No. 130 A NEW SECONDARY SELENODETIC TRIANGULATION

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March 15, 1968

ABSTRACT

Coordinate measures on 25 Yerkes star-trailed lunar photographs are used in conjunction with ephemeris values of the moon's libration to derive the selenodetic coordinates of 47 secondary points. The coordinates and their standard errors are listed together with the absolute altitudes.

1. Introduction

In the previous paper (Arthur, 1967) the results were given of measures on 25 star-trailed photographs obtained by E. Moore using the 40-in. refractor of the Yerkes Observatory of the University of Chicago.

These photographs form a relatively small part of a much larger collection of star-trailed plates, which were intended to be used in a determination of the moon's constants of rotation. Unfortunately there was an oversight in the processing that did not come to light until some years later when the plates were placed in a comparator for measurement. Other plates, without star-trails, were exposed on the same nights and all plates were processed in the usual manner. Because the star-trailed plates were open to the sky for times of up to 1 hr., they were always more or less sky-fogged and should have been developed for maximum contrast, but were not. Consequently, the bulk of the star-trailed plates are much too flat for measurement even though they appear quite satisfactory in a naked-eye inspection.

When the limitations of the collection were realized, and it became known that only two dozen plates could be measured without some form of contrast enhancement, the purpose of the plates was changed. If previous determinations of the moon's constants of rotation and the place of Mösting A are accepted then the demands on the data are considerably reduced. Despite the rather poor quality of some of the star-trailed plates, I judged the data to be good enough for a new secondary triangulation of the lunar surface. The results will show that a useful precision has been achieved.

2. The Older Secondary Positions

The extensiveness of selenodetic literature may lead the reader to believe that selenodetic positions rest on a broad observational basis. In one respect this is not true. Most of the literature is concerned with numerous measures connecting the fundamental point Mösting A to the limb. These determine the moon's constants of rotation and the place of Mösting A, but they provide nothing more toward a network of fixed positions.

Therefore, starting from the fundamental point, a furthur series of *secondary* measures must be made connecting this point to a number of secondary points dispersed over the disk. Only two such sets of such secondary measures appear to have been made and published in full. In the years 1890–94 Franz (1898) used the famous Königsberg heliometer to connect Mösting A to eight other points, two in each quadrant. Each connection was made on 12 evenings. Hayn (1904) at Leipzig used a micrometer to connect Mösting A to four other points, one in each quadrant. However, one of these points, the central peak of Tycho, is quite unsuitable for selenodetic purposes. Both the Franz and Hayn measures were rigorously reduced by Schrutka-Rechtenstamm (1956). The Hayn triangulation is the more precise of the two, with standard height errors at the center of face of about 1 km.

3. *The Overall Characteristics of the New Triangulation*

For the purposes of calculating the rectangular selenodetic coordinates (E, F, G) from the measures, a position must be assumed for the fundamental point, in this case the crater Mösting A. The most recent determination

$$\begin{aligned} E_A &= -.08990 = (1 + H_A)\sin\lambda_A\cos\beta_A, \\ F_A &= -.05549 = (1 + H_A)\sin\beta_A, \\ G_A &= +.99483 = (1 + H_A)\cos\lambda_A\cos\beta_A, \end{aligned}$$

of Koziel (1963) was used for the determinations of this paper. While Koziel's position appears to be the best available, it is emphasized that any reasonably precise position will serve the purpose. For convenience the results have been given as rectangular coordinates, but it should be noted that the quantities that are really determined are the coordinate differences $E - E_A, F - F_A, G - G_A$.

The orientation of the new triangulation is virtually independent of the older work and comes from the star-trails. It thus provides a valuable check on the results of Franz and Hayn. It is preferable, of course, to make a new secondary triangulation completely independent of the older triangulation, but this has not proved possible in this instance. The scale of the new triangulation still depends, through the Breslau photographic triangulation (Schrutka-Rechtenstamm 1958), on the secondary net of Franz. The work at LPL (Arthur 1966*b*) has shown that this Breslau triangulation has scale errors that are quite small, perhaps no larger than 0.0001. In any case a slight scale error in the triangulation is not a very serious defect, since it is easy to remedy when better data become available.

Each plate is scaled by using the Breslau points as controls and by estimating the factor μ as detailed in *Comm. LPL*, 4 (60). It should be noted, however, that because of the rather low resolution of the star-trailed plates, μ is often poorly determined. Details of the adjustments to the Breslau controls and the calculations for μ for the 25 plates are given in *Comm. LPL* No. 129.

4. *The Computations*

Comm. LPL No. 129 gives the coordinates (x, y) for each secondary point on each plate on which it was measured. The (x, y) system in this case is a refraction-free photographic system with the origin at the center of the circle of the limb and with the y-axis directed along the north part of the hour-circle through the origin. Let (E, F, G) be the rectangular selenodetic coordinates

$$\left.\begin{aligned} E &= (1 + H)\sin\lambda\cos\beta \\ F &= (1 + H)\sin\beta \\ G &= (1 + H)\cos\lambda\cos\beta \end{aligned}\right\}, \tag{1}$$

where λ, β are the selenographic longitude and latitude respectively, and H is the absolute altitude in units of the mean radius. Let (l', b') be the topocentric librations, and let C' and s' be the topocentric values of the position angle of the moon's axis and the angular semidiameter. Note that l', b', C' and s' were computed from the ephemeris values as detailed in *Comm. LPL*, 4 (60).

The matrix for rotating from the orientation of the (E, F, G) system to that of the (x, y) system as defined above is

$$\mathbf{M} = \mathbf{NL}, \tag{2}$$

where

$$\mathbf{L} = \begin{pmatrix} \cos l' & , 0 & , -\sin l' \\ -\sin l'\sin b', & \cos b', & -\cos l'\sin b' \\ \sin l'\cos b', & \sin b', & \cos l'\cos b' \end{pmatrix}, \tag{3}$$

and

$$\mathbf{N} = \begin{pmatrix} \cos C', & -\sin C', & 0 \\ \sin C', & -\cos C', & 0 \\ 0 , & 0 , & 1 \end{pmatrix} \tag{4}$$

For brevity we write

$$\mathbf{M} = \mathbf{NL} = \begin{pmatrix} l , & m , & n \\ l' , & m' , & n' \\ l'', & m'', & n'' \end{pmatrix} \tag{5}$$

We now have the rigorous scheme

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \mathrm{M} \begin{pmatrix} E \\ F \\ G \end{pmatrix} \tag{6}$$

$$\left.\begin{aligned} x &= X / (1 - Z \sin s') \\ y &= Y / (1 - Z \sin s') \end{aligned}\right\} \tag{7}$$

$$\left.\begin{aligned} x' &= (x - x_A) / \mu \\ y' &= (y - y_A) / \mu \end{aligned}\right\}, \tag{8}$$

where x' and y' are merely the values of x and y in millimeters after reduction to Mösting A as origin. Corresponding to the above, we have the approximate scheme

$$x' \approx L\ (E - E_A) + M\ (F - F_A) + N\ (G - G_A),$$
$$y' \approx L'\ (E - E_A) + M'\ (F - F_A) + N'\ (G - G_A),$$

where we have put

$$\left.\begin{aligned} L &= l / \mu, \quad M = m / \mu, \quad N = n / \mu, \\ L' &= l' / \mu, \quad M' = m' / \mu, \quad N' = n' / \mu. \end{aligned}\right\}. \tag{9}$$

From the approximate scheme, we have,

$$\left.\begin{aligned} \frac{\partial x'}{\partial E} \approx L, \frac{\partial x'}{\partial F} \approx M, \frac{\partial x'}{\partial G} \approx N, \\ \frac{\partial y'}{\partial E} \approx L', \frac{\partial y'}{\partial F} \approx M', \frac{\partial y'}{\partial G} \approx N'. \end{aligned}\right\}. \tag{10}$$

In these, x' and y' are the coordinate steps on the plate from the image of Mösting A to the image of the secondary point. Let (x'_0, y'_0) represent the observed values of (x', y'), and let (x'_c, y'_c) represent the computed values, calculated from (E, F, G) using (6), (7) and (8). In general these disagree. Let $\delta E, \delta F, \delta G$ be the corrections applied to the assumed values E, F, G to bring x'_c and y'_c into accord with the observed values x'_0, y'_0. Then to the first order,

$$x'_0 = x'_c + \frac{\partial x'}{\partial E} \cdot \delta E + \frac{\partial x'}{\partial F} \cdot \delta F + \frac{\partial x'}{\partial G} \cdot \delta G,$$

and

$$y'_0 = y'_c + \frac{\partial y'}{\partial E} \cdot \delta E + \frac{\partial y'}{\partial F} \cdot \delta F + \frac{\partial y'}{\partial G} \cdot \delta G.$$

That is, to the first order,

$$\left.\begin{aligned} L\ \delta E + M\ \delta F + N\ \delta G &= x'_0 - x'_c = \delta x \\ L' \delta E + M' \delta F + N' \delta G &= y'_0 - y'_c = \delta y \end{aligned}\right\}. \tag{11}$$

Thus there is one equation pair for each plate on which a point is observed. Treating all such equations as having equal weight and collecting together the equations referring to one point, we obtain the normal equations

$$\delta E\,[LL + L'L'] + \delta F\,[LM + L'M'] + \delta G\,[LN + L'N'] = [L\delta x + L'\delta y]$$

$$\delta E\,[LM + L'M'] + \delta F\,[MM + M'M'] + \delta G\,[MN + M'N'] = [M\delta x + M'\delta y]$$

$$\delta E\,[LN + L'N'] + \delta F\,[MN + M'N'] + \delta G\,[NN + N'N'] = [N\delta x + N'\delta y]$$

For these we write

$$[LL + L'L'] = \Sigma LL + \Sigma L'L'$$
$$[LM + L'M'] = \Sigma LM + \Sigma L'M'$$
....

....
$$[N\delta x + N'\delta y] = \Sigma N\delta x + \Sigma N'\delta y$$

for brevity. The summations are over all values relating to one secondary point. To facilitate the estimations of the precisions, the normal equations are solved by inverting the matrices.

The computation is iterative but converges extremely rapidly. Experience shows that when E and F are estimated to about 0.002, with G taken to be $+\sqrt{(1 - E^2 - F^2)}$, the first iteration is enough to correct E, F and G. Subsequent iterations produce negligible changes. After correcting E, F, G with the first values of δE, δF and δG, we calculate (x'_c, y'_c) a second time to derive the new values of

$$\left.\begin{aligned} \delta x &= x'_0 - x'_c \\ \delta y &= y'_0 - y'_c \end{aligned}\right\}. \tag{13}$$

These will not be diminished by further increments δE, δF and δG and, hence, must now be regarded as residuals.

Table 1 of this paper gives the final coordinates E, F and G, and also the absolute altitude of the secondary points,

$$H = \sqrt{(E^2 + F^2 + G^2)} - 1. \tag{14}$$

5. *The Precision of the Relative Coordinates*

As already noted the computations determine for each point the inverse normal matrix

$$\begin{pmatrix} r_{11}, r_{12}, r_{13} \\ r_{21}, r_{22}, r_{23} \\ r_{31}, r_{32}, r_{33} \end{pmatrix}$$

in which $r_{ik} = r_{ki}$. The estimated error-variances of E, F and G are then

$$\left.\begin{aligned} \sigma^2_E &= r_{11}\,\sigma^2 \\ \sigma^2_F &= r_{22}\,\sigma^2 \\ \sigma^2_G &= r_{33}\,\sigma^2 \end{aligned}\right\}, \tag{15}$$

where σ^2 is the variance of x' and y'. For each point σ^2 is computed from

$$\sigma^2 = \frac{\Sigma\,(\delta x^2 + \delta y^2)}{2n - 3}. \quad (16)$$

The correlation introduced by reducing the coordinates (x, y) to Mösting A as origin has not been overlooked. Its effect is to introduce error-correlation between all the selenodetic coordinates (E, F, G) of all the secondary points. However, since the observation equations *for one point* all come from different plates, there is no correlation between these, and, therefore, no need to modify the normal equations.

The standard errors of the secondary coordinates, or rather of the differences $E - E_A, F - F_A, G - G_A$, are given in Table 1. The most poorly determined point is Grimaldi B with a standard error of 4.5 km in G. The corresponding figure for the craters Bruce and Blagg at the center of face is about 1 km.

6. *Comparison with Previous Triangulations*

Almost all modern selenodetic triangulations derive their scale, orientation and origin from the Franz secondary net as reduced by Schrutka-Rechtenstamm (1956), but they do this via the Breslau triangulation (Schrutka-Rechtenstamm, 1958). Hence, it is legitimate and convenient to reduce the new secondary triangulation to the older work by relating it to the Breslau net, since the latter can justly be regarded as a smoothed and amplified form of the Franz secondary net. Fourteen common points are available for this comparison, although there are doubts about the usefulness of the point Damoiseau E.

Let (ξ, η, ζ) be the rectangular selenodetic coordinates in the Breslau net, and let (E, F, G) be the corresponding values in the new secondary triangulation. Assuming that the two systems are connected by a rotation, shift, and change of scale, then

$$\begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} = (1 + \epsilon) \cdot \Theta \cdot \begin{pmatrix} E \\ F \\ G \end{pmatrix} + \begin{pmatrix} e \\ f \\ g \end{pmatrix}. \quad (17)$$

In this transformation of rectangular coordinates, ϵ represents a small change of scale and the orthogonal matrix Θ represents a small rotation. Since the matrix is orthogonal and is limited to small rotations, its first order form may be written as

$$\Theta = \begin{pmatrix} 1, & \alpha, & \beta \\ -\alpha, & 1, & \gamma \\ -\beta, & -\gamma, & 1 \end{pmatrix}. \quad (18)$$

This is orthogonal only when the second order terms (such as α^2) are completely negligible. Substituting (18) in (17), expanding and rejecting terms containing products of α, β, γ and ϵ, we get

$$\left.\begin{aligned} e + \epsilon E + \alpha F + \beta G &= \xi - E \\ f + \epsilon F - \alpha E + \gamma G &= \eta - F \\ g + \epsilon G - \beta E - \gamma F &= \zeta - G \end{aligned}\right\}. \quad (19)$$

There is one such triplet of observation equations for each point common to the Breslau triangulation. The first two equations of each triplet have about the same weight throughout, while the third equation has a much lower weight. In the formation of the normals, weights of 10.0 were assigned to the first two equations and unit weights to the third equation of all triplets. This weighting is rather rough but is probably good enough for the purpose. The observation equations are displayed in Table 2, the resulting values of the parameters in Table 3, and the residuals in Table 4. The equations were solved twice, with and without Damoiseau E, and both solutions are given in Table 3. Also in Table 4 the residuals of both solutions are given in alternate lines, those with Damoiseau E being given first.

It will be noted that neither solution improves the ζ or G residuals, which implies perhaps that the weights of the G equations are perhaps too low. In the solution without Damoiseau E, only the rotation α and the scale change ϵ appear to be significant. The former corresponds to a rotation about the earth-moon axis of 0.00016 radians. This leaves positions uncertain to about 200 meters in the north-south directions in the limb regions.

The discrepancy ϵ is larger than expected and indicates a small scale discrepancy between the LPL secondary triangulation and Breslau, despite the derivation of the scale of the former from the Breslau points. The results indicate that selenodesy would benefit from further work with star-trailed lunar photographs.

Acknowledgment. This work is supported by the U.S. Air Force under Contract F 1928-67-C-0240.

REFERENCES

Arthur, D. W. G. 1967, "Selenodetic Measures on 25 Star-Trailed Photographs," *Comm. LPL*, 129.

————. 1966, "The Validity of Selenodetic Positions," *Comm. LPL*, 5 (75), 19.

Franz, J. 1898, "Die Figur des Mondes," *Astr. Beob., Konigsberg*, 38.

Hayn, F. 1904, *Selenographische Koordinaten* (Leipzig: Sächs. Gesell. der Wiss.), II Abh.

Koziel, K. 1963, "Crater Mösting A as the First-Order Point of Triangulation on the Moon," *Fourth International Space Science Symposium, Warsaw,* 1963.

Schrutka-Rechtenstamm, G. 1956, "Neureduktion der acht von J. Franz und der vier von F. Hayn gemessenen Mondkraterpositionen," *Sitz. Österr. Akad. Wiss.* [*Math.-Naturw. Kl.*], Abt. II, 165 Bd.

————. 1958, "Neureduktion der 150 Mondpunkte der Breslauer Messungen von J. Franz," *Sitz. Österr. Akad. Wiss.* [*Math.-Naturw. Kl.*], Abt. II, 167 Bd.

TABLE 1

THE DETERMINATION OF μ

PLATE	10^7p	10^7q	D	H	$10^7\mu$	n
Y– 57	+106036	+14728	–1.447506	–1.510569	107055	17
Y– 100	+106542	– 5553	–1.431327	–1.282533	106686	15
Y– 158	+ 93446	–59263	–1.722985	–0.398769	110654	10
Y– 268	+108500	+23281	–1.279124	–1.746947	110969	8
Y– 526	+108821	+47248	–0.896699	–2.006855	118636	7
Y– 556	+110949	–24345	–1.467339	–1.250611	113588	9
Y– 616	+ 96851	+64458	–0.226582	–1.925880	116340	8
Y– 624	+108412	+45304	–0.739954	–1.790437	117497	6
Y– 720	+102163	+54968	–0.650095	–2.105345	116012	17
Y– 753	+ 79919	+87640	+0.267755	–2.106032	118608	10
Y– 898	+102487	+46627	–0.681235	–1.834505	112595	14
Y– 910	+110884	+ 70	–1.471862	–1.356246	110885	7
Y– 958	+105520	+12079	–1.263803	–1.551661	106209	11
Y–1158	+ 96228	–44113	–1.807601	–0.612011	105857	13
Y–1190	+106459	+13070	–1.303780	–1.548414	107258	17
Y–1288	+107431	+ 5652	–1.245587	–1.417206	107580	8
Y–1348	+ 81615	–79425	–2.243597	+0.273484	113883	7
Y–1502	+103254	+55249	–0.604664	–2.044323	117107	8
Y–1579	+102487	+48413	–0.737525	–2.017865	113346	17
Y–1638	+102035	+49837	–0.687804	–2.107749	113559	6
Y–1694	+ 82541	+70784	–0.195394	–1.999734	108735	12
Y–1771	+ 94508	+46931	–0.688438	–1.866014	105519	7
Y–1778	+104365	+15232	–1.156191	–1.518051	105470	9
Y–1784	+105395	+ 4633	–1.295500	–1.400450	105497	7
Y–1799	+101964	+26859	–1.006770	–1.659666	105442	17

See Comm. LPL No. 60 for notation.

TABLE 2-a

YERKES PHOTOGRAPH NO. 57

POINT	u'	RESIDUAL	v'	RESIDUAL
987 10950	–0.31801	0.00018	0.87731	0.00042
622 13421	0.08419	–0.00022	0.54779	–0.00010
857 10002	–0.07321	–0.00040	0.06508	–0.00001
3683 43100	0.25903	–0.00002	0.05941	–0.00012
4004 42697	0.47885	–0.00009	–0.48435	0.00013
183 16315	0.38174	–0.00020	0.59267	0.00007
3667 44001	0.31850	–0.00009	0.18522	0.00007
1145 20497	–0.33064	0.00026	0.43675	–0.00042
2933 30095	–0.13413	0.00049	–0.04267	0.00019
3055 31397	–0.11043	–0.00050	–0.38250	–0.00011
1614 24671	–0.71704	0.00028	0.41387	0.00054
1529 24197	–0.58573	0.00002	0.01081	0.00016
2481 25010	–0.54216	0.00034	–0.15513	–0.00009
2417 36278	–0.56427	0.00031	–0.49103	0.00019
2419 35356	–0.43424	–0.00014	–0.52021	–0.00015
1833 27078	–0.79916	–0.00041	–0.18413	–0.00023
1992 38049	–0.78825	0.00019	–0.38098	–0.00053
RMS Residuals		0.00028		0.00026
With Bessel		0.00031		0.00029

TABLE 2-b

YERKES PHOTOGRAPH NO. 100

POINT	u'	RESIDUAL	v'	RESIDUAL
987 10950	–0.24953	0.00030	0.86181	0.00008
622 13421	0.22245	–0.00011	0.51151	0.00023
183 16315	0.49649	–0.00005	0.56154	0.00027
1145 20497	–0.18666	0.00010	0.40031	–0.00005
1529 24197	–0.44563	–0.00014	–0.02454	–0.00021
1614 24671	–0.61340	0.00045	0.38949	0.00001
2481 25010	–0.40106	–0.00000	–0.19119	–0.00013
1833 27078	–0.69677	–0.00009	–0.20916	–0.00013
2933 30095	0.02905	–0.00026	–0.08573	–0.00018
3055 31397	0.03799	0.00009	–0.42220	0.00028
2419 35356	–0.31193	–0.00011	–0.55220	–0.00006
2417 36278	–0.45171	0.00003	–0.52000	–0.00001
4004 42697	0.58822	0.00008	–0.51566	–0.00010
3683 43100	0.41348	–0.00019	0.01745	0.00009
3667 44001	0.46685	–0.00011	0.14483	–0.00007
RMS Residuals		0.00018		0.00015
With Bessel		0.00020		0.00017

TABLE 2-c

YERKES PHOTOGRAPH NO. 158

POINT	u'	RESIDUAL	v'	RESIDUAL
857 10002	0.12806	–0.00002	–0.07247	0.00022
987 10950	0.24935	–0.00006	0.84655	–0.00022
622 13421	0.48712	0.00008	0.28634	–0.00028
183 16315	0.74076	0.00003	0.19544	0.00008
1145 20497	0.08253	–0.00027	0.39877	0.00002
2933 30095	0.01971	–0.00014	–0.13265	0.00003
3055 31397	–0.15010	–0.00022	–0.42112	0.00022
4004 42697	0.26098	0.00007	–0.77171	–0.00004
3683 43100	0.39840	0.00015	–0.23850	0.00007
3667 44001	0.50823	0.00039	–0.15512	–0.00009
RMS Residuals		0.00018		0.00016
With Bessel		0.00022		0.00019

TABLE 2-d

YERKES PHOTOGRAPH NO. 268

POINT	u'	RESIDUAL	v'	RESIDUAL
987 10950	0.11791	0.00046	0.92749	0.00003
622 13421	0.27849	–0.00022	0.45241	0.00007
3683 43100	0.18763	–0.00005	–0.05801	0.00011
183 16315	0.56884	0.00020	0.34739	–0.00028
3667 44001	0.30268	–0.00027	0.02332	0.00036
2933 30095	–0.20698	0.00002	0.04128	0.00018
2481 25010	–0.60636	–0.00028	0.13584	–0.00005
1992 38049	–0.90588	0.00014	0.05067	–0.00042
RMS Residuals		0.00024		0.00023
With Bessel		0.00031		0.00030

TABLE 2-e

YERKES PHOTOGRAPH NO. 256

POINT	u'	RESIDUAL	v'	RESIDUAL
987 10950	–0.07679	0.00020	0.85917	0.00012
622 13421	0.26192	0.00000	0.37458	0.00032
3683 43100	0.32205	–0.00027	–0.15615	–0.00021
183 16315	0.55384	–0.00011	0.36928	0.00010
3667 44001	0.40760	–0.00014	–0.04278	–0.00017
2933 30095	–0.08172	0.00013	–0.16874	0.00010
2481 25010	–0.51361	0.00019	–0.15789	–0.00027
RMS Residuals		0.00017		0.00020
With Bessel		0.00022		0.00027

TABLE 2-f
YERKES PHOTOGRAPH NO. 556

POINT	u′	RESIDUAL	v′	RESIDUAL
857 10002	−0.05528	0.00008	0.11842	−0.00007
987 10950	0.53667	0.00027	0.75307	−0.00003
622 13421	0.44598	−0.00011	0.24666	0.00019
183 16315	0.65380	−0.00015	0.01944	0.00012
1145 20497	0.12717	0.00003	0.53337	−0.00010
2933 30095	−0.17817	−0.00001	0.11048	−0.00026
4004 42697	−0.19671	−0.00012	−0.64937	−0.00011
3683 43100	0.12261	−0.00005	−0.16342	0.00002
3667 44001	0.26291	0.00007	−0.14510	0.00024
RMS Residuals		0.00012		0.00015
With Bessel		0.00015		0.00019

TABLE 2-g
YERKES PHOTOGRAPH NO. 616

POINT	u′	RESIDUAL	v′	RESIDUAL
857 10002	−0.11123	0.00036	−0.02303	0.00041
987 10950	−0.12903	−0.00011	0.88599	−0.00008
622 13421	0.16082	−0.00018	0.42455	−0.00018
1145 20497	−0.26807	−0.00014	0.41272	−0.00011
2933 30095	−0.19466	−0.00003	−0.11253	0.00008
4004 42697	0.31470	−0.00025	−0.65512	−0.00035
3683 43100	0.21204	0.00025	−0.10245	0.00018
3667 44001	0.30060	0.00011	0.00934	0.00006
RMS Residuals		0.00020		0.00022
With Bessel		0.00026		0.00028

TABLE 2-h
YERKES PHOTOGRAPH NO. 624

POINT	u′	RESIDUAL	v′	RESIDUAL
987 10950	0.02844	0.00008	0.88429	0.00010
622 13421	0.24447	−0.00046	0.37004	−0.00006
857 10002	−0.09681	0.00009	−0.02840	0.00042
183 16315	0.54235	0.00010	0.30873	−0.00015
3667 44001	0.31385	0.00012	−0.06409	−0.00004
2933 30095	−0.19435	0.00007	−0.10225	−0.00028
RMS Residuals		0.00021		0.00022
With Bessel		0.00029		0.00031

TABLE 2-i
YERKES PHOTOGRAPH NO. 720

POINT	u′	RESIDUAL	v′	RESIDUAL
987 10950	−0.59001	0.00030	0.65085	−0.00014
622 13421	−0.00266	0.00009	0.53005	−0.00022
857 10002	0.09740	−0.00033	0.02703	−0.00040
3683 43100	0.38711	−0.00008	0.16711	−0.00006
4004 42697	0.76986	0.00025	−0.22588	0.00030
183 16315	0.21755	0.00015	0.70035	0.00004
3667 44001	0.37834	0.00014	0.30624	0.00001
1145 20497	−0.31997	−0.00027	0.24886	−0.00035
2933 30095	0.08889	−0.00004	−0.09652	−0.00001
3055 31397	0.24169	−0.00003	−0.39185	−0.00009
1614 24671	−0.70418	−0.00045	0.05917	0.00037
1529 24197	−0.36700	−0.00011	−0.24603	0.00026
2481 25010	−0.25404	0.00002	−0.37627	0.00027
2417 36278	−0.16015	0.00029	−0.68826	0.00002
2419 35356	−0.01924	0.00004	−0.65752	0.00012
1833 27078	−0.51733	−0.00032	−0.51503	0.00022
1992 38049	−0.43993	0.00035	−0.68747	−0.00033
RMS Residuals		0.00023		0.00023
With Bessel		0.00026		0.00025

TABLE 2-j
YERKES PHOTOGRAPH NO. 753

POINT	u′	RESIDUAL	v′	RESIDUAL
857 10002	0.09258	0.00023	−0.04813	0.00038
987 10950	−0.62477	0.00003	0.57907	−0.00008
183 16315	0.17859	−0.00010	0.65369	−0.00002
1145 20497	−0.33549	0.00013	0.15765	−0.00003
1529 24197	−0.35657	0.00002	−0.33593	−0.00032
1614 24671	−0.70858	−0.00016	−0.03435	0.00007
2481 25010	−0.23698	−0.00003	−0.45928	−0.00010
2933 30095	0.09052	0.00016	−0.17122	0.00014
3055 31397	0.25879	−0.00046	−0.44878	0.00008
3683 43100	0.37489	0.00016	0.11326	−0.00012
RMS Residuals		0.00019		0.00018
With Bessel		0.00023		0.00021

TABLE 2-k
YERKES PHOTOGRAPH NO. 898

POINT	u′	RESIDUAL	v′	RESIDUAL
987 10950	−0.61585	0.00018	0.65659	0.00004
622 13421	−0.03413	−0.00030	0.56742	0.00010
3683 43100	0.36285	−0.00013	0.21758	−0.00008
4004 42697	0.76187	−0.00001	−0.17940	−0.00015
183 16315	0.18552	−0.00007	0.73549	−0.00022
3667 44001	0.35153	0.00013	0.35464	0.00021
1145 20497	−0.34601	−0.00004	0.28175	0.00010
2933 30095	0.06927	0.00008	−0.04943	−0.00003
3055 31397	0.23146	0.00005	−0.34531	0.00012
1614 24671	−0.72088	0.00007	0.07337	0.00010
1529 24197	−0.38076	−0.00008	−0.21442	−0.00003
2481 25010	−0.26460	0.00001	−0.34191	−0.00017
2417 36278	−0.15918	−0.00016	−0.65947	−0.00019
2419 35356	−0.02005	0.00025	−0.62321	0.00016
RMS Residuals		0.00014		0.00014
With Bessel		0.00016		0.00015

TABLE 2-l
YERKES PHOTOGRAPH NO. 910

POINT	u′	RESIDUAL	v′	RESIDUAL
857 10002	0.08756	0.00008	0.06205	0.00025
987 10950	−0.29596	−0.00007	0.86224	−0.00005
622 13421	0.19317	−0.00011	0.55173	0.00009
183 16315	0.46441	0.00010	0.61038	−0.00017
2933 30095	0.03164	−0.00005	−0.04849	−0.00009
3683 43100	0.41010	−0.00007	0.07197	−0.00006
3667 44001	0.45666	0.00012	0.20044	0.00004
RMS Residuals		0.00009		0.00013
With Bessel		0.00012		0.00017

TABLE 2-m
YERKES PHOTOGRAPH NO. 958

POINT	u′	RESIDUAL	v′	RESIDUAL
857 10002	−0.00726	0.00028	0.11936	0.00036
183 16315	0.47342	−0.00000	0.58967	0.00002
1145 20497	−0.24072	0.00035	0.50439	0.00014
2481 25010	−0.50288	−0.00008	−0.06602	−0.00022
1833 27078	−0.77643	−0.00012	−0.08428	−0.00029
2933 30095	−0.07749	0.00054	0.01734	0.00003
3055 31397	−0.08775	−0.00051	−0.32647	0.00009
2417 36278	−0.56373	−0.00012	−0.40615	−0.00031
4004 42697	0.47836	−0.00032	−0.48925	0.00037
3683 43100	0.32079	0.00020	0.08259	−0.00011
3667 44001	0.38851	−0.00021	0.20062	−0.00008
RMS Residuals		0.00030		0.00022
With Bessel		0.00035		0.00026

TABLE 2-n
YERKES PHOTOGRAPH NO. 1158

POINT	u'	RESIDUAL	v'	RESIDUAL
857 10002	0.09481	0.00026	0.09068	−0.00010
987 10950	0.42272	0.00005	0.84546	−0.00008
622 13421	0.52641	0.00003	0.32604	0.00007
183 16315	0.75645	0.00017	0.15089	0.00020
1145 20497	0.15526	−0.00008	0.53686	0.00045
1529 24197	−0.32568	0.00026	0.41165	−0.00014
1614 24671	−0.16303	0.00029	0.79268	−0.00058
2481 25010	−0.40892	−0.00004	0.26269	−0.00010
1833 27078	−0.63766	−0.00034	0.42353	−0.00009
2933 30095	−0.02417	−0.00007	0.05646	−0.00012
2419 35356	−0.59645	−0.00038	−0.07262	0.00027
2417 36278	−0.67566	0.00006	0.03964	0.00006
3683 43100	0.32350	−0.00021	−0.14540	0.00015
RMS Residuals		0.00021		0.00024
With Bessel		0.00024		0.00027

TABLE 2-o
YERKES PHOTOGRAPH NO. 1190

POINT	u'	RESIDUAL	v'	RESIDUAL
857 10002	−0.07143	−0.00018	0.07208	0.00020
987 10950	0.24800	0.00068	0.88554	−0.00045
622 13421	0.35052	0.00013	0.37004	0.00005
183 16315	0.62238	−0.00006	0.23181	−0.00013
1145 20497	−0.04890	0.00019	0.52767	0.00038
1529 24197	−0.50557	0.00019	0.33803	−0.00013
1614 24671	−0.35847	−0.00026	0.74589	−0.00026
2481 25010	−0.57009	0.00018	0.17844	−0.00013
1833 27078	−0.78042	−0.00027	0.31422	−0.00034
2933 30095	−0.18412	0.00001	0.02196	0.00011
3055 31397	−0.36331	−0.00022	−0.26327	0.00046
2419 35356	−0.69575	−0.00007	−0.17678	−0.00023
2417 36278	−0.77910	−0.00022	−0.07425	−0.00009
1992 38049	−0.88410	−0.00018	0.15176	−0.00017
4004 42697	0.05869	−0.00012	−0.69148	0.00002
3683 43100	0.19349	0.00041	−0.12955	0.00038
3667 44001	0.31800	−0.00021	−0.06301	0.00032
RMS Residuals		0.00026		0.00026
With Bessel		0.00028		0.00029

TABLE 2-p
YERKES PHOTOGRAPH NO. 1288

POINT	u'	RESIDUAL	v'	RESIDUAL
857 10002	0.05837	−0.00028	0.13334	−0.00016
987 10950	−0.06605	0.00007	0.94504	0.00016
1145 20497	−0.12386	0.00002	0.54613	−0.00011
2481 25010	−0.46481	0.00026	0.02100	−0.00033
2933 30095	−0.02578	−0.00003	0.04282	0.00019
4004 42697	0.43623	−0.00034	−0.54374	−0.00006
3683 43100	0.37472	0.00007	0.04806	−0.00008
3667 44001	0.45659	0.00023	0.15414	0.00040
RMS Residual		0.00020		0.00022
With Bessel		0.00026		0.00027

TABLE 2-q
YERKES PHJTJGRAPH NO. 1348

POINT	u'	RESIDUAL	v'	RESIDUAL
857 10002	−0.11282	0.00028	0.07860	0.00002
987 10950	0.78858	−0.00030	0.35676	0.00006
2933 30095	−0.22050	0.00003	0.13470	0.00008
3055 31397	−0.54096	−0.00000	0.09607	0.00020
2419 35356	−0.65872	0.00010	0.41264	0.00019
2417 36278	−0.62313	−0.00015	0.54347	−0.00003
3683 43100	−0.09901	0.00005	−0.25332	−0.00052
RMS Residuals		0.00017		0.00023
With Bessel		0.00022		0.00030

TABLE 2-r
YERKES PHOTOGRAPH NO. 1502

POINT	u'	RESIDUAL	v'	RESIDUAL
987 10950	−0.67375	−0.00015	0.61934	−0.00011
622 13421	−0.11282	0.00031	0.55534	0.00001
857 10002	0.01041	−0.00011	0.06651	0.00037
3683 43100	0.29559	0.00001	0.22514	0.00003
183 16315	0.10774	0.00001	0.73630	−0.00043
3667 44001	0.27990	0.00006	0.36194	0.00008
2933 30095	0.01124	0.00001	−0.05739	0.00033
1992 38049	−0.43636	−0.00013	−0.69951	−0.00028
RMS Residuals		0.00014		0.00026
With Bessel		0.00018		0.00032

TABLE 2-s
YERKES PHOTOGRAPH NO. 1579

POINT	u'	RESIDUAL	v'	RESIDUAL
857 10002	−0.01398	−0.00018	0.14492	−0.00011
987 10950	−0.59424	0.00031	0.73488	−0.00001
622 13421	−0.06220	0.00018	0.62975	0.00015
183 16315	0.18634	−0.00001	0.76597	−0.00008
1145 20497	−0.40341	−0.00007	0.37682	−0.00012
1529 24197	−0.48179	0.00005	−0.11270	0.00059
1614 24671	−0.77365	0.00015	0.18406	0.00006
2481 25010	−0.37813	0.00023	−0.24840	0.00032
1833 27078	−0.62492	−0.00013	−0.39756	−0.00017
2933 30095	−0.03037	−0.00017	0.02253	0.00009
3055 31397	0.11295	−0.00003	−0.29123	0.00010
2419 35356	−0.14885	−0.00010	−0.55685	−0.00008
2417 36278	−0.28556	−0.00005	−0.58528	0.00015
1992 38049	−0.54810	−0.00008	−0.58558	−0.00085
4004 42697	0.68094	0.00029	−0.19127	0.00031
3683 43100	0.29262	−0.00009	0.25727	−0.00012
3667 44001	0.29750	−0.00030	0.39095	−0.00022
RMS Residuals		0.00017		0.00029
With Bessel		0.00019		0.00032

TABLE 2-t
YERKES PHOTOGRAPH NO. 1638

POINT	u'	RESIDUAL	v'	RESIDUAL
987 10950	−0.35338	0.00014	0.78380	0.00029
622 13421	0.07966	−0.00017	0.39987	−0.00026
857 10002	−0.04711	0.00001	−0.11284	0.00005
3683 43100	0.28381	0.00003	−0.09418	0.00011
3667 44001	0.33545	−0.00028	0.03953	0.00009
2933 30095	−0.10238	0.00028	−0.22135	−0.00029
RMS Residuals		0.00018		0.00021
With Bessel		0.00026		0.00029

TABLE 2-u
YERKES PHOTOGRAPH NO. 1694

POINT	u'	RESIDUAL	v'	RESIDUAL
357 10002	0.07428	−0.00013	0.07168	0.00013
987 10950	−0.34481	0.00010	0.84772	0.00027
183 16315	0.42537	0.00023	0.63728	0.00003
1145 20497	−0.24002	0.00009	0.41442	−0.00045
1529 24197	−0.45276	0.00007	−0.03510	−0.00012
1614 24671	−0.66023	0.00030	0.35005	−0.00032
2481 25010	−0.39096	−0.00031	−0.19589	−0.00031
2933 30095	0.02420	0.00024	−0.04164	0.00005
3055 31397	0.06976	−0.00040	−0.37874	−0.00019
2419 35355	−0.26268	−0.00034	−0.55022	0.00020
2417 36278	−0.40440	0.00020	−0.53493	0.00005
3667 44001	0.43691	−0.00006	0.22859	0.00067
RMS Residuals		0.00023		0.00030
With Bessel		0.00027		0.00034

TABLE 2-v
YERKES PHOTOGRAPH NO. 1771

POINT	u′	RESIDUAL	v′	RESIDUAL
857 10002	−0.04055	−0.00037	−0.10760	0.00018
3683 43100	0.27447	0.00042	−0.20397	0.00024
183 16315	0.54884	0.00006	0.30933	−0.00020
3667 44001	0.36852	−0.00020	−0.09612	−0.00028
2933 30095	−0.13098	0.00014	−0.18984	0.00013
2481 25010	−0.55561	−0.00012	−0.14524	0.00027
1992 38049	−0.86133	0.00006	−0.21970	−0.00034
RMS Residuals		0.00024		0.00024
With Bessel		0.00031		0.00032

TABLE 2-w
YERKES PHOTOGRAPH NO. 1778

POINT	U	RESIDUAL	V	RESIDUAL
987 10950	0.26415	−0.00016	0.81556	0.00026
622 13421	0.39542	−0.00020	0.23130	−0.00020
857 10002	−0.00713	−0.00015	−0.10340	0.00017
3683 43100	0.26182	0.00039	−0.28700	0.00018
183 16315	0.66339	−0.00021	0.12559	−0.00038
3667 44001	0.38194	0.00003	−0.21115	−0.00008
2933 30095	−0.11879	0.00038	−0.15534	−0.00027
2481 25010	−0.52066	0.00004	0.01341	0.00039
1992 38049	−0.85723	−0.00011	0.03613	−0.00006
RMS Residuals		0.00022		0.00025
With Bessel		0.00027		0.00030

TABLE 2-x
YERKES PHOTOGRAPH NO. 1784

POINT	U	RESIDUAL	V	RESIDUAL
622 13421	0.41978	0.00012	0.18505	0.00017
857 10002	−0.01811	−0.00000	−0.10173	0.00036
3683 43100	0.22818	0.00021	−0.31472	−0.00016
183 16315	0.67389	−0.00028	0.04955	−0.00035
2933 30095	−0.13497	0.00013	−0.14066	0.00018
2481 25010	−0.51515	−0.00008	0.07263	−0.00010
1992 38049	−0.84720	−0.00010	0.13337	−0.00010
RMS Residuals		0.00015		0.00023
With Bessel		0.00020		0.00030

TABLE 2-y
YERKES PHOTOGRAPH NO. 1799

POINT	U	RESIDUAL	V	RESIDUAL
987 10950	0.09467	−0.00014	0.84514	0.00032
622 13421	0.32075	−0.00046	0.29240	−0.00034
857 10002	−0.02764	0.00004	−0.10157	0.00010
3683 43100	0.26945	0.00029	−0.24050	−0.00004
4004 42697	0.23108	0.00004	−0.77055	−0.00017
183 16315	0.60744	−0.00014	0.23012	−0.00007
3667 44001	0.37694	0.00000	−0.14665	−0.00034
1145 20497	−0.10162	−0.00006	0.36721	−0.00016
2933 30095	−0.12910	0.00021	−0.17053	0.00011
3055 31397	−0.25479	0.00019	−0.46790	−0.00010
1614 24671	−0.46336	0.00015	0.56228	0.00018
1529 24197	−0.51568	−0.00025	0.10118	0.00003
2481 25010	−0.54759	0.00006	−0.06761	0.00003
2417 36278	−0.71375	0.00021	−0.32542	0.00050
2419 35356	−0.60818	0.00020	−0.41955	0.00016
1833 27078	−0.79481	−0.00027	0.06353	0.00008
1992 38049	−0.87055	−0.00006	−0.09893	−0.00031
RMS Residuals		0.00020		0.00022
With Bessel		0.00022		0.00024

TABLE 3

SELENODETIC COORDINATES

No.	Designation	*E*	*F*	*G*	*H*
0	Mösting A	−.08990	−.05549	+ .99483	+.00042
1	Bruce	+.00685± 7	+.02096± 7	+1.00106± 61	+.00130
2	Blagg	+.02591± 7	+.02178± 7	+ .99990± 57	+.00047
3	Triesnecker J	+.04287± 9	+.05726± 9	+ .99893± 78	+.00149
4	W. Bond B	+.05536± 9	+.90596± 9	+ .41662± 79	−.00130
5	Democritus A	+.25509± 9	+.87921± 9	+ .39848± 79	−.00157
6	Arago B	+.35448± 7	+.06041± 7	+ .93247± 63	−.00060
7	Bessel A	+.32517± 7	+.41852± 7	+ .84785± 61	−.00013
8	Hall K	+.45746± 8	+.58123± 8	+ .67277± 73	−.00014
9	Macrobius B	+.61050± 8	+.35734± 8	+ .70723± 73	+.00029
10	Tralles B	+.68721± 8	+.45821± 9	+ .56430± 74	+.00032
11	Archimedes A	−.09844± 8	+.47013± 8	+ .87778± 70	+.00060
12	Gambart G	−.20816± 6	+.03455± 7	+ .97687± 57	−.00060
13	Laplace A	−.32655± 9	+.69060± 9	+ .64269± 81	−.00170
14	Milichius	−.49500±11	+.17398±11	+ .85218± 97	+.00075
15	Diophantus B	−.46917± 8	+.48577± 9	+ .73633± 74	−.00086
16	Mairan E.	−.47728±11	+.61223±11	+ .62793±101	−.00154
17	Harpalus E	−.46911±10	+.79532±10	+ .37960± 93	−.00165
18	Lansberg A.	−.51647± 7	+.00351± 7	+ .85648± 66	+.00155
19	Wollaston	−.62810±11	+.50831±12	+ .58630±113	−.00168
20	Rümker E	−.65485±16	+.62367±18	+ .42405±169	−.00120
21	Reiner E	−.76016+12	+.03360±12	+ .64711±108	−.00114
22	Reiner A	−.77819±13	+.08961±12	+ .61883±125	−.00172
23	Galilaei A	−.87160±10	+.20277±10	+ .44238± 91	−.00175
24	Casatus C	−.15333±14	−.95041±14	+ .26301±136	−.00202
25	Turner F	−.24345± 7	−.02762± 7	+ .96955± 64	+.00003
26	Nicollet	−.19986±10	−.37261± 9	+ .90600± 85	−.00019
27	Fra Mauro B	−.36793± 8	−.06949± 8	+ .92753± 69	+.00026
28	Schiller A	−.41447± 9	−.73246± 9	+ .53853± 81	−.00085
29	Gassendi J	−.55932±10	−.36748±10	+ .74225± 91	−.00059
30	Doppelmayer J	−.59800± 9	−.41390± 9	+ .68573± 81	−.00043
31	Gassendi G	−.67178± 9	−.28749± 9	+ .68217± 82	−.00035
32	Schickard H	−.64178±17	−.68812±17	+ .33744±158	−.00037
33	Flamsteed D	−.70381± 9	−.05492± 9	+ .70749± 79	−.00055
34	Zupus A	−.76755±10	−.29546±10	+ .56759± 88	−.00071
35	Damoiseau E	−.84705± 8	−.09081± 8	+ .52098± 72	−.00142
36	Grimaldi B	−.93322±29	−.05055±28	+ .35324±263	−.00088
37	Seeliger	+.05249± 6	−.03810± 6	+ .99885± 54	+.00095
38	Schomberger A	+.07954±15	−.98099±17	+ .18137±156	+.00078
39	Nicolai A	+.29565±10	−.67439±10	+ .67611± 85	−.00033
40	Alfraganus C	+.30917± 6	−.10606± 6	+ .94620± 50	+.00106
41	Rothmann K	+.36170± 6	−.48218± 6	+ .79986± 55	+.00155
42	Boussingault R	+.32472±11	−.90165±11	+ .28445± 98	−.00034
43	Moltke	+.40933± 6	−.00973± 6	+ .91198± 55	−.00032
44	Reimarus H	+.57689±17	−.75690±16	+ .30271±141	−.00133
45	Bohnenberger G	+.61496± 8	−.29493± 8	+ .73056± 73	−.00056
46	Furnerius A	+.71498±18	−.55238±18	+ .43027±159	+.00072
47	Langrenus C	+.86232±16	−.09731±16	+ .49733±137	+.00020
48	Gilbert D	+.97010±23	−.04483±23	+ .23272±213	−.00137

TABLE 4
COMPARISON WITH BRESLAU
TABULAR OBSERVATION EQUATIONS

	e	f	g	ϵ	α	β	γ	l
	1	0	0	+ .610	+.357	+ .707	0	+.00016
Macrobius B	0	1	0	+ .357	−.610	0	+ .707	−.00015
	0	0	1	+ .707	0	− .610	− .357	−.00070
	1	0	0	+ .007	+.021	+1.001	0	+.00011
Bruce	0	1	0	+ .021	−.007	0	+1.001	−.00022
	0	0	1	+1.001	0	− .007	− .021	−.00020
	1	0	0	+ .055	+.906	+ .417	0	−.00007
Bond B	0	1	0	+ .906	−.055	0	+ .417	−.00006
	0	0	1	+ .417	0	− .055	− .906	+.00080
	1	0	0	+ .325	+.419	+ .848	0	+.00020
Bessel A	0	1	0	+ 4.19	−.325	0	+ .848	−.00014
	0	0	1	+ .848	0	− .325	− .419	−.00160
	1	0	0	− .098	+.470	+ .878	0	−.00019
Archimedes A	0	1	0	+ .470	+.098	0	+ .878	−.00002
	0	0	1	+ .878	0	+ .098	− .470	−.00050
	1	0	0	− .495	+.174	+ .852	0	+.00010
Milichius	0	1	0	+ .174	+.495	0	+ .852	+.00007
	0	0	1	+ .852	0	+ .495	− .174	−.00050
	1	0	0	− .477	+.612	+ .628	0	+.00003
Mairan E	0	1	0	+ .612	+.477	0	+ .628	−.00001
	0	0	1	+ .628	0	+ .477	− .612	+.00170
	1	0	0	− .778	+.090	+ .619	0	+.00040
Reiner A	0	1	0	+ .090	+.778	0	+ .619	+.00023
	0	0	1	+ .619	0	+ .778	− .090	+.00210
	1	0	0	− .847	−.091	+ .521	0	+.00033
Damoiseau E	0	1	0	− .091	+.847	0	+ .521	+.00067
	0	0	1	+ .521	0	+ .847	+ .091	+.00250
	1	0	0	− .672	−.287	+ .682	0	−.00006
Gassendi G	0	1	0	− .287	+.672	0	+ .682	+.00019
	0	0	1	+ .682	0	+ .672	+ .287	+.00150
	1	0	0	− .559	−.367	+ .742	0	+.00013
Gassendi J	0	1	0	− .367	+.559	0	+ .742	+.00012
	0	0	1	+ .742	0	+ .559	+ .367	+.00140
	1	0	0	− .090	−.055	+ .995	0	−.00002
Mösting A	0	1	0	− .055	+.090	0	+ .995	−.00002
	0	0	1	+ .995	0	+ .090	+ .055	+.00040
	1	0	0	+ .296	−.674	+ .676	0	−.00005
Nicolai A	0	1	0	− .674	−.296	0	+ .676	+.00003
	0	0	1	+ .676	0	− .296	+ .674	−.00090
	1	0	0	+ .309	−.106	+ .946	0	−.00002
Alfraganus C	0	1	0	− .106	−.309	0	+ .946	−.00010
	0	0	1	+ .946	0	− .309	+ .106	+.00040

TABLE 5
COMPARISON WITH BRESLAU
LPL Secondary to Breslau

With Damoiseau E

$e = -0.00017 \pm 22,\quad \alpha = +0.00020 \pm 8,$
$f = +0.00044 \pm 23,\quad \beta = +0.00031 \pm 28,$
$g = +0.00046 \pm 49,\quad \gamma = -0.00054 \pm 29,$
$\epsilon = -0.00018 \pm 8$

Without Damoiseau E

$e = -.00022 \pm 20,\quad \alpha = +0.00016 \pm 7,$
$f = +.00023 \pm 20,\quad \beta = +0.00036 \pm 25,$
$g = +.00029 \pm 41,\quad \gamma = -0.00030 \pm 25,$
$\epsilon = .00012 \pm$

TABLE 6
COMPARISON WITH BRESLAU
Residuals after Adjustment to Breslau

		E	F	G
9	Macrobius B	−16	+ 3	+104
		−14	+ 2	+ 79
1	Bruce	+ 2	+11	+ 49
		+ 3	+14	+ 37
4	W. Bond B	+19	+10	+ 6
		+14	+ 4	− 31
7	Bessel A	−10	− 2	+203
		− 9	+ 1	+180
11	Archimedes A	+ 1	− 8	+109
		− 1	− 6	+ 86
14	Milichius	+11	− 3	+105
		+ 7	− 4	+ 91
16	Mairan E	+19	+ 9	− 87
		+13	+ 5	−113
22	Reiner A	−22	+ 1	−147
		−29	− 8	−158
35	Damoiseau E	−21	−33	−192
		—	—	—
31	Gassendi G	+16	+ 6	− 92
		+12	− 3	− 94
29	Gassendi J	− 4	+ 9	−120
		− 7	+ 2	−121
0	MöstingA	+16	− 5	− 12
		+16	− 3	− 22
39	Nicolai A	−10	+11	+108
		− 7	+ 3	+ 11
40	Alfraganus C	+ 6	− 1	− 26
		+ 8	0	− 37
	RMS	+20	+16	+160
		+19	+ 8	+148