The Gravity of the Situation

PTYYS206-2
4 Mar 2008
Upcoming Events

• Exam 1 next Tuesday, March 11.
• Essays due next Thursday, March 13.
• Review session, Thursday, March 6.
• New Homework will be posted today, due next Tuesday.
• Come to office hours this week!!!!
Exam 1

- Will cover everything so far
- Bring your calculator!
- You are allowed 1 side of an 8.5x11” paper with your own notes.
- You will be given any numerical values that you need, but not equations.
- The exam is a mix of multiple choice, short answer and long answer questions.
Essay

• You must pass in an essay next Thursday.
• It will be assigned a provisional grade.
• I will make comments on it and you will get to revise it to improve the grade.
• If you do not pass in the essay your grade will be an E and you will not be able to improve it.
List of Symbols

- $F$, force
- $a$, acceleration (not semi-major axis in this lecture)
- $v$, velocity
- $M$, mass of Sun
- $m$, mass of planet
- $d$, general distance
- $r$, radius of circle, semi-major axis of orbit
- $R$, radius of Earth
Mass and Weight

Mass - an intrinsic property of matter. Formally, the m in F=ma. The amount of “stuff” in an object.

Weight - the force experienced by an object in a gravitational field. Weight is equal to mass multiplied by gravitational acceleration, W=mg.

Mass and weight are often confused because the gravitational acceleration is the same everywhere on the surface of the Earth and so mass and weight are directly proportional.

However,
Mass and Weight Continued

In deep space an object still has mass but no weight because there is no gravity.

Lonely object, a zillion AU from anything, experiencing negligible gravitational attraction. This is almost true weightlessness.
There is a feeling of weightlessness when falling, but only because the acceleration is equal to the gravitational acceleration. This is not true weightlessness because there is still a gravitational force.
Weight on the Moon

An astronaut weighs less on the moon than on Earth, but has the same mass.

Mass of Moon = $7.35 \times 10^{22}$ kg

Radius of Moon = $1.74 \times 10^6$ m

Gravity on Moon

\[ g = \frac{GM}{R^2} \]

\[ g = \frac{(6.67 \times 10^{-11}) (7.35 \times 10^{22})}{(1.74 \times 10^6)^2} \]

\[ g = 1.6 \text{ m s}^{-2} \]

This is 1/6 th the value on Earth.
Units for Mass and Weight

SI: Mass - kilograms, Force - Newtons (kg m s\(^{-2}\))

English: Mass - slugs, Force - pounds

Technically, it is incorrect to say that someone weights X kilograms (that’s a mass) or to say that someone has a mass of Y pounds (that’s a weight).

In Europe (metric system) kilograms are incorrectly used to describe weight. In the US pounds are incorrectly used to describe mass.
The velocity needed to escape the gravitational pull of a planet or star is

\[ V_{\text{esc}} = (2\text{GM}/R)^{1/2} \]
Notice, that $V_{\text{esc}}$ does not depend on direction. A rocket fired at 45° will escape a planet just as well as a rocket fired vertically. (Of course, firing a rocket downward doesn’t work.)
Notice, that $V_{\text{esc}}$ depends on $R$, the distance from the center of the planet or star. Thus, smaller velocities are needed for escape, the further one is from the attracting body.

**Escape Velocity on Earth**

$$V_{\text{esc}} = \left( \frac{2GM}{R} \right)^{1/2}$$

$$V_{\text{esc}} = \left( 2 \cdot 6.67 \times 10^{-11} \cdot 5.97 \times 10^{24} / 6.37 \times 10^{6} \right)^{1/2}$$

$$V_{\text{esc}} = 1.12 \times 10^{4} \text{ m/s} = 11.2 \text{ km/s}$$

Or roughly 25,000 miles per hour
**Approach**

Using algebra put the unknown quantity on the left hand side of the equation and all the known quantities on the right hand side.

Write down an algebraic expression before inserting any numbers.

If you are given quantities as ratios (ratio of stars mass to sun’s mass, etc.) then use them as such. It’ll make the math easier.

Think about the solution before you calculate it. What do you expect the answer to be, big or small, fast or slow?
Using Newton’s Form of Kepler’s Third Law: Example 1

Planet Gabrielle orbits star Xena. The semi major axis of Gabrielle's orbit is 1 AU. The period of its orbit is 6 months. What is the mass of Xena relative to the Sun?

\[
\frac{GM_*}{4\pi^2} = \frac{r^3}{P^2}
\]

Start with

\[
r^3 = \left(\frac{GM_*}{4\pi^2}\right)P^2
\]

Rearrange

\[
\frac{GM_*}{4\pi^2} = \frac{r^3}{P^2}
\]

Insert \( r = 1 \text{ AU}, \ P = 0.5 \text{ years} \)

\[
\frac{GM_*}{4\pi^2} = \frac{1^3}{(0.5)^2} = 4 \text{ AU}^3/\text{year}^2
\]

We know

\[
\frac{GM_*}{4\pi^2} = 1 \text{ AU}^3/\text{year}^2
\]

So we must have

\[
M_* = 4 M_\odot
\]

\( M_* \) - Mass of Xena, \( M_\odot \) - Mass of Sun
Using Newton’s Form of Kepler’s Third Law: Example 2

Planet Linus orbits star Lucy. The mass of Lucy is twice the mass of the Sun. The semi-major axis of Linus' orbit is 8 AU. What is the orbital period for Linus?

\[
\text{Start with: } \quad \frac{GM_*}{4\pi^2} \cdot \frac{1}{p^2} = \frac{1}{r^3}
\]

Rearrange:

\[
p^2 = \left(\frac{4\pi^2}{GM_*}\right) r^3
\]

We know \( M_* = 2M_\odot \), \( r = 8 \text{ AU} \)

So:

\[
p^2 = \left(\frac{4\pi^2}{G \times 2M_\odot}\right) 8^3
\]

\[
p^2 = \frac{4\pi^2}{GM_\odot} \cdot 512 \quad \text{Years}^2
\]

\[
p^2 = 256 \text{ Years}^2
\]

\[
p = 16 \text{ Years}
\]
Using Newton’s Form of Kepler’s Third Law: Example 3

Planet Osiris orbits star Isis. The mass of Isis is half the mass of the Sun. The period of Osiris’ orbit is 16 years. What is the semi-major axis of Osiris’ orbit?

Start with

\[ r^3 = \frac{G M_A P^2}{4\pi^2} \]

No need to rearrange! we know \( M_A = \frac{1}{2} M_\odot \)

and \( P = 16 \) years

So

\[ r^3 = \frac{G \frac{1}{2} M_\odot (16)^2}{4\pi^2} \]

\[ r^3 = \frac{G M_\odot}{4\pi^2} \frac{256}{2} \]

\[ r^3 = 128 \text{ } \text{AU}^3 \]

\[ r \approx 5 \text{ } \text{AU} \]
Using Newton’s Form of Kepler’s Third Law: Example 4

Jupiter's satellite (moon) Io has an orbital period of 1.8 days and a semi-major axis of 421,700 km. What is the mass of Jupiter?

\[
\frac{r^3}{P^2} = \frac{GM_J}{4\pi^2}
\]

Solve for \(M_J\)

\[
M_J = \frac{4\pi^2 r^3}{G P^2}
\]

Need to use SI units!

\[
\begin{align*}
\Gamma &= 421,700 \text{ km} = 4.2 \times 10^8 \text{ meters} \\
P &= 1.8 \text{ days} = 1.6 \times 10^5 \text{ seconds}
\end{align*}
\]

Now

\[
M_J = \frac{4 (3.14)^2 \left( \frac{4.2 \times 10^8}{1.6 \times 10^5} \right)^3}{6.67 \times 10^{-11} \left( 1.6 \times 10^5 \right)^2}
\]

\[
M_J = 1.8 \times 10^{27} \text{ kg}
\]
Using Newton’s Form of Kepler’s Third Law: Example 5

The moon has an orbit with a semi-major axis of 384,400 km and a period of 27.32 days. What is the mass of the Earth?

\[
\text{Start with} \quad r^3 = \left(\frac{GM_E}{4\pi^2}\right)p^2
\]

Solve for \(M_E\)

\[
M_E = \frac{4\pi^2}{G} \frac{r^3}{p^2}
\]

Use SI units

\[r = 384,400 \text{ km} = 3.84 \times 10^8 \text{ meters}\]

\[p = 27.32 \text{ days} = 2.36 \times 10^6 \text{ seconds}\]

Insert values

\[
M_E = \frac{4 \left(3.14\right)^2 \left(3.84 \times 10^8\right)^3}{6.67 \times 10^{-11} \left(2.36 \times 10^6\right)^2}
\]

\[
M_E = 6.0 \times 10^{29} \text{ kg}
\]