NO. 27. WAVELENGTH DEPENDENCE OF THE POLARIZATION OF THE SUNLIT SKY*

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ABSTRACT

The blue sky was observed near 90° from the sun, in the vertical plane through the direction of the sun, with the sun above and just below the horizon. The observations were made with six filters ranging from 3250 to 9400 Å. For a normal clear sky at the McDonald Observatory with the sun 11°.5 above the horizon, 75% polarization was found at 5500 Å. The agreement of the Rayleigh–Chandrasekhar theory with the observations is good. The difference of the above 75% with 100% polarization appears to be due to multiple scattering (6%), molecular anisotropy (6%), and reflection by the ground (5%), while a residual 8% is presumably due to aerosols. At the McDonald Observatory, the polarization of the daytime sky has a maximum near 5500 Å, with a decrease toward longer as well as toward shorter wavelengths. The decline in the ultraviolet is caused by multiple scattering, while in the infrared it is mostly due to ground reflection, which is especially strong when green plants are present.

I. Introduction

REVIEWS of the study of skylight polarization have been published by Sekera[1,2] and by Rosenberg,[3] with many references to observational and theoretical work. Kalatin[4] and Sekera observed various sky conditions at wavelengths ranging from 4380 to 6400 Å.

A general discussion of the blue daylight sky has been made by van de Hulst.[5] Van de Hulst also enumerated the reasons why a close agreement between observations and theory has not yet been established. The reasons are: unfavorable conditions (haze or dust in the air, unknown reflection by the ground, the possibility of "blue clouds"), inadequate measurements (wavelength region too wide, unknown plane of polarization), and inaccurate computations (incorrect law of scattering, the omission of higher-order scattering, curvature of the earth).

The purpose of the present paper is to compare the Rayleigh–Chandrasekhar theory with observations of the clearest-possible blue sky. The paper is part of a study of planetary atmospheres, the symbols used are

* Reprinted from *J. Opt. Soc. America, 52,* 1164-1173, October, 1962; also *Pub. Goethe Link Obs.,* No. 47. This program is supported by the Office of Naval Research.

[1] Z. Sekera, *Advances in Geophysics,* edited by H. E. Landsberg, (Academic Press Inc., New York, 1956) Vol. 3.

[2] Z. Sekera, *Handbuch der Physik,* edited by S. Flügge, (Springer-Verlag, Berlin, 1957), Vol. 48, Pt. II.

[3] G. V. Rosenberg, Usp. Fiz. Nauk. 71, 173 (1960); transl. Sov. Phys. Usp. 3, 346 (1960).

[4] N. N. Kalitin, Meteor. Z. 43, 132 (1926).

[5] H. C. van de Hulst, *The Atmospheres of the Earth and Planets,* edited by G. P. Kuiper (The University of Chicago Press, Chicago, 1952), Chap. III.

defined in Secs. III and IV. Plane polarization only was measured; the observations were limited to approximately 90° from the sun, and close to the plane through sun, observer, and zenith. However, the range of wavelengths was extended with filters from 3250 to 9400 Å, and an attempt is made to take ground reflections and molecular anisotropy into account.

II. Equipment

The observations were made with the 82- and 36-in. reflectors of the McDonald Observatory situated in west Texas at nearly 6800-ft altitude. On November 2, 1959, observations were also made at the Goethe Link Observatory, which is situated some 15 miles south–southwest of Indianapolis, at approximately 900-ft altitude. The McDonald telescopes were used at Cassegrain focus, while the one at the Goethe Link Observatory was used at prime focus but with a Barlow lens; the f ratio was near $f/13$ for all observations. Diaphragms between 0.5 and 2 mm were used so that the aperture was between 3 and 31 sec of arc.

The photometer, filters, and the techniques of observing and of making the reductions have been described by Gehrels and Teska,[6] and they are the same as used for papers on the wavelength dependence of instrumental polarization,[7] the interstellar polarization,[8] and the polarization of Venus[9] and of the poles of Jupiter.[10] Observations are made with and without a Lyot depolarizer at seven discrete steps of analyzer rotation. From the seven photometer orientations a least-squares solution of a cosine curve is made to determine both the position angle (of electric vector maximum) and the amount of polarization. The effects of instrumental polarization and depolarization are small; corrections have been applied.

The effective wavelengths of the filters were determined, for the blue skylight, and they are: $1/\lambda=3.08$, 2.82, 2.29, 1.89, 1.46, 1.06; 2.80, 1.92, and 1.26 μ^{-1} (in Å, respectively: $\lambda=3250$, 3550, 4370, 5290, 6850, 9430; 3570, 5210, and 7940). It is estimated that the determinations of the effective wavelengths have a probable error of ± 0.02 in $1/\lambda$; the determinations are based on tracings of the actual filters and on the manufacturer's sensitivity curves for the tubes, allowing, however, for the blue color of the skylight. The filters generally are some 700 Å wide (380 Å for the one at 3250 Å).

III. Polarization Measurements

Table I gives the dates of the observations, in Universal Time. Also listed are the right ascension and the declination; the telescope was set at these coordinates with the drive on, at sidereal rate, during the observations. The *phase angle* is defined as the angle between the sun, the point of sky, and the observer. It is computed with

$$\cos i = -\sin\delta \sin\delta_{\odot} - \cos\delta \cos\delta_{\odot} \cos\Delta\alpha,$$

where i is the phase angle, δ the declination listed in Table I, $\delta_{\odot}$ the declination of the sun's center interpolated from the entries in the Nautical Almanac, and $\Delta\alpha$ is the difference between the two right ascensions.

TABLE I. DATA ON ASPECT AND SKY CONDITIONS

Date UT	Tel.[a]	R.A. (1960)	Dec. (1960)	Phase	Sky conditions
1959					
May 2	36″	20h35m5	+30°40′	97°6	Clear.
Aug. 10	82″	3 18.0	30 40	97.8	Clear.
Nov. 2	GL	18 22.2	56 00	94.1	Clear.
Dec. 24	36″	14 50.0	56 00	90.1	Varying, thin clouds in the field and over the sun. Rather cloudy during first observation at $1/\lambda=1.89$.
Dec. 26*A*	36″	11 26.0	47 00	64.5	Dust and slight haze, transparency not good; but fairly uniform.
Dec. 26*B*	36″	13 42.0	30 40	94.7	
Dec. 26*C*	36″	15 37.0	12 00	127.1	
1960					
Jan. 1	36″	13 00.0	30 40	81.7	Thin clouds about, and some dust. Clouds in the field during the observation at $1/\lambda=2.82$.
Jan. 6*A*	36″	14 00.0	30 40	89.4	Perfectly clear.
Jan. 6*B*	36″	15 30.0	20 00	112.1	
Jan. 6*C*	36″	12 30.0	41 00	68.9	
Apr. 6	82″	19 06.4	30 00	94.2	Clear, but sky not hard blue.
Apr. 9	82″	19 10.0	30 00	93.2	Hazy; clouds about. Clouds in the field only for the first observation at $1/\lambda=1.46$.
Aug. 31	82″	4 38.9	+30 40	94.3	Clouds at start, for the first observation at $1/\lambda=2.29$; clear otherwise.

[a] 82″ = McDonald 82 in.; 36″ = McDonald 36 in.; and GL = Goethe Link 36 in.

[6] T. Gehrels and T. M. Teska, Publs. Astron. Soc. Pacific **72**, 115 (1960).
[7] T. Gehrels, Astron. J. **65**, 466 (1960).
[8] T. Gehrels, Astron. J. **65**, 470 (1960).
[9] T. Gehrels and R. E. Samuelson, Astrophys. J. **134**, 1022 (1961).
[10] T. Gehrels, Astron. J. (to be published).

TABLE II. POLARIZATION MEASURES OF THE DAYTIME SKY

UT Date	$1/\lambda$	Sun's elev.	P	θ_r
			%	
1959 May 2	1.06	+5°2	40.3	179°2
		6.6	40.1	179.0
	1.89	3·7	72.3	178.4
		8.8	68.2	178.4
	2.82	2.2	63.9	178.4
		10.3	63.1	178.5
Aug. 10	1.06	7.7	64.9	178.4
		9.4	63.8	178.4
	1.46	24.7	69.5	179.6
		26.5	69.3	179.5
	1.89	5.2	71.3	179.8
		11.2	68.4	179.7
	2.29	23.0	64.4	179.2
		28.2	64.1	179.4
	2.82	2.7	60.9	179.2
		12.8	59.4	179.4
	3.08	21.7	53.2	179.3
		29.9	52.8	179.2
Nov. 2	1.06	34.8	42.8	0.5
		35.7	42.7	0.4
	1.46	35.8	63.8	179.0
		35.0	62.7	179.3
	1.89	35.2	71.0	179.7
	2.29	35.6	68.9	179.7
	2.82	35.4	61.6	178.5
	3.08	35.4	55.9	179.1
Dec. 24	1.06	32.1	29.1:	178.9
	1.46	35.3	50.4	0.4
	1.89	31.6	52.1	179.5
		33.3	66.5	179.8
	2.29	35.7	66.5	0.4
	2.82	32.7	56.8	179.8
	3.08	35.8	56.3	0.5
Dec. 26*A*	1.06	25.1	36.6	178.8
	1.46	7.8	56.7	179.2
		19.8	52.8	...
	1.89	27.0	51.2	0.5
	2.29	8.9	51.8	179.4
		19.5	49.8	...
	2.82	27.6	42.8	2.3
	3.08	9.7	40.4	179.0
		19.3	39.8	...
Dec. 26*B*	1.06	29.0	36.1:	178.8
	1.46	11.3	71.8	179.3
	1.89	29.7	71.0	0.1
	2.29	12.3	75.9	179.7
	2.82	30.2	67.0	0.4
	3.08	+12.9	61.9	179.2

UT Date	$1/\lambda$	Sun's elev.	P	θ_r
			%	°
1959 Dec. 26*C*	1.06	+23°8	22.2	...
		+31.4	16.0	177.8
	1.46	+17.8	30.1	177.9
	1.89	+24.0	35.2	...
		+31.9	34.4	178.6
	2.29	+16.9	34.6	179.1
	2.82	+24.2	34.1	...
		+32.5	36.1	177.8
	3.08	+15.9	29.1	178.1
1960 Jan. 1	1.06	+ 5.4	52.3	178.9
	1.89	− 8.1	56.6	179.1
		+ 6.3	75.9	0.0
	2.82	+ 7.6	62.8	179.7
Jan. 6*A*	1.06	+ 9.6	72.1	178.8
	1.26	+11.1	76.5	...
	1.89	− 6.4	71.1:	0.2
		+10.6	79.6	...
	2.82	+10.8	68.7	...
Jan. 6*B*	1.06	+ 2.8	47.4	178.2
		+14.8	42.4	...
	1.26	+ 4.2	52.0	...
		+13.9	50.1	...
	1.89	+ 3.7	59.0	...
		+14.3	58.8	...
	2.82	+ 4.0	51.9	...
		+14.1	53.7	...
Jan. 6*C*	1.06	+ 6.4	64.0	179.1
		+12.9	58.7	...
	1.26	+ 7.8	64.2	...
		+12.2	62.1	....
	1.89	+ 7.2	62.1	...
		+12.5	61.1	...
	2.82	+ 7.6	50.9	...
		+12.3	50.3	...
Apr. 6	1.06	+22.2	50.4	179.7
	1.89	+19.3	66.8	179.4
	2.82	+20.8	61.1	179.4
Apr. 9	1.06	+44.1	45.4	178.2
	1.46	+51.7	49.0:	179.1
		+55.6	53.9	179.6
	1.89	+45.1	58.0	178.4
	1.92	+54.6	58.0	178.8
	2.29	+52.6	57.1	178.9
	2.82	+45.9	53.3	178.5
	3.08	+53.7	50.5	178.8
Aug. 31	1.06	+ 4.1	61.2	178.7

Table II gives the polarization measurements on the daytime sky. The *sun's elevation* is computed with the relation

$$\sin E = \sin\delta_\odot \sin\gamma + \cos\delta_\odot \cos\gamma \cos HA,$$

where E is the sun's elevation, γ the latitude of the observatory, and HA is the hour angle of the center of the sun. The elevation of the sun also is an indication of the time of observation; all observations were made in the morning, except those of November 2 at $1/\lambda = 1.46$, 2.29, and 3.08, which fell immediately after the middle of the day.

The *percentage polarization* is defined by

$$P = 100(I_1 - I_2)/(I_1 + I_2),$$

where I_1 and I_2 are the intensities of the light at the position angle of electric vector maximum and minimum, respectively. The *position angle* of the electric vector maximum θ_r is given with respect to the normal to a plane through the centers of the sun and the earth, and through the direction of observation (the right ascension and declination given in Table I). The direction of θ_r is from north through east, etc.; position angles are between 0° and 180° only. The direction of observation was nearly in the plane through sun, observer, and zenith (the *sun's vertical*), which, incidentally, is confirmed by the angles θ_r in Table II being close to 180°. The Stokes parameter U therefore is nearly equal to zero, and there is no elliptical polarization[11] so that the Stokes parameter V also is equal to zero.

[11] V. G. Fesenkov, Astron. Zhur. **37**, 785 (1960); transl. Sov. AJ 4, 741 (1961).

TABLE III. POLARIZATION MEASURES FOR THE CREPUSCULAR SKY

Sun's elev.	$1/\lambda$	P	Sun's elev.	$1/\lambda$	P	Sun's elev.	$1/\lambda$	P
			January 1, 1960; observations at $\theta=16°3$					
		%			%			%
−7°7	1.89	47.3:	−4°9	1.89	64.2	+1°2	1.89	67.3
−7.3	1.06	38.3	−4.7	1.06	64.2	+1.7	2.82	57.0
−7.0	1.89	51.4:	−4.2	1.06	69.2	+2.0	1.26	58.0
−6.7	2.82	44.1:	−4.0	1.89	66.6	+2.2	1.26	55.9
−5.9	1.26	56.9:	−3.9	2.82	58.8	+2.6	2.82	57.0
−5.7	1.26	57.0	−3.0	1.26	68.8:	+3.0	1.89	66.3
−5.1	2.82	54.5	+1.0	1.06	62.2:	+3.3	1.06	44.0
			January 6*A*, 1960; observations at $\theta=177°1$					
		%			%			%
−5°9	1.89	73.9	−3°5	1.06	84.3	−0°7	1.89	81.6
−5.7	1.06	75.9:	−3.0	1.89	80.0	−0.3	2.82	68.9
−5.2	1.89	76.3:	−2.7	2.82	70.2	0.0	1.26	82.4
−5.0	2.82	66.5	−2.5	1.26	84.1	+0.5	1.26	81.6
−4.8	1.26	82.3	−2.0	1.26	84.0	+0.7	2.82	68.8
−4.2	1.26	83.2	−1.8	2.82	69.8	+1.0	1.89	81.2
−4.0	2.82	69.1	−1.7	1.89	81.0	+1.1	1.06	77.7
−3.8	1.89	79.2	−1.3	1.06	84.5			
−3.6	1.06	84.5	−0.8	1.06	85.3			
			August 31, 1960; observations at $\theta=176°3$					
		%			%			%
−4°6	1.06	83.0	−1°5	1.06	78.0	+1°5	1.26	73.9
−4.2	1.89	77.4	−1.3	1.06	76.5	+1.8	1.06	67.0
−4.0	2.82	63.2	−1.0	1.26	78.8	+2.0	1.06	66.3
−3.8	1.26	80.9	−0.7	1.89	79.1	+2.4	1.26	73.2
−3.2	1.06	81.8	−0.4	1.06	72.3	+2.6	1.89	77.6
−2.6	2.82	67.0	0.0	2.82	66.1	+3.0	2.82	65.6
−2.2	1.89	79.0	+0.5	2.82	65.9	+3.5	1.06	63.0
−2.0	1.26	81.2	+1.1	1.89	78.5	+4.6	1.06	62.2

Table III lists additional measurements especially made for the study of the sky with the sun just below the horizon. At the start of these observations, checks were made for the absence of stars within the diaphragm, as well as the usual checks for the absence of light-leaks of the photometer. Because of the fast changes of the sky at this time of the day, the observations were made, instead of at the usual seven analyzer orientations, at a single preset orientation, θ in Table III, close to that of cosine-curve maximum.

IV. Color Measurements

A few measurements of colors were made; they are listed in Table IV together with the sun's elevation at the time of observation. All color observations were made with the depolarizer in the incoming beam. The *difference in magnitude* [for instance: $(B-R)=2.5 \times\log_{10}(i_{\mathrm{red}}/i_{\mathrm{blue}})$] is given of the intensities observed with the *I*nfrared, *R*ed, *G*reen, *B*lue, and *U*ltraviolet filters, of which the effective wavelengths are, respectively, $1/\lambda=1.06$, 1.46, 1.89, 2.29, and 2.82 μ^{-1}. The

TABLE IV. OBSERVATIONS OF COLORS FOR THE DAYTIME SKY AND STANDARD STARS

Object	Date UT	$B-R$	$G-I$	$U-G$	Sun's elevation for B, R	Sun's elevation for U, G, I
Daytime sky	Dec. 24	$-1^{m}09$	$-1^{m}75$	$-1^{m}60$	35°0	33°9
	Dec. 26*A*	−1.29	−1.84	−1.52	10.5	28.1
	Dec. 26*B*	−1.31	−1.73	−1.70	13.8	30.6
	Dec. 26*C*	−1.14	−1.55	−1.61	18.6	33.0
	Jan. 1	···	−1.52	−0.68	···	4.5
	Jan. 6*B*	···	−1.48	−0.78	···	5.2
	Jan. 6*C*	···	−2.15	−1.17	···	9.0
	Apr. 9	···	−1.67	−1.91	···	42.3
α Vir; *B*1 V	···	−1.12	−1.38	−2.06	···	···
β Lib; *B*8 V	···	−1.00	−1.02	−1.24	···	···
α Lyr; *A*0 V	···	···	−0.98	−0.84	···	···
τ CrB; *K*0 III	···	+0.37	+0.35	+1.15	···	···
α Ser; *K*2 III	···	+0.52	+0.47	+1.68	···	···
γ Dra; *K*5 III	···	···	+1.12	+2.56	···	···

effective wavelengths are comparable to those of the six-color photometry of Stebbins *et al.*,[12] and the calibration was therefore conveniently made with stellar values of that system. For these stars, Table IV lists the spectral types and the averages of color observations made immediately prior to those on the daytime sky. The U, G, I (and B, R) reductions were made as usual for stellar photometry, applying atmospheric-extinction corrections for the stars but *not* for the blue sky. The colors of the sky therefore are, while observed at the bottom of the atmosphere, referred to a system outside the atmosphere. After the colors of the sun ($B-R=-0.03$; $G-I=-0.21$; $U-G=-0.16$ mag.; also outside the atmosphere) are taken into account, the blue sky colors of Table IV are directly comparable to the theory. We shall see in Sec. VIII how well the Rayleigh–Chandrasekhar theory accounts for the extinction of the skylight.

V. *Precision of the Observations*

The precision of the percentage polarization is estimated from the residuals in the least-squares solution of the cosine curves; the average residual is ±0.56%, nearly independent of the wavelength. In measurements on stars,[8] using the same techniques, the average residual was ±0.142%, while, from repetition on the same star but in different nights, a probable error of ±0.069% of the published result of one night was found. It then follows that the blue-sky results have a probable error of ±0.27% in P. The increase in the scatter, compared to that of measures on stars, is mostly due to the change of the polarization with change of sun's elevation during the measurements. Colons are used in Table II when the average residual in the least-squares solution is greater than ±1.1%. The P values in Table III were obtained from two measurements, at only one analyzer orientation, and the average is listed. Colons are used in Table III when only one measurement is available, and also when the difference between the two determinations is greater than ±1.38%.

The probable error of the right ascension is about $\pm0^{m}.1$, of the declinations it is $\pm1'.5$, and it is $\pm0^{\circ}.15$ for the sun's elevations as well as for the phase angles. The precision of the position angles θ_r was determined from 14 repetitions of measurements made within a few minutes with the same filter and tube combination, and a probable error of $\pm0^{\circ}.1$ was found. The probable error of the colors in Table IV is estimated to be of the order of ±0.05 mag.

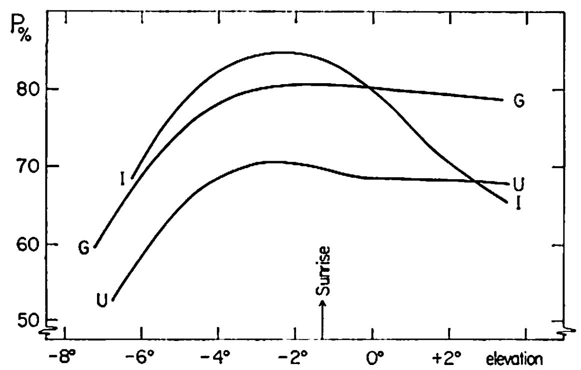

FIG. 1. The polarization of the blue sky, near quarter-phase, about the time of sunrise. Percentage polarization obtained with *I*nfrared, *G*reen, and *U*ltraviolet filters, as a function of geometrical sun's elevation. The start of sunrise is actually observed earlier due to refraction, etc.

VI. *Wavelength and Time Dependence*

The degree of polarization of the sunlit sky strongly depends on the elevation of the sun. About the time of sunrise, the changes are spectacular as is seen in Fig. 1. Figure 1 was obtained by taking the smooth average of the data in Table III, corrected to exactly $90^{\circ}.0$ from the sun and to polarization maximum (see below), for the sunrise on January 1 and 6, and August 31, 1960, at the McDonald Observatory. The telescope was pointed nearly 90° away from the direction of the sun, in the sun's vertical, and the drive was on at sidereal rate. The polarization is given for the filters in the *I*nfrared (9430 Å), *G*reen (5290 Å), and *U*ltraviolet (3550 Å), as a function of the sun's elevation. The elevations are computed as stated in Sec. III, without corrections for refraction, dip of the horizon, nor for the sun having a disk rather than being a point source. The start of sunrise actually was seen about 7 min earlier than the time of 0° elevation, that is, at about $-1^{\circ}.3$ elevation. The principal features of Fig. 1 are, on the left, the steep change of all polarizations, and on the right, the steep drop of the infrared polarization with respect to the others. There is a peculiar "bulge" in the ultraviolet, with the maximum near $-2\frac{1}{2}°$ elevation. An approximate idea of the extension of Fig. 1 to the right may be obtained from the numbers in Table II. Until about +20° elevation all polarizations decrease, and beyond +20° until midday there is a further slow decrease in the infrared and perhaps a very small increase in the ultraviolet. These descriptions are in agreement with the detailed observations made by Robley of the crepuscular sky,[13,14] and in general with those made by Sekera.[15]

In Table V, on the left, an attempt is made to establish the polarizations at a mean time, from the actual observations made at slightly different times. The mean is taken by interpolation in Tables II and III, and the mean elevation of the sun is also found. For the purpose of making intercomparisons of the wavelength dependence on different occasions, the percentages are normalized, on the right in Table V,

[12] J. Stebbins and G. E. Kron, Astrophys. J. 126, 266 (1957).

[13] See reference 2, p. 324.
[14] R. Robley, Ann. Géophys. 8, 1 (1952).
[15] See reference 1, Fig. 22.

TABLE V. PERCENTAGE POLARIZATION, INTERPOLATED IN ELEVATION, AND NORMALIZED TO 61.1% AT λ = 1.89

Date UT	Phase	Sun's elev.	Interpolated polarization at 1/λ 1.06	1.46	1.89	2.29	2.82	3.08	Est. Pol.[a] at 90°0	Line	Normalized polarization at 1/λ 1.06	1.46	2.29	2.82	3.08
1959			%	%	%	%	%	%	%		%	%	%	%	%
May 2	97°6	+ 6°5	40.1	···	70.0	···	63.5	···	72.6	1	35.0	···	···	55.4	···
Aug. 10	97.8	+15.0	61.2:	71.6:	67.3	65.0	59.2	54.0	69.4	2	55.6:	65.0:	59.0	53.7	49.0
Nov. 2	94.1	+35.8	42.7	63.6	71.0	68.9	61.5	55.9	71.7	3	36.7	54.7	59.3	52.9	48.1
Dec. 24	90.1	+34.0	29.1:	50.4:	65.5:	66.5:	56.8:	56.3:	65.5:	4	27.1:	47.0:	62.0:	53.0:	52.5:
Dec. 26*A*	64.5	+18.0	38.8	53.0	53.4	49.9	42.9	39.9	···	5	44.4	60.6	57.1	49.1	45.7
Dec. 26*B*	94.7	+18.0	44.1:	69.5	72.5	74.9	66.5	61.2	73.3	6	37.2:	58.6	63.1	56.0	51.6
Dec. 26*C*	127.1	+18.0	28.2:	30.0	36.8	34.6	32.1	29.5	···	7	46.8:	49.8	57.4	53.3	49.0
		− 5.8	*53.0*	(*56.7*)	*59.9*	···	*50.8*	···	74.7	8	54.1	(57.8)	···	51.8	···
1960		− 4.0	*71.0*	(*65.5*)	*66.8*	···	*58.6*	···	83.2	9	64.9	(59.9)	···	53.6	···
Jan. 1	81.7	+ 1.0	*62.2*:	(*63.0*:)	*67.4*	···	*57.1*	···	83.9	10	56.4:	(57.1:)	···	51.8	···
		+ 2.6	*49.6*	(*53.9*)	*66.5*	···	*57.0*	···	82.8	11	45.6	(49.5)	···	52.4	···
		+ 6.3	52.0	···	75.9	···	62.9	···	78.6	12	41.9	···	···	50.6	···
		− 4.0	*83.0*	(*83.5*)	*78.9*	···	*69.1*	···	79.3	13	64.3	(64.7)	···	53.5	···
Jan. 6*A*	89.4	− 1.3	*85.3*	(*83.6*)	*81.3*	···	*69.4*	···	81.7	14	64.1	(62.8)	···	52.2	···
		+ 1.0	*78.4*	(*80.9*)	*81.4*	···	*68.7*	···	81.8	15	58.8	(60.7)	···	51.6	···
		+10.6	71.8	(76.6)	79.6	···	68.7	···	79.8	16	55.1	(58.8)	···	52.7	···
Jan. 6*B*	112.1	+ 4.0	46.8	(52.0)	59.0	···	51.9	···	···	17	48.5	(53.9)	···	53.7	···
		+14.0	42.6	(50.1)	58.8	···	53.7	···	···	18	44.3	(52.1)	···	55.8	···
Jan. 6*C*	68.9	+ 7.5	63.0	(64.4)	62.0	···	50.9	···	···	19	62.1	(63.5)	···	50.2	···
		+12.4	59.0	(62.0)	61.1	···	50.3	···	···	20	59.0	(62.0)	···	50.3	···
Apr. 6	94.2	+20.8	50.7	···	66.5	···	61.1	···	67.1	21	46.6	···	···	56.1	···
Apr. 9	93.2	+49.9	45.4:	53.9	58.0	57.1	53.3	50.5	58.4	22	47.8:	56.8	60.2	56.1	53.2
		− 4.0	*82.8*	(*80.5*)	*77.5*	···	*63.2*	···	78.5	23	65.3	(63.5)	···	49.8	···
Aug. 31	94.3	− 1.3	*76.9*	(*79.5*)	*79.1*	···	*67.6*:	···	80.2	24	59.4	(61.4)	···	52.2:	···
		+ 1.0	*68.5*	(*74.4*)	*78.6*	···	*66.0*	···	79.6	25	53.2	(57.8)	···	51.3	···
		+ 2.6	*65.2*	(*73.2*)	*77.6*	···	*65.6*	···	78.5	26	51.3	(57.6)	···	51.7	···

[a] The amount of polarization at 1/λ = 1.89, if the observation had been made at exactly 90°0 phase, is estimated under "Est. Pol. at 90°0." The values in *italics* are smaller than the amplitude of the polarization cosine curve. Parentheses are used for observations at 1/λ = 1·26.

by multiplication of each line with such a factor that the polarization in the green is 61.1%. (The value 61.1 is arbitrary; it was chosen for convenience of comparison with other results in this program.[8]) *Italic* print has been used in Table V for the values taken from Table III; parentheses are used for observations with the red filter at 1/λ = 1.26, instead of the usual combination of filter and tubes that has 1/λ = 1.46. The middle column of Table V, "Est. Pol. at 90°0," gives the percentage polarization that would have been observed, near the sun's vertical, if the measures for 1/λ = 1.89 had been made at exactly 90°0 phase; the modification is based on a figure published by Sekera.[16] The values from Table III now are changed to the cosine-curve maximum, by division by cos2($\Delta\theta$), where I estimated $\Delta\theta$ = 16°8, 1°7, and 2°4 for January 1, January 6, and August 31, respectively.

It is noted that, except in the infrared, the wavelength dependence is nearly constant, even when there is an appreciable change from day to day in the actual amount of polarization.

For further intercomparisons, the normalized polarizations of Table V are averaged in Table VI, in selected groups that are indicated by the first and second columns. First, the difference of the sun below and above the horizon is given by the averages on three mornings: with the sun about 5°3 below the apparent horizon, and about half an hour later, for the sun immediately above the horizon. The contrast between the sun above and just below the horizon is further

TABLE VI. AVERAGES OF PERCENTAGE POLARIZATION, NORMALIZED TO 61.1% AT 1/λ = 1/89

Conditions	Lines used (of Table V)	Average polarization at 1/λ 1.06	1.26	1.46	2.29	2.82	3.08
		%	%	%	%	%	%
Sun below horizon	9,13,23	64.8	62.7	···	···	52.3	···
Sun above horizon	11,16,26	50.7	55.3	···	···	52.3	···
Clear sky	1,2,3,16,26	45.8:	58.2:	58.1:	59.2:	53.3	48.6:
Sky not perfectly clear	4,6,11,12,21,22	42.3	···	55.6	61.7	54.1	52.4
Infrared polarization high	2,8,9,10,13,14,15,16, 19,20,23,24,25,26	59.0	60.7	···	···	51.9	···
Infrared polarization low	1,3,4,5,6,7,11,12,17, 18,21,22	42.2	51.8	55.3	59.7	53.7	49.8

[16] See reference 1, Fig. 3.

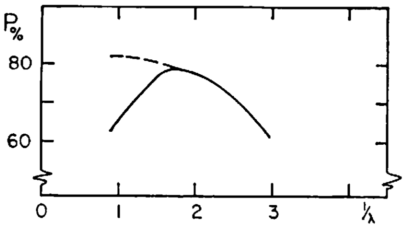

FIG. 2. Observed percentage polarization, of the clear blue sky near 90° phase, as a function of the inverse of the wavelength in microns. *Solid line*, for the sun 4° above the horizon. *Broken line*, for the sun 3° below the horizon. A typical case, as observed on the morning of August 31, 1960, at the McDonald Observatory.

illustrated by Fig. 2; the difference in the ultraviolet is small, but in the infrared it is large.

Next in Table VI, a comparison is made of observations on perfectly clear and on less-perfect sky, near 90° phase and with the sun well above the horizon. For the hard-blue sky, the red and infrared polarizations are somewhat stronger, and the blue and ultraviolet weaker than those of the less-perfect sky. There are too few observations to study the effects of a large change of phase. The first impression from Table V is that at slightly different distances from the sun, near 90°, there still is about the same wavelength dependence of polarization.

Finally in Table VI, two groups are intercompared combining the polarizations that are strong and those that are weak in the infrared; the division is chosen rather arbitrarily, with the separation at 50% polarization. In Fig. 3, the two diagrams are given for strong and weak infrared polarizations. We should keep in mind, however, that there is no firm distinction into two groups, and that all intermediate cases, and even more extreme cases, may occur. The dispersion of the polarization is highly variable in the infrared, but not so at shorter wavelengths. This conclusion, and all the work of this paper, pertains to the nearly perfectly clear sky; different conclusions are found from observations made near industrial centers, near the ocean, etc.

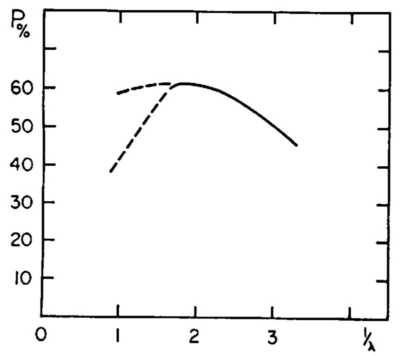

FIG. 3. Typical patterns for different days and seasons of blue-sky polarization, normalized to 61.1% in the green, as a function of the inverse of the wavelength in microns. Arbitrarily divided into two groups, namely with weak and strong infrared polarizations.

VII. Interpretations

The polarization in the red and infrared is largely due to single scattering. It strongly depends on ground reflections which, in Texas as well as in Indiana, appear to be greater in the infrared than at other wavelengths. Water-vapor absorption and aerosols apparently affect the observations, causing day-to-day variations.

The polarization at shorter wavelengths still is affected by ground reflections, for, even in the ultraviolet, the ground is a bright and near-by source of light. More characteristic for this part of the spectrum, however, is the multiple scattering. The multiple scattering, increasing with $(1/\lambda)^4$, brings into the observed beam increasingly more light from all directions, rather than directly from the sun. The sharp 90°-phase situation is thereby spoiled and the polarization diluted. (An illustration of the multiple scattering is given by Strong.[16a])

The variations from day to day (for a "clear" sky on a continental mountaintop), at constant sun's elevation, are of the order of ±3% polarization (±14% in the infrared); they may be due to aerosols and to, partly seasonal, changes of the ground reflectivity.

The dependence on sun's elevation of the polarization values can also be understood. Starting at noon and going back in time, the sun is getting lower, the relative brightness of the earth's surface decreases and all polarizations, therefore, slightly increase. With the sun setting below the horizon, still going back in time in Fig. 1, the illumination of the ground, of course, terminates, with drastic effect on the polarization in the infrared. For it is in the infrared that ground reflections are especially strong (Sec. IX).

As the sun sets still lower below the horizon, the polarization values decrease sharply, possibly for lack of direct illumination into the observed beam so that the multiple scattering becomes more important. One may well argue that the explanation must be more complex. If there simply were only an increase of multiple scattering, the slopes of the three curves, on the left in Fig. 1, should differ appreciably. However, the multiple scattering also affects the transmission of the grazing light from the low sun. As a further complication it is noted that the observations were made on the top of a steep mountain so that the atmosphere locally was not plane parallel. The problem of the crepuscular sky merits a separate study as has been made by Robley,[14] Dave,[17] and others. The variations of the ground reflectivity just before sunset[17a] should also be taken into account.

[16a] J. Strong, *Concepts of Classical Optics* (W. H. Freeman and Company, Inc., San Francisco and London, 1958), p. 108.
[17] J. V. Dave, Proc. Indian Acad. Sci. 43A, 336 (1956).
[17a] E. V. Ashburn and R. G. Weldon, J. Opt. Soc. Am. 46, 583 (1956).

TABLE VII. COMPARISON OF THEORY WITH OBSERVATIONS OF POLARIZATION

Site	1/λ	Obs. P	Opt. Depth	Anisotr. factor	Computed at R 0%	25%	80%	R	For this site Comp. P	O−C
		%			%	%	%	%	%	%
	1.11	55.0	0.01	0.940	93.9	82.9	65.4	51	73.8	−18.8
	1.31	63.5	0.02	0.941	93.0	82.1	65.1	32	79.9	−16.4
McDonald[a]	1.66	74.8	0.05	0.942	90.2	80.3	64.2	14	84.4	− 9.6
Observatory	1.96	74.9	0.10	0.942	87.3	78.1	62.5	11	83.0	− 8.1
6800-ft alt	2.17	74.0	0.15	0.943	84.6	75.8	60.6	7	81.9	− 7.9
	2.46	70.9	0.25	0.943	79.7	71.7	57.0	5	78.3	− 7.4
	2.92	63.3	0.50	0.944	69.5	62.3	48.0	4	68.4	− 5.1
	1.62	73.4	0.05	0.942	90.2	80.3	64.2	28	79.5	− 6.1
Cactus Peak[a]	1.92	74.6	0.10	0.942	87.3	78.1	62.5	20	79.8	− 5.2
(Sekera 1956)	2.13	74.9	0.15	0.943	84.6	75.8	60.6	16	78.5	− 3.6
5400-ft alt	2.40	73.0	0.25	0.943	79.7	71.7	57.0	12	75.6	− 2.6
	2.86	64.0	0.50	0.944	69.5	62.3	48.0	11	66.5	− 2.5
	1.04	42.1	0.01	0.940	92.1	66.5	41.2	41	57.9	−15.8
	1.24	52.4	0.02	0.941	91.0	65.8	41.0	36	58.9	− 6.5
	1.56	67.1	0.05	0.941	88.5	64.9	40.4	8	78.2	−11.1
Goethe Link[b]	1.84	71.1	0.10	0.942	85.3	63.7	39.7	10	74.7	− 3.6
Observatory	2.04	70.4	0.15	0.943	82.2	62.0	39.0	5	77.4	− 7.0
900-ft alt	2.30	68.9	0.25	0.943	76.8	59.4	36.9	4	73.4	− 4.5
	2.75	62.9	0.50	0.944	66.4	52.8	33.0	3	64.7	− 1.8
	3.27	50.6	1.00	0.946	50.6	41.1	25.3	3:	49.4	+ 1.2

[a] For 90° phase, in the sun's vertical, sun's elevation 11°.5.
[b] For 94° phase, in the sun's vertical, sun's elevation 37°.

VIII. Comparison with Theory

Chandrasekhar developed the theory of multiple molecular scattering. Chandrasekhar and Elbert,[18] and recently Coulson, Dave, and Sekera,[19] have published convenient tables for the application of the theory. The tables were computed for scattering by isotropic molecules, while the effect of arbitrary shapes of the molecules should also be taken into account.[20] Only recently, however, has the numerical value of the anisotropy correction for air been accurately determined, and even now only for single scattering. From the work of de Vaucouleurs[21] and Pulido,[22] for single scattering at 90° phase, a depolarization of 6.0% (±0.2% m.s.e.) may be derived.

The depolarization in multiple scattering was obtained from calculations made by Sekera; I am indebted to Dr. Sekera for sending me the results before publication. The calculations were made several years ago; they are based on 5.0% depolarization in single scattering (optical thickness 0.00); the computations are for various values of μ_0, of the optical thickness, and of distance from the sun. From Sekera's calculations I have derived, after scaling by a factor 6/5 to the results obtained by Pulido and de Vaucouleurs, the correction factors listed in the fifth column of Table VII. The factors are strictly valid only for $R=0\%$; at other albedo values the same factors probably are applicable for the present purpose, since the depolarization corrections are relatively small. Incidentally, it is of interest to note that the anisotropy factors in Table VII are nearly the same for different values of optical thickness.

A comparison of theory and observations is made in Table VII. The observed amounts of polarization are first listed for the present observations made at the McDonald Observatory; actually taken is the case of the lower part of Fig. 3 normalized to 75% at $1/\lambda=1.89$, which is the average for sun's elevation near 11°.5. Secondly, Table VII lists observations made by Sekera[15] at sun's elevation 11°.5 on the morning of July 22, 1954, at Cactus Peak, California (they are shown in the present Fig. 4). Thirdly, the polarization curve of November 2, 1959, at the Goethe Link Observatory is compared with the theory. The relations between wavelength and optical thickness, as a function of the altitude of the observatory, are based on the work of

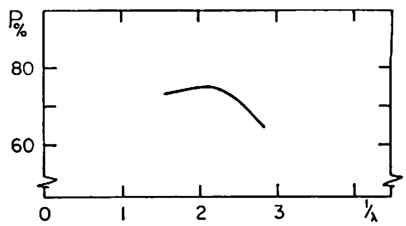

FIG. 4. Percentage polarization, observed by Sekera (1956) at Cactus Peak, as a function of the inverse of the wavelength in microns.

[18] S. Chandrasekhar and D. D. Elbert, Trans. Am. Phil. Soc. 44, 643 (1954).

[19] K. L. Coulson, J. V. Dave, and Z. Sekera, *Tables Related to Radiation Emerging from a Planetary Atmosphere with Rayleigh Scattering* (University of California Press, Berkeley and Los Angeles, 1960).

[20] Rayleigh (J. W. Strutt), Phil. Mag. 35, 373 (1918); also *Scientific Papers* (Cambridge University Press, Cambridge, England, 1920), Vol. 6, p. 540.

[21] G. de Vaucouleurs, Ann. Phys. 6, 213 (1951).

[22] A. A. Pulido, dissertation, Chemistry Department, Indiana University (1961).

Deirmendjian,[23] which is for molecular scattering without anisotropy correction. Following the anisotropy corrections are three columns with theoretical polarizations (from Table 4 of reference 19, after multiplication by the correction factors). The values taken from the tables are for various reflectivities, namely 0, 25, and 80%; near the sun's vertical (so that $\varphi-\varphi_0=180°$); for the McDonald Observatory and for Cactus Peak at sun's elevation 11°.5 ($\mu_0=0.20$) and at 90° phase (so that $\mu=0.98$, or nearly so); and for the Goethe Link Observatory at sun's elevation 37° ($\mu_0=0.60$) and at 94° from the sun (so that $\mu=0.755$). The values for optical thickness 0.01 were obtained by graphical extrapolation. The comparison of theory with observations is completed in the last three columns. The reflectivity of the ground R is taken from Fig. 5 (Sec. IX), and the computed amount of polarization is obtained by interpolation in the preceding columns. The last column of Table VII gives the difference, namely the *O*bserved minus the thus *C*omputed polarization.

The reduction from 100% polarization, for single scattering on isotropic molecules, to the 75% observed in the visual, on the average at the McDonald Observatory, apparently is due to multiple scattering (6%), molecular anisotropy (6%), while the remaining 13% is due to aerosols and ground reflections. Typical for the McDonald Observatory apparently is, in visual light, a division of some 5% dilution due to ground reflections, and perhaps about 8% due to aerosols. Of these percentages, 75% varies from day to day because of the varying contribution of the aerosols; the other contributions should be fairly constant, except for seasonal changes in reflectivity. The case of Dr. Sekera's observation at Cactus Peak was for 74% in the visual, and the dilutions are 6% by multiple scattering, 6% because of the molecular anisotropy, 9% by the ground reflections, and 5% due to aerosols. At the Goethe Link Observatory in visual light the dilutions are 9% by multiple scattering, 6% because of the molecular anisotropy, 10% by the ground reflections, and 4% due to aerosols. The lower altitude of the Goethe Link Observatory accounts for the larger dilution by multiple scattering and by ground reflection. November 2 was not a typical but rather an unusually clear day.

The uncertainties in the above percentages are small as far as the multiple scattering and molecular anisotropy are concerned. The uncertainties in aerosol effects and in the determinations and adopted law of ground reflections, however, still are of the order of 4%, and much more uncertain in the infrared. The next improvement needed is to determine the ground reflectivity simultaneously with the measures on the blue sky (Sec. X). As for the percentages in the infrared, the filters at $1/\lambda=1.06$ and 1.46 include strong water-vapor bands, and it remains to be seen how much the *theory* is affected by the absorptions.

The colors of Table IV are compared with the Rayleigh–Chandrasekhar theory, again by interpolation in the tables by Coulson, Dave, and Sekera.[24] The following colors were found, at 90° from the sun, in the sun's vertical, using the values of reflectivity and optical thickness for the McDonald Observatory (Table VII), without applying anisotropy corrections: at sun's elevation 5°.7: $B-R=-1.39$, $G-I=-2.42$, $U-G=-0.86$ magnitude, while at sun's elevation 37°: $B-R=-1.71$, $G-I=-2.25$, $U-G=-1.66$ [the colors for the sun (Sec. IV) have been taken into account]. The strong extinction effect is noted; as the setting sun gets redder, the 90°-phase sky becomes less blue. The observed $U-G$ colors of Table IV are well represented by these computed values, especially when one considers that the observations have limited precision and that the theory is for a plane-parallel atmosphere. The greatest differences between observations and theory are at longer wavelengths. In the infrared, the observed brightness appears larger than computed. There apparently is a nonmolecular component in the atmosphere that scatters with λ^x law ($x\cong 1$, at $0.6<\lambda<1.1\ \mu$), and that causes a few percent depolarization. From the present data, however, such a conclusion can only be very tentative. For a further study of aerosols reference is made to the work of Deirmendjian and others.[25]

IX. *Reflectivity of the Ground*

A classical investigation of reflectivities, for different types of ground surface and at different wavelengths, was made by Krinov.[26] For a few of Krinov's samples, Fig. 5 of the present paper gives the percentage of the incident (on the material) sunlight reflected, as a function of $1/\lambda$ in μ^{-1}. I have taken averages of Krinov's measurements, made at various phase angles, so as to obtain the best possible agreement with the albedo, based on Lambert's law, that is used in the calculations.[19]

The ground around the McDonald Observatory is covered with usually dry grass and with deciduous as well as coniferous trees, in between scattered rocks, so that curves (2), (3), and (4) are applicable. Cactus Peak, on the other hand, has bare rocks and sand with only sparse vegetation,[27] and curve (2) of Fig. 5 applies there. For the Goethe Link Observatory in November, the reflectivities lie between those of curves (4) and (3). There was no snow on the ground during any of the observations.

[23] See reference 1, Fig. 1.

[24] See reference 19, Table 1.

[25] D. Deirmendjian, Quart. J. Roy. Meteorol. Soc. 86, 371 (1960) and references given.

[26] *Handbook of Geophysics*, rev. ed. (The Macmillan Co., New York, 1960), Chap. 14; the original Krinov reference is at *Laboratoriia Aerometodov*, Akad. Nauk SSSR, (1947).

[27] J. B. Irwin (personal communication).

The rise in reflectivity in the near infrared on healthy plants is due to chlorophyll. It is a protective mechanism in order to reflect more where the earth's atmosphere attenuates less, and where the infrared radiation would heat the plant too much. The mechanism appears in detail in the extended curve for plants, between $1/\lambda = 0.4$ and $2.5\ \mu^{-1}$, published by Kuiper.[28]

X. Concluding Remarks

We should try to improve further the comparison of theory and observations on the clearest possible sky. One should keep in mind, however, that a sky that is seen as "clear" from below, may still have "blue clouds" in it, as was recently confirmed by Packer and Lock.[29] Of the causes of incomplete agreement discussed by van de Hulst (see Sec. I), the unknown ground reflection now appears the worst. Observations made from a low-flying balloon could yield the reflectivity of the ground, simultaneously with the observations of the sky overhead. The treatment of the earth's atmosphere as being plane parallel does not cause serious errors when the comparison is made at appreciable sun's elevation. In any case, the theory may be modified to that for a spherical atmosphere.[30] Narrower filters should be used, avoiding the water vapor absorption bands in the infrared.

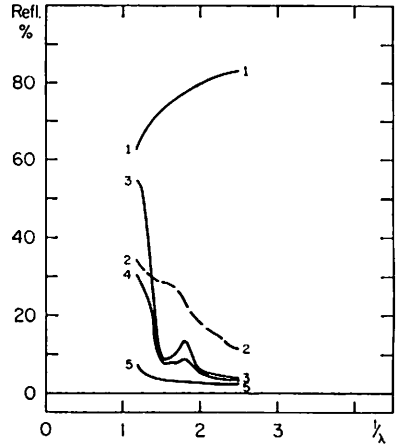

FIG. 5. The percentage of incident light reflected, as a function of the inverse of the wavelength in microns, determined by E. L. Krinov (see *Handbook of Geophysics*, 1960). Curve (1) is for fresh snow, (2) for bare desert and rocks, (3) for green grass and deciduous trees, (4) for dry grass and coniferous trees, and (5) is for black soil.

In conclusion, I would like to thank Dr. J. V. Dave, Dr. D. Deirmendjian, and Dr. G. A. Newkirk for valuable criticism and suggestions. I am also indebted to Mrs. D. Owings and to J. R. Carrasco for their assistance with reductions and observations, respectively.

[28] *The Atmospheres of the Earth and Planets*, rev. ed. edited by G. P. Kuiper (The University of Chicago Press, Chicago, 1952), Fig. 88.

[29] D. M. Packer and C. Lock, J. Opt. Soc. Am. 41, 478 (1951).

[30] J. Lenoble and Z. Sekera, Proc. Natl. Acad. Sci. 47, 372 (1961).