NO. 38. ON THE DISTRIBUTION OF LUNAR CRATER DIAMETERS

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ABSTRACT

In the first two lunar quadrants, the diameters (*D*) of both "young" and "old" craters follow the frequency distribution, d (log *F*)/d (log *D*) $\simeq -2.1$. For the "youngest" craters, this function remains valid down to $D = 8$ km. It is probable that throughout the period of crater formation this relationship was valid for newly-formed craters; certainly this is the case at the larger diameters. There is a deficiency of small craters which increases toward the "oldest" classes; there appears to be a process that has eroded very old small craters. The assumption that lunar craters were formed by impacts of bodies having the presently-observed asteroidal or meteoritic mass distribution closely predicts the observed lunar crater diameter distribution.

1. Observed Diameter Distribution

The compilation of lunar crater diameters (D) for craters of $D > 3.5$ km. by Arthur and his associates (1963, 1964) provides basic data for many studies of lunar history. This work is more complete, especially at small diameters, than the work by Young (1953). In the present analysis of Arthur's data on quadrants I and II (northern hemisphere) the distributions of different "age" classes of craters are compared and the implications for crater and mare formation are discussed.

The first task is a determination of the frequency distribution of D among the youngest craters. It is important to select a sample of fresh craters, because the battered craters have been deformed or partly obliterated.

The category of "youngest" craters, as used in this paper, includes all class 1 craters plus the post-mare craters of the other four classes, as defined by Arthur *et al.* (1963, p. 76). The Arthur classes (1 = freshest, 5 = most battered) are based on appearance alone, and are not entirely equivalent to those used by Baldwin (1963, p. 189). In the "young" category of the present paper, effects such as overlapping and flooding are minimal, so that essentially the complete initial population is preserved.

The following steps were taken in the reduction of the data:

(a) Craters in the limb regions were excluded because of the difficulty of seeing the small craters there. A comparison of distribution for the whole first quadrant with the central regions used here confirmed that when all craters out to the limb were included, a relative deficiency in small craters was introduced. The regions included here are shown in Figure 1.

(b) The remaining craters were sorted into increments of log D.

(c) The distribution was normalized by dividing the number of craters in each increment by the number with diameters larger than 35 km, on the assumption that in all classes, the initial population larger than 35 km would be essentially intact, so that all populations could be directly compared through use of this normalizing factor. Thus the incremental frequency parameter F is defined as:

$$F = \frac{\text{craters in log } D \text{ increment}}{\text{craters of } D > 35 \text{ km}} \qquad (1)$$

(d) Log F was plotted against log D.

The result for the youngest craters is shown in Figure 2. The observational effect of missing the smallest craters is clearly the main reason for the

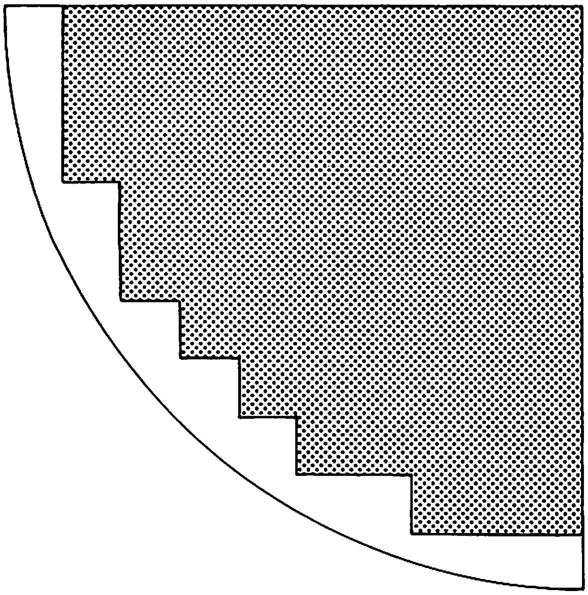

Fig. 1. Area of lunar quadrant studied in this paper. Only the shaded area (quadrants I and II) was included to reduce the effects of foreshortening.

turn-down found at diameters $D < 4$ km. (When all craters out to the limb were included, the turn-down occurred at about 9 km, confirming the need for omitting the limb regions.) We conclude that *the youngest craters, mostly post-mare, were formed with a diameter distribution obeying*

$$\frac{d(\log F)}{d(\log D)} = -S, \qquad (2)$$

with S constant, about 2.1.

We next compare the "oldest" craters with the "youngest." The category of "oldest" craters is defined to include all craters of Arthur classes 3, 4, and 5 which were catalogued as "continental." Craters on the continental borders were rejected if they were classified as postmare. Craters in the mare regions were not included because flooding might have affected the observed distribution. The same four steps [(*a*) to (*d*) above] were followed in treating the oldest craters.

A plot of the diameter distribution of the "old" craters is included in Figure 2. The curve is similar to that of the "young" craters in the following respects: *(a)* There is a linear portion at large diameters [eq. (2) is obeyed]; *(b)* the slopes of the linear portions are similar; *(c)* there is a turn-down at small diameters; and *(d)* the near coincidence of the F values for $D > 16$ km indicates that normalization to $D > 35$ km was safe and that all original craters of $D > 35$ km are detected even in the oldest populations.

2. Interpretation of Distribution Curves

The most prominent difference between young and old craters is the relative absence of small, old craters. Below 16 km, markedly fewer occur than expected if the initial distributions were the same and all craters were preserved. This issue is further considered below.

We next compare the slopes of the linear portions of Figure 2. Taking into account the limited accuracy of the statistics, one may conclude from Figures 2–4 that a value $S \simeq 2.1$ fits crater families of all ages.

If *(a)* the deficiency of small diameters is not a characteristic of the original population of older craters, and *(b)* crater families of all ages show linear branches of nearly the same slope, then *there is no evidence for a major secular change in the diameter distribution of newly-formed craters during lunar history.* This statement contradicts the commonly held opinion that large craters formed first and small ones last, which has been incorrectly inferred from the fact that small craters frequently overlap large craters while the reverse is rarely seen. The data used in Figure 2 show two relevant facts: *(a)* The number of small craters being formed at any time greatly exceeds the number of large ones; and *(b)* most presently-observed small craters are "young" and most large craters are "old."

These two facts explain the overlap problem. The high formation rate of small craters produces frequent overlap of small on large craters. The formation of large craters leads inevitably to the destruction of underlying objects. Moderate-sized underlying craters may partly survive, as is the case in Phocylides, Maurolycus, Aristoteles, and Janssen.

We now undertake a more detailed study of the depletion of small old craters, plotting each of the five classes separately by the same procedures as used in Figure 2. The results are shown in Figure 3. Two important conclusions follow: *(a)* All classes together define a linear branch which fits the relation

$$\frac{d(\log F)}{d(\log D)} = -2.1. \qquad (3)$$

(b) The depletion of small craters is greater among the more battered (presumably older) classes of craters.

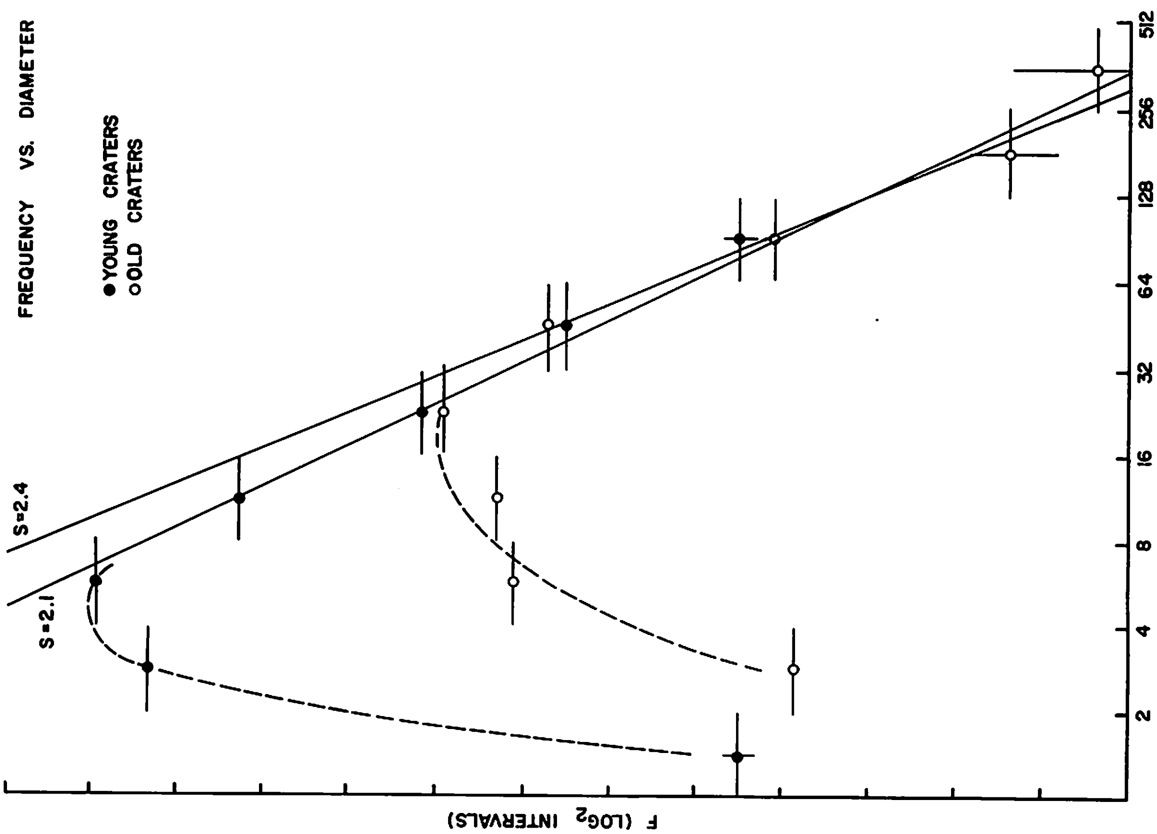

Fig. 2. Comparison of diameter distributions of "young" and "old" craters (for definitions see text, Sec. 1). Vertical bars show uncertainty in F corresponding to $\pm 1/\sqrt{n}$. F as defined in Eq. (1). Total number of "young" craters, 1448; of "old" craters, 520.

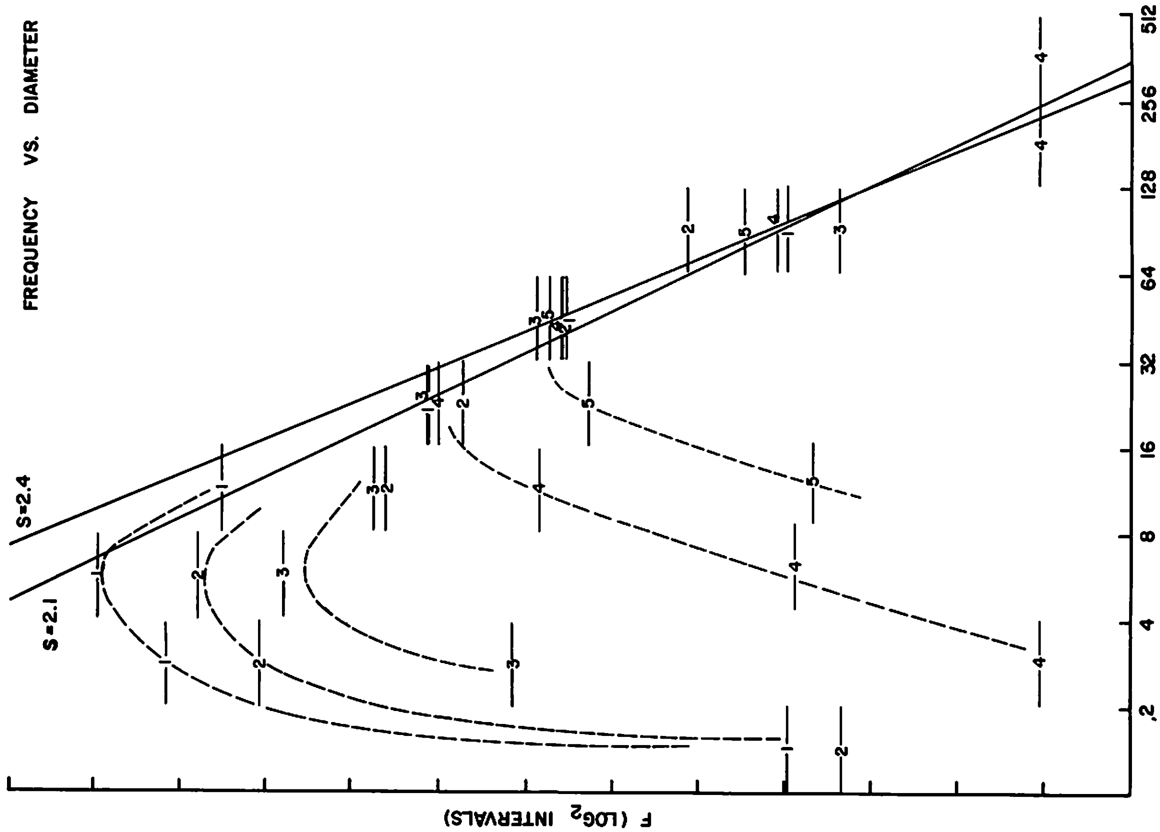

Fig. 3. Comparison of diameter distributions of five classes (1 = fresh; 5 = most damaged). All classes show the same slope at large diameters. F as defined in Eq. (1). Total number of craters in each class: 1, 1058; 2, 716; 3, 357; 4, 241; 5, 70.

We have already suggested the interpretation: *At all epochs, craters were formed with the distribution given approximately by Equation (3) (a straight line in Figure 3,* and *the depletion of small, old craters measures the partial or complete obliteration of these objects.*

This interpretation is favored over the alternative that the five different curves of Figure 3 show a secular change in the distributions of craters formed during the five different periods, and is supported by at least three arguments: *(a)* Observations indicate that a log-log plot of the distribution of both meteorite and asteroid masses gives a linear curve (Brown, 1960). Assuming an impact origin for craters, a linear crater distribution of slope −2.4 may be derived (see below), in good agreement with the observations. It is unlikely that the linearity of the curves for meteorites, asteroids, and young craters is a property only of the present period of the solar system. *(b)* All curves show linearity in the portions where we expect the crater data to be most complete. *(c)* The most natural interpretation of the Arthur erosion classes 1–5 is a progression in age, with the erosion caused by crater-forming impacts. A single hypothesis then accounts for both the formation of the craters and their gradual destruction.

There are, however, some questions of procedure that still need review:

(a) Classification errors. Could the deficiency of small craters be an observational effect whereby small craters were classified too "young"? There was indeed a tendency to do this, but it cannot account for the depletion in the more damaged classes. If the classification only were in error, the total number of craters would be unaffected, and there would be no net loss of craters. Under this assumption the distribution function F for all craters should be of a form similar to the distribution for some undisturbed sample selected by a criterion independent of class. Figure 4 compares all craters with post-mare craters. It shows that the two F functions differ, with a marked deficiency in small craters in the comprehensive sample. Therefore, unless we are mistaken in rejecting a secular change in the diameter distribution, a large number of craters have disappeared since the earliest craters were formed.

The magnitude of the deficiency of small craters is shown by the following two examples: (1) Craters of $D > 35$ km have been well preserved. In Figure 2, there are 18 "young" craters and 77 "old" ones with $D > 35$ km. Thus, *the "old" group was initially more numerous by a factor of about four.* Yet the present total numbers (all D values) are reversed: "young" 1448, and "old" 520, apparently because of the destruction of old, small craters. (2) In our (undepleted) sample of "young" craters (Fig. 2), the ratio of the number of $D > 35$ km craters to the total number is 1:80. In the studied area there are 132 craters with $D > 35$ km. Therefore, in this area there should be of the order of 10,000 craters, whereas only 2442 were actually counted. Thus, some 8000 craters must have been lost in the portion of the moon here considered.

The five classes cannot be strictly age classes, because the oldest small craters have already been lost, while the oldest large craters still remain. At any given diameter, each Arthur damage class has its own average age, but in classes 2 through 5 these average ages are not necessarily the same for different diameters. Nevertheless, the comparison in Figure 2 of "old" with "young" groups is meaningful.

(b) Flooding. One might suggest that our data have been biased by inclusion of flooded areas. Actually, a comparison of all craters of classes 3 and 4 with the continental craters of classes 3 and 4 shows little difference. We conclude that distributions of $F(D)$ in this paper are little affected by the flooding itself.

(c) Tectonic adjustments. An increasingly large body of evidence, such as studies of the lineament "grid" systems, supports the view that the entire lunar surface has undergone tectonic activity, probably accompanying the flooding that produced the maria. These adjustments will indeed have contributed to the difficulties of tracing the oldest craters. Fielder (1963) attaches major importance to this effect.

(d) Isostatic adjustments. Baldwin (1963, p. 193) attributes much of the modification of old craters to isostatic adjustments. He shows that the depths of the floors have been reduced faster than rim heights, pointing out that this phenomenon is in accord with isostatic adjustment.

(e) Overlapping. Smaller craters are preferentially destroyed by overlapping of successive generations of craters. This may be seen from the following simplified model:

Define D_0 = crater diameter in an existing distribution of craters,

D_1 = crater diameter in a new generation of overlapping craters,

A = area of the moon,

n = the average number of craters of diameter D_0 per unit interval ΔD_0 destroyed by a single overlapping crater of diameter D_1,

N = the number of craters of diameter D_0 per unit interval ΔD_1 destroyed by all overlapping craters of diameter D_1,

$\mathcal{N}$ = the number of craters of diameter D_0 destroyed by all overlapping craters.

Suppose that a crater of diameter D_1 overlaps a crater of diameter D_0 (i.e., the D_1 impact follows the D_0 impact.) Assume, as a first approximation, that if $D_1 > D_0$ the latter crater is obliterated, while if $D_1 < D_0$ both craters remain visible.

The *fraction* of the D_0 craters destroyed by a single D_1 impact, if $D_1 > D_0$, is the area of the D_1 crater divided by the area of the moon, assuming a random uniform distribution of the D_0 craters. Therefore, the *average number n* of D_0 craters (per unit interval ΔD_0) destroyed per D_1 impact is, by Equation 2,

$$n = \frac{\pi D_1^2}{4A} \text{Const.}\, D_0^{-S-1} \qquad \text{if } D_1 > D_0\,,$$

and

$$n = 0 \qquad \text{if } D_1 < D_0\,. \quad (4)$$

Therefore, the number N of D_0 craters destroyed by all new craters of diameter D_1 (per unit interval ΔD_1) is

$$N = \frac{\pi D_1^2}{4A} \text{Const.}\, D_0^{-S-1}\, \text{Const.}\, D_1^{-S-1} \qquad \text{if } D_1 > D_0\,,$$

and

$$N = 0 \qquad \text{if } D_1 < D_0\,, \quad (5)$$

and therefore, the total number of D_0 craters destroyed by *all* larger, overlapping D_1 craters is

$$\mathcal{N} = \frac{\pi\, \text{Const.}}{4AD_0^{S+1}} \int_{D_0}^{D_{\text{Max}}} D_1^{1-S}\, dD_1$$

$$= \frac{\pi\, \text{Const.}}{4A\,(2-S)} \frac{D_{\text{Max}}^{2-S} - D_0^{2-S}}{D_0^{S+1}} \qquad (\text{excluding } S = 2) \quad (6)$$

Therefore, the fraction of craters D_0 destroyed (per unit interval ΔD_0) is

$$\frac{\mathcal{N}}{F} = \frac{\pi\, \text{Const.}}{4A\,(2-S)} \left(D_{\text{Max}}^{2-S} - D_0^{2-S}\right) \quad (7)$$

In the case of $S = 2.1$, as found in Figures 2 and 3, we have

$$\mathcal{N} = \text{Const.} \left[\left(\frac{D_{\text{Max}}}{D_0}\right)^{0.1} - 1\right] D_0^{1/3.1}\,, \quad (8)$$

and

$$\frac{\mathcal{N}}{F} = \text{Const.} \left[\left(\frac{D_{\text{Max}}}{D_0}\right)^{0.1} - 1\right] \quad (9)$$

The fraction of destroyed craters thus increases toward small diameters, and the equations predict that each successive generation of impacts increases the departure from F at small diameters. Dr. Kuiper has pointed out an additional cause for this effect. In reality a given D_1 impact destroys an area greater than $\frac{\pi D_1^2}{4}$. For large D_0's this area of effective destruction approaches $\frac{\pi D_1^2}{4}$, but for small D_0's it is appreciably greater. Therefore in Equation (4) the exponent of D_0 might better be considered $-S-1-\epsilon$. This exponent carries through to Equation (6) so that the denominator in (6) becomes $D_0^{S+1+\epsilon}$ and in (7), D_0^{ϵ}. The model is of the first order in Δt (time), and makes various assumptions as listed, but it does predict qualitatively the observed effect. The second generation craters follow the diameter distribution of equation (2) by assumption, so that only the losses need be considered in a first-order theory.

Further study will be needed, especially in the heavily-cratered quadrant IV (where Δt is obviously not small), to determine whether the area covered by craters and the degree of apparent overlap are compatible with the hypothesized crater losses, e.g., some 90 percent for 10-km craters.

(f) Erosion by external agents. Several authors have discussed mechanisms by which material is constantly eroded and redistributed on the lunar surface. Meteorite impacts may result in either accretion or mass loss by the moon, depending on impacting mass and the surface structure. Such processes, if of sufficient magnitude, would account for the softened appearance of the larger class 4 and 5 craters and could obliterate smaller craters with lower total relief. Quantitative estimates of the present rate of erosion make it too small to have the necessary effect (Fielder, 1963), but erosion, especially by meteoritic material, may have been significantly greater in early lunar history. If it is assumed that a 20 km crater can be lost by erosion the disturbed layer must be 1–2 km in depth.

In summary, it appears likely that craters are obliterated in time by several processes, with the smallest craters being most affected. More light will be shed on this problem by data from the other quadrants, especially IV, where unflooded continents predominate. Meanwhile, it is held that the distribution of craters forming at any epoch is characterized by equation (3).

Young (1940) published diagrams similar to Figure 2, using measures of some 1300 craters. He noted a discontinuity or maximum curvature of slope at about 20 km, when all craters were included, and at about 12 km for post-mare craters alone. The latter corresponds to the bend near 10 km in the "young" crater curve of Figure 2. In Figure 4, the plot for all craters confirms a departure from linear near 20 km. Young (1940, p. 316) suggested the possibility that his discontinuity in slope related to differences between "walled-plain and ringed-plain types of craters," and Fielder (1961, p. 219), who confirmed the discontinuity, also implied a possible anomaly in the original population of craters. According to the present paper, the discontinuity is the result of including samples deficient in observable small craters and does not represent the original distribution. This conclusion is supported by the facts that (*a*) the "young" curve shows very little curvature, (*b*) each class shows a different departure from linearity (Fig. 3), and (*c*) Young's discontinuity is best seen when all craters are mixed before plotting.

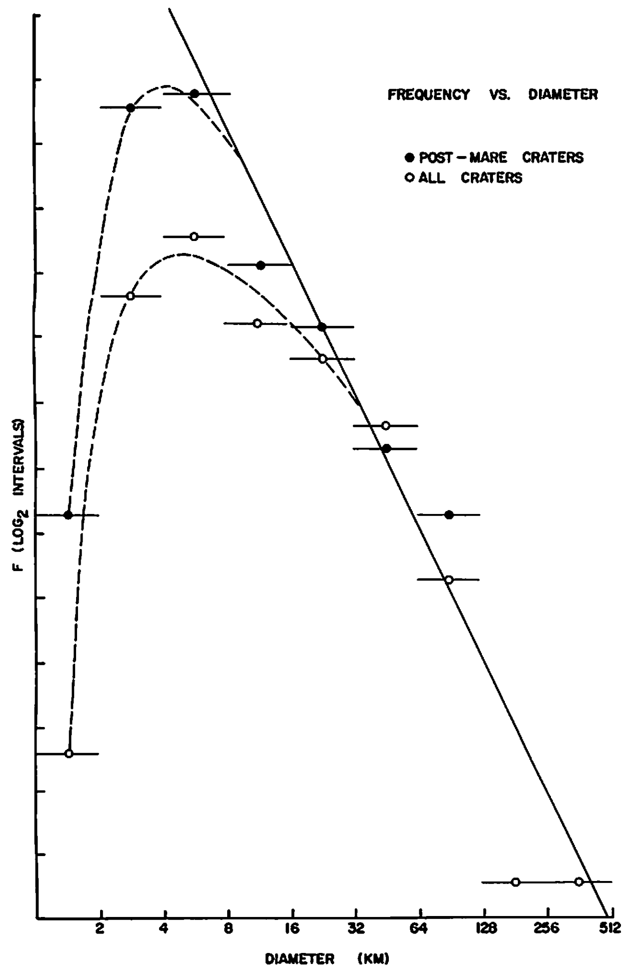

Fig. 4. Comparison of diameter distributions of all craters (2442) with post-mare craters alone (390), demonstrating that the deficiencies in small craters are not classification errors (see text, Sec. 2). *F* as defined in Eq. (1). Slope of the line $S = 2.1$.

The last point to be discussed involves the slope of the log *F*–log *D* curve. In the undisturbed parts this slope is approximately -2.1. Young's (1940, p. 315) value was -2.5. Assuming that the lunar craters are formed by impacts, one can theoretically predict this slope. The following discussion is based on that given by Shoemaker, Hackman, and Eggleton (1962). We let

E = energy available to form crater,
M = mass of impacting body,
V = velocity of impacting body,
D = diameter of crater,
$f(M)$ = distribution function of impacting bodies in log M increments,
$F(D)$ = distribution function of lunar craters in log D increments.

Brown (1960) has pointed out that the mass distributions of meteorites and asteroids are of the same form, namely,

$$\frac{d[\log f(M)]}{d[\log M]} = -.77 . \quad (10)$$

According to Shoemaker, Hackman, and Eggleton, the crater diameter *D* is proportional to a power of *E* less than 1/3, on the basis of theoretical considerations, and they give 1/3.4 as a value derived from studies of terrestrial explosion craters. Baldwin (1963, chap. 8) gives an extensive discussion of this problem, and his extrapolation to large diameters based on semiquantitative arguments on the nature of cratering gives

$$D = \text{Const.}\, E^{1/3.06}. \quad (11)$$

If the full kinetic energy is applied to crater formation,

$$E = \frac{1}{2} MV^2 . \quad (12)$$

Substituting (11) into (12), and the result into (10), and assuming that variations in impact velocity have negligible effect on F, we have

$$d[\log F(D)] = -.77(3.06)\, d[\log D] \quad (13)$$

or

$$\log F = -2.4 \log D + \text{Const.} \quad (14)$$

The observed equation is

$$\log F = -2.1 \log D + \text{Const.} \quad (15)$$

In spite of the uncertainties in Brown's data, the cratering theory, and the crater statistics, the observed curve is thus predicted closely.

Jaschek (1960) made an analysis very similar to this and found the same results for asteroidal data, though he concluded that the meteoritic data did not agree. However, he used the older crater counts of Young, Baldwin's 1949 cratering theory, and, as Shoemaker, Hackman, and Eggleton point out, the unlikely exponent of 1/2.5 in equation (11). Shoemaker, Hackman, and Eggleton predicted a slope of -2.7 but their crater counts did not confirm this, and they suggested that the mass distribution of the crater-forming objects was significantly different from that presently observed among small solar system objects. The improved data used in this paper reduce the need for this assumption.

3. *Conclusions*

In terms of the impact hypothesis, the implications of the presently available data are:

(a) The diameter distribution of the impacting bodies did not appreciably change during the interval of crater formation.

(b) The typical overlap of small on large craters is not due to a change in the distribution of diameters of the impacting bodies.

(c) The period of mare formation must have been relatively short, since the mare surfaces have relatively similar post-mare crater densities. Mare formation must have occurred after most craters had formed, when the rate of crater formation was too low to produce numerous post-mare craters.

(d) Among the oldest generation of craters, small craters are not detected in their original numbers.

(e) The bodies that formed the craters by impact came from a population whose diameter distribution was essentially the same as that presently observed among the asteroids and meteorites.

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Note added in proof. This paper was completed before the first successful Ranger flight to the moon. In the first analysis of the Ranger VII photographs the cratering curve for post-mare craters was extended over three more orders of magnitude down to diameters of two meters. The incremental diameter distribution is apparently linear over the entire range. There are some irregularities in a small range around 125 meters, but these are probably due to clusterings of secondary craters. A second interpretation is that large numbers of secondary craters modify the shapes of the primary crater curve for diameters less than 500 meters. The best fit to the curve, either down to 2 meters or up to 500 meters, depending on the interpretation, is $S = 2.4$, in exact agreement with the calculations in this paper.

REFERENCES

Arthur, D. W. G., Agnieray, A. P., Horvath, R. A., Wood, C. A., and Chapman, C. R. 1963, *Comm. L.P.L.*, *2*, 71.

———. 1964, *Comm. L.P.L.*, *3*, 1.

Baldwin, R. B., 1963, *The Measure of the Moon* (Chicago: University of Chicago Press).

Brown, H. 1960, *J. Geophys. Res.*, *65*, 1679.

Fielder, G. 1961, *Structure of the Moon's Surface* (New York: Pergamon Press).

———. 1963, *Planet. Space Sci.*, *11*, 1335.

Jaschek, C. O. R. 1960, *The Observatory*, *80*, 119.

Shoemaker, E. M., Hackman, R. J., and Eggleton, R. E. 1962, *Advances in the Astronautical Sciences*, (New York: Plenum Press), vol. 8.

Young, J. 1940, *J. B. A. A.*, *50*, 309.

———. 1953, *A Catalogue of Lunar Craters*, (Privately printed by D. W. G. Arthur).