1. Direct and Diffuse Component

"Don't let schooling interfere with your education."
- *Mark Twain*

The illumination of an atmosphere by sunlight causes two somewhat distinct radiation fields, where the intensity can be considered as a sum of two components, the direct, I_s^-, and diffuse, I_d^-, intensities,

$$I^-(\tau, \mu, \phi) = I_s^-(\tau, \mu, \phi) + I_d^-(\tau, \mu, \phi).$$

Assume that sunlight is illuminating the top of a plane-parallel atmosphere, with an intensity of $F_\odot$, and a slant path defined by ($\mu_\circ$, $\phi_\circ$). Then I_s^- is the downward radiation that has survived without any scattering event:

$$I_s^-(\tau, \mu, \phi) = F_\odot e^{-\tau/\mu_\circ}\delta(\mu - \mu_\circ)\delta(\phi - \phi_\circ). \quad (1)$$

Here $\delta(\mu - \mu_\circ)$ is an example of the delta function:

$$\delta(\mu - \mu_\circ) = 1 \quad \text{if} \quad \mu = \mu_\circ$$

$$\delta(\mu - \mu_\circ) = 0 \quad \text{otherwise.}$$

The diffuse component I_d^- is radiation that has been scattered at least once. On Earth, about 25% of the sunlight is scattered before reaching the surface. Of this scattered light 2/3 reaches the surface. On Titan, all of the radiation is diffuse, and 10% reaches the surface.

Consider an atmosphere with no emission or reflection from the surface, as might be applicable for the ultraviolet illumination of Titan's atmosphere. Thus, there is no upward diffuse or direct intensity coming from the surface. The surface boundary conditions are:

$$I_s^+(\tau_{tot}, \mu, \phi) = 0,$$
$$I_d^+(\tau_{tot}, \mu, \phi) = 0.$$

where τ_{tot} is the optical depth at the surface. The equations of radiative transfer can be written in terms of these components. The downward intensity is:

$$-\mu\frac{dI_d^-(\tau,\theta,\phi)}{d\tau} - \mu\frac{dI_s^-(\tau,\theta,\phi)}{d\tau}$$
$$= I_d^-(\tau,\Omega) + I_s^-(\tau,\Omega) - (1 - a_\nu)B_\nu$$
$$-\frac{a_\nu}{4\pi}\int_+ d\omega'\ p(+\Omega', -\Omega)I_d^+(\tau,\Omega')$$
$$-\frac{a_\nu}{4\pi}\int_- d\omega'\ p(-\Omega', -\Omega)I_d^-(\tau,\Omega')$$
$$-\frac{a_\nu}{4\pi}\int_- d\omega'\ p(-\Omega', -\Omega)I_s^-(\tau,\Omega'). \quad (2)$$

Here, we have used the shorthand $(-\Omega', -\Omega)$ to designate the polar and azimuthal angles of the radiation in the incoming (in this case downward) direction $(-\Omega')$, and the scattered (in this case downward) radiation $(-\Omega)$. Note that there is no direct upward component of radiation $(p(+\Omega', -\Omega)I_s^-(\tau, \Omega'))$, because that light would have had to have come from the surface. Yet our boundary conditions indicate that no light was reflected and none emitted from the surface.

Also note that the direct component of light is not affected by scattering nor by emission. Therefore, I_s^- obeys the a radiative transfer equation with no source function:

$$-\mu\frac{dI_s^-(\tau,\Omega)}{d\tau} = I_s^-(\tau,\Omega)$$

and these two terms in Eq. 2 cancel out. As a result, this equation can be expressed in terms of the diffuse component of radiation, only, if we hide the one last term in Eq. 2 that depends on I_s^- in an extra term, S^*. Equation 2 becomes:

$$-\mu\frac{dI_d^-(\tau,\Omega)}{d\tau} = I_d^-(\tau,\Omega) - (1 - a_\nu)B_\nu - S^*(\tau, -\Omega)$$
$$-\frac{a_\nu}{4\pi}\int_+ d\omega'\ p(+\Omega', -\Omega)I_d^+(\tau,\Omega')$$
$$-\frac{a_\nu}{4\pi}\int_- d\omega'\ p(-\Omega', -\Omega)I_d^-(\tau,\Omega'). \quad (3)$$

Where the direct scattering term, hidden is S^*, simplifies as:,

$$S^*(\tau, -\Omega) = \frac{a_\nu}{4\pi}\int_- d\omega'\ p(-\Omega', -\Omega)F_\odot e^{-\tau/\mu_\circ}\delta(\Omega' - \Omega_\circ).$$

$$S^*(\tau, -\Omega) = \frac{a_\nu}{4\pi}\ p(-\Omega_\odot, -\Omega)F_\odot e^{-\tau/\mu_\circ}, \quad (4)$$

and acts like an additional source function term.

Also, for the upward component of radiation:

$$\mu\frac{dI_d^+(\tau,\Omega)}{d\tau} = I_d^+(\tau,\Omega) - (1 - a_\nu)B_\nu - S^*(\tau, +\Omega)$$
$$-\frac{a_\nu}{4\pi}\int_+ d\omega'\ p(+\Omega', +\Omega)I_d^+(\tau,\Omega')$$
$$-\frac{a_\nu}{4\pi}\int_- d\omega'\ p(-\Omega', +\Omega)I_d^-(\tau,\Omega'). \quad (5)$$

where,

$$S^*(\tau, +\Omega) = \frac{a_\nu}{4\pi}\, p(-\Omega_\odot, +\Omega) F_\odot e^{-\tau/\mu_\circ} \qquad (6)$$

The equation for the diffuse radiation, I_d^-, has the same form as the equation for the full intensity, I_ν^- with the exception that the source function is now has an extra term: $S^*(\tau, -\Omega)$. The second term arises from the direct beam; in the radiative transfer equation it becomes the radiation that is scattered once into the outgoing beam direction, which is downward in former equation and upward in the latter equation. This term is thus called the *single scattering source function*.

The equation for the diffuse intensity $I_d = I_d^+ + I_d^-$ can then be written as:

$$\mu \frac{dI_d}{d\tau_s} = I_d - [1 - a_\nu] B_\nu(T) - S^*(\tau, \Omega)$$

$$-\frac{a_\nu}{4\pi} \int_0^{2\pi} d\phi' \int_{-1}^{+1} d\mu'\; p(\mu', \phi' : \mu, \phi) I_d(\tau, \mu', \phi'), \qquad (7)$$

where the signs arise from the decreasing optical depth with increasing altitude. This expression resembles that for the full intensity (Eq. 6, last lecture) with the direct beam represented as an extra source term, S^*. We can now include more effects in our radiative transfer model, as long as we represent the direct intensity as an extra source term in the equation for the diffuse intensity.

Let's change the lower boundary condition so that now there is radiation coming from the surface, both thermal radiation and reflected radiation. The incident radiation field at the surface, $I^-(\tau^*, \Omega'')$ is reflected in the direction Ω' with a reflectance of $\rho(\Omega'', \Omega')$. Radiation is emitted from the surface with an emittance, ϵ. The intensity of the upward radiation at the base of the atmosphere is then:

$$I(\tau^*, \Omega') = \int_- d\omega'' \rho(-\Omega'', \Omega') \cos(\theta'') I^-(\tau^*, \Omega'')$$

$$+\epsilon(\Omega') B(T_s) \qquad (8)$$

and the source function must be modified to include an extra term, S_b^*, which quantifies the direct intensity coming from the surface and scattered once up to a level defined by τ:

$$S_b^*(\tau, \pm\Omega) = \frac{a_\nu}{4\pi} \int_+ d\omega' p(+\Omega', \pm\Omega) e^{-(\tau^* - \tau)/\mu} I(\tau^*, \Omega'). \qquad (9)$$

Both the down going direct and diffuse components are reflected from the surface, so, in Eq. 8:

$$I^-(\tau^*, \Omega'') = F_\odot e^{-\tau^*/\mu} \delta(\Omega_\circ - \Omega'') + I_d^-(\tau^*, \Omega'') \qquad (10)$$

2. An Example of Diffuse and Direct components.

Last year a paper was published that announced the first observation of specular reflection from Titan's atmosphere. The Cassini spacecraft was flying over the north polar region of Titan, and saw a glint of sunlight from Titan's surface. An entire spectrum of this glint, from 0.8 to 5.0 μm, was measured. Yet the glint was observed at only 5 μm. Why is that? Why was it not seen at other wavelengths?

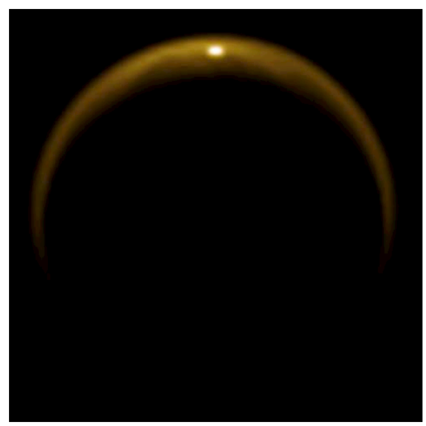

Fig. 1.— **Specular reflection off Titan.**

We can figure this out with a radiative transfer calculation. For specular reflection to occur the direct light illuminates a mirror surface that directs the light to the observer. Only the direct component of light participates in specular reflection. Titan has a very hazy atmosphere. We therefore need to calculate the direct to diffuse component of light to see which wavelengths the direct component is dominant. Figure 2 shows our calculation. We find that only at 5 μm does non-scattered light dominate over diffuse light eminating from Titan's atmosphere.

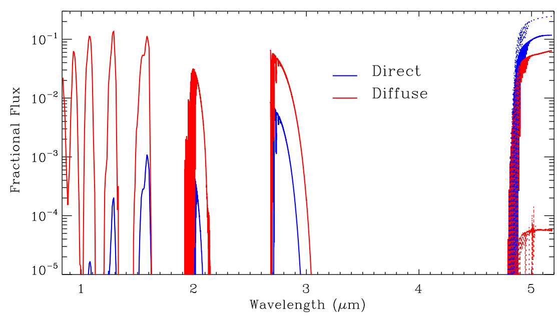

Fig. 2.— **The diffuse (red) and direct (blue) flux that reaches Titan's surface for radiation incident on Titan with an angle of 73.64°. Shown at 5 μm are two atmospheric models; one is free of particulates (dotted lines), and one adopts the particulate distribution measured by Huygens at -10° latitude (solid lines).**

2.1. Another Two-Stream Example

We now entertain a realistic model of Venus's, Titan's or Saturn's atmospheres, at short wavelengths. We assume that of a beam of sunlight illuminates the top of the atmosphere. We'll assume that no light emerges from the lower boundary. Again assume isotropic scattering. This problem is best to approach by solving radiative transfer equation for the *diffuse intensities*, I_d^+ and I_d^-, from which we can derive the full intensity. Let the angle of the incident beam be $\theta = \cos^{-1}(\mu_0)$. Again approximate the intensities by their hemispheric averages I^+ and I^- and assume no thermal emission, so that we can omit the $B_\nu(T)$ term. From Eqs. 3 and 5, the RT equations are:

$$\bar{\mu}\frac{dI_d^+}{d\tau} = I_d^+ - \frac{a}{2}(I_d^+ + I_d^-) - \frac{a}{4\pi}F_\odot e^{-\tau/\mu_0} \quad (14)$$

$$-\bar{\mu}\frac{dI_d^-}{d\tau} = I_d^- - \frac{a}{2}(I_d^+ + I_d^-) - \frac{a}{4\pi}F_\odot e^{-\tau/\mu_0} \quad (15)$$

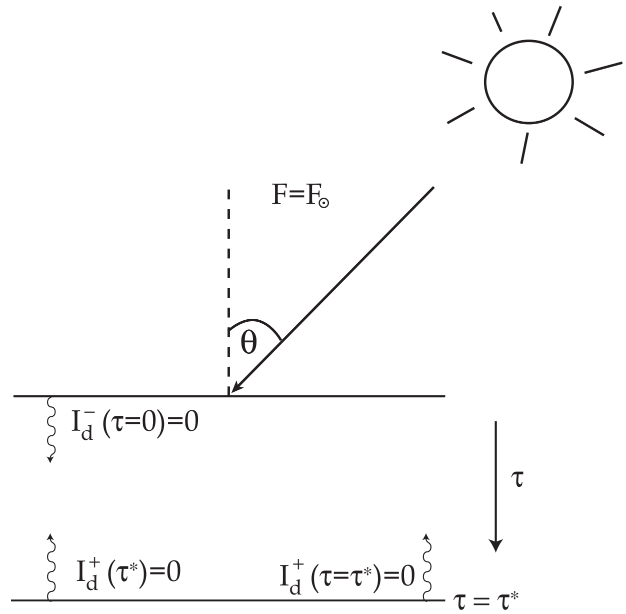

Fig. 3.— **The boundary conditions for our second simplified problem.**

As before, we sum and difference these equations and take the derivative w.r.t. τ of the resulting equations. The result, as before, is two equations that depend on a single variable each.

$$\bar{\mu}^2\frac{d^2(I_d^+ + I_d^-)}{d\tau^2} = (1-a)(I_d^+ + I_d^-) - \frac{a}{2\pi}F_\odot e^{-\tau/\mu_0}$$

Similarly,

$$\bar{\mu}^2\frac{d^2(I_d^+ - I_d^-)}{d\tau^2} = (1-a)(I_d^+ - I_d^-) + \frac{a\bar{\mu}}{2\pi\mu_0}F_\odot e^{-\tau/\mu_0}$$

The solution has the same form as that of the homogeneous solution plus an extra exponential term that accounts for the addition exponential term:

$$I_d^+(\tau) = Ae^{\Gamma\tau} + \rho_\infty De^{-\Gamma\tau} + Z^+e^{-\tau/\mu_0},$$

$$I_d^-(\tau) = \rho_\infty Ae^{\Gamma\tau} + De^{-\Gamma\tau} + Z^-e^{-\tau/\mu_0}.$$

The constants Z^+ and Z^- can be determined by inserting these expressions into the equation of radiative transfer:

$$Z^+ = \frac{aF_\odot\mu_0(\bar{\mu}-\mu_0)}{4\pi\bar{\mu}^2(1-\Gamma^2\mu_0^2)}$$

$$Z^- = -\frac{aF_\odot\mu_0(\bar{\mu}+\mu_0)}{4\pi\bar{\mu}^2(1-\Gamma^2\mu_0^2)}.$$

The boundary conditions for the diffuse component are:

$$I_d^-(\tau = 0) = 0, \tag{16}$$

$$I_d^+(\tau = \tau^*) = 0. \tag{17}$$

These expressions give us the values for A and D:

$$A = \frac{aF_\odot \mu_0[\rho_\infty(\bar{\mu} + \mu_0)e^{-\Gamma\tau^*} + (\bar{\mu} - \mu_0)e^{-\tau^*/\mu_0}]}{4\pi\bar{\mu}^2(1 - \Gamma^2\mu_0^2)(e^{\Gamma\tau^*} - \rho_\infty^2 e^{-\Gamma\tau^*})}$$

$$D = \frac{aF_\odot \mu_0[(\bar{\mu} + \mu_0)e^{\Gamma\tau^*} + \rho_\infty(\bar{\mu} - \mu_0)e^{-\tau^*/\mu_0}]}{4\pi\bar{\mu}^2(1 - \Gamma^2\mu_0^2)(e^{\Gamma\tau^*} - \rho_\infty^2 e^{-\Gamma\tau^*})}.$$

Now that we know I_d^+ and I_d^-, we can determine the intensities, source function, and flux. The full intensities are the sum of the direct and diffuse intensities:

$$I^+ = I_d^+ \tag{18}$$

$$I^- = I_d^- + \frac{\mu_0}{2\pi}F_\odot e^{-\tau/\mu_0}. \tag{18}$$

The flux can be expressed in terms of the intensities as

$$F(\tau) = 2\pi \int_0^1 [I^+(\tau, \mu) - I^-(\tau, \mu)] \ \mu \ d\mu$$

or,

$$F(\tau) = 2\pi\bar{\mu}(I_d^+ - I_d^-) - \mu_0 F_\odot e^{-\tau/\mu_0}. \tag{20}$$

Recall that the equation of transfer for the diffuse field differs from that of the total field in that it has an extra term that for single scattering. The source function can be represented as the sum of the multiple scattering radiation and the single scattered radiation:

$$S(\tau) = \frac{a}{2}(I_d^+ + I_d^-) + \frac{aF_\odot}{4\pi}e^{-\tau/\mu_0} \tag{21}$$

In summary, we find that by separating the direct and diffuse components we are able to improve upon our former two stream model, which assumed that the sky is illuminated (at the top of the atmosphere) by diffuse radiation only. Here we were able illuminate the sky with a beam of sunlight. The final intensities (upward and downward), flux and source function are determined by including the effects of both the diffuse and direct beams.