Deep jets on gas-giant planets

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Abstract

Three-dimensional numerical simulations of the atmospheric flow on giant planets using the primitive equations show that shallow thermal forcing confined to pressures near the cloud tops can produce deep zonal winds from the tropopause all the way down to the bottom of the atmosphere. These deep winds can attain speeds comparable to the zonal jet speeds within the shallow, forced layer; they are pumped by Coriolis acceleration acting on a deep meridional circulation driven by the shallow-layer eddies. In the forced layer, the flow reaches an approximate steady state where east–west eddy accelerations balance Coriolis accelerations acting on the meridional flow. Under Jupiter-like conditions, our simulations produce 25 to 30 zonal jets, similar to the number of jets observed on Jupiter and Saturn. The simulated jet widths correspond to the Rhines scale; this suggests that, despite the three-dimensional nature of the dynamics, the baroclinic eddies energize a quasi-two-dimensional inverse cascade modified by the $\beta$ effect (where $\beta$ is the gradient of the Coriolis parameter). In agreement with Jupiter, the jets can violate the barotropic and Charney–Stern stability criteria, achieving curvatures $\partial^2 u/\partial y^2$ of the zonal wind $u$ with northward distance $y$ up to $2\beta$. The simulations exhibit a tendency toward neutral stability with respect to Arnol’d’s second stability theorem in the upper troposphere, as has been suggested for Jupiter, although deviations from neutrality exist. When the temperature varies strongly with latitude near the equator, our simulations can also reproduce the stable equatorial superrotation with wind speeds greater than 100 m s$^{-1}$. Diagnostics show that barotropic eddies at low latitudes drive the equatorial superrotation. The simulations also broadly explain the distribution of jet-pumping eddies observed on Jupiter and Saturn. While idealized, these simulations therefore capture many aspects of the cloud-level flows on Jupiter and Saturn.

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1. Introduction

Tracking of clouds in Voyager, Galileo, and Cassini spacecraft images shows that Jupiter and Saturn exhibit numerous zonal (east–west) jet streams with speeds that reach 180 m s$^{-1}$ on Jupiter and about 400 m s$^{-1}$ on Saturn. On Jupiter, the zonal jets are located on the boundaries between dark belts and bright zones, with the westward jets on the poleward edges of belts and the eastward jets on the poleward edges of zones (Limaye, 1986). The Galileo probe entered Jupiter’s atmosphere at a latitude of 7.4° N in 1995 and measured winds that increased from 0.4–5 bars and became constant from 5–22 bars (Atkinson et al., 1997). Despite these measurements, the deep wind structure on Jupiter and Saturn—and the mechanisms that drive the winds—remain unknown. Two popular jet structure models have been proposed: the shallow model, in which jets are confined to a thin region near cloud level, and the deep model, in which the jets penetrate the molecular hydrogen envelope (~$10^4$ km depth). Two popular jet driving mechanisms have also been proposed: shallow forcing, which includes injection of turbulence, absorption of solar radiation and latent heat release in the cloud layer, or deep forcing, in which convection within the deep interior drives the jets (see Vasavada and Showman, 2005, for a review). Analysis of Voyager and Cassini images (Beebe et al., 1980; Ingersoll et al., 1981; Salyk et al., 2006; Del Genio et al., 2007) provide evidence for pumping of the jets by eddies at cloud level, although this does not rule out the possibility that deep forcing contributes as well. However, the vast majority of studies assume that shallow forcing would produce only shallow jets and that deep jets can result only from deep forcing (Vasavada and Showman, 2005). In contrast to this view, recent investigations with idealized lin-
ear models show that shallow forcing can drive deep winds that extend far below the forcing level (Showman et al., 2006). However, such studies neglect potentially important nonlinear interactions and parameterize, rather than explicitly resolve, the eddy-induced jet pumping in the forced layer.

In addition to the numerous mid-latitude zonal jets, the circulations on Jupiter and Saturn contain a strongly superrotating (eastward) jet at the equator. This feature is notable because zonally symmetric circulations produce westward equatorial flow. Upgradient momentum transport by waves or eddies is necessary to produce this feature. Many studies have shown that convection in spherical shells can easily produce equatorial superrotation (Christensen, 2001, 2002; Aurnou and Olson, 2001; Heimpel et al., 2005; Heimpel and Aurnou, 2007). On the other hand, two-dimensional shallow turbulence models usually produce westward equatorial flow, although eastward equatorial flow can occur in a small fraction of cases (Nozawa and Yoden, 1997; Huang and Robinson, 1998). Likewise, one-layer shallow-water turbulence on a sphere produces westward equatorial flow under jovian conditions of small deformation radius (Iacono et al., 1999; Cho and Polvani, 1996; Showman, 2007). Interestingly, however, three-dimensional models of shallow atmospheres have shown that, under some conditions, eastward equatorial flow can occur (Williams, 2002, 2003a, 2003b, 2003c, 1978, 1979; Panetta, 1993). Here, we parameterize the complex latent-heating and solar-energy absorption processes by applying a simple latitude- and height-dependent heating that is confined above a pressure $p_T$ of $\sim 2$–10 bars. To isolate the physical mechanisms, our model includes only shallow forcing; deep forcing mechanisms (such as convection cells that penetrate the molecular envelope) are excluded.

We use a global circulation model, the MITgcm, to solve the three-dimensional, hydrostatic primitive equations in pressure coordinates on a sphere. The equations are given by (Marshall et al., 2004)

$$\frac{Dv}{Dt} + f \hat{k} \times v + \nabla_p \Phi = 0,$$

$$\frac{\partial \Phi}{\partial p} = -\frac{1}{\rho},$$

$$\nabla_p \cdot v + \frac{\partial \omega}{\partial p} = 0,$$

$$\frac{D\theta}{Dt} = Q_0,$$

where $v$ is the horizontal wind vector, comprised of zonal wind $u$ and meridional wind $v$, $\omega = dp/dt$ is vertical wind in pressure coordinates, $\Phi$ is geopotential, $\hat{k}$ is the unit vector in the vertical direction (positive upward), $\rho$ is density, $\nabla_p$ is horizontal gradient operator at a given pressure level, $D/Dt$ is the total derivative operator given by $D/Dt = \partial/\partial t + v \cdot \nabla$, and $\theta = T(p_0/p)\kappa$ is potential temperature. Here $T$ is temperature and $\kappa \equiv R/c_p$, which is a specified constant, is the ratio of the universal gas constant to the specific heat at constant pressure. The reference pressure $p_0$ is taken as 1 bar (note, however, that the dynamics are independent of the choice of $p_0$). $Q_0$ is rate of heating (expressed in K s$^{-1}$) due to diabatic processes such as radiation or latent heating associated with the condensation of water vapor. $f = 2\Omega \sin \phi$ is the Coriolis parameter, where $\phi$ is latitude and $\Omega$ is the rotation rate of the planet. The dependent variables $v$, $\omega$, $\Phi$, $\rho$, and $\theta$ are functions of longitude $\lambda$, latitude $\phi$, pressure $p$, and time $t$.

In our numerical model, the upper boundary is zero pressure and the lower boundary is impermeable, and both boundaries are free-slip in the horizontal direction. The mean pressure at the bottom is a free parameter that we vary from 25 to 1000 bars. The equations contain no large-scale momentum sources or sinks (e.g., drag). We represent the forcing with Newtonian cooling/heating

$$\frac{d\theta}{dt} = Q_0 = -\frac{\theta - \theta_{eq}}{\tau},$$

where $\theta_{eq}$ is the equilibrium potential temperature and $\tau$ is a relaxation time. We define $\theta_{eq}$ as

$$\theta_{eq}(\phi, p) = \theta_{ref}(p) + \delta \theta(\phi, p),$$

where $\delta \theta$ parameterizes the latitudinal difference in equilibrium potential temperature associated with differences in solar-energy absorption or latent heat release of water vapor. The reference potential temperature profile $\theta_{ref}$, which is guided by
Fig. 1. Schematic vertical reference potential temperature profile (a) and temperature profile (b). Solid lines show profiles for Jupiter cases and dashed lines show profiles for the parameter variations with a smaller radius corresponding to that of Uranus.

radio occultation and Galileo-probe results (Lindal et al., 1981; Seiff et al., 1998), corresponds to a temperature that is isothermal in the stratosphere and transits smoothly (over a pressure range of 0.4–1.5 bars) to neutrally stable (constant $\theta_{\text{ref}}$) in the interior (Fig. 1), as expected for a giant planet. The initial condition contains no winds and has a temperature structure equal to $\theta_{\text{ref}}$ with the addition of a weak ($\sim 10^{-4}$ K) thermal perturbation to break the longitudinal symmetry.

Three modes of $\delta \theta$ are tested in our simulations (Fig. 2). Mode A produces a smooth equator-to-pole temperature variation with a hot equator and cold poles: $\delta \theta = \Delta \theta \cos^2 \phi$. Mode B produces alternating hot-and-cold latitude bands, as expected for Jupiter and Saturn: $\delta \theta = \Delta \theta \cos^2(8\phi)$ when $-78.75^\circ < \phi < 78.75^\circ$ and otherwise $\delta \theta = 0$. Mode C produces smooth equator-to-pole temperature variations with a cold equator and hot poles: $\delta \theta = \Delta \theta (1 - \cos^2 \phi)$. $\Delta \theta$ is constant, with values ranging from 2.6–8 K at $p \leq p_T$ and zero at $p > p_T$. Our mode-B cases use $\Delta \theta = 2.6$ K while our modes-A and -C cases use $\Delta \theta = 8$ K. At 1 bar, these values imply latitudinal temperature contrasts of 5 K for mode B and 8 K for modes A and C; these contrasts are chosen to be roughly consistent with the modest latitudinal temperature differences observed on the giant planets (Simon-Miller et al., 2006).

On Jupiter and Saturn, sunlight penetrates to a few bars pressure (Sromovsky et al., 1996, 1998; Pérez-Hoyos and Sánchez-Lavega, 2006), and for three-times solar water abundance, water condenses at ~8 bars on Jupiter and ~15 bars on Saturn. These mechanisms could therefore produce latitudinal temperature contrasts extending to at least 5–10 bars. To bracket this range, we vary the pressure at the base of the forced layer, $p_T$, between 2.5–20 bars.

Our adopted relaxation time (40 Earth days in the nominal simulations) is much shorter than the typical values for giant planets (~one Earth year for Jupiter) but allows us to perform simulations in reasonable time while preserving the quasi-isentropic behavior of atmospheric motions over typical dynamical timescales of 1–10 days. We reran some simulations with $\tau$ in the forced layer ranging from 29 to 215 Earth days and found very similar end states, although the spin-up timescale with $\tau$. In most simulations, we also applied a weak thermal damping in the neutrally stable interior ($p > p_T$) with a timescale of 400 Earth days to help maintain numerical stability. The simulations include no explicit viscosity, but a fourth-order Shapiro filter (Shapiro, 1970) (analogous to eighth-order hyperviscosity) and zonal polar filter are added to maintain numerical stability.

We run the simulations on the whole sphere using a longitude–latitude grid. For our Jupiter cases, the horizontal resolution is $512 \times 256$ gridpoints ($0.703125^\circ$) and $N_L = 22–30$ vertical layers. The bottom $N_L - 1$ layers have interfaces that are evenly spaced in log pressure between 0.033 bars and the basal pressure; the top layer extends from 0 to 0.033 bars. These simulations use a planetary radius $a = 71,492$ km, rotation rate $\Omega = 1.7585 \times 10^{-4}$ s$^{-1}$, specific heat $c_p = 13,000$ J K$^{-1}$ kg$^{-1}$, $\kappa = 0.29$ and gravity $g = 22.88$ m s$^{-1}$, all appropriate to Jupiter. Our high horizontal resolution is necessary to resolve the internal deformation radius, which is crucial because baroclinic instabilities inject energy into the flow predominantly at this scale (a few thousand km). We found that simulations with Jupiter parameters and a lower horizontal resolution of $256 \times 128$ exhibited much weaker eddy activity because they...
We obtain (Karoly et al., 1998) these definitions into the zonal momentum equation and average over the line. The equatorial flow remains zero or weakly westward (depending on pressure), unlike Jupiter. Nevertheless, because simulations at 512 × 256 with >20 layers are computationally demanding, we performed some parameter variations using a smaller planetary radius while attempting to maintain wind speeds and static stabilities similar to that in our Jupiter cases. The idea is that, for a given mean wind speed and static stability, the key length scales—the Rhines scale, Rossby deformation radius, and jet width—have values in these parameter studies comparable to those in our Jupiter cases. Because the radius is smaller, this means the deformation radius and jet width are a larger fraction of the planetary radius than is the case on Jupiter. Using a smaller planetary radius therefore allows us to numerically resolve the jets and deformation radius while using fewer total gridpoints, hence allowing faster runtimes. These simulations used a horizontal resolution of 256 × 128 gridpoints (1.40625°) and Nt = 22–30 vertical layers, again evenly spaced in log pressure; we used parameters a = 25,559 km, Ω = 1.0124 × 10^{-4} s^{-1}, and g = 7.77 m s^{-1}.

2.2. Diagnostics

Before presenting our results, it is useful to describe the formalism we use to diagnose our simulations. For any quantity A, we can define $\tilde{A} = [A] + A^*$ where [A] denotes the zonal mean and $A^*$ denotes the deviation from the zonal mean. Likewise, we can define $\overline{A} = \overline{A} + A'$, where $\overline{A}$ denotes the time average and $A'$ denotes the deviation from the time average. Inserting these definitions into the zonal momentum equation and averaging in longitude and time, we obtain (Karoly et al., 1998)

$$\frac{\partial [\overline{A}]}{\partial t} = -\frac{\partial}{\partial y} \left( [u^* v^*] + [\overline{u^*} \overline{v^*}] \right) - \frac{\partial}{\partial p} \left( [\overline{u^*} \overline{\omega^*}] + [u^* \overline{\omega^*}] \right).$$

This equation contains the most important terms in Eq. (7) are the first and last terms. In this equation, $[u^* v^*]$ and $[\overline{u^*} \overline{\omega^*}]$ are the latitudinal and vertical fluxes of eastward momentum, respectively, associated with traveling eddies, and $[\overline{u^*} \overline{v^*}]$ and $[u^* \overline{\omega^*}]$ are the latitudinal and vertical fluxes of eastward momentum, respectively, associated with stationary eddies. On the terrestrial planets, stationary eddies (i.e., spatial variations in the dynamical fields that do not change in time) are primarily associated with topography or continent–ocean contrasts, which are not expected to be important on the giant planets. In the absence of topography, baroclinic instabilities primarily manifest as traveling eddies. Here, we group them together and generally plot the total eddy-induced acceleration, for example, $-\partial([u^* v^*] + [\overline{u^*} \overline{v^*}]) / \partial y$.

Equation (7) states that accelerations of the time- and zonal-mean zonal wind result from latitudinal convergence of the latitudinal eddy-momentum flux, vertical convergence of the vertical eddy-momentum flux, latitudinal and vertical advection, and Coriolis acceleration on the mean-meridional wind. In geostrophic balance, the latitudinal convergence of latitudinal eddy-momentum flux generally dominates over the vertical convergence of vertical eddy-momentum flux, and for our simulations the advection terms are generally small too. Therefore, the most important terms in Eq. (7) are the first and last terms. We will illustrate the magnitude of these terms for several of our simulations.

3. Results

3.1. Overview of basic flow regime

Our simulations show that the shallow thermal forcing generates multiple mid-latitude jets in the forced layer, with a mean

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Fig. 3. Zonal mean zonal winds for simulations with mode A at 24 Earth days (a), 278 Earth days (b), 347 Earth days (c) and 1157 Earth days (d). The forcing is confined above the dotted line, but deep jets develop that extend to the bottom of the domain. The fluid below the dotted line contains ten times as much mass as that above the line. The equatorial flow remains zero or weakly westward (depending on pressure), unlike Jupiter.
simulated jet width (and total number of jets) in approximate agreement with Jupiter. This is illustrated in Figs. 3 and 4, which show the zonally averaged zonal wind at four times for Jupiter simulations with modes A and B, respectively. At first, the thermal forcing quickly produces latitudinal temperature variations at pressures less than 10 bars, and the resulting pressure gradients drive a transient, zonally symmetric meridional flow in the forced layer. The east–west Coriolis acceleration on this meridional circulation drives weak, wide zonal flows in the forced layer by ∼30 days as the fluid achieves thermal-wind balance with the imposed temperature contrasts in the forced layer. As shown in Figs. 3a and 4a, these zonal flows have a horizontal length scale comparable to that of the thermal forcing (∼80° and 20° latitude in modes A and B, respectively). However, this flow is dynamically unstable, and these wide flows break up into numerous narrow east–west jets by ∼500 days (Figs. 3b–3c and 4b–4c). By 1000 days, the simulations in Figs. 3 and 4 each contain ∼26 jets at the 1-bar level. We emphasize that the resulting latitudinal jet width is controlled by the inherent dynamics of the system; the jets are much narrower than the latitudinal length scale of the thermal forcing. The number of jets in these simulations is similar to that on Jupiter.

Intriguingly, these shallow jets gradually develop deep barotropic components that extend to the bottom of the model at 100 bars and reach speeds comparable to the top-level jets by ∼1000 days (Figs. 3d and 4d). These deep jets are driven entirely by the shallow forcing which in our simulations is confined to the top 3–10% of the fluid mass. This demonstrates, in agreement with Showman et al. (2006), that deep jets can result from shallow forcing.

In our Jupiter simulations, the total number of jets and the development of deep jets are relatively insensitive to the details of the thermal forcing; simulations with modes A and B produce similar behavior. However, the behavior of the equatorial jet depends strongly on the form of the thermal forcing. Despite having a Jupiter-like number of jets, our mode-A simulation exhibits equatorial flow that is westward at most pressures (Fig. 3); this simulation therefore fails to explain the equatorial flow on Jupiter. On the other hand, our mode-B simulation produces a superrotating equatorial jet with local maxima at latitudes of ∼7° and pressures of ∼0.4 bars, near the maximum in the eddy-induced forcing.

Figs. 5a and 5b show the time evolution of the shallow (0.9-bar) and deep (50-bar) zonal-mean zonal wind for our Jupiter simulation with mode B. Because the deep winds are barotropic in this simulation, Fig. 5b represents the winds at any pressure below ∼10 bars. In the forced layer, the jets at times earlier than ∼200 days have a width comparable to the wavelength of the imposed thermal forcing (∼20° latitude in this case). Fig. 5b shows that the deep winds remain extremely weak during this period, which results from the fact that the eddy forcing and meridional circulation are also weak at early times. Baroclinic instabilities become active by ∼200 days, however. As shown in Fig. 5a, this causes a narrowing of the jet spacing in the forced layer by ∼500 days (the number of jets increases from 15 to ∼25 as eddies become active). Panel (b) shows that the pumping of deep jets begins in earnest only after the eddies become active at ∼200 days.

Our basic results—the formation of multiple zonal jets that develop deep barotropic components extending far below the forcing region—occur also in our simulations with a smaller planetary radius. This is illustrated in Fig. 6, which shows the outcome at 4630 days of three simulations where the pressure at the bottom of the forced layer is \( p_T = 2.5, 10, \) and 20 bars in (a), (b), and (c), respectively. The bottom pressure in each case

![Fig. 4. Zonal mean zonal winds for simulations with mode B at 12 Earth days (a), 104 Earth days (b), 926 Earth days (c) and 6481 Earth days (d). The forcing is confined above the dotted line, but deep jets develop that extend to the bottom of the domain. The fluid below the dotted line contains ten times as much mass as that above the line. This simulation also develops an equatorial superrotation whose profile at the ∼1-bar level is similar to that on Jupiter.](image-url)
is ten times $p_T$, which implies that the ratio of forced-layer mass to total mass is the same for all three simulations. All three simulations exhibit deep winds whose speeds approach that of the shallow jets. This result, together with the behavior shown in Figs. 3–5, illustrates that development of deep winds from shallow forcing is a robust phenomenon that occurs over a wide range of forcing parameters. Interestingly, the trend from (a) to (c) provides clues about the mechanism. The deep jets are strongest in (c) and weakest in (a), and the eddy activity is also strongest in (c) and weakest in (a) [a result of the thicker baroclinically unstable zone in (c) than (a)]. This correlation hints that shallow-layer eddy activity plays an important role in pumping the deep jets. To investigate the issue further, we now turn to detailed diagnostics.

### 3.2. How do deep jets arise from shallow forcing?

In our simulations, the dynamics are predominantly driven by turbulence injected into the flow by baroclinic instabilities. The effect of this turbulence is shown in Fig. 7, which depicts the acceleration of the mean zonal wind caused by the horizontal eddy-momentum convergence $-\frac{\partial}{\partial y} \left( \langle u'v' \rangle + \langle \bar{u} \bar{v} \rangle \right)$, the horizontal eddy heat flux, and the vertical eddy heat flux for our mode-B Jupiter simulations. Key points are as follows. First, the eddies (and their accelerations) are confined primarily to the thermally forced layer between 1–10 bars. This vertical confinement makes sense: at great pressures ($>0.2$ bars), the horizontal thermal contrasts are zero, so no baroclinic eddies are generated. At small pressures ($<0.2$ bars), the large stratospheric static stability ensures that isentropes have shallow slopes despite the existence of horizontal temperature contrasts, so the stratosphere is baroclinically stable and again no eddies are generated at most latitudes. Only the 0.3–10 bar layer is baroclinically unstable and generates eddies. Second, the eddy-induced jet accelerations have latitudinal length scales much smaller than that of the thermal forcing that caused the baroclinic instabilities. Baroclinic instabilities inject energy primarily at the first internal deformation radius, which is expected to be a few thousand km on Jupiter; in agreement, we find that the eddies generated by the instabilities have typical horizontal length scales of $\sim 5000$ km. This small eddy length scale is a prerequisite for obtaining closely spaced jets; indeed, the mean latitudinal jet spacing is $5000$–$10,000$ km in Figs. 3–4. Third, the eddy accelerations have magnitudes of $\sim 10^{-5}$ m s$^{-2}$, which implies that, acting alone, eddies would cause jets with Jupiter-like speeds to grow over fast time scales of 10–50 Earth days. The fact that the actual jets accelerate on much longer time scales implies that some other process counteracts these eddy accelerations.

The eddy accelerations induce a meridional circulation that has a major effect on the flow. We find that, in the forced layer, the time-averaged meridional flow is equatorward at eastward jets and poleward at westward jets, and the east–west Coriolis acceleration on this meridional flow resists the jet acceleration caused by eddies. Fig. 8 compares the magnitudes of these two accelerations at 0.37 bars. The solid curves show the eddy acceleration $-\frac{\partial}{\partial y} \left( \langle u'v' \rangle + \langle \bar{u} \bar{v} \rangle \right)$ and the dash–dot curves show the Coriolis acceleration $f \langle \bar{v} \rangle$ for three cases. Interestingly, there is a strong anticorrelation between the two. They sum to nearly zero away from the equator, leading to a net acceleration that is much smaller than either acceleration acting alone. Thus, the forced layer reaches a quasi-steady state where the mean jet speed evolves slowly (on $\sim 1000$ day time scales) despite the fact that either acceleration acting in isolation would induce order-unity changes in the jet speed over time scales of $\sim 30$ days. This explains the fact that the jets in the forced layer
evolve slowly (Figs. 3–4) despite the large magnitude of the individual accelerations (Fig. 7).

Showman et al. (2006) showed that the meridional circulation provides a mechanism whereby shallow forcing can drive deep jets. They performed a linear investigation with an imposed parameterization of eddy accelerations in the weather layer, and they solved for the meridional circulation and the rate at which the deep jets are accelerated. We first construct an appropriate metric for the effect of the forced-layer eddies. A characteristic value of the eddy-induced acceleration in the forced layer is the mass-weighted root-mean-square value of \(-\partial([\vec{u}\vec{v}^*]+[\vec{v}^*\vec{v}^*])/\partial y\) averaged over the forced layer:

\[
a_{\text{eddy}} = \left( \int_{-90^\circ}^{90^\circ} \int_{0}^{P_T} \frac{\partial}{\partial y} \left( [\vec{u}^*\vec{v}^*] + [\vec{v}^*\vec{v}^*] \right) \right)^{1/2} \times \frac{dp}{g} \cos \phi \, d\phi \left( \int_{0}^{P_T} \frac{dp}{g} \cos \phi \, d\phi \right)^{-1/2}.
\]  

Fig. 10 demonstrates that the Coriolis acceleration on these deep meridional flows pumps the deep jets. The figure illustrates Jupiter mode-A (top row) and two time frames for Jupiter mode-B (middle and bottom rows). In each case, there is a correlation at most latitudes between the Coriolis acceleration \(f[\vec{r}]\) and the zonal-wind profile \([\vec{u}]\). The acceleration is weak, \(\sim 10^{-7} \text{ m s}^{-2}\), implying that jets with speeds of \(\sim 10 \text{ m s}^{-1}\) would be pumped over 1000 days, consistent with the results in Figs. 3–5. This weak acceleration results from the fact that the typical meridional wind in the deep layer is small, \(\sim 3 \times 10^{-4} \text{ m s}^{-1}\), when averaged over hundreds of days, in comparison to peak speeds of \(\sim 0.01 \text{ m s}^{-1}\) in the forced layer.

To further understand the conditions that lead to deep winds, we next examine the relationship between forced-layer eddies and the rate at which the deep jets are accelerated. We first construct an appropriate metric for the effect of the forced-layer eddies. If the scenario in Showman et al. (2006) is correct, then we can relate \(a_{\text{eddy}}\) to the expected rate of deep jet acceleration. As shown above (Fig. 8), the east–west eddy and Coriolis accelerations approximately balance in the forced layer, which can be written to order-of-magnitude as \(f v_{\text{shallow}} \sim a_{\text{eddy}}\), where \(v_{\text{shallow}}\) is a characteristic meridional velocity in the forced layer. The continuity equation, Eq. (3), implies that \(v_{\text{deep}} \sim v_{\text{shallow}} \Delta p_{\text{shallow}}/\Delta p_{\text{deep}}\), where \(v_{\text{deep}}\) is a characteristic meridional velocity in the deep (unforced) layer and

nearly barotropic jets that can extend far below the depth of the forcing.

We find that this mechanism also explains the deep jets in our numerical simulations. Fig. 9 shows contours of the streamfunction associated with the mean meridional circulation for our Jupiter mode-B case. Here, the streamfunction is defined as \([u] = -\partial \psi / \partial p\) and \([\omega] = \partial \psi / \partial y\). The meridional flow generated by the forcing in the shallow layer causes horizontal convergence in anticyclonic regions (equatorward sides of eastward jets) and divergence in cyclonic regions (poleward sides of eastward jets). Mass conservation requires the existence of a meridional return flow at other pressures. The small static stability in the interior implies that vertical motion can easily occur, so the meridional cells readily penetrate downward. On the other hand, the large stratospheric static stability prevents significant penetration of the meridional circulation into the stratosphere. As a result, the tops of the meridional overturning cells occur at \(\sim 0.2–0.5\) bars, near the top of the forced region, and the cells extend to the bottom of the domain. The number of deep jets equals the number of meridional overturning cells.

Fig. 6. Zonally averaged zonal winds for mode-A parameter variations with a smaller planetary radius \(a = 25559\) km, rotation rate \(\Omega = 1.0124 \times 10^{-4} \text{ s}^{-1}\), gravity \(g = 7.77 \text{ m s}^{-2}\), and horizontal resolution of 256 \times 128. (a) \(P_T = 2.5\) bars, (b) \(P_T = 10\) bars, (c) \(P_T = 20\) bars. In all three cases, the bottom pressure is ten times \(P_T\). The simulation time for all three cases is 4630 Earth days. Dotted line denotes zero speed; contour interval is 5 m s\(^{-1}\) in (a) and 20 m s\(^{-1}\) in (b) and (c). In all three cases, multiple jets appear in the shallow layer and deep winds develop from the shallow forcing.
\( \Delta p_{\text{shallow}} \) and \( \Delta p_{\text{deep}} \) are the pressure thicknesses of the forced and unforced layers, respectively. If the Coriolis acceleration on \( v_{\text{deep}} \) drives the deep jets, we therefore expect that over a time interval \( \Upsilon \) the deep jets would attain a characteristic speed \( u_{\text{deep}} \sim f v_{\text{deep}} \Upsilon / \Delta p_{\text{shallow}} / \Delta p_{\text{deep}} \sim a_{\text{eddy}} \Upsilon / \Delta p_{\text{shallow}} / \Delta p_{\text{deep}} \). Therefore, an appropriate metric to compare against \( u_{\text{deep}} \) is \( a_{\text{eddy}} \Delta p_{\text{shallow}} / \Delta p_{\text{deep}} \).

Fig. 11 plots this quantity, averaged over the first 926 days, against the root-mean-square deep wind at 926 days for many simulations. The figure demonstrates that the two quantities are correlated: simulations with strong eddy activity experience fast pumping of the deep jets; conversely, when eddy activity is weak, deep jets do not develop. Nevertheless, the above scaling suggests a linear relationship between the deep winds and \( a_{\text{eddy}} \Delta p_{\text{shallow}} / \Delta p_{\text{deep}} \), so the fact that the points do not fall on a straight line is odd. Several factors could cause such deviations. For example, although the shallow eddies generally act to accelerate the jets, instabilities could occasionally occur that instead remove energy from the jets; this would lead to a slower deep jet speed for a given \( a_{\text{eddy}} \). Variation in this phenomenon between simulations could potentially produce scatter around an expected linear correlation. Despite these complications, the figure provides strong support that the mechanism proposed by Showman et al. (2006) occurs in our simulations.

The flow’s time evolution also broadly supports the above picture of deep-jet pumping. Fig. 12 plots the root-mean-square latitudinal eddy flux of eastward momentum, latitudinal eddy heat flux, streamfunction of the mean-meridional flow, and deep winds versus time for our Jupiter mode-B simulation. The top two panels, which provide different measures of eddy activity, show similar qualitative behavior: from an initial state with no eddies, a spike in eddy activity occurs at \( \sim 100-200 \) days followed by a gradual decline to an approximately steady, nonzero value. The reason for the spike and subsequent decline is unclear; one possibility is that the flow becomes more baroclinically stable (leading to weaker eddy activity) as the deep barotropic jets develop, a phenomenon that is well known in the Earth context (James, 1987). The streamfunction also shows an early spike, a rapid decline, and subsequent attainment of an approximately steady, nonzero value. Interestingly, the spike in the streamfunction occurs at \( \sim 50-100 \) days of simulated time, which is earlier than the spike in eddy activity. This early spike in streamfunction probably results from the initial transient development of a meridional circulation in response to the applied, zonally symmetric thermal forcing (as can be seen in Fig. 9a); the transient ends when the fluid achieves approximate thermal-wind balance with the imposed temperature distribution. The nonzero streamfunction at times later than \( \sim 200 \) days results from the mean-meridional circulation caused by the shallow-layer eddies. The final panel indicates that the deep jets accelerate rapidly over the first \( \sim 200 \) days and that the growth rate declines at later times. This behavior is broadly consistent with the expectation that strong eddy activity and streamfunction lead to rapid pumping of the deep jets.

To summarize, our simulations show that production of deep jets from shallow forcing requires continual generation of eddy activity in the forced layer, which in our study results from
baroclinic instability. When the thermal forcing produces shallow jets that are baroclinically unstable, the resulting forced-layer eddies drive a mean-meridional circulation that extends to the bottom of the domain, and the east–west Coriolis acceleration on this circulation generates deep jets. On the other hand, when the thermal forcing produces shallow jets that are stable, no eddies form, so the mean-meridional circulation remains weak and no deep jets form.

Fig. 8 shows the ratio of the deep wind speed to shallow wind speed for a variety of simulations. In this figure, the deep wind is defined as the mass-weighted mean speed integrated over pressures exceeding $p_T$, whereas the shallow wind is defined as the mass-weighted mean speed integrated over pressures less than $p_T$. Initially, the shallow winds develop more rapidly than the deep winds (e.g., Figs. 3 and 4), so the deep-to-shallow wind ratio is small. However, as the barotropic component of the flow increases, the ratio increases, reaching 0.5–1 for a wide variety of simulations. This implies that the deep winds are ~50–100% the strength of the shallow winds in these simulations. The decrease in the deep-to-shallow ratio in the mode-A Jupiter case from ~1700–2700 days results from an increase in the shallow equatorial jet speed during this time, not from a decrease in the deep jet speeds.

Williams (2002, 2003c) performed simulations that, like ours, were forced by baroclinic instabilities in a shallow weather layer overlying a deep abyssal layer; however, in contrast to our results, his simulations did not develop deep barotropic jets. There are several possible explanations. Williams states “near the lower surface, a weak linear drag with a timescale $\tau_D$ helps equilibrate some flows.” He sets $\tau_D = 300$ days. The same value of $\tau_D$ is used for the Newtonian relaxation in his forced layer, which represents only the top ~2% of his domain. One therefore expects that the meridional circulations induced by shallow eddies pump the deep jets over a timescale of $\sim \tau_D \Delta p_{\text{deep}}/\Delta p_{\text{shallow}} \sim 10^4$ days, where for his simulations $\Delta p_{\text{deep}}/\Delta p_{\text{shallow}} \sim 50$. Because this deep-jet-pumping time greatly exceeds the timescale of the imposed drag in the deep layer, the steady-state barotropic jet speed in the abyssal layer of his simulations must be extremely weak (see Showman et al., 2006). Note that even if the drag extends only partway through the abyssal layer, the barotropic nature of the deep jets forces the entire abyssal-layer zonal wind to be nearly zero. The $10^4$-day timescale for pumping deep jets in his simulations also implies that, even if deep jets could form (e.g., if the drag were turned off), these deep winds would only manifest in long-term simulations integrating $>10^4$ days.
Fig. 9. Overturning streamfunction for mode-B Jupiter simulation at 58 days (a) and 4630 days (b). Only the southern hemisphere is shown since the meridional stream function is nearly symmetric about the equator. Solid lines imply the overturning is clockwise and dash lines mean the overturning is counter-clockwise. Forcing occurs only at \( p < 10 \) bars, but the overturning cells extend to the bottom of the domain. The number of meridional circulation cells is doubled in (b).

3.3. Horizontal aspects of jet dynamics: Jet spacing, curvature, and potential vorticity

We next turn to a consideration of the meridional jet spacing. Two-dimensional turbulence theory predicts that jets forced by small-scale turbulence have widths close to the Rhines scale, \( \pi (2U/\beta)^{1/2} \), where \( U \) is the characteristic jet speed and \( \beta = df/dy \) is the meridional gradient of planetary vorticity (Rhines, 1975; Williams, 1978; Nozawa and Yoden, 1997; Huang and Robinson, 1998; Vasavada and Showman, 2005). Simple theories predict that even in a three-dimensional fluid, baroclinic turbulence can energize a barotropic mode that still follows approximately two-dimensional dynamics (Salmon, 1998). Although our simulations are fully three-dimensional, it is therefore of interest to compare our jet widths with the Rhines scale. To do so, we choose a deep region well below \( p_T \), where jets are nearly barotropic, to calculate the jet speeds and widths from our simulations.

Fig. 14 shows that the measured jet widths in our simulations are similar, within a factor of \( \sim 2 \), to the Rhines scale calculated from the jet speeds. Interestingly, our Jupiter mode-A simulation produces jets that are \( \sim 1.5-3 \) times wider than the Rhines scale, whereas the Jupiter mode-B simulation produces jets with widths very similar to the Rhines scale. Most published two-dimensional turbulence investigations also produce jets somewhat wider than predicted by Rhines scaling (or equivalently, the latitudinal curvature of the jets \( \partial^2 u/\partial y^2 \) has a magnitude substantially less than \( \beta \)) (Marcus et al., 2000; Huang and Robinson, 1998; Nozawa and Yoden, 1997; Williams, 1978). We also find that our eastward jets are generally narrower than our westward jets, also in agreement with
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Fig. 10. Left column (a, c, and e) shows zonal-mean zonal wind at 50 bars (in the deep barotropic region) for three cases. Right column (b, d and f) shows the Coriolis accelerations on the mean-meridional wind $f[v]$ at 50 bars for those same cases. Top row (a and b) shows mode A at 926 days; middle row (c and d) shows mode B at 926 days; bottom row (e and f) shows mode B at 4630 days. The averaging time interval is 231 Earth days. At most latitudes, peaks in the jets occur near the same latitudes as peaks in the Coriolis acceleration, indicating that the Coriolis acceleration pumps the deep jets.

many two-dimensional investigations. Fig. 14 supports the conclusion that our jet widths are controlled by the Rhines scale.

We now compare our simulated jet profiles to simple stability criteria for zonal jets. A long-term puzzle is that Jupiter’s cloud-level winds violate the barotropic stability criterion (Ingersoll et al., 1981), which states that, in a two-dimensional fluid, jets are stable provided that

$$\frac{\partial^2 u}{\partial y^2} < \beta.$$  \hspace{1cm} (9)

A more general criterion is the Charney–Stern criterion, which states that jets are stable provided that their potential vorticity profile is monotonic in latitude (Dowling, 1995a). Read et al. (2006) recently evaluated the potential vorticity versus latitude on Jupiter by combining cloud-top zonal-wind profiles with estimates of the static stability from thermal infrared measurements, and they showed that Jupiter also violates the Charney–Stern stability criterion. These violations are puzzling because almost all two-dimensional turbulence calculations produce jets

that satisfy these criteria (Nozawa and Yoden, 1997; Huang and Robinson, 1998; Williams, 1978). Ingersoll and Pollard (1982) proposed that, if the jets form Taylor columns that penetrate the
that $\partial^2 u/\partial y^2$ exceeds $\beta$. This is a viable hypothesis, but it remains unclear whether the jets penetrate the envelope as coherent Taylor columns. Gierasch (2004) proposed an alternate 3D model that may allow shallow jets to remain stable while having curvatures $\partial^2 [u]/\partial y^2$ that exceed $\beta$ at the cloud level. However, he made several assumptions (e.g., geostrophy) that may influence the results; furthermore, his results showed that only eastward jets were stabilized; westward jets with curvatures exceeding $\beta$ remained unstable. To summarize, the fact that $\partial^2 [u]/\partial y^2$ exceeds $\beta$ at several latitudes may have implications for whether Jupiter’s jets penetrate deeply, but these implications remain unclear.

On the other hand, Dowling (1993) and Read et al. (2006) provided evidence that Jupiter’s upper troposphere is close to neutral stability with respect to Arnol’d’s second stability theorem. This theorem relates the mean zonal wind $|u|$ to the meridional gradient of potential vorticity (Dowling, 1995a; Stamp and Dowling, 1993). Essentially, the cloud-level flow (at $\sim 0.5$ bars) is stabilized because it is underlain by fast zonal jets at a few bars pressure, and inferences suggest that those jets also have curvatures exceeding $\beta$ (Dowling, 1995b). Although these are exciting discoveries, work performed to date has focused on demonstrating the stability of the cloud-level jets under the assumption that the inferred deeper jets are stable; the question of how those deeper jets can remain stable with sharp curvatures remains unaddressed. Effectively, the uncertainty has simply been moved one layer deeper into the atmosphere. Thus, while the importance of Arnol’d’s second stability theorem for the upper troposphere cannot be overemphasized, we would still appear to lack a satisfying explanation for why Jupiter’s jets have such sharp curvatures.

We are thus interested in two issues: (i) whether our simulations produce jets with curvatures exceeding $\beta$, and (ii) whether our simulations approach neutrality with respect to Arnol’d’s second stability theorem.

To address the first issue, we compare in Fig. 15 the profiles of zonal wind, $\partial^2 u/\partial y^2$ and potential vorticity versus latitude for Jupiter and our simulations. The top row shows the Jupiter observations, and the middle and bottom rows show results from our Jupiter mode-B and mode-A simulations, respectively. The winds are shown at a pressure of 0.66 bars and potential vorticity is at 0.12 bars. In the panels containing $\partial^2 u/\partial y^2$, we include $\beta$ (dashed curve) for comparison. The potential vorticity $q_G$ is the quasi-geostrophic version calculated following Read et al. (2006):

$$ q_G = f + \zeta - f \frac{\partial}{\partial p} \left[ \frac{p \Delta T(\lambda, \phi, p)}{s(p) T_a(p)} \right], $$

where $\zeta$ is relative vorticity calculated on isobars, $T_a(p) = (T(\lambda, \phi, p) - \Delta T(\lambda, \phi, p))$ is the horizontal mean temperature calculated on isobars, $\Delta T(\lambda, \phi, p) = T(\lambda, \phi, p) - T_a(p)$ is deviation of temperature from its horizontal mean, and $s(p)$ is a stability factor defined as

$$ s(p) = -\frac{\partial (\ln(\theta))}{\partial p}, $$

where $\theta$ is the horizontal mean potential temperature calculated on isobars.

Interestingly, our Jupiter mode-A case satisfies the barotropic and Charney–Stern stability criteria (consistent with two-dimensional turbulence calculations), but our Jupiter mode-B case violates both criteria at many latitudes, in agreement with Jupiter (Fig. 15). In the mode-B case, the wind curvature $\partial^2 u/\partial y^2$ exceeds $2\beta$ at some latitudes. The potential vorticity profile shows that the potential vorticity is nonmonotonic at many latitudes, again similar to Jupiter. The jets produce eddies at many latitudes where they violate the criteria; however, the time-averaged jets are still stable which suggests a stable configuration between the eddies and the zonal jets. The strongly
nonmonotonic potential vorticity distribution seems to result partly from the nonmonotonic profile of temperature versus latitude in the mode-B case. (The nonmonotonic temperature produces a nonmonotonic $\partial\theta/\partial p$ and hence readily produces a nonmonotonic potential vorticity via Eq. (10).)

The results described above demonstrate that stable, three-dimensional jets can produce curvatures exceeding $\beta$ even when the jets do not penetrate the molecular envelope (and hence thus do not feel the full spherical geometry as postulated by Ingersoll and Pollard, 1982). These results hence provide an alternative to the scenario of Ingersoll and Pollard (1982) for explaining the violations of the criteria on Jupiter.

As already mentioned, evidence suggests that Jupiter’s cloud-level winds are close to neutrally stable with respect to Arnol’d’s second stability criterion (Dowling, 1993; Read et al., 2006; Dowling, 1995a, 1995b). The neutral stability criterion can be expressed as a relationship between zonal wind and potential vorticity gradient (Stamp and Dowling, 1993, 1995a, 1995b):

$$[u] = \frac{\partial[qG]}{\partial y} L_d^2,$$  \hspace{1cm} (12)

where $L_d$ is the deformation radius. Fig. 16 displays $[u]$ versus $\partial[qG]/\partial y$ at 0.12 bars for mode-A (top) and mode-B (bottom) cases. Our simulations develop a positive correlation between these two quantities, consistent with expectations of neutrality to Arnol’d’s second theorem. Under the assumption of neutrality, Eq. (12) implies that the deformation radius can be estimated from the slope of the correlation. We performed a least-squares linear fit to the points in each panel, which yields estimates of the deformation radius of $1523 \pm 150$ km and $2626 \pm 870$ km for mode-A and mode-B simulations, respectively. These values are broadly consistent with the deformation radius estimated for our simulations from the relation $L_d \sim NH/f$, where $N$ is Brunt–Vaisala frequency, $H$ is scale height, and $f$ is Coriolis parameter. Thus, our simulations exhibit a tendency toward neutral stability with respect to Arnol’d’s second theorem. Nevertheless, the points in Fig. 16 exhibit considerable scatter; this probably results in part from the fact that $L_d$...
varies with latitude, but it also suggests that deviations from neutrality occur at some latitudes. Note that the Jupiter observations also exhibit considerable scatter (see Fig. 12 in Read et al., 2006).

### 3.4. Equatorial superrotation

As described in Section 3.1, our Jupiter mode-B case develops an equatorial superrotation analogous to that occurring on Jupiter and Saturn (Figs. 4, 5, and 15d). Additional simulations with a smaller planetary radius and forcing analogous to mode B (but with fewer sinusoids across the planet) also develop an equatorial superrotation. In contrast, our simulations with mode-A or mode-C forcing have near-zero or subrotating (westward) equatorial flow. The key feature that triggers the superrotation is the steep latitudinal temperature gradient near the equator caused by the mode-B-like forcing, which is absent in the mode-A and mode-C cases.

Several previous studies have investigated superrotation in the Earth and Jupiter contexts (Saravanan, 1993; Suarez and Duffy, 1992; Yamazaki et al., 2005; Williams, 2003a, 2003c, 2003b, 2006). Of these, Williams (2003a, 2003c, 2003b, 2006) used a forcing most similar to ours, namely, a zonally symmetric heating that induces latitudinal temperature contrasts. Like ours, his simulations developed superrotation only when the latitudinal temperature gradient near the equator is strong. In these papers, Williams suggested that the strong latitudinal temperature contrasts near the equator lead to low-latitude eastward jets, and when these jets are sufficiently close to the equator, they become barotropically unstable, which drives eddy energy and momentum equatorward and causes the equatorial superrotation. Here, we diagnose our mode-B Jupiter simulation to determine whether this mechanism also explains the superrotation in our case.

Fig. 17 presents the equatorial configuration of our mode-B Jupiter simulation at 231 days (left column) and 2310 days (right column). From top to bottom, the six rows show the zonal-mean zonal wind \( \bar{u} \), the latitudinal eddy flux of eastward momentum \( \bar{u}'\bar{v}' \) + \( \bar{u}\bar{v}'^* \), the latitudinal eddy heat flux \( \bar{v}'\bar{\theta}' \) + \( \bar{v}\bar{\theta}'^* \), the acceleration of the zonal wind caused by latitudinal eddy-momentum convergence \( \frac{1}{\rho_0} (\bar{u}'\bar{v}') + \bar{u}\bar{v}'^* \), the acceleration of the zonal wind caused by Coriolis acceleration on the mean-meridional flow \( f\theta \), and the acceleration of the zonal wind caused by meridional advection \( f \delta (\bar{u}) / \partial y \), respectively (see Eq. (7) for the role of these terms in the Eulerian-mean momentum equation).

As shown in Fig. 17 (top row), the equatorial wind consists primarily of two jet cores peaking at latitudes of \( \sim 7^\circ \) and a pressure of 0.5 bars. These maxima experience a modest acceleration from \( \sim 90 \) m s\(^{-1}\) at day 231 to \( \sim 120 \) m s\(^{-1}\) by day 2310. However, the wind at the equator itself undergoes a more significant acceleration from 10 to 80 m s\(^{-1}\) (which is also evident in Fig. 5a), leading to a wind profile by 3000 days that is extremely similar to that of Jupiter. At early times, the latitudinal jet profile \( \bar{u}(\phi) \) has a “horned” structure where the wind maxima occur off the equator at all pressures. By \( \sim 2000 \) days, however, this horned structure holds only at pressures of \( \sim 0.1 \)–3 bars; at greater pressures of 3–10 bars, the maximum jet speed occurs on the equator.

In the simulation, the shallow heating source occurs between latitudes of \( \pm 10^\circ \) and peaks at the equator (see Fig. 2). At first, this heating induces a zonally symmetric meridional circulation, and the Coriolis acceleration on this flow drives the two off-equatorial jets at \( 7^\circ \) latitude as the fluid achieves thermal wind balance. During this phase, which is best viewed in Fig. 5a, the equatorial jet speed remains zero. By day \( \sim 200 \), however, the flow becomes unstable, and the resulting eddies strongly modify the jet profile. Figs. 17c and 17i show that these eddies are strongly baroclinic at pressures less than 0.2 bars. At early times, they transport thermal energy poleward across the jet cores; at later times, this poleward heat transport becomes weaker and is accompanied by an equatorward transport at pressures of 0.1–0.2 bar. As shown in Figs. 17b and 17h, the momentum transport associated with these baroclinic eddies is initially strongest on the poleward flanks of the jet cores, where they transport zonal momentum equatorward into the jets. At low pressure (<0.2 bars), these baroclinic eddies also transport eddy momentum away from the equator throughout the simulation, which drives a westward equatorial subrotation with wind speeds reaching \( \sim 80 \) m s\(^{-1}\) at pressures <0.1 bars as shown in Fig. 17g. The Coriolis and advective accelerations counteract each other on the equatorward sides of the jet cores but both accelerate the winds on the poleward sides of the jet cores.

At pressures of 0.2–10 bars, however, the equatorial eddies have nearly zero heat flux, which suggests that the eddies in this pressure range are predominantly barotropic. Although their eddy momentum transport is complicated (Figs. 17b and 17h), the direction of transport is largely equatorward between latitudes of \( \pm 3^\circ \) and pressures of 0.2–10 bars. The resulting accelerations of the zonal wind in Figs. 17d and 17j show that these barotropic eddies induce an equatorial superrotation at most latitudes and pressures within this range; these eddy accelerations dominate over the Coriolis and advective accelerations (Figs. 17e, 17f, 17k, and 17l) which leave the convergence of eddy momentum alone to drive the wind at the equator. This eddy-momentum convergence fills in the local minimum wind between the two jet maxima at \( 7^\circ \) latitude (Fig. 5a), leading to the local maximum in \( \bar{u}(\phi) \) on the equator at pressures of 3–10 bars, and supports equatorward migration of the jet cores. Interestingly, Fig. 17j shows that the acceleration produced by convergence of eddy momentum at the equator has reduced 50% in about 2000 days. Combined with the fact that the equatorial jet becomes nearly steady after \( \sim 3000 \) days, this suggests that the jet may be near an equilibrated state.

Taken together, the above considerations suggest that barotropic instabilities drive the equatorial superrotation in the region between 0.2 bars and 10 bars. This entire process is very similar to that proposed by Williams (2006, 2003c, 2003b, 2003a).

In contrast, simulations with thermal forcing mode A and mode C have equatorial subrotation in the forced region. These simulations exhibit vigorous eddy activity in the mid-latitudes but very weak eddy activity near the equator, consistent with their lack of equatorial superrotation.
3.5. Statistics of eddies

Observational cloud-tracking analyses provide evidence that small eddies pump the jets at the cloud level on Jupiter and Saturn (Salyk et al., 2006; Del Genio et al., 2007), but the source of those eddies remains unclear. They could result from baroclinic instabilities, thunderstorms, or other convective processes. Detailed eddy statistics provide a useful diagnostic that, when compared with models, may help to discriminate between these possibilities. We here present some measures of eddy statistics and compare them to available observations.

Our simulations produce patterns of eddy activity similar to those of cloud tracking results on Jupiter and Saturn. Fig. 18 shows the distribution of eddy-momentum flux $u^*v^*$ values at a particular latitude on the south (Fig. 18a) and north (Fig. 18b) flank of an eastward jet in the northern hemisphere of our Jupiter mode-B simulation. The distributions peak at $u^*v^* = 0$, indicating that, most commonly, the eddies are weak and do not transport momentum into the jet. However, the distribution is also broad; at any given latitude, most of the eddies have $u^*v^*$ values between $-50$ and $50$ $\text{m}^2\text{s}^{-2}$, although some have values as large as $\sim 100$ $\text{m}^2\text{s}^{-2}$. This indicates that, at a given latitude, there exist many eddies that pump momentum up-gradient into the jets and many others that pump momentum down-gradient out of the jets. Fig. 18 also shows, however, that the distribution is skewed such that, south of an eastward jet, the number of eddies with positive $u^*v^*$ outweighs the number with negative $u^*v^*$; north of an eastward jet, the number of eddies with negative $u^*v^*$ outweighs those with positive $u^*v^*$. This means that, on average, the eddies pump momentum up-gradient into the jets. This is true on both sides of the jet.

Our simulated distributions broadly agree with the distribution of $u^*v^*$ values observed on Jupiter and Saturn. In Fig. 19, we reproduce observed eddy statistics from jet flanks on Jupiter...
Fig. 18. Frequency distribution of $u^* v^*$ for mode-B Jupiter simulation at 4630 Earth days, where $u^*$ and $v^*$ are deviations of zonal and meridional wind, respectively, from their zonal means. (a) is on the south flank of the eastward jet peaked at 41° S. (b) is on the north flank of the same eastward jet. Both are shown at 0.9 bars.

and Saturn from Salyk et al. (2006) and Del Genio et al. (2007). As in our simulations, the observed distributions at a given latitude on both planets peak near zero, have broad wings with positive and negative $u^* v^*$ values, and have a skewed shape such that the average eddy pumps momentum up-gradient into the jet. The primary difference between the planets is the width of the distribution: on Jupiter, the observed eddy $u^* v^*$ values range from $-100$ to $100 \, \text{m}^2 \text{s}^{-2}$ whereas on Saturn they range from $-10$ to $10 \, \text{m}^2 \text{s}^{-2}$. Eddies therefore appear to be weaker on Saturn. Our simulations (Fig. 18) produce distributions with widths qualitatively similar to that on Jupiter but wider than that on Saturn. On the other hand, the distributions in our simulations appear to be less sharply peaked at $u^* v^* = 0$ than the observations of either planet.

The correlation between latitudinal wind shear $\partial [\bar{u}] / \partial y$ and eddy momentum flux $[\bar{u}^* \bar{v}^* + \bar{u}' \bar{v}']$, which evaluates the conversion rate from eddy kinetic energy to zonal mean kinetic energy, is also consistent with observations. Fig. 20 shows that eddy momentum flux and wind shear have positive correlation at most latitudes, which indicates that eddies are pumping energy into the zonal mean flow. This should be compared to Del Genio et al. (2007), Figs. 7–8.

On balance, the agreement between the observed and simulated eddy statistics supports the hypothesis that baroclinic instabilities contribute to jet pumping on Jupiter and Saturn. An advantage of the baroclinic-instability mechanism over thunderstorms is that baroclinic instabilities can pump momentum up-gradient on both the cyclonic and anticyclonic flanks of a jet. In contrast, as pointed out by Del Genio et al. (2007), thunderstorms might be expected to pump momentum up-gradient primarily on the cyclonic flanks, since most thunderstorms occur in cyclonic regions (Del Genio et al., 2007). Nevertheless, detailed modeling studies that explore alternate sources of turbulence are needed before any possible source can be ruled out.

Fig. 19. Frequency distributions of $u^* v^*$ from cloud-tracking observations of Jupiter (a and b; from Salyk et al., 2006) and Saturn (c and d; from Del Genio et al., 2007). Here, $u^*$ and $v^*$ are deviations of zonal and meridional wind, respectively, from their zonal means. (a) is on the south flank of the eastward jet peaked at 17.9° S. (b) is on the north flank of the eastward jet peaked at −5.1° S. (c) is on the north flank of the westward jet at −48.5°, and (d) is on the south flank of the same jet.

4. Conclusion

We presented numerical models with a simple Newtonian heating to simulate the zonal jets and their vertical extent on Jupiter. Our simulations show that shallow forcing can produce barotropic deep winds that extend from the level of thermal
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Fig. 20. Correlation between latitudinal wind shear $\partial u/\partial y$ and northward flux of eastward momentum for Jupiter mode-B at 0.9 bars and 4630 Earth days. In (a), the winds and eddy fluxes have been time-averaged for 231 days. In (b), the winds and eddy fluxes are instantaneous (zonally averaged) snapshots. The triangles and pluses show values equatorward and poleward, respectively, of 8.8$^\circ$ latitude. The tendency for $\partial u/\partial y$ and the eddy-momentum flux to have the same sign indicates that eddies pump momentum up-gradient into the jets.

forcing all the way down the bottom of the simulated domain. On Jupiter and Saturn, this base might represent an internal statically stable layer or potentially the base of the molecular envelope. In agreement with Showman et al. (2006), our simulations therefore support the idea that the zonal jets may penetrate deeply on Jupiter and Saturn even if the forcing is shallow. Development of such deep jets occurs approximately over a timescale $\tau \Delta p_{\text{deep}}/\Delta p_{\text{shallow}}$ (Showman et al., 2006), where $\tau$ is the timescale to pump the shallow jets and $\Delta p_{\text{deep}}$ and $\Delta p_{\text{shallow}}$ are the pressure thicknesses of the shallow (forced) layer and the deeper abyssal layer. This timescale is $\sim 1000$ Earth days in our simulations but could be longer if the layer containing the deep jets extends deeper than in our simulations (as is probable).

In our simulations, the shallow thermal forcing produces eddies that drive the shallow zonal jets, and this induces an ageostrophic meridional circulation whose Coriolis acceleration closely counteracts the eddy accelerations in the forced layer. Such a balance leads to jets that evolve slowly and could explain the steadiness of the observed jets on Jupiter and Saturn despite the fast rate of observed jet pumping (Del Genio et al., 2007; Salyk et al., 2006). Our simulations show that, because of the low static stability in the interior, these meridional circulations extend deeply, and it is the Coriolis acceleration on these
deep-meridional circulations that pump the deep jets. In our simulations, there are thus two mechanisms of jet pumping—eddy-convergence in the shallow layer and Coriolis acceleration in the deep layer.

Our Jupiter simulations develop 25–30 zonal jets, which closely agrees with the number of jets observed on Jupiter. Despite the inherently three-dimensional nature of our simulated flows, the meridional jet spacing in the barotropic region is controlled by the Rhines scale \( \pi(2U/\beta)^{1/2} \). This suggests, in agreement with simple theories, that the three-dimensional eddies energize a barotropic mode that obeys quasi-two-dimensional dynamics.

Interestingly, we find that some of our simulations violate the barotropic and Charney–Stern stability criteria. These violations preferentially occur when we utilize mode-B forcing, which creates alternating hot-and-cold latitude bands in the forced layer. This agreement provides a possible explanation for the violation of these criteria on Jupiter. Our simulations also exhibit a tendency toward neutral stability with respect to Arnol’d’s second stability theorem, as has been suggested for Jupiter; nevertheless, deviations from neutrality do occur in our simulations.

We also find that some of our simulations develop a Jupiter-like superrotation, while others do not. Our mode-B Jupiter simulation, while adopting a prescribed thermal forcing, shows encouraging agreement with the Voyager and Cassini cloud tracking results. This simulation reproduces the dynamic features of winds on Jupiter such as the multiple cloud-level zonal jets, equatorial superrotation, comparable wind speeds and behaviors of eddy activities. On the other hand, our simulations do not reproduce the decay of the zonal jets with altitude in the upper troposphere inferred from thermal observations; future work is needed to address this issue.

Our thermal forcing, which relaxes the temperature in a shallow upper layer toward an assumed latitudinally varying equilibrium temperature, is intended as a crude parameterization of latitudinally varying solar-energy absorption or latent heat released by condensation of water vapor. This has the advantage of simplicity and, more importantly, allows a systematic exploration of how the dynamics depends on the thermal structure. Our simulations show that the development of deep winds is not sensitive to the latitudinal temperature gradient, nor is the jet spacing as long as the winds have Jupiter-like speeds. On the other hand, our forcing scheme has the disadvantage that the length scales and vertical structure of the latitudinal temperature contrast are externally imposed rather than self-consistently evolving with (and being determined by) the circulation. More advanced models that directly include radiative transfer and the advection, condensation, and latent-heating of water vapor will allow such coupling and will build on the work presented here. In particular, it will be interesting to see whether the interaction of radiation and moist convection can naturally produce alternating hot and cold bands as assumed in our mode-B forcing.

Although our simulations show that weather-layer dynamics can influence the deep interior, the primitive equations adopted here do not account for convective motions in the interior. Several studies of convection in rotating spherical shells have shown that the convection can lead to Reynolds stresses that produce zonal jets (Heimpel et al., 2005; Heimpel and Aurnou, 2007; Aurnou and Olson, 2001; Christensen, 2001, 2002). We expect the true dynamics to involve a complex interplay between the interior convection and the weather-layer mechanisms identified here. A new generation of coupled interior–atmosphere circulation models will be needed address this interaction.

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