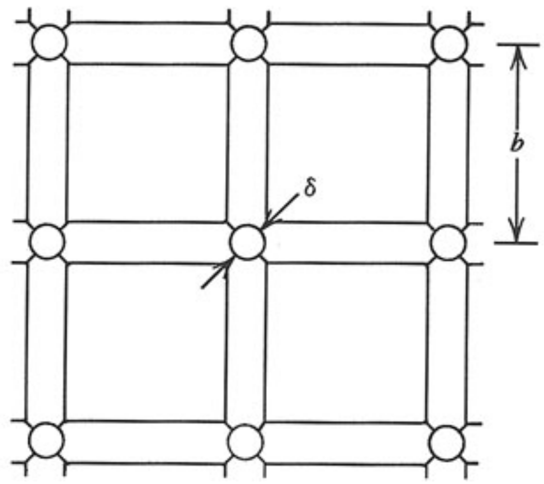
1) Groundwater flow: Flow of a fluid through a porous medium depends on the permeability of that medium. There aren't any sure fire ways to relate porosity and permeability, but some simple schemes give decent results. Let's think of our permeable medium as a solid rock that's been fractured into approximately cubic fragments with cylindrical cracks along the cube edges. The cube centers are spaced a distance 'b' apart and the size of fractures (diameter of the cylinders) is δ. e.g. the figure on the right.



Show that the porosity of such a rock is: $\phi = \frac{3\pi}{4}\left(\frac{\delta}{b}\right)^2$

When a fluid flows through a pipe its mean velocity is given by: $\langle u \rangle = -\frac{\delta^2}{32\eta}\frac{dp}{dx}$

where dp/dx is the along-pipe pressure gradient, η is the fluid viscosity and δ is the pipe diameter. Let's assume that groundwater flows through this rock parallel to one set of these orthogonal cracks. Using Darcy's law (check the notes), which describes the flux of ground water per unit area, show that the permeability is given by: $k = \frac{\pi}{128}\frac{\delta^4}{b^2}$

If the cracks are 1mm wide and crack junctions are spaced 10cm apart, what are the numerical values for porosity and permeability?

What is the hydrologic conductivity ($k\rho g/\eta$) if the fluid is water on Mars? What is it for methane on Titan? What do you think this says about the effectiveness of groundwater flow on Mars vs Titan?

2) *Freezing of a Martian Ocean.* Given how hard it is to produce and maintain large greenhouse atmospheres on Mars any standing body of water was probably ice-covered. So long as the ice surface is colder than the freezing temperature then the ice layer will get thicker with time. Here we'll figure out how long it would take for this ocean to freeze through. Let's assume a liquid ocean of constant temperature and that the whole thing cooled to 0^oC (i.e. forget about the density variations in near-freezing water). Freezing begins at the surface and the ice-water interface gradually descends until the entire ocean is frozen. At some point the ice thickness increases from z to $z+\Delta z$ in a time Δt.

How much heat is released by this phase change per unit area?
How much heat can be conducted through a unit area of the ice slab in this time?
Write these two expressions in terms of the latent heat (L), ice density (ρ), conductivity (k), mean surface temperature (T_s) and ice thicknesses (z).

Equate these two quantities to show that the ice thickness (at time t after freezing starts) is given by: $z = \sqrt{\frac{2k}{L\rho}(273 - T_S)}\ t^{1/2}$

If the surface temperature is 240K, how many years does it take to freeze an ocean 1km thick? [$L=3.2\times10^5$ J kg^{-1}, $\rho=920$ kg m^{-3}, k =2 W K^{-1} m]

3) *Porosity and Martian groundwater:* Porosity is highest at the surface and gets smaller with increasing depth as the increasing pressure closes pore spaces and cracks. Based on some (pretty noisy) seismic data from the Moon, porosity is thought to decrease, from its surface value, exponentially with depth in a regolith i.e.

$$\phi(z) = \phi(z=0)e^{-z/H}$$

where H is the e-folding depth (similar to a scale height in the equation for atmospheric pressure). Usually the pressure in the near surface is just the weight of the overlying rock ρgZ.

In this case, show that the pressure is given by: $P = \rho g\left[z + H\phi_{z=0}\left(e^{-z/H} - 1\right)\right]$. If porosity decreases in such a way so that it is 1% of the surface value when the pressure reaches 100 MPa, show that $H = \dfrac{100\,MPa}{\rho g\left[\ln(100) - 0.99\phi_{z=0}\right]}$.

If surface porosity is 35%, what is H on Mars?
Show that the regolith can store (per square meter) a volume of water given by: $H\phi_{z=0}$. If this water were on the surface how deep would it be?

The pore-space water freezes from the surface downwards as the planetary heat flux declines. The current martian heat flux is about 30mW m^{-2}, how deep is the ice-water interface today if the mean surface temperature is ~240K and the regolith conductivity is 2 W K^{-1} m.

4) River profiles: The shear stress at the base of a flow is given by: $\tau = \rho g h \, sin(slope)$. Rivers adjust their beds (by eroding or depositing material) so that this shear stress tends to be held constant. If river depth increases linearly downstream then show that the elevation of the river bed is given by:

$$z = c \, ln\left(\frac{L}{x}\right)$$

where c is a constant and L is the distance between the drainage divide and the ocean.

Let's say that c=50 and L=100km in one example.

If the river depth is 0.5m, 25km from the ocean then what is the shear stress on the river bed? What size particle (assume the sediment is quartz) do you expect to find on the bed at that location? What would these numbers be for the same river on Titan (i.e. liquid methane with water ice sediment)?

This is for fun – #5 is for optional extra credit.

5) *Martian Methane*. The global average concentration of methane on Mars was measured at 10 ppb by volume (number of particles). How many methane molecules are in the Martian atmosphere?
[Hint: a unit-area column of atmosphere has a mass P_s/g, where P_s is the surface pressure, g is gravity and the atmosphere is dominated by CO_2].

If a methane molecule has a life-time of $2x10^{10}$s before being photo-dissociated then what is the methane production rate? (In Kg per year)

[For Fun] A terrestrial cow produces an incredible 600 liters of CH_4 a day (at room temperature and pressure)! Convert this to kg of methane per year and compare to the Martian production rate…. Based on this analysis, how many cattle are needed on Mars to maintain the planet's methane concentrations?