Exoplanet Atmospheres

- Motivation (brown dwarfs, observations, etc.)
- Equilibrium and non-equilibrium chemistry
- Radiative transfer, opacities, and spectra
- Simplified model atmosphere problems
- Retrieval techniques
**Atmosphere:** Hydrogen & Helium, or other stuff.

**Interior:** solid (rock, ice) or convective fluid (Hydrogen, Helium)

**Bottom Layer:** abrupt solid/liquid surface or deep continuous transition region.
Atmosphere [noun]:
“a transition region between the stellar interior and the interstellar medium” (Grey 1992)
Atmosphere [noun]: a transition region between the planet interior and the interplanetary medium.

interior models

upper BC (Teff/log(g)/Z)
spectra (Teff/log(g)/Z)

colors & Mags (age/mass/Z)
L & R (mass/age/Z)

atmosphere models

spectra (Teff/log(g)/Z)

Comparisons to observations
We will focus first on giants
The atmospheres we can study are generally warm to hot.
Lessons Learned from Brown Dwarfs
(~ 500K < $T_{\text{eff}}$ < 2500K)

- $M/M_{\text{Jup}} > 80$ (Star)
- $13 < M/M_{\text{Jup}} < 80$ (Brown Dwarf)
- $M/M_{\text{Jup}} < 13$ (planet)
Lessons Learned from Brown Dwarfs
(\(\sim 500 \text{K} < T_{\text{eff}} < 2500 \text{K}\))

- observable atmosphere:
  \(T_{\text{gas}} \sim 1000 \text{K}, P_{\text{gas}} \sim 0.1 \text{ to } 1 \text{ bar}.\)

- relatively thin atmospheres (\(H_p \sim 12 \text{ km}\))

- Major sources of opacity: Water, CIA, “dust”, Alkali (Na, K) doublets
Giant Planet / Brown Dwarf overlap:

- mass: $\sim 1$ to $< 80 \times M_{\text{Jupiter}}$
- radius: $\sim 1$ to $< 5 \times R_{\text{Jupiter}}$
- ages: $\sim$ millions to billions of years old
- gravity: $\sim$ 2 orders of magnitude
- effective temperature: $\sim$ 100K to 2500K
- clouds: broad range of grains, ices (complex mixtures)
- non-equilibrium chemistry
- dynamics and “weather”
- BUT: different formation ... (composition & early evolution)
“Solar Abundances” (Asplund et al. 2009)

- solar abundances defines our baseline elemental composition

- solar C,N,O are often debated values (check the reference!)

- starting point for equilibrium chemistry calculations

- the relative values are “initial conditions” of atmosphere models (and usually conserved quantities).
<table>
<thead>
<tr>
<th>Chemical Formulas</th>
<th>Structures</th>
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<tbody>
<tr>
<td><strong>Al\textsubscript{2}O\textsubscript{3}(a)</strong></td>
<td><img src="image" alt="Al\textsubscript{2}O\textsubscript{3}(a)" /></td>
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Chemical Equilibrium (in a box)

- independent of time
- independent of box history
- all fluctuations are damped out
- independent of position in box
- \( p_i = f(T, P, a_j) \) (partial pressure)

\[
p_i = \left( \frac{n_i}{n_{tot}} \right) P_{gas} = x_i P_{gas}
\]

\( x_i = \) mole fraction

Important references:
Fegley & Lodders (1994)
Burrows & Sharp (1999)
Equilibrium Chemistry (in a nutshell) More on this later ... 

- Simplified example: Iron in Jupiter’s atmosphere

\[ \sum Fe = P_{Fe} + P_{Fe(OH)_2} + 2P_{Fe_2Cl_4} \]

- mass balance:

- Expressed in terms of thermodynamic quantities:

\[ \sum Fe = a_{Fe} \left[ K_{Fe} + K_{Fe(OH)2}(f_{H2})(f_{O2}) + 2a_{Fe}K_{Fe_2Cl_4}(f_{Cl2})^2 \right] \]

- system of equations for each element (each equation can be very long, e.g., hydrogen can have 100s of terms)

- solved for f, numerically, with some initial guesses and fixed element abundances. Equivalent to minimizing the Gibbs potential

- Alternative methods used (examples discussed later)
Temperature Structures of sub-stellar mass objects:

Results of model atmosphere calculations (See also Allard et al. 2001)

- chemical equilibrium
- hydrostatic equilibrium
- radiative+convective equilibrium
- one-dimension (radial)
- time-independent
- fixed abundances (e.g. “solar”)
A quick introduction to *spectral types* of brown dwarfs

![Graph showing spectral types of brown dwarfs across different wavelengths.](image-url)
A quick introduction to *spectral types* of brown dwarfs

![Diagram showing normalized flux ($F_\lambda$) + constant vs. wavelength (\(\mu\text{m}\)) for different spectral types. The graph indicates absorption features at around 2500 K and 700 K.](image-url)
Jupiter

HD 189733

Y T L M

chemical eq.
(at 1 bar)
Results of model atmosphere calculations (See also Allard et al. 2001)

- chemical equilibrium
- hydrostatic equilibrium
- radiative+convective equilibrium
- one-dimension (radial)
- time-independent
- fixed abundances (e.g. “solar”)

*Temperature Structures* of sub-stellar mass objects:
A single object evolves through the Spectral Type (SpT(Teff) also not exactly a unique function at low T)
major molecular absorbers
(NOT “continuous” -- 100s of billions of lines)

This is actually cm² / molecule

Zahnle & Marley (2014)
The basic model atmosphere recipe:

1. **Initial $T(r)$ structure**
2. **Hydrostatic Equilibrium Constraint:**
   \[
   \frac{dP}{dr} = -\rho g \Rightarrow P(r), \rho(r)
   \]
3. **Solve LTE EOS**
   \Rightarrow occupation numbers
4. **Get the absorption coefficient** $\kappa_\lambda(r)$
5. **Solve the RTE (PPRTE or SSRTE)**
   \Rightarrow $I(\lambda, r)$
   Moments: $J(\lambda, r), H(\lambda, r), K(\lambda, r)$
6. **Energy Conserved?**
   Total Flux = $\sigma T_{\text{eff}}^4$ ?
7. **Yes**
   Finished $\Rightarrow T(r), P(r)$
   Spectrum $\Rightarrow F(\lambda, R)$
Major distinction between most brown dwarfs and giant planets -- incident stellar flux.