No. 2. A PHOTOGRAPHIC MAP OF THE MOON*  
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The cartography of the lunar surfaces can be split into two operations which can be carried on quite independently. The first, which is also the most laborious, is the interpretation of the lunar photographs into the symbolism of the map, with the addition of fine details from telescopic sketches. An example of this kind of work is contained in Johann Krieger's Mond Atlas which consists of photographic enlargements in which Krieger has sharpened up the detail to accord with his telescopic impressions. Krieger did not go on either to convert the photographic picture into the line symbolism of a map, or to place this picture on any definite map projection.

The second task then is the conversion of the innumerable photographic positions into map positions, according to some definite map projection. The basic step in this process is the placing of some map grid on the photograph, thus establishing a one-to-one correspondence between the distorted squares of the photographic grid and the rigorously true squares on the map grid. Thereafter it is a relatively simple task for a trained cartographic draughtsman to transfer the detail square by square. The choice of the map projection and the interval of the rectangular map grid are arbitrary, but of course both should be such that both the gridding and subsequent cartographic work are made as simple as possible.

Our group is a small one and is interested mainly in the scientific aspects of selenography, so that the compilation of a large-scale map of the entire lunar surface would not be an appropriate task for us. Hence we have limited ourselves to the production of what may be regarded as a basic cartographic document, that is, an atlas of large-scale photographs carrying the standard orthographic map grid at intervals of 0.01 of the lunar radius.

The choice of the orthographic projection is dictated by the considerations already mentioned. Lunar photographs approximate very closely to orthographic projections of the lunar surface, so far as their geometry is concerned. Hence on the photographs the orthographic grid appears with but slight distortion of the squares, except near the limb. The characteristics of this grid are fairly well-known, but for completeness they are now described. The standard rectangular coordinates of a lunar surface point may be written

\[ \begin{align*}
X &= R\xi = R \cos \beta \sin \lambda, \\
Y &= R\eta = R \sin \beta, \\
Z &= R\zeta = R \cos \beta \cos \lambda,
\end{align*} \]

where \( R \) is the distance of the point from the origin or center, \((\lambda, \beta)\) are the selenographic longitude and latitude, and \((\xi, \eta, \zeta)\) are the standard direction-cosines. In the present state of our knowledge of the lunar surface \( R \) is not known with any certainty; indeed we have only very approximate values for relatively few points. We do know, however, that \( R \) does not depart by more than a few kilometers from a mean value of 1738 km. Hence it is usual to assume \( R = \) 1738 for all points so that \((\xi, \eta, \zeta)\) then become the standard rectangular coordinates in units of the Moon's radius. Our photomap carries the lines of \( \xi = \) constant and \( \eta = \) constant at intervals of 0.01.

To facilitate production, the atlas of gridded photographs has been divided into two parts. Part 1 contains all sheets which do not involve the Moon's limb, while Part 2 deals with the limb regions. The entire disk is covered by 60 sheets, of which 29 are in Part 1.

The grid is based on the position catalogs of Franz and Saunder, and also on some unpublished lists by Arthur. An examination of these catalogs revealed that Saunder's contains very few errors while that of Franz contains numerous misprints and mistakes. It was also known from previous work that many of the Franz positions are rather rough since Franz derived them from photographs of inferior quality. In view of this and the lower random errors of the Saunder positions, we have

always used the Saunder positions in preference to those of Franz. Generally speaking, the Franz positions were preferred to those of Arthur but occasionally the Franz values were completely rejected. Gridding operations in the limb regions indicate rather large random errors in many of the Franz values, so the grids in these regions are unavoidably less accurate than those of the central areas which depend almost entirely on the Saunder positions.

Generally speaking, it should be possible to interpolate selenographic positions with an accuracy of at least 0.001 in either coordinate in the central area. In the limb regions the grid errors may reach 0.002 in places, but this cannot be helped at present.

Preliminary experiments soon showed that, in the central area, the grid could be plotted by linear interpolation between the controls. The plotting was performed on the paper sheets of the Photographic Lunar Atlas. The controls were identified and pricked and the pricks circled on the reverse side and annotated with the \((\xi, \eta)\) values. They were then connected up into horizontal and vertical chains or open polygons, by joining each point to its neighbors by straight lines. Each such line was interpolated linearly for the cuts of the \(\xi\) or \(\eta\) grid lines, the horizontal lines being used for the cuts of the \(\xi\) curves and the vertical lines for the cuts of the \(\eta\) curves. Cuts with the same value were then connected by smooth curves which were ruled in with the aid of flexible splines.

These manuscripts were then sent to the Aeronautical Chart and Information Center where the grids were copied and engraved for reproduction. Part I of the photographic map was published in November, 1960 by the University of Arizona Press.

The preparation of Part 2, dealing with the limb regions, is still in progress. In these areas we cannot use linear interpolation and the gridding procedure is much more laborious. The method is based on the following considerations. The instantaneous orthographic coordinates of the point \((\xi, \eta, \zeta)\) when the axes are defined by the apparent center of face and apparent first meridian, are \((x, y, z)\), where

\[
\begin{pmatrix}
  x \\
  y \\
  z
\end{pmatrix} =
\begin{pmatrix}
  \cos l & 0 & -\sin l \\
  -\sin l & \cos b & \cos l \sin b \\
  \sin l \cos b & \sin b & \cos l \cos b
\end{pmatrix}
\begin{pmatrix}
  \xi \\
  \eta \\
  \zeta
\end{pmatrix},
\]

in which \((l, b)\) are the topocentric librations. These are converted to conical coordinates by

\[
X = x + xz \sin s, \\
Y = y + yz \sin s,
\]

where \(s\) is the apparent lunar semidiameter. The topocentric librations, the semidiameter, and the rotation matrix are computed with desk machines, but the larger task of computing \((X, Y)\) for the controls and grid intersections is performed at ACIC using the DATATRON high-speed computer.

\((X, Y)\) are still in units of the lunar radius and so have to be scaled to the photographic points. To get over the uniaxial dilation or shrinkage of these prints, separate scale factors were derived for \(X\) and \(Y\). These were obtained from the relation

\[
\mu = \frac{(\text{Photo Distance})}{(\Delta X^2 + \Delta Y^2)^{1/2}}
\]

as applied to two well separated controls. The \((X, Y)\) values for the controls and grid intersections are then scaled and the points plotted and finally picked up on a trace. This is then placed over the photograph, fitted by means of the controls, and the grid intersections pricked through. The pricked positions are joined by smooth curves as before and the manuscripts forwarded to ACIC for engraving.

The gridding of the limb sheets has proved fairly troublesome. In these areas we have to depend almost entirely on Franz and it is clear that many of his positions have relatively large random errors. In addition to this, the Atlas sheets went through one stage as paper enlargements which were subsequently glazed and mounted. Hence these too have sensible distortions and it is frequently difficult to decide on the origin of a poor fit at a particular point, and to decide whether to leave a residual at this point or to distort the grid and leave no residual.

This photographic lunar map should be completed in the first half of 1961. So far as positional accuracy is concerned, it is much superior to any previous maps. It should be useful for any research requiring fairly accurate lunar positions, such as the determination of relative altitudes. In addition, it has the advantage of being a completely objective representation of the lunar surface.

It is hoped also that this map will provide a firm basis for a new lunar nomenclature, since it will make it possible to assign fairly accurate positions to all surface features.