Define Local Thermodynamic Equilibrium
IR spectra of planetary atmospheres
First realistic RT model of an atmosphere
Local Thermodynamic Equilibrium

- The population of excited states, e.g. the number of molecules $n_i$ in state $i$ and that $n_j$ in state $j$ obey the Boltzmann distribution:

$$\frac{\bar{n}_j}{\bar{n}_i} = \frac{g_j}{g_i} e^{-(E_j - E_i)/k_B T}$$

- In that case, even if the temperature varies we have LTE, and the Planck function is the source function, but not the intensity.
Earth’s Spectrum: sum of 2 parts
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What is the source function on this side?

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Planetary emission
Earth’s surface temperature

New et al. 2002, Reynolds et al. 2002
Constant Source Function

\[ I_\nu(\tau_\nu) = I_\nu(0) \ e^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu e^{-(\tau_\nu - \tau'_\nu)} \ d\tau' \]

How do you solve this equation for a constant source function?
Constant Source Function

\[
I_\nu(\tau_\nu) = I_\nu(0) \ e^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu e^{-(\tau_\nu - \tau_\nu')} d\tau'
\]

\[
I_\nu(\tau_\nu) = I_\nu(0) \ e^{-\tau_\nu} + S_\nu (1 - e^{-\tau_\nu})
\]

As tau goes to infinity, I \rightarrow ???
Constant Source Function

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\[ I_\nu(\tau_\nu) = I_\nu(0) \ e^{-\tau_\nu} + S_\nu \ (1 - e^{-\tau_\nu}) \]

\[ I_\nu(\tau_\nu) = S_\nu + e^{-\tau_\nu} (I_\nu(0) - S_\nu) \]

Note: as tau goes to infinity, I \to S!
Familiar Source Function

\[ B_\nu = I_\nu = \frac{c}{4\pi} \mu = \frac{2\hbar \nu^3}{c^2(\nu h/k_B T) - 1} \]
Emission Spectra: which planets are these?
Primary vs Secondary Transits:

Light curve of brightest “Hot Jupiters”:

- ~0.2%

Planet approaches **secondary eclipse**
enabling measurements
of emission spectra of
the dayside hemisphere

Planet in **primary eclipse**
enables measurements
of transmission spectra
of the terminator

- ~0.2%

- ~1.5%

Secondary Transit provides information on Temperature & Composition Profiles
RT Equation

- Assume IR radiation only, in an atmospheric slab of constant temperature. Simplify this:

\[ I_\nu(\tau_\nu) = I_\nu(0) e^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu e^{-(\tau_\nu - \tau'_\nu)} d\tau'. \]
Assume IR radiation only, in an atmospheric slab of constant temperature. Simplify this:

\[ I_{\nu}(\tau_{\nu}) = I_{\nu}(0) \, e^{-\tau_{\nu}} + \int_{0}^{\tau_{\nu}} S_{\nu} e^{-(\tau_{\nu}-\tau')} d\tau'. \]

\[ I_{\nu}(\tau_{\nu}) = I_{\nu}(0) \, e^{-\tau_{\nu}} + B_{\nu} (1 - e^{-\tau_{\nu}}). \]
To calculate the intensity emerging from the top of an atmosphere, what do we do?

\[ I_\nu(\tau_\nu) = I_\nu(0) e^{-\tau_\nu} + B_\nu(1 - e^{-\tau_\nu}) \]
Assumption: Atmospheric scale height is small compared to the radius of the planet.
To calculate the intensity emerging from the top of an atmosphere, what do we do?

\[ I_{\nu}(\tau_{\nu}) = I_{\nu}(0) \ e^{-\tau_{\nu}} + B_{\nu}(1 - e^{-\tau_{\nu}}). \]
\[ I_v^1 = I_{\text{observed}} \quad \tau_1 \ll 1 \]

\[ T_1, P_1, \rho_1, \Delta \tau_1 \quad Z_2, \tau_2 \]

\[ I_v^2 \]

\[ \vdots \]

\[ I_v^i = I_v^{i+1} e^{-\Delta \tau_v} + B_v(T_i)(1 - e^{-\Delta \tau_v}) \quad Z_i, \tau_i \]

\[ T_i, P_i, \rho_i, \Delta \tau_i \quad Z_{i+1}, \tau_{i+1} \]

\[ I_v^{i+1} \]

\[ \vdots \]

\[ I_v^N = B_v(T_N) \quad Z_N, \tau_N \gg 1 \]

or surface
Consider a few atmospheres

- Constant temperature, variable tau
- Variable temperature, constant tau
- Variable temperature, variable tau

What kind of a spectrum would you get from these?
Why are these absorption features?
Contribution functions

- RT Eq. \( I_{\nu}(\tau_{\nu}) = B_{\nu}(\tau_{surf}) \ e^{-\tau/\mu} + \frac{1}{\mu} \int_{\tau_{surf}}^{0} B_{\nu} e^{-\tau/\mu} \ d\tau. \)

- No surface \( I_{\nu}(\tau_{\nu}) = \frac{1}{\mu} \int_{\infty}^{0} B_{\nu} e^{-\tau/\mu} \ d\tau. \)

- Weighting & Contribution Functions:
  \( I_{\nu}(\tau_{\nu}) = \int_{\infty}^{0} B_{\nu} \ WF(P) \ d\ln P, \quad I_{\nu}(\tau_{\nu}) = \int_{\infty}^{0} CF(P) \ d\ln P. \)

  \( WF(P) = e^{-\tau/\mu} \frac{d(\tau/\mu)}{d\ln P} \)

  \( CF(P) = B_{\nu} e^{-\tau/\mu} \frac{d(\tau/\mu)}{d\ln P} \)
Earth’s IR spectrum

[Graph of intensity vs. wavenumber]

[Graph of intensity vs. wavelength]

[Graph of pressure vs. temperature]

Mesosphere

EARTH

Stratopause

Stratosphere

Tropopause

Troposphere
Example: CO in Titan’s atmosphere
Determine TP profile with CH4 features
Results:
47 ± 8 ppm of CO, uniform with latitude.
Vertical abundance constant above condensation level
Consistent with long chemical lifetime (500 Myr)
Origin: unknown
Possibly from Comets (like CO2 and H2O)
Possibly from interior
composition from data
Start with model & then fit the data
Atmospheres are tricky

- We are still trying to understand the composition & structure of the well measured 8 Solar System planets.
- Take Titan. We don’t know where the oxygen and carbon species in the atmosphere come from.
- For exoplanets, we are mostly looking at Hot Jupiters which has no surface. This helps because we don’t have to worry about subsurface sources and surface chemistry...