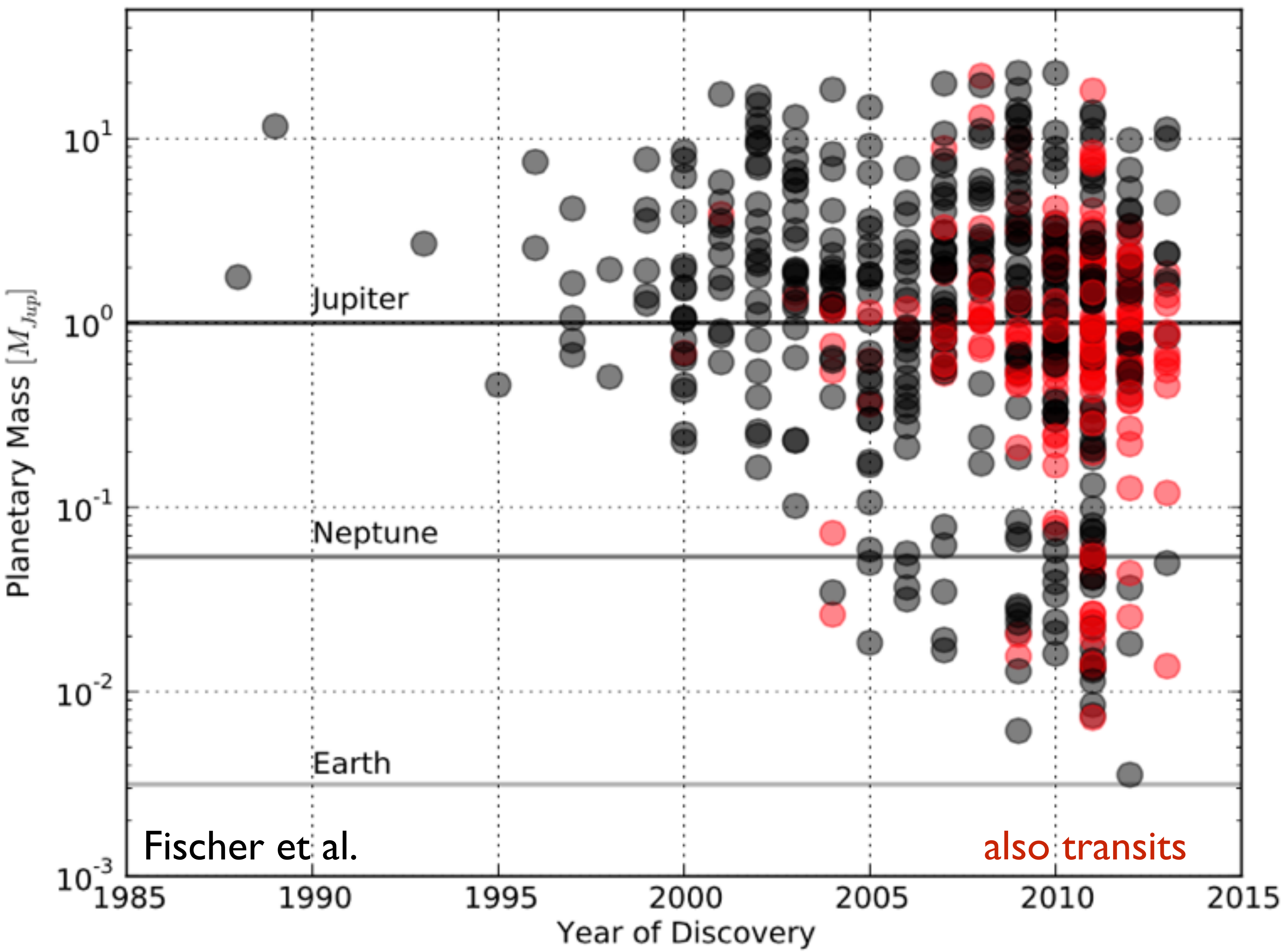
Reading list ...

- Fischer et al. chapter in PPVI, *Exoplanet Detection Techniques*
- Chapter 2 in Exoplanets (Fischer & Lovis) pg 27 - 53. (RV method)
- Wright 2018 (chapter in 2018 Handbook)

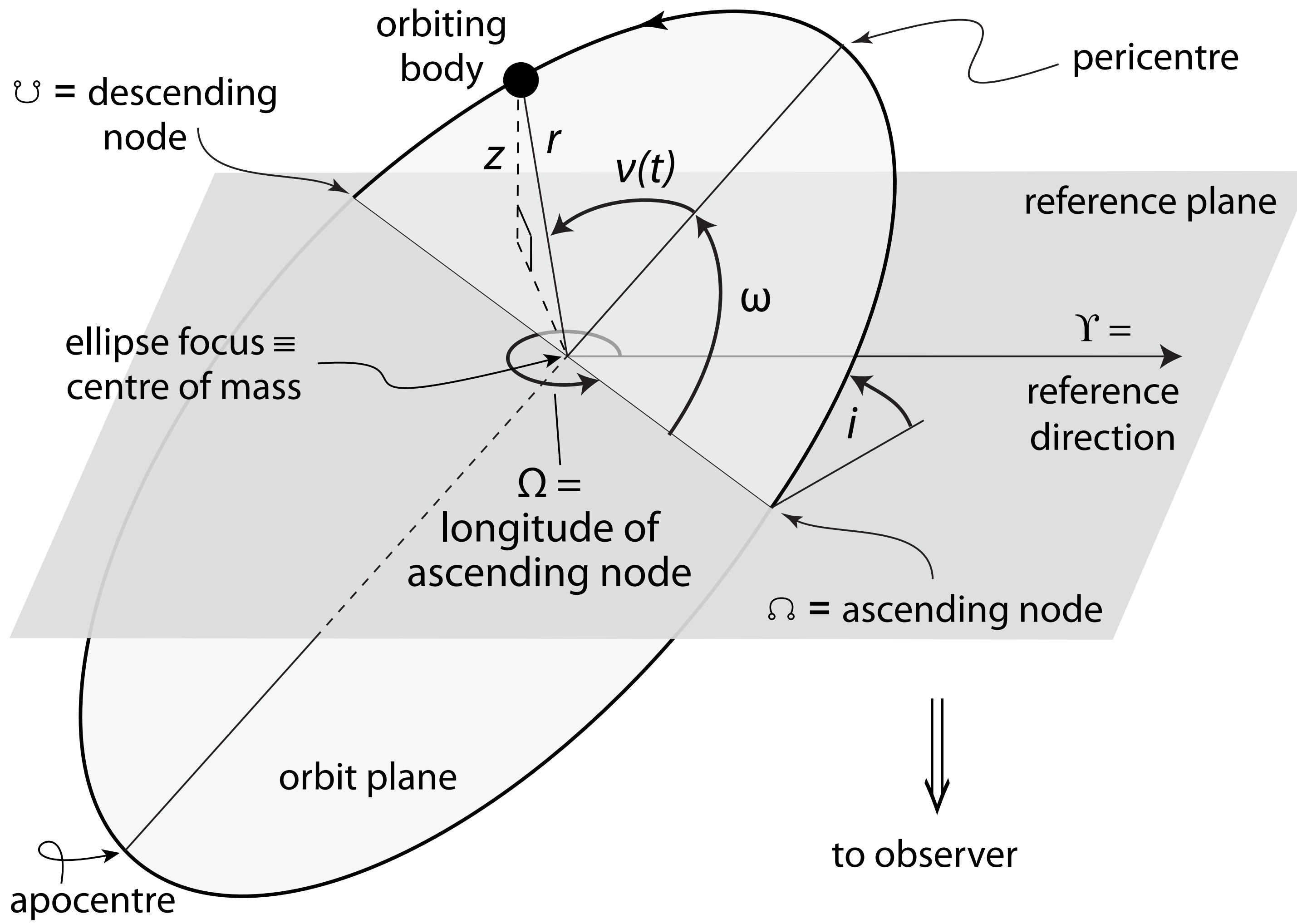
Radial Velocity (RV) Detection:

- Early 20th century RV precision ~ 1 km/s
- By late 80s / early 90s ---> 10 m/s (3 m/s by 1995, detection of 51 Peg b)
- By 2005, ~ 1 m/s.

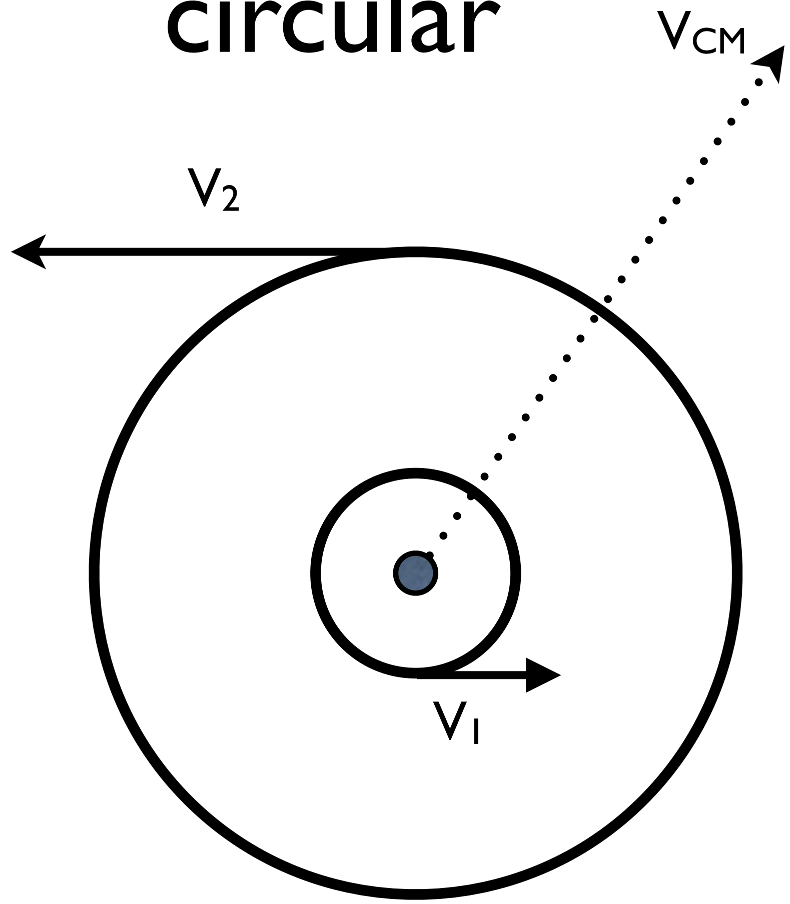
RV discoveries (up to 2014)

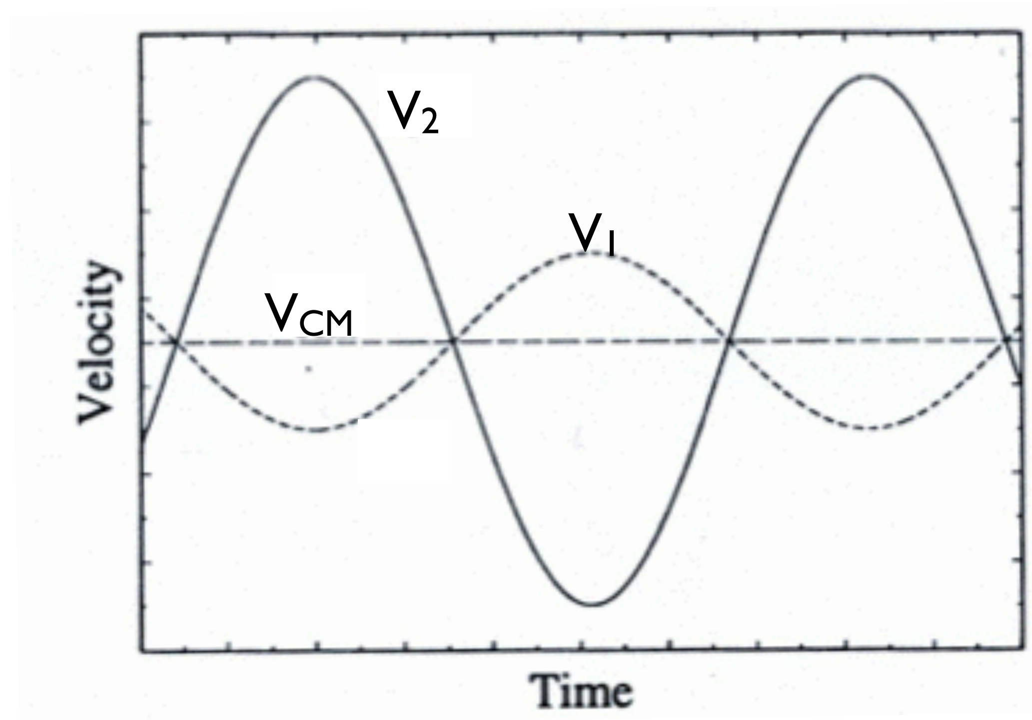


General orientation of orbit:



reference plane tangent to celestial sphere circular





1. $$V_1 = \frac{2\pi a_1}{P} \text{ and } V_2 = \frac{2\pi a_2}{P}$$

2. $$M_1 a_1 = M_2 a_2$$

3. $$\frac{M_1}{M_2} = \frac{V_2}{V_1} = \frac{V_2 \sin(i)}{V_1 \sin(i)} = \frac{V_{2,rad}}{V_{1,rad}}$$

4. $$a = a_1 + a_2 = \frac{P}{2\pi}(V_1 + V_2)$$

5. $$M_1 + M_2 = \frac{4\pi^2}{G}\frac{a^3}{P^2}$$

$$1 \rightarrow \star \text{ and } 2 \rightarrow p \;(M\star >> m_p)$$

...

7. $$m_p^3 \simeq \frac{P}{2\pi G}\left(\frac{V_{\star,rad}}{\sin(i)}\right)^3 M_\star^2$$

↑ ↑

unknowns

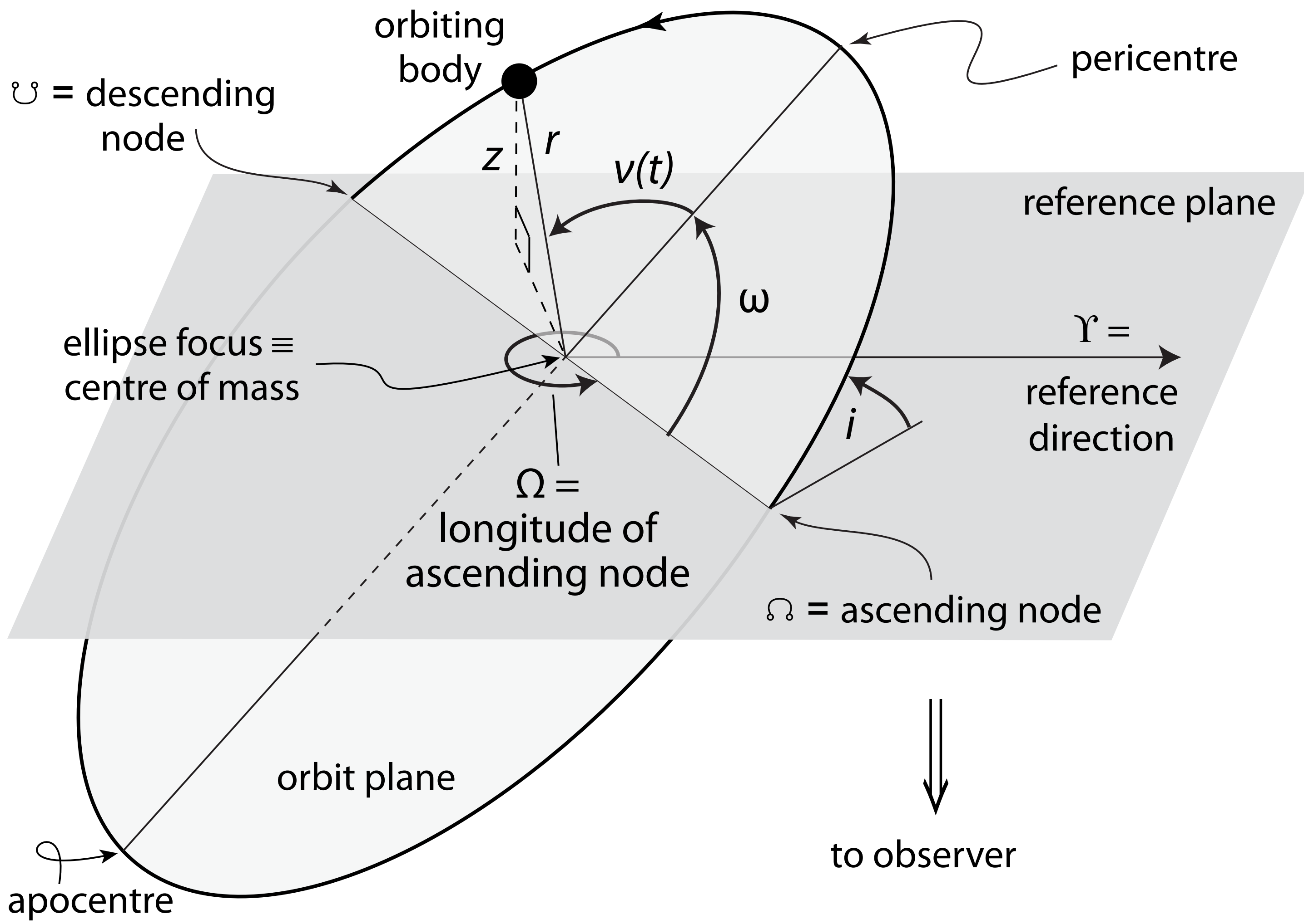
$$m_p^3 \simeq \frac{P}{2\pi G}\left(\frac{V_{\star,rad}}{\sin(i)}\right)^3 M_\star^2$$

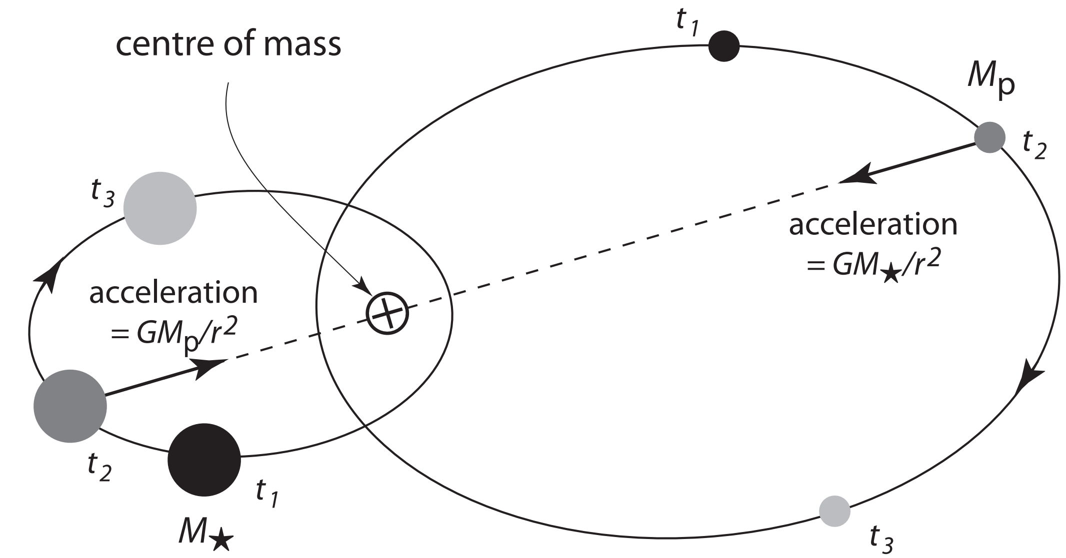
Still the circular (e = 0) case

Usually express the observable (RV semi-amplitude) *K* as function of *P* (or *a*), M_p*sin*(*i*), M_*.

$$K_\star = 28.4\mathrm{ms}^{-1}\left(\frac{P}{1yr}\right)^{-1/3}\left(\frac{M_p\sin(i)}{M_J}\right)\left(\frac{M_\star}{M_\odot}\right)^{-2/3}$$

K is directly proportional to M_p*sin*(*i*).





$$v_{r,\star} = K(\cos(\omega + \nu)) + e\cos(\omega))$$

(rad. vel. semi-amplitude) $$K = (v_{r,max} - v_{r,min})/2$$

note: reference plane tangent to celestial sphere

systemic vel. + "systematics"

$$v_{r,\star} = K(\cos(\omega + \nu)) + e\cos(\omega)) + (\gamma + d(t))$$

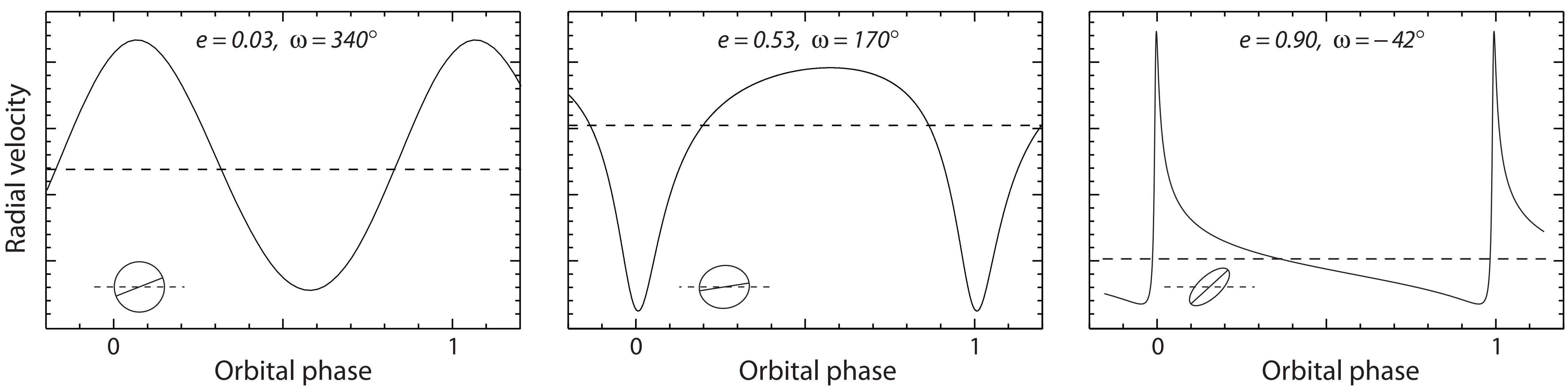
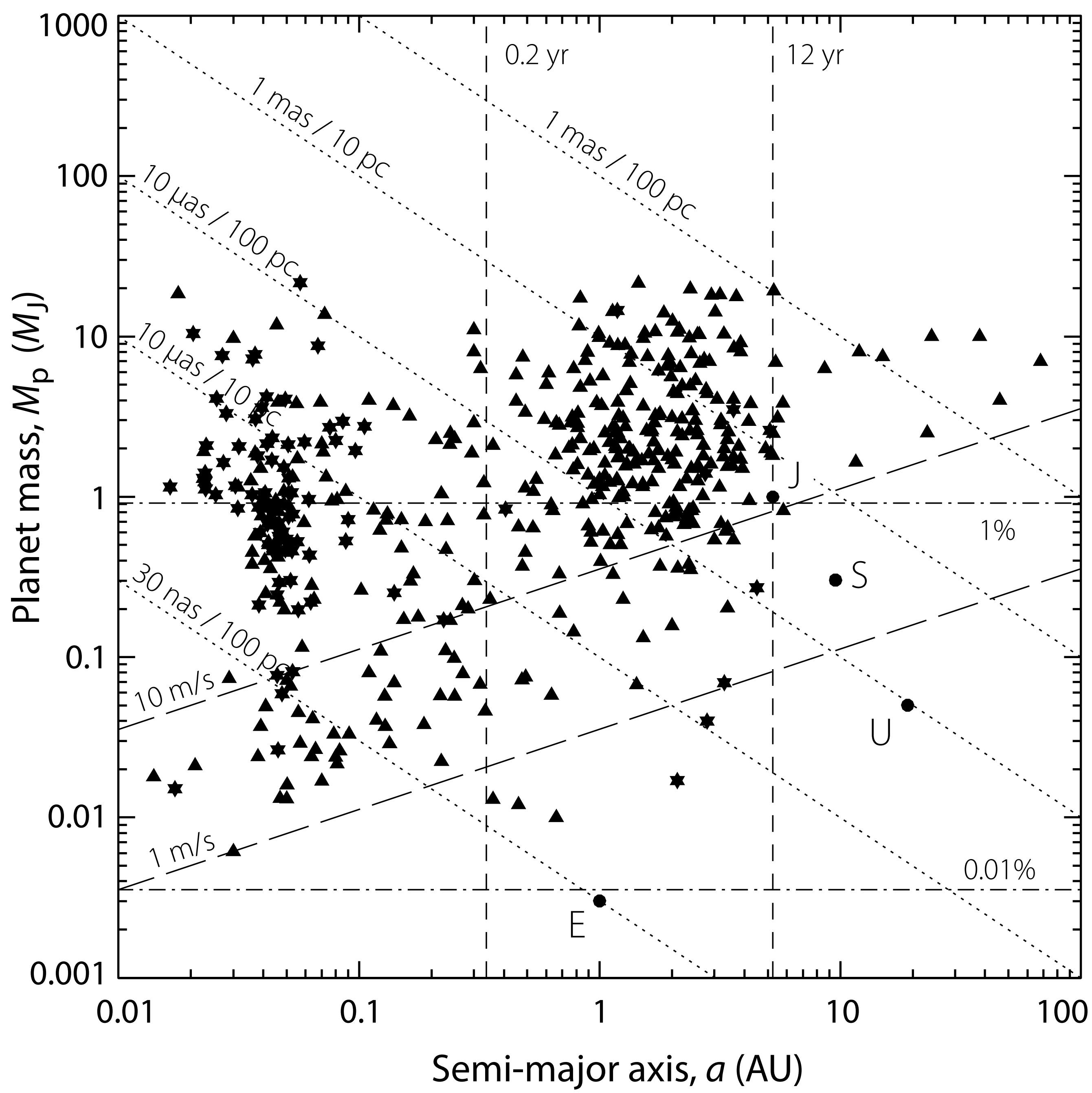


TABLE 1. Radial velocity signals for different kinds of planets orbiting a solar-mass star.

Planet	a (AU)	K_1 (m s^{-1})
Jupiter	0.1	89.8
Jupiter	1.0	28.4
Jupiter	5.0	12.7
Neptune	0.1	4.8
Neptune	1.0	1.5
Super Earth (5 $M_\oplus$)	0.1	1.4
Super Earth (5 $M_\oplus$)	1.0	0.45
Earth	0.1	0.28
Earth	1.0	0.09



detection limits obtained from earlier equations.

minimum mass $m_p sin(i)$

- If planetary systems are randomly oriented (i between 0 and 90°):

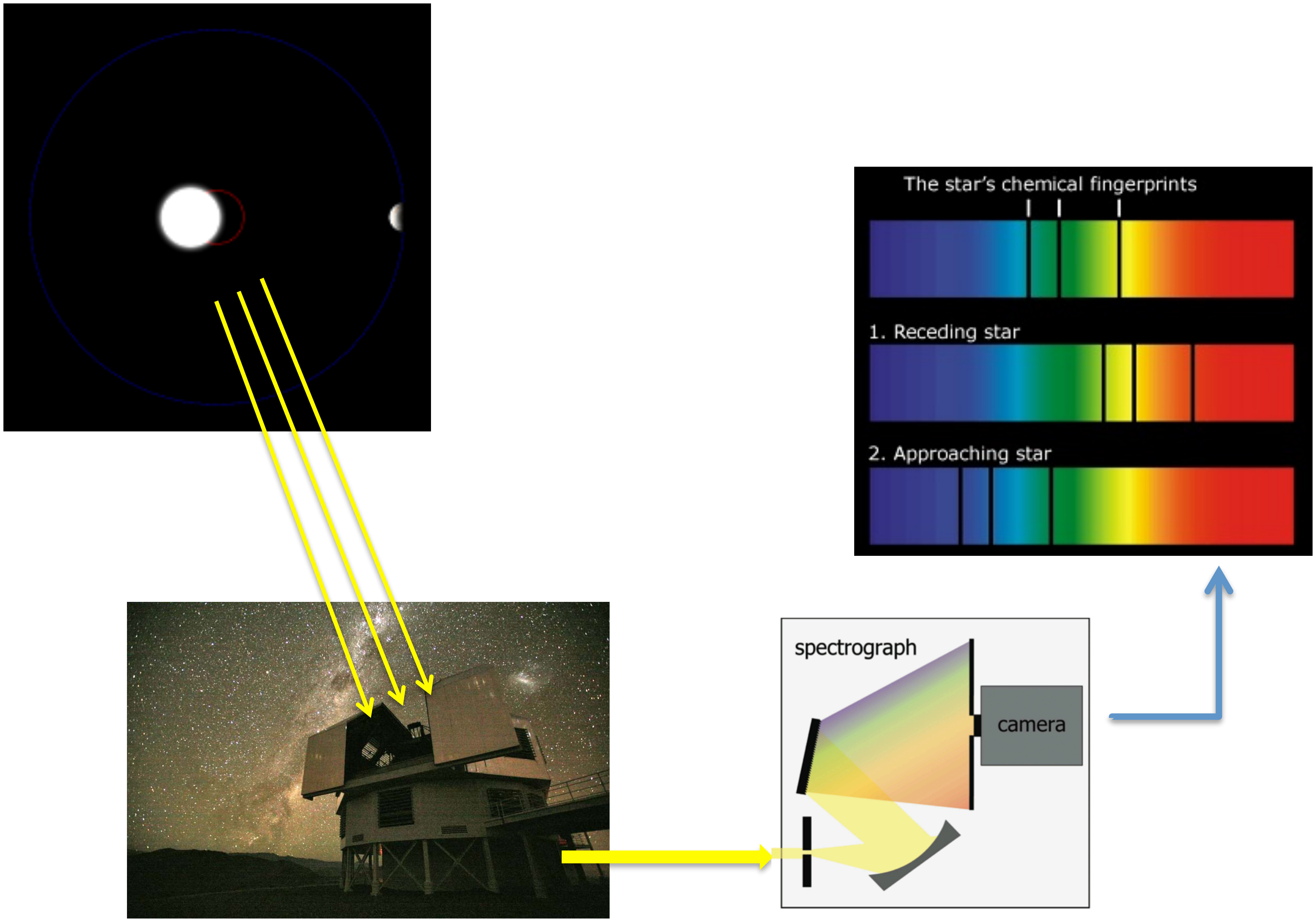
  Average of sin(i) = 2/pi ~ 0.6

- probability of i being between two values:

$$P = |cos(i_2) - cos(i_1)|$$

- 87% chance that $m_p sin(i)$ is within a factor of 2 of actual m_p

Measuring RVs (basic idea)



Doppler Shift

relativistic part is grav. potential (at the observer) and this changes over a year due to earth's eccentricity.

- the usual beta^2 part is the 1 - v^2/c^2 part

$$\lambda = \lambda_0 \frac{1 + \frac{1}{c}\hat{k}\cdot\vec{v}_{obs}}{1 - \frac{\Phi_{obs}}{c^2} - \frac{v_{obs}^2}{2c^2}}$$

Relativistic terms are usually ignored. However, they can vary by ~ 0.1 m/s over a year (Earth's orbit).

$$\lambda = \lambda_0(1 + \frac{v}{c})$$

note: vel. > 0 for motion away from observer

- must correct for Earth’s motion (barycentric correction)
- This requires precise clocking of observations (including precise knowledge of where your telescope is located).

Measuring RVs (basic idea)

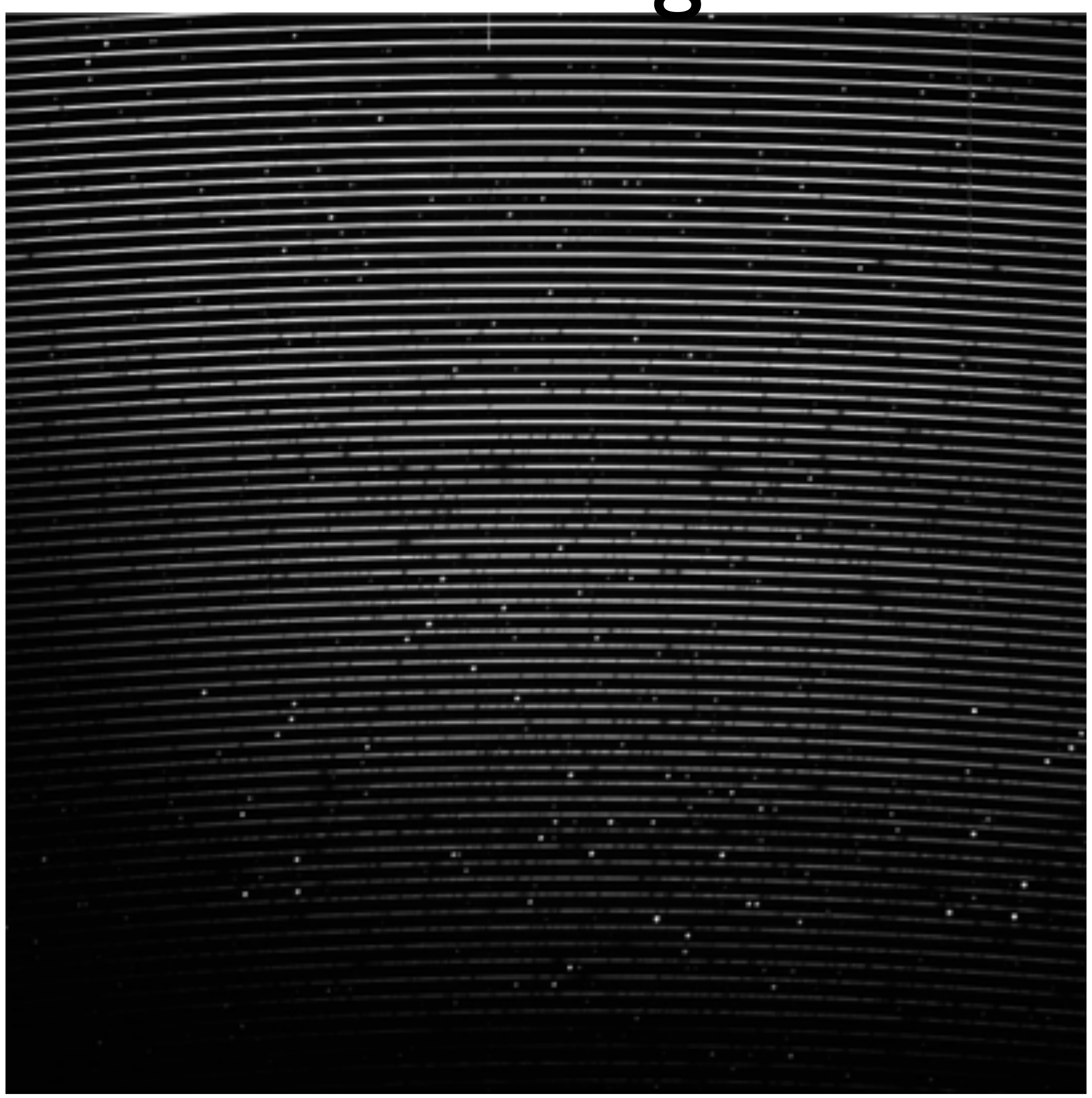
You must have a reference spectrum to determine the wavelength calibration.

- use telluric lines (Griffin & Griffin, 1973)
- gas-cell (Walker & Campbell, 1979, HF), Iodine is the modern choice. HIRES/Keck
- simultaneous references spectrum, Thorium-argon lamp (lamp + star spectrum recorded simultaneously) ELODIE, SOPHIE, CORALIE, HARPS
- other "fancy" methods ...

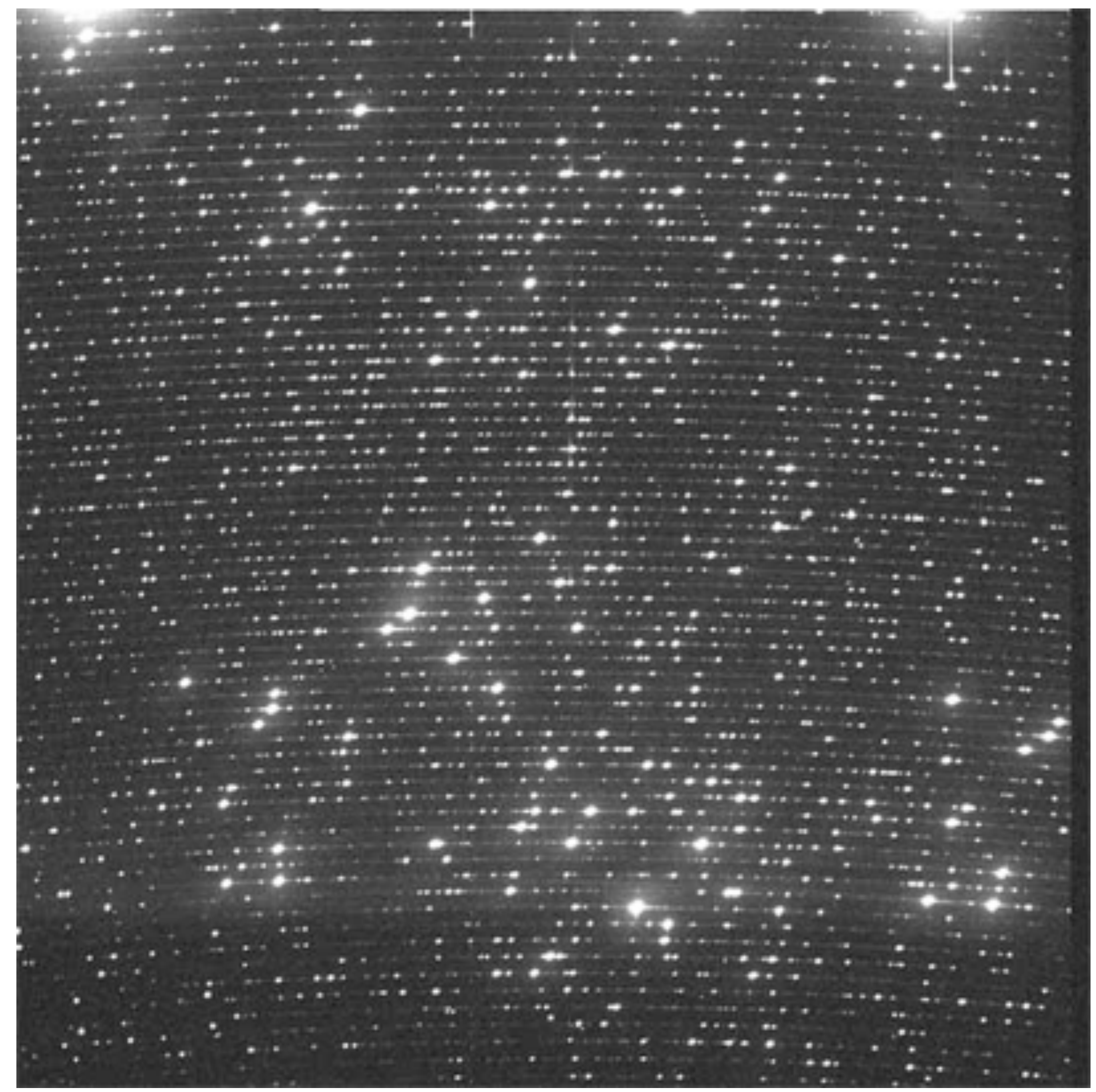
cross-dispersed echelle spectrograph

wavelength

orders

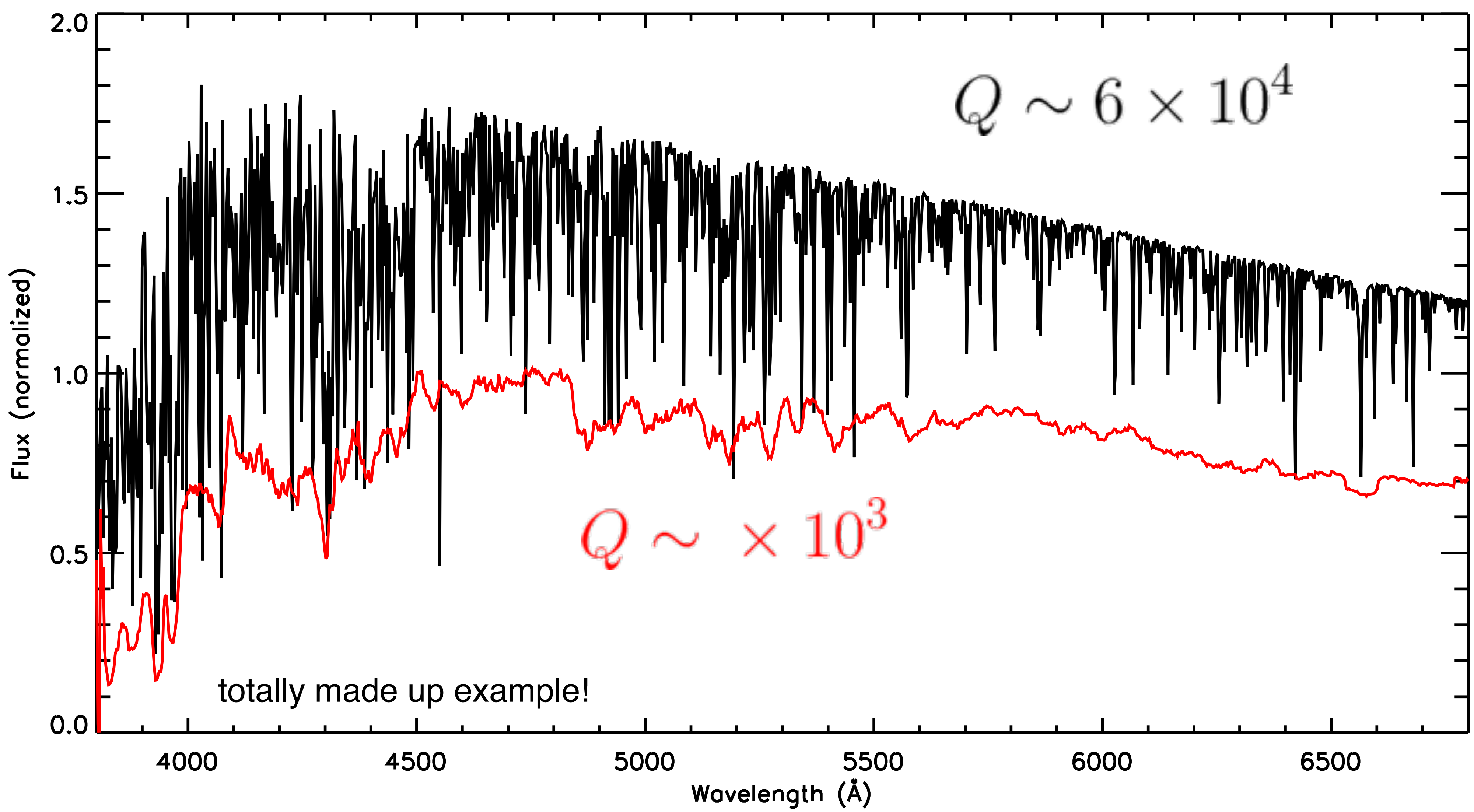


star+lamp spectrum
(51 Peg)



Lamp Spectrum

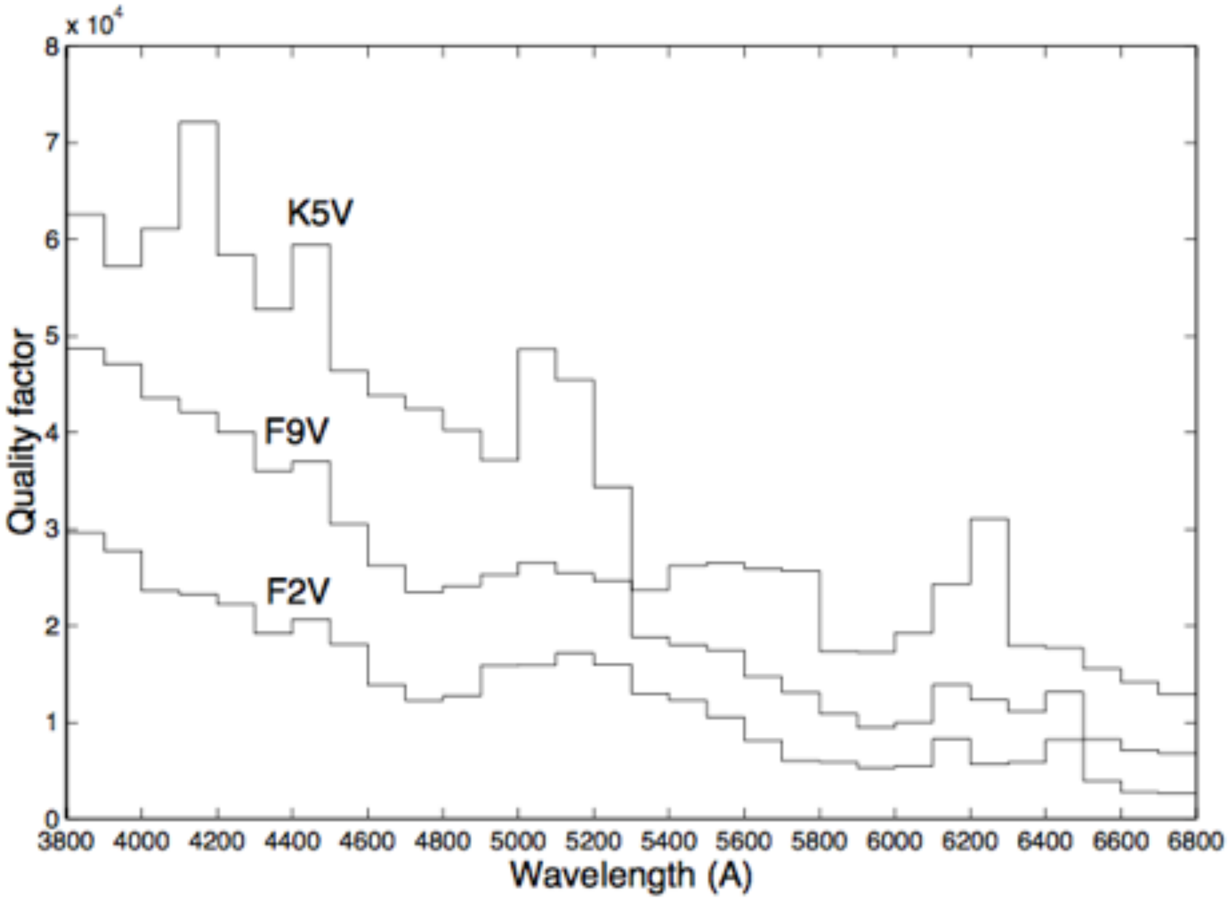
Quality Factor and theoretical RV limit

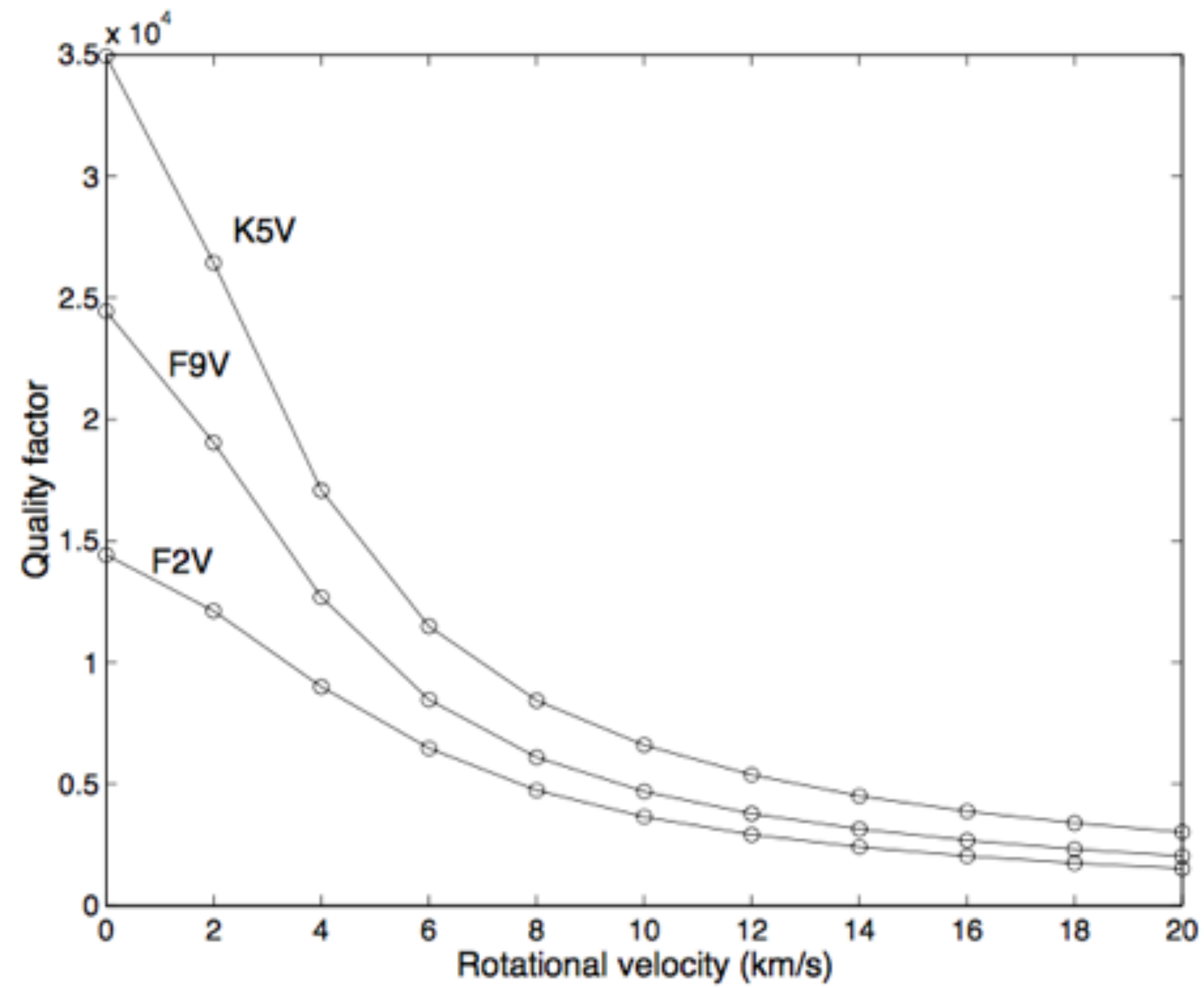


$$\delta V_{\mathrm{RMS}} = \frac{c}{Q N_e}$$

Bouchy et al. 2001

Quality Factor and theoretical RV limit



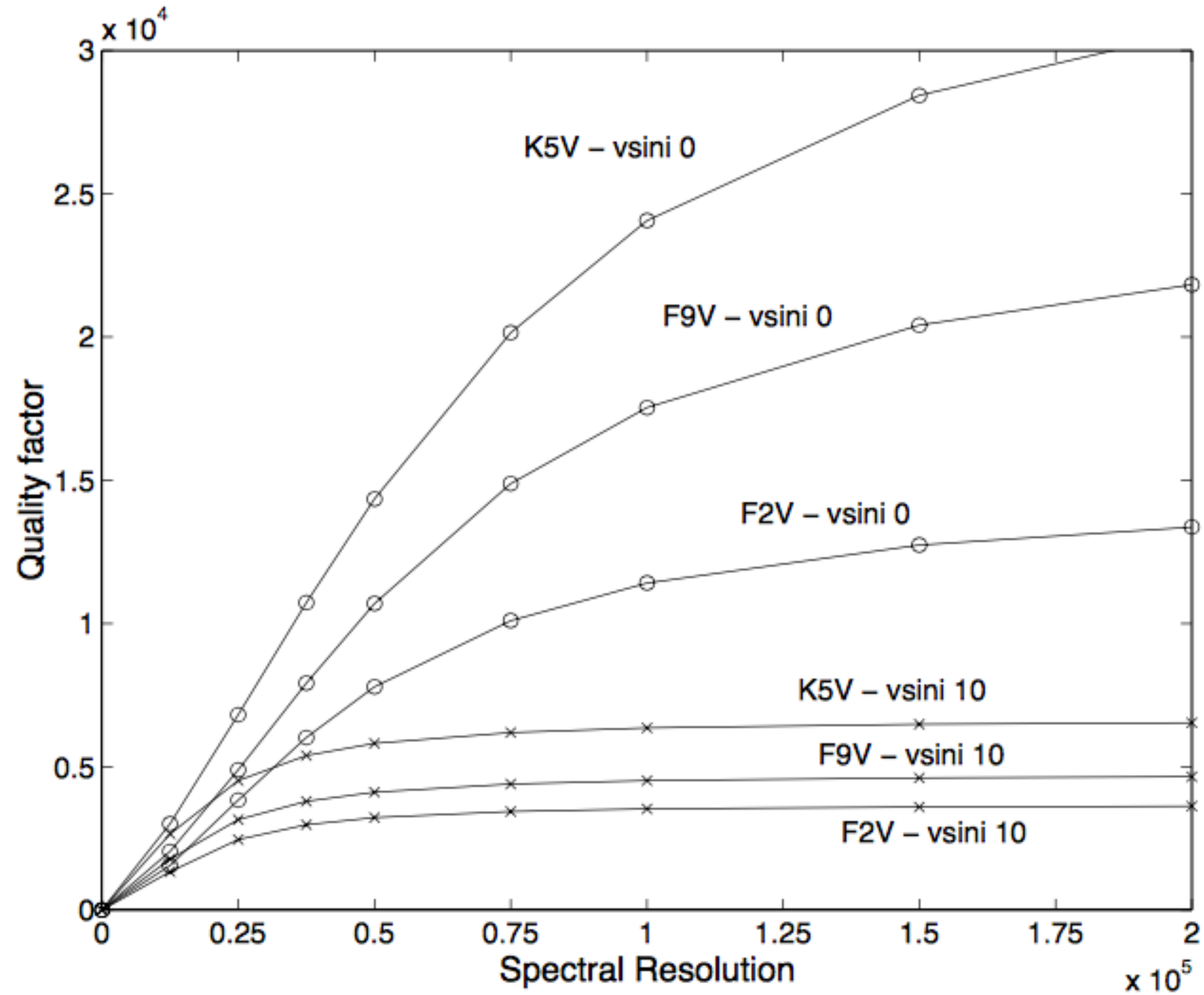


Bouchy et al. 2001

Spectral Resolution:

- Resolving Power: $R = \lambda / \Delta\lambda = c / \Delta v$
- R =100,000 --> dlam ~ 0.05 angst, in the optical.
- 1 m/s shift ---> 1/3000 of a resolution element, for R ~ 100,000.
- Typical RV instruments disperse light such that 1 m/s doppler shift corresponds to a shift on the detector of ~ 1/1000 of a pixel.

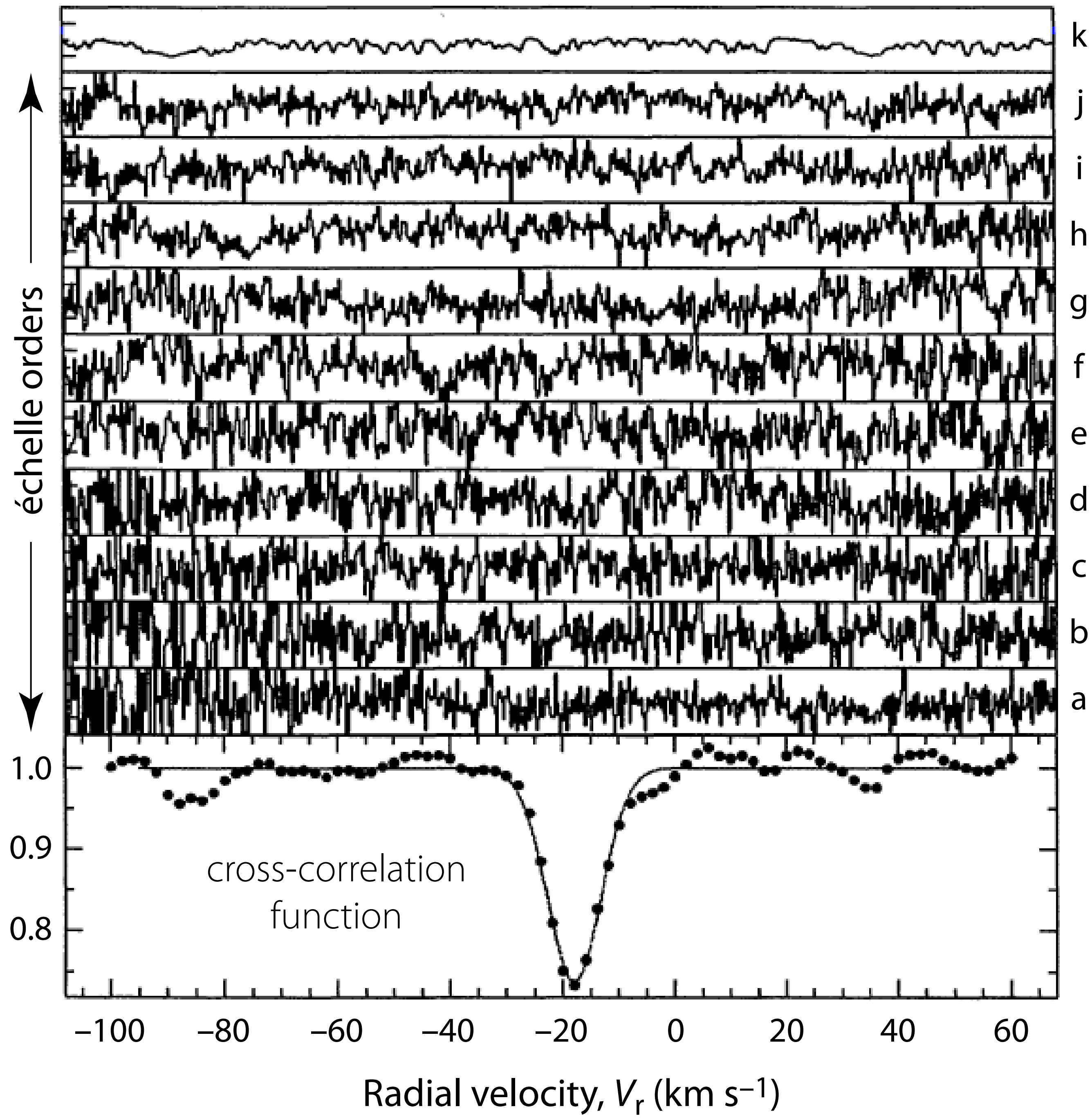
Quality Factor and theoretical RV limit



Q versus Spectral Resolving Power $R = \frac{\lambda}{d\lambda}$

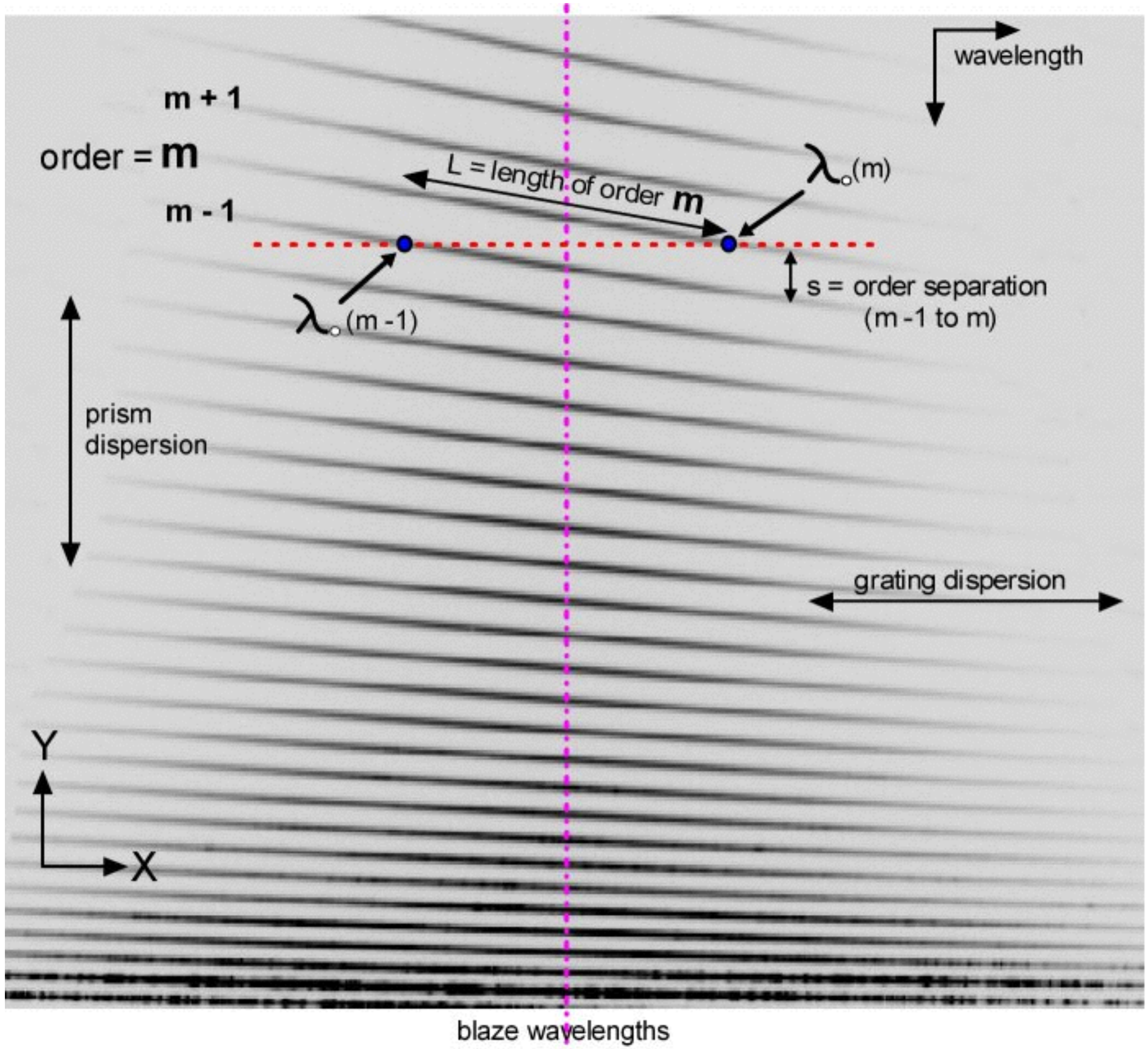
- Measuring tiny fractions of a pixel requires many spectral lines. Usual spectrum might contain ~ 1000 good lines.
- Modeling and cross-correlation techniques allow greater RV precision than is achieved on a single line.
- 30 m/s per line can result in 1 m/s (for high SNR spectrum with many lines).

cross-correlation of many orders

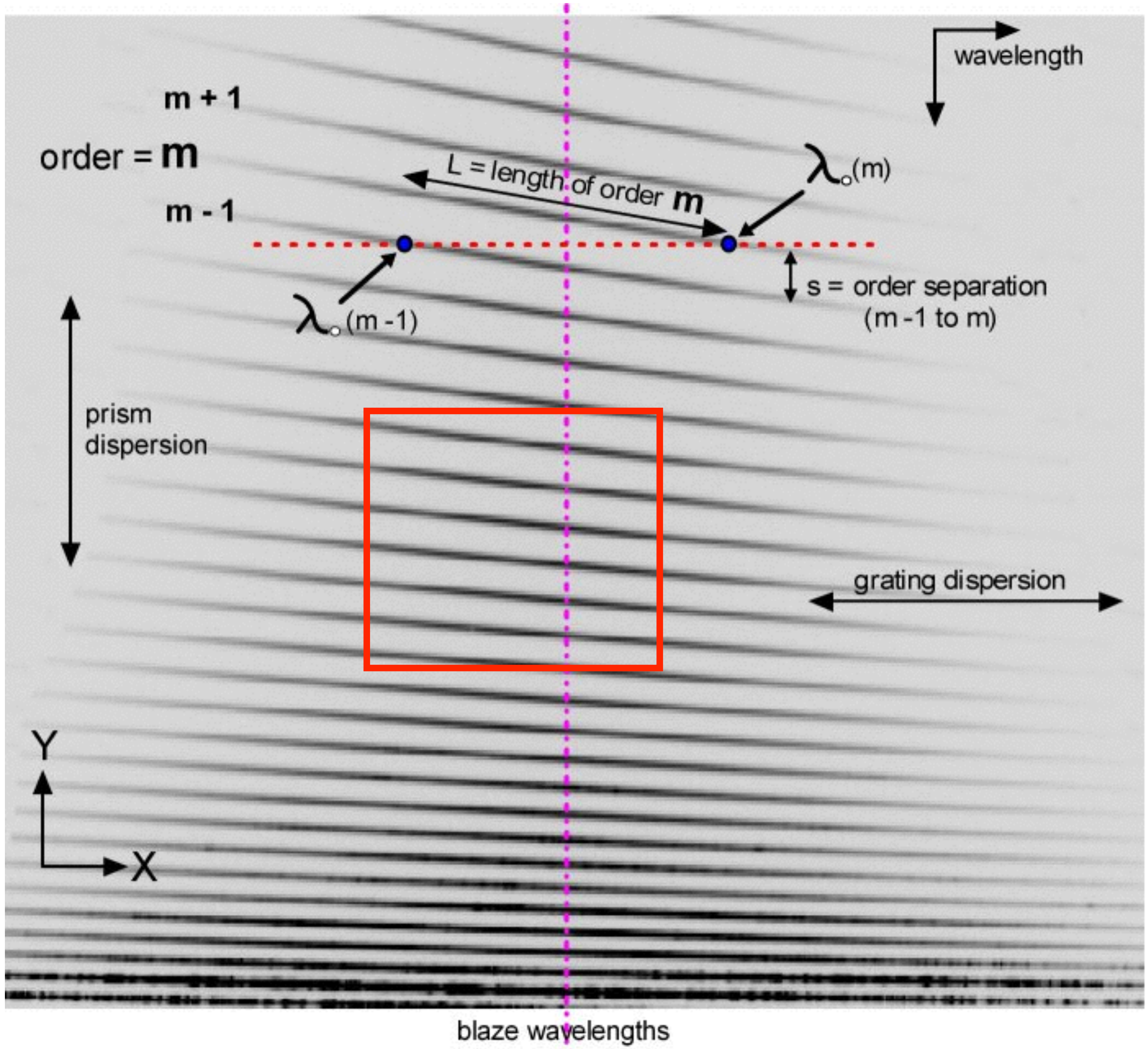


low SNR

high SNR



wavelength
m + 1
order = m
m - 1
L = length of order m
λ₀(m)
λ₀(m -1)
s = order separation
(m -1 to m)
prism
dispersion
grating dispersion
Y
X
blaze wavelengths


wavelength
m + 1
order = m
m - 1
L = length of order m
λ₀(m)
λ₀(m -1)
s = order separation
(m -1 to m)
prism
dispersion
grating dispersion
Y
X
blaze wavelengths

RV precision depends on:

- number of spectral lines / wavelength
- FWHM of the lines (contrast with continuum)
- signal-to-noise (SNR) of the data
- stability of the instrument and wavelength reference

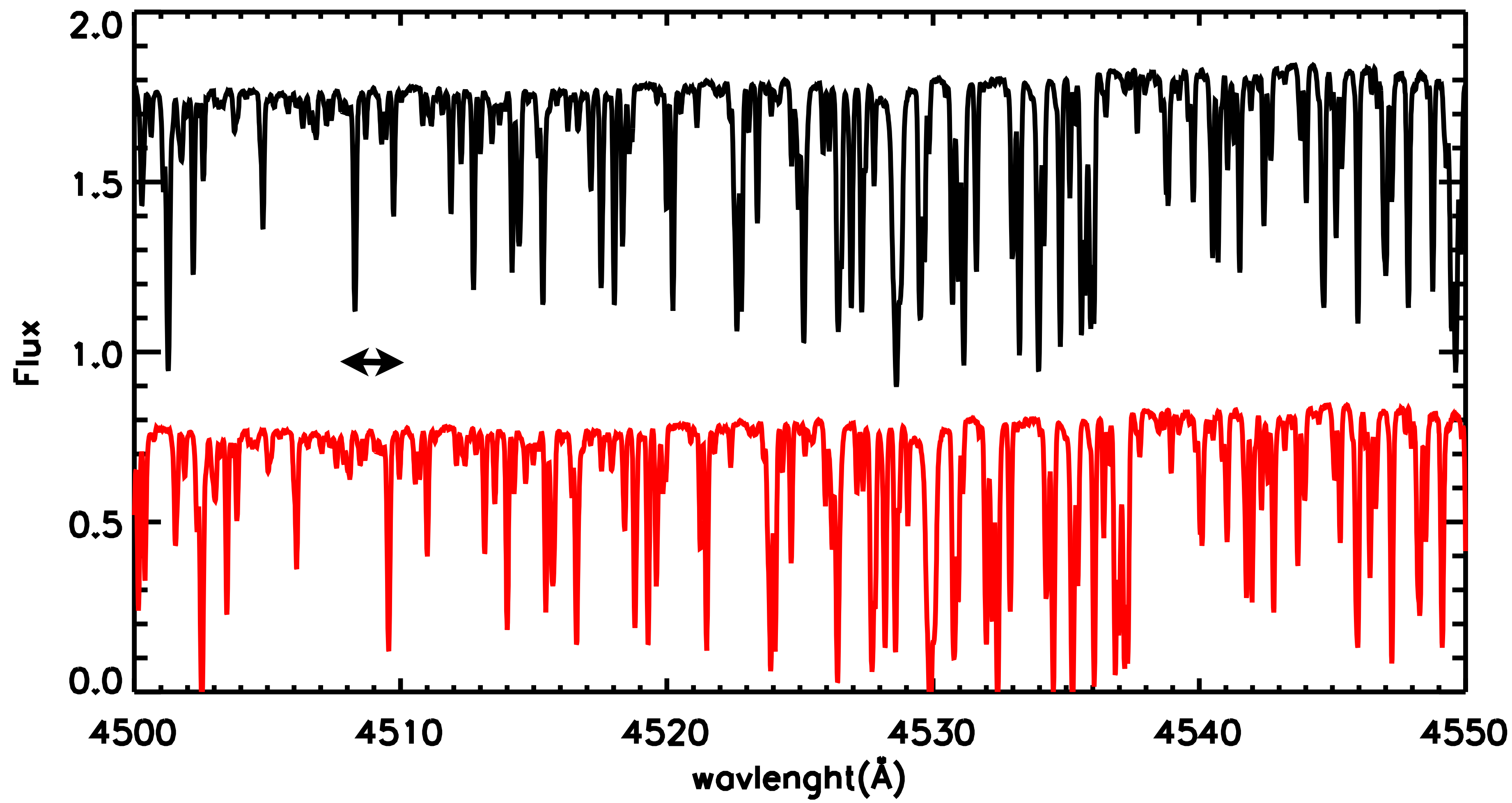
- The goals are to measure relative RV (not absolute) to high precision and have repeatability, night-to-night, for many years.

- The instrument must be ultra-stable or the calibration near-perfect and repeatable (preferably both).

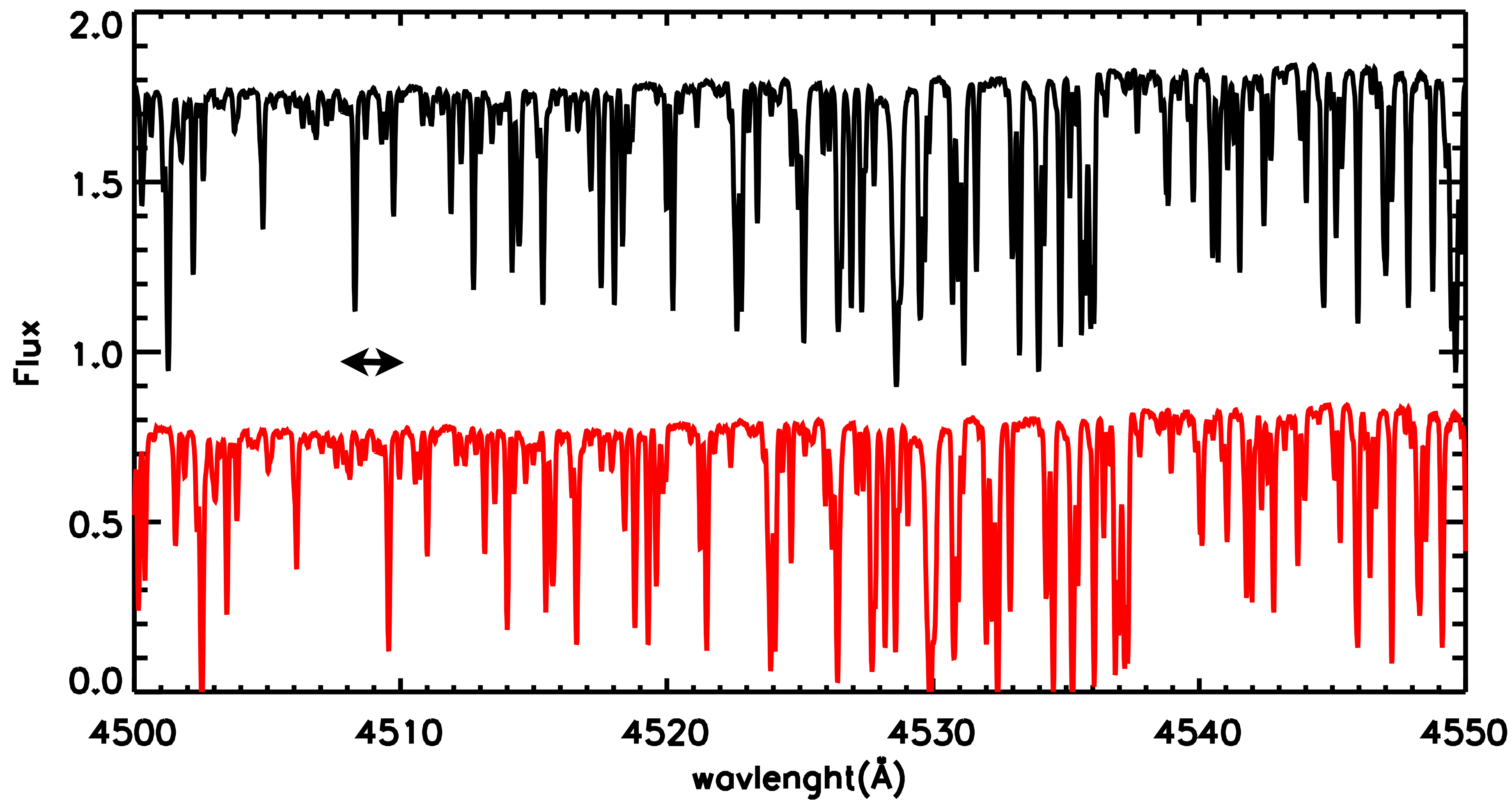
What are the things you might control?

- changes in spectrograph
- variation in instrument illumination

What is the source of this wavelength shift (~ 1.3 angst.)?

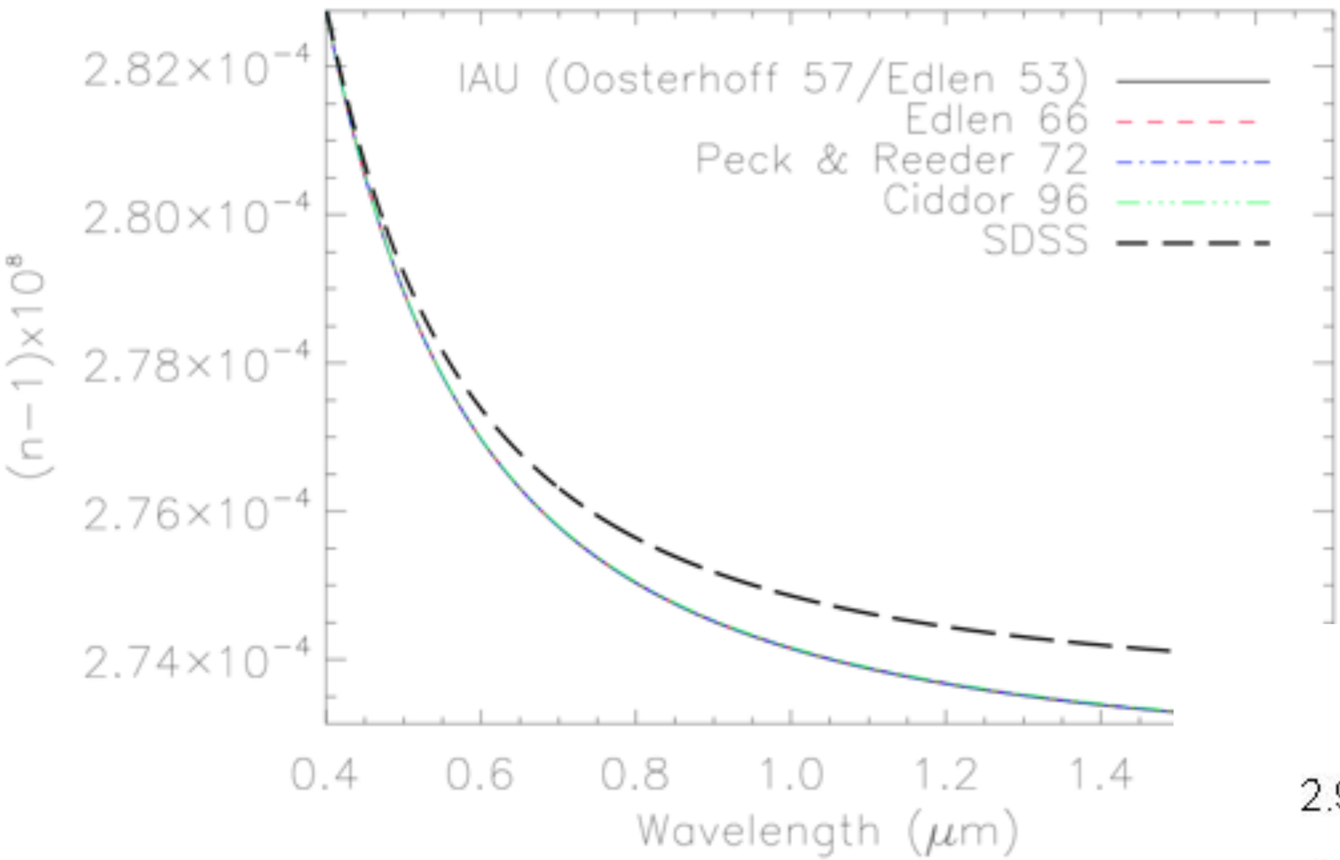


What is the source of this wavelength shift (~ 1.3 angst.)?

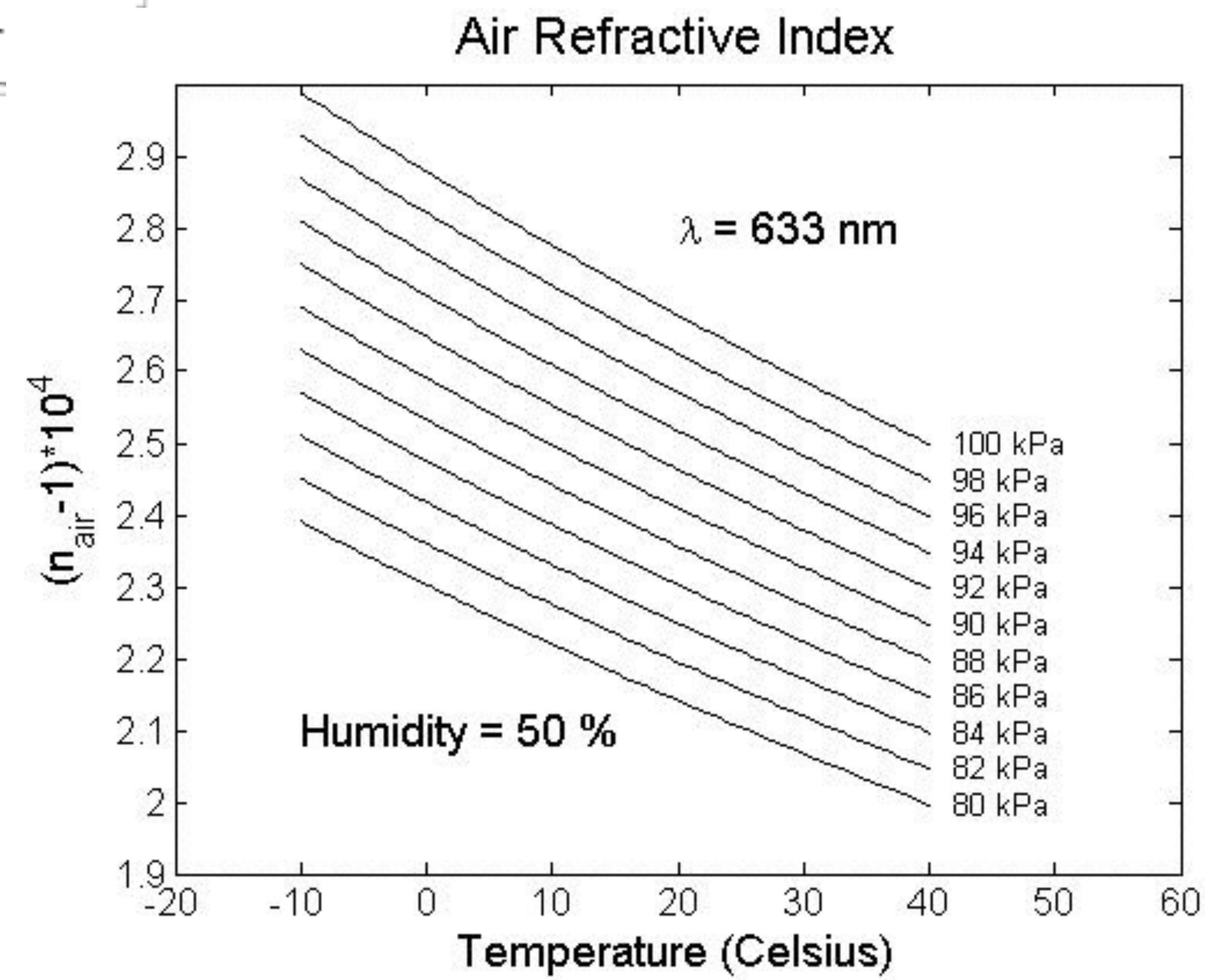


$$\frac{\lambda_0 - \lambda}{\lambda} = n - 1 = a + \frac{b1}{c1 - 1/\lambda_0^2} + \frac{b2}{c2 - 1/\lambda_0^2}$$

n(air) ~ 1.0003



Wavelength shifts are around 200 to 300 m/s / K change in temperature (at fixed P) and ~ 100 m/s / mbar change in pressure (at fixed T).

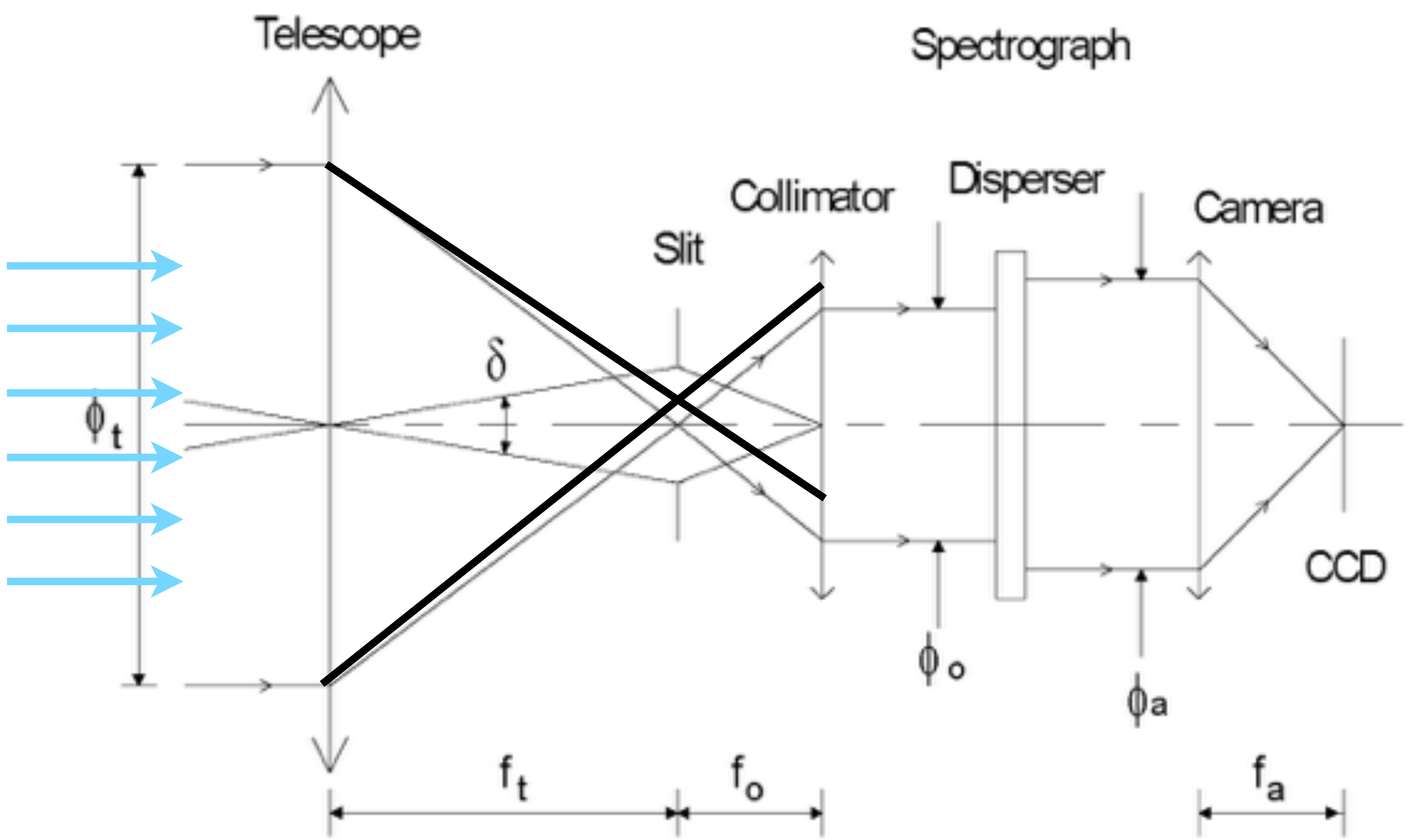


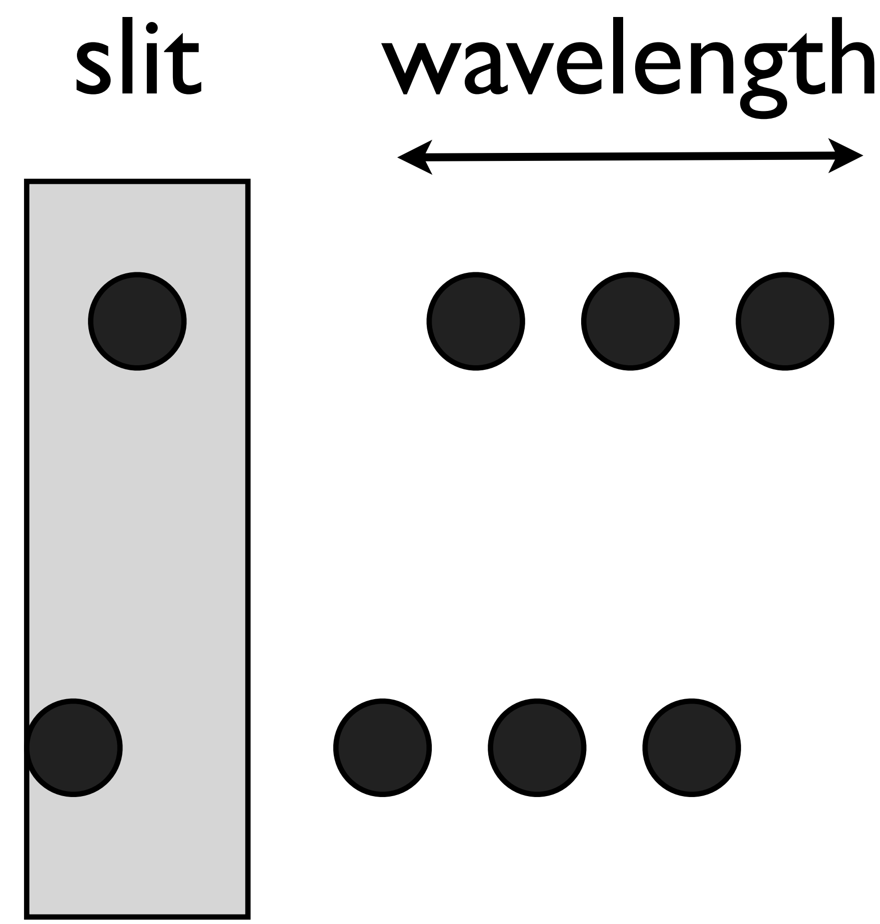
need climate-controlled vacuum chamber

Thermal stability

- A 1-meter optical bench made of aluminum expands or contracts by ~ 20 microns for every 1 K change in temperature.
- 20 microns is about the size of one CCD pixel.
- Such shifts are comparable to, or larger than, the RV change one wants to measure.

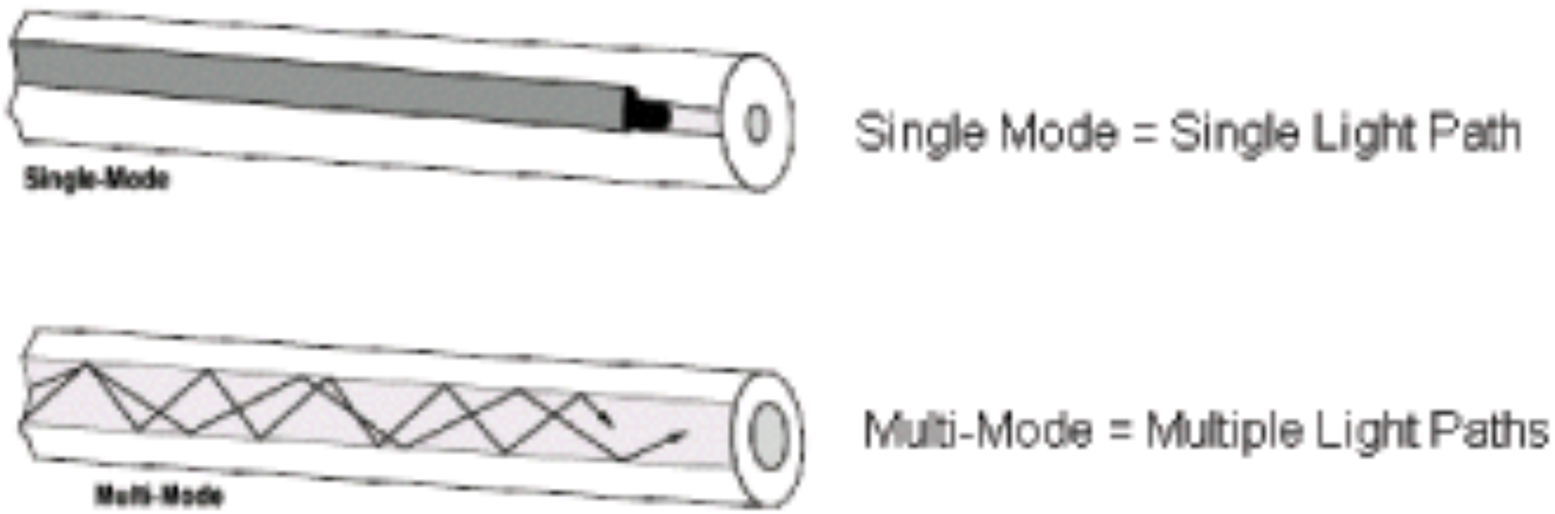
Image Stability





- A changes in the illumination across the entrance of the spectrograph can produce wavelength shifts exceeding 100 m/s. Telescope guiding never good enough.

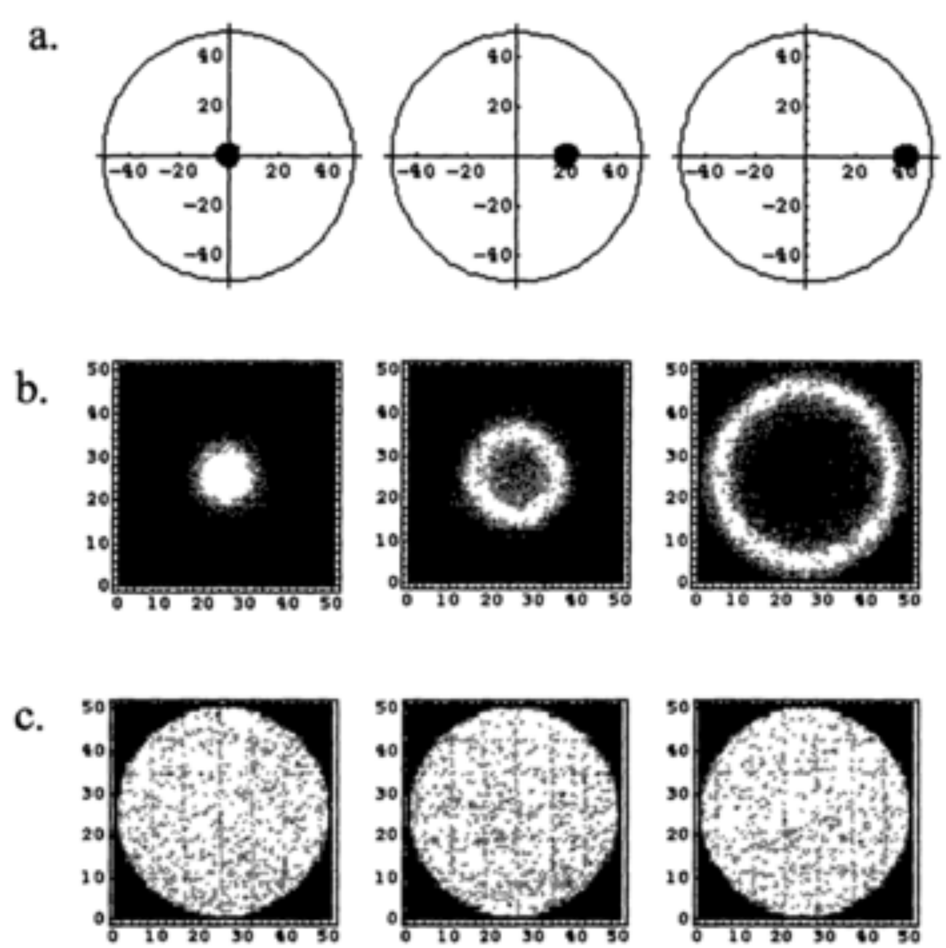
Fiber-scramblers:



< 10 um
(perfect scramble)

50 -- 500 um
(good scramble)

"perfect"
multi-mode fiber
(1 is good, 2 is better,
aka double-scrambler)



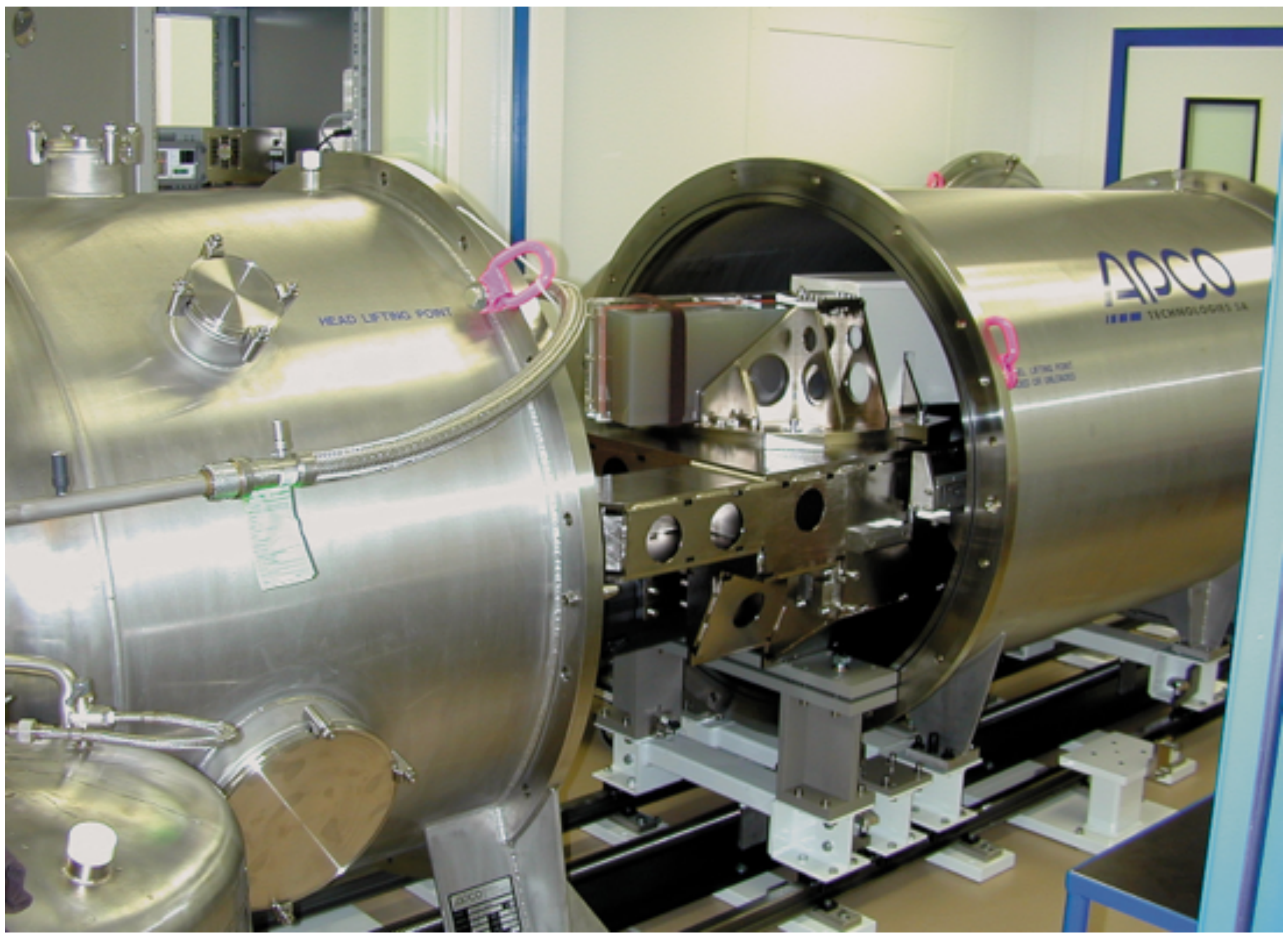
input

output
(intermediate)

output (to
spectrograph)

(Queloz 1999)

Very stabilized instruments:
HARPS @ 3.6m at ESO/Chile



Spectrograph on a rigid bench, which is housed in a vacuum tank.

Very stabilized instruments: HARPS @ 3.6m at ESO/Chile

Spectrograph on a rigid bench, which is housed in a vacuum tank.

The tank itself is housed in a climate controlled room that is never opened.

- Pressure controlled to 10^{-3} mbar
- Optical bench controlled to 1 mK

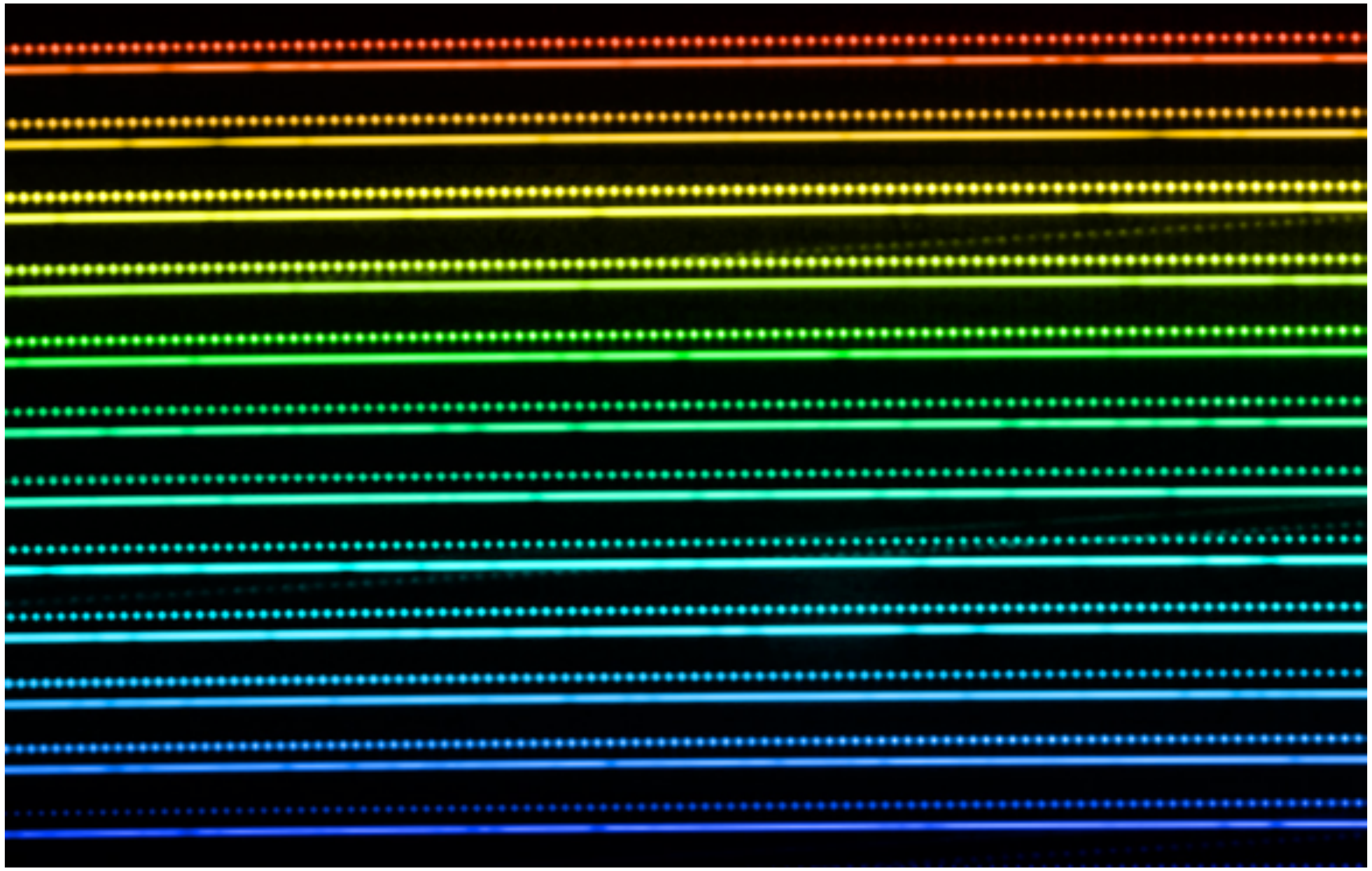
Very stabilized instruments: HARPS @ 3.6m at ESO/Chile

Spectrograph on a rigid bench, which is housed in a vacuum tank.

The tank itself is housed in a climate controlled room that is never opened.

Light is coupled from the telescope with fiber optics that “scramble” the light.

Very stabilized instruments: HARPS @ 3.6m at ESO/Chile



Spectrograph on a rigid bench, which is housed in a vacuum tank.

The tank itself is housed in a climate controlled room that is never opened.

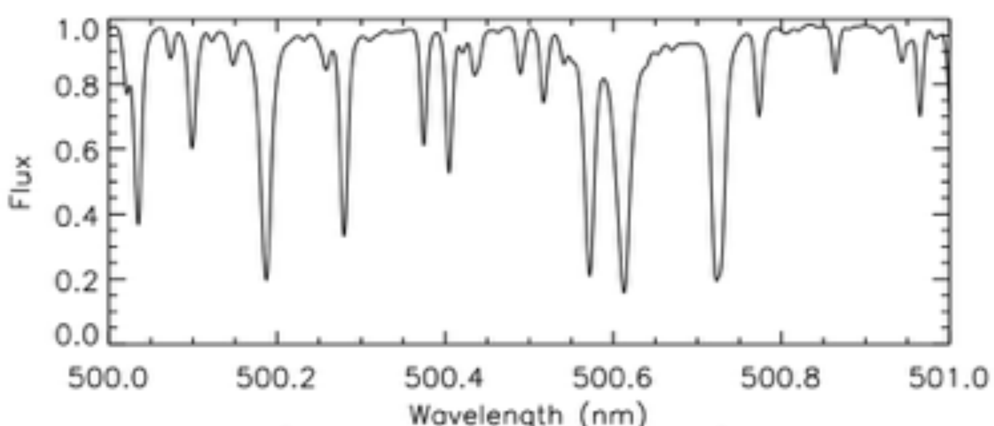
Light is coupled from the telescope with fiber optics that “scramble” the light.

A second fiber feed a simultaneous calibration source.

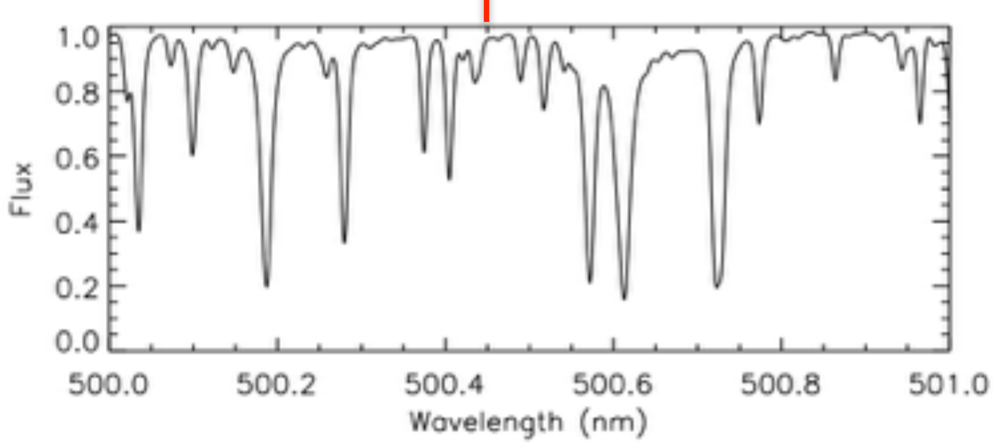
Very stabilized instruments: HARPS @ 3.6m at ESO/Chile

Cross-Correlation Technique

observed spectrum

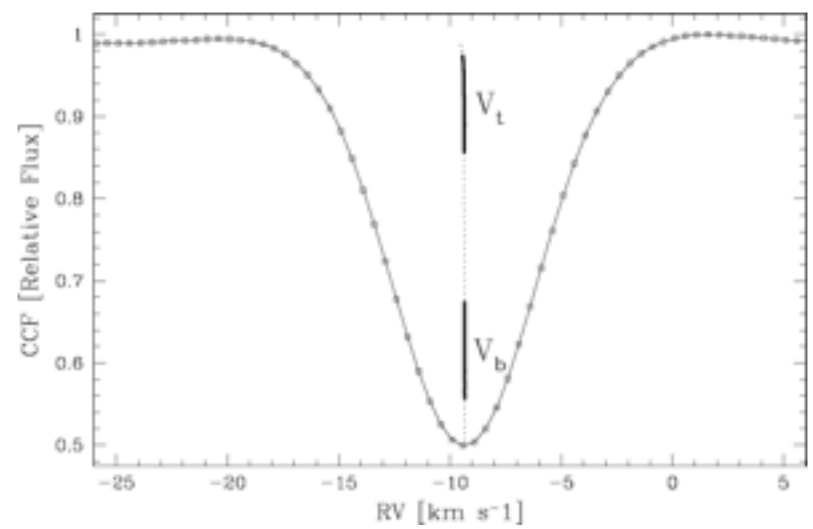


CCF



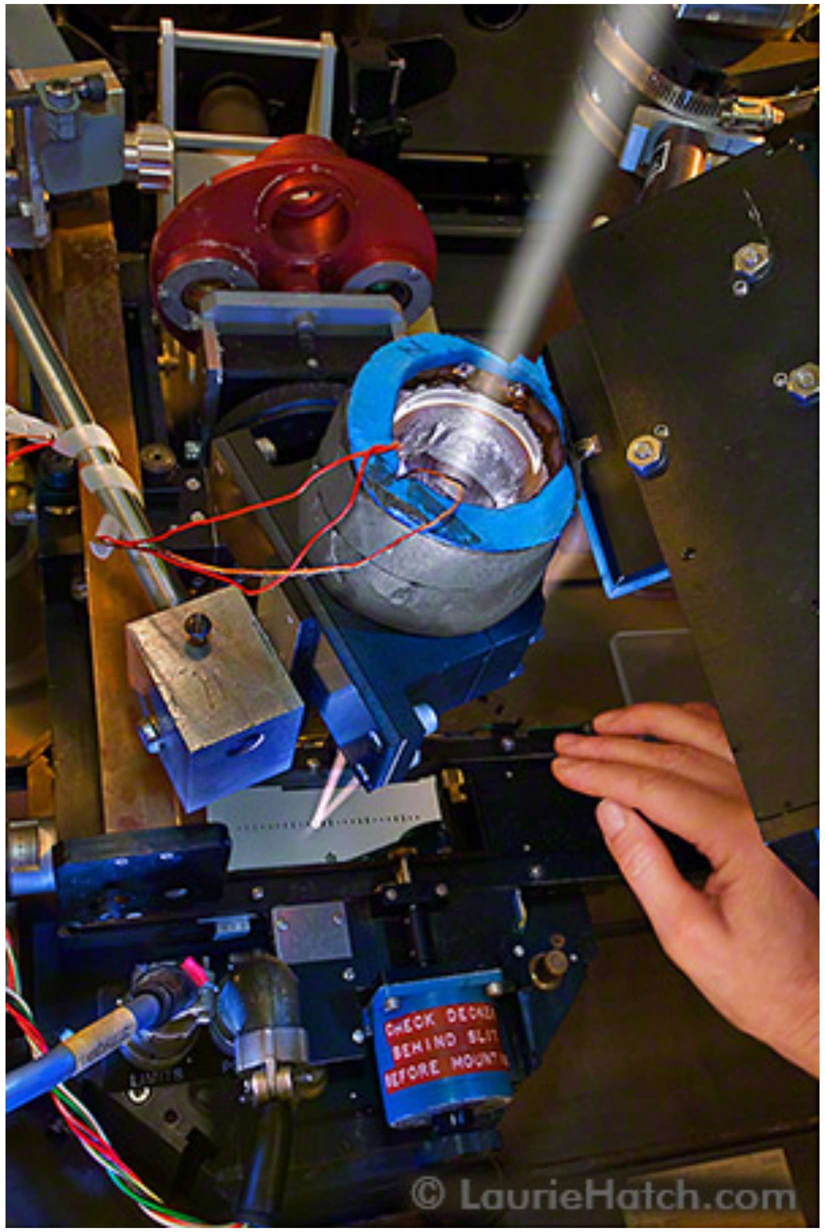
template spectrum

Original reference:
Tonry & Davis 1979, AJ, 84, 10

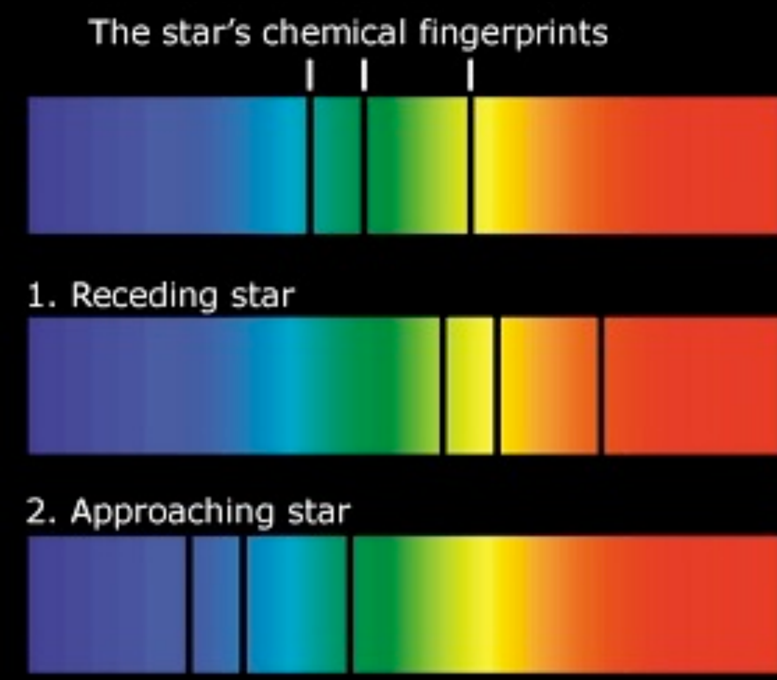


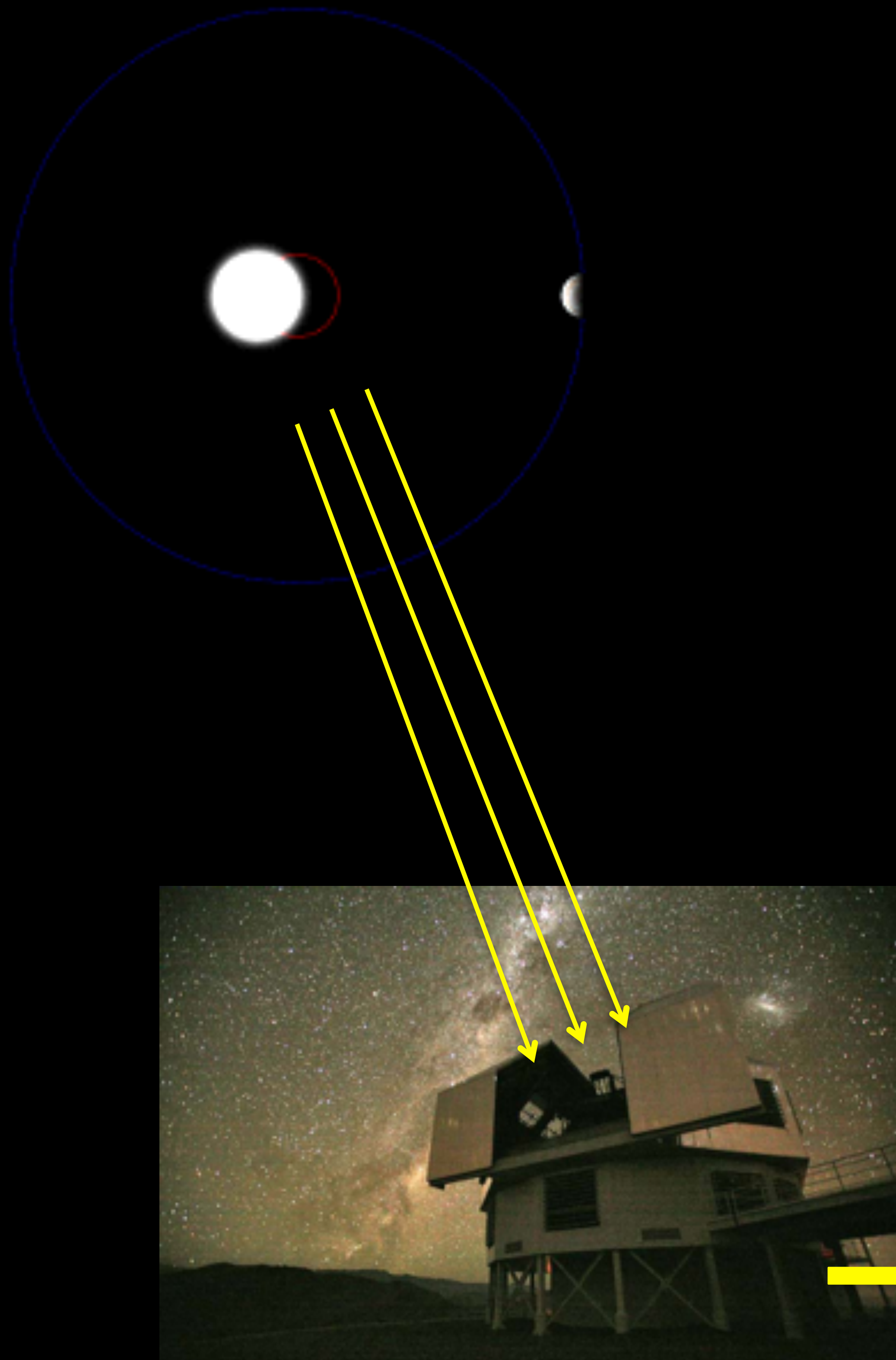
radial velocity

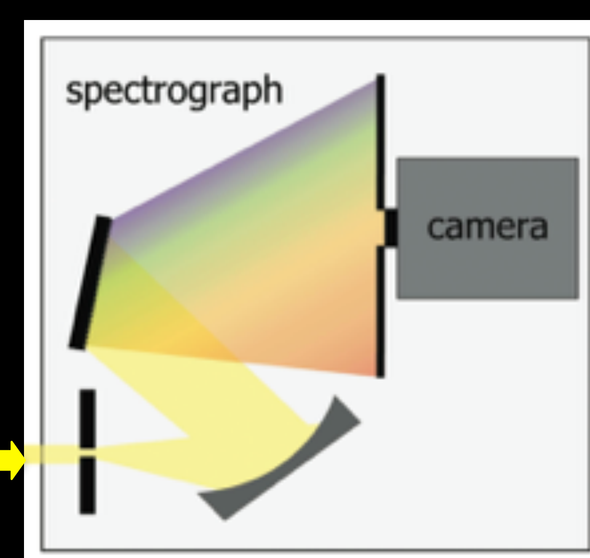
Gas cell technique



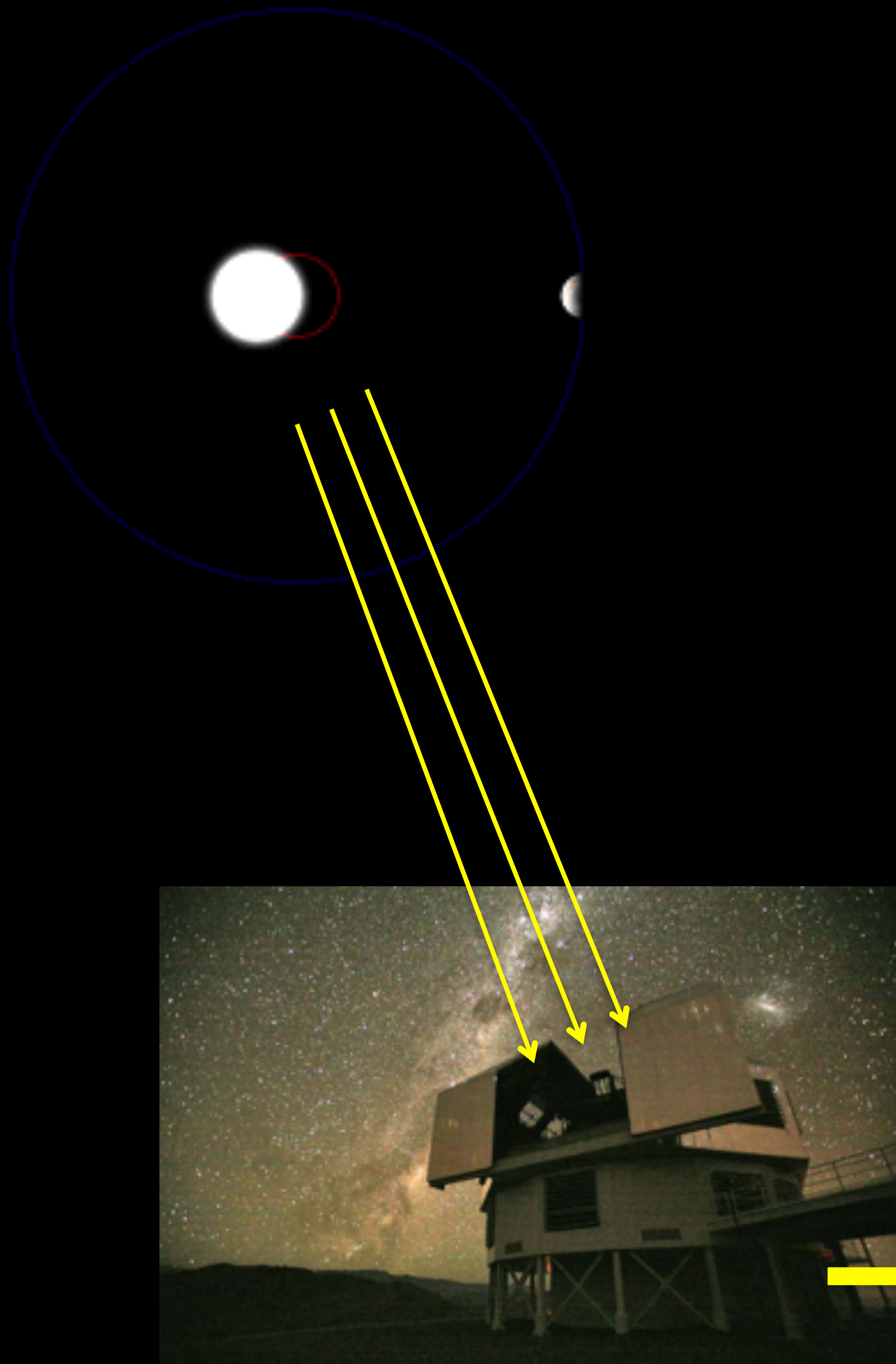
Radial Velocity Technique

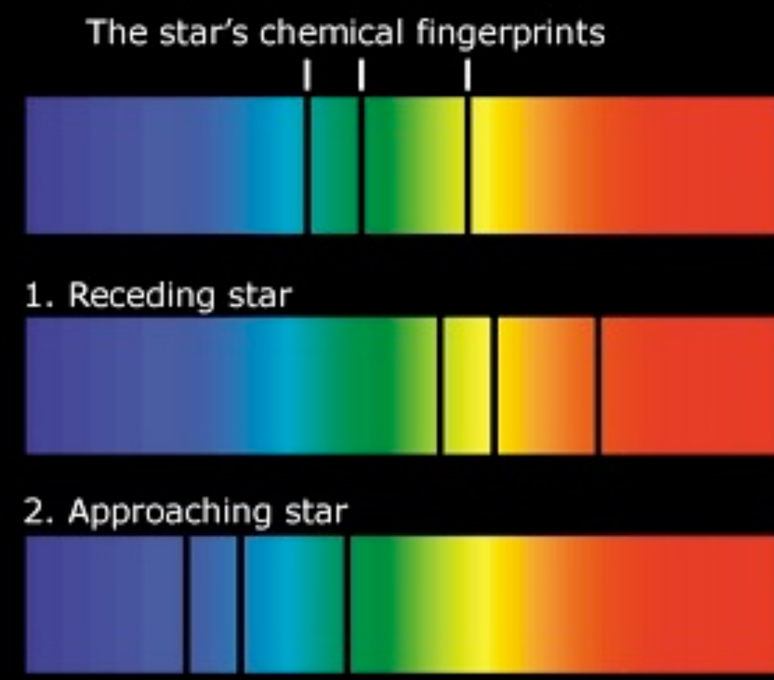


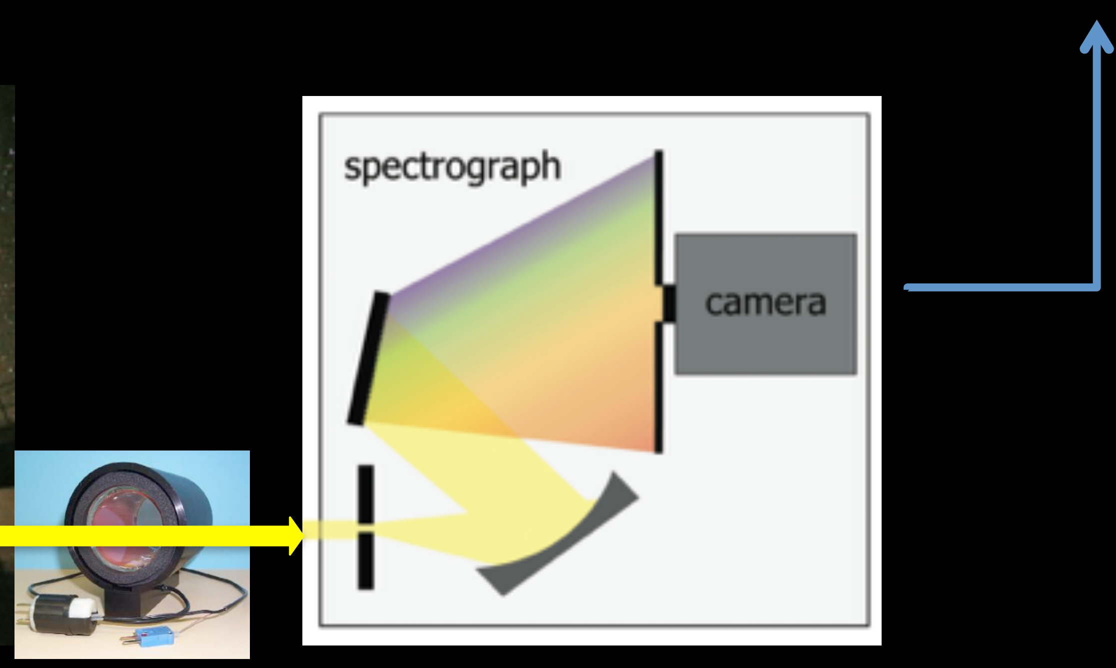




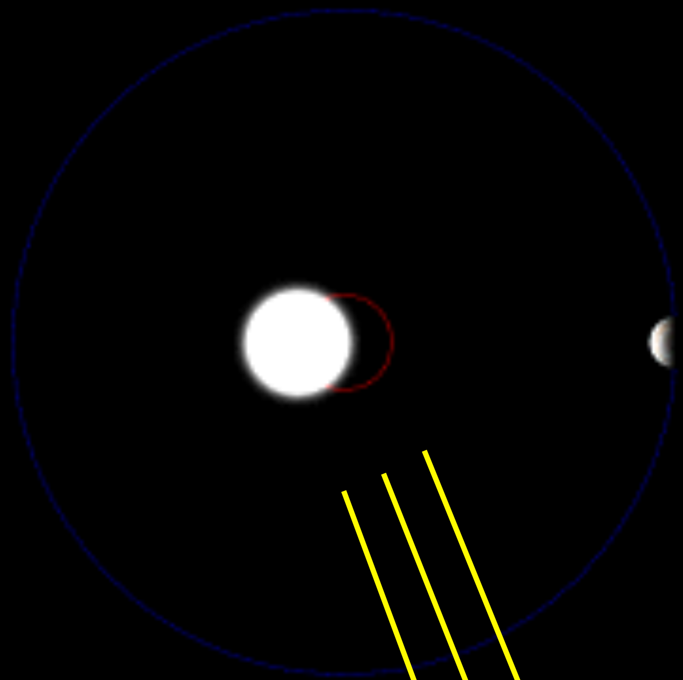
Radial Velocity Technique

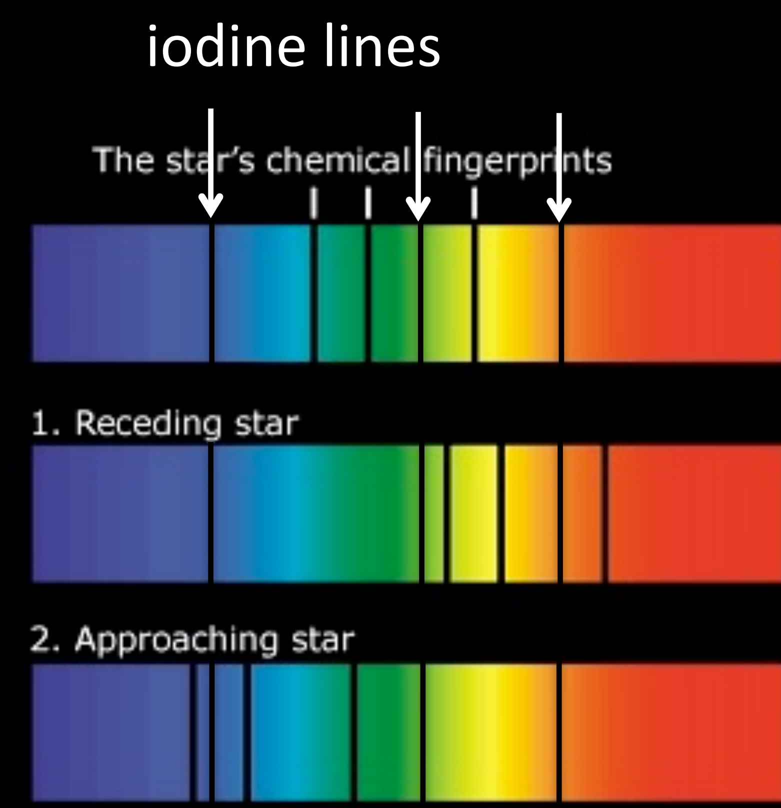


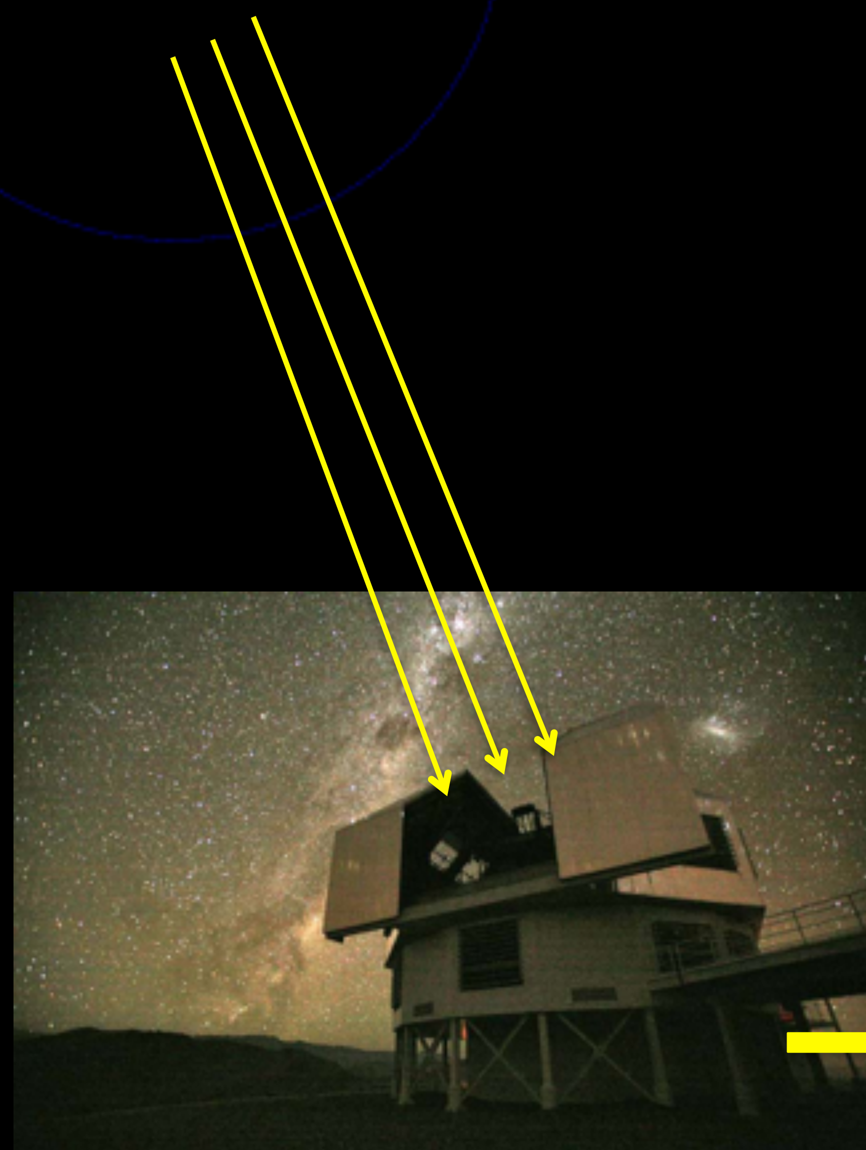


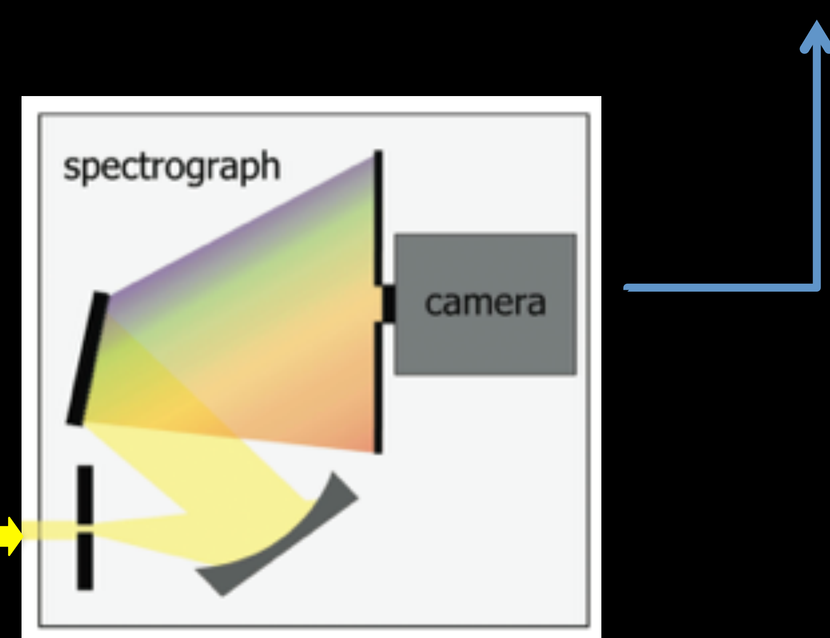


Radial Velocity Technique





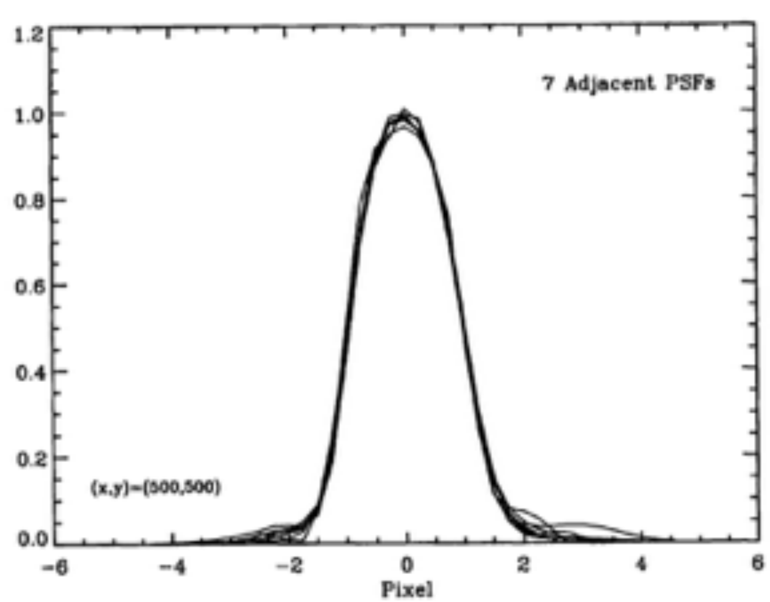


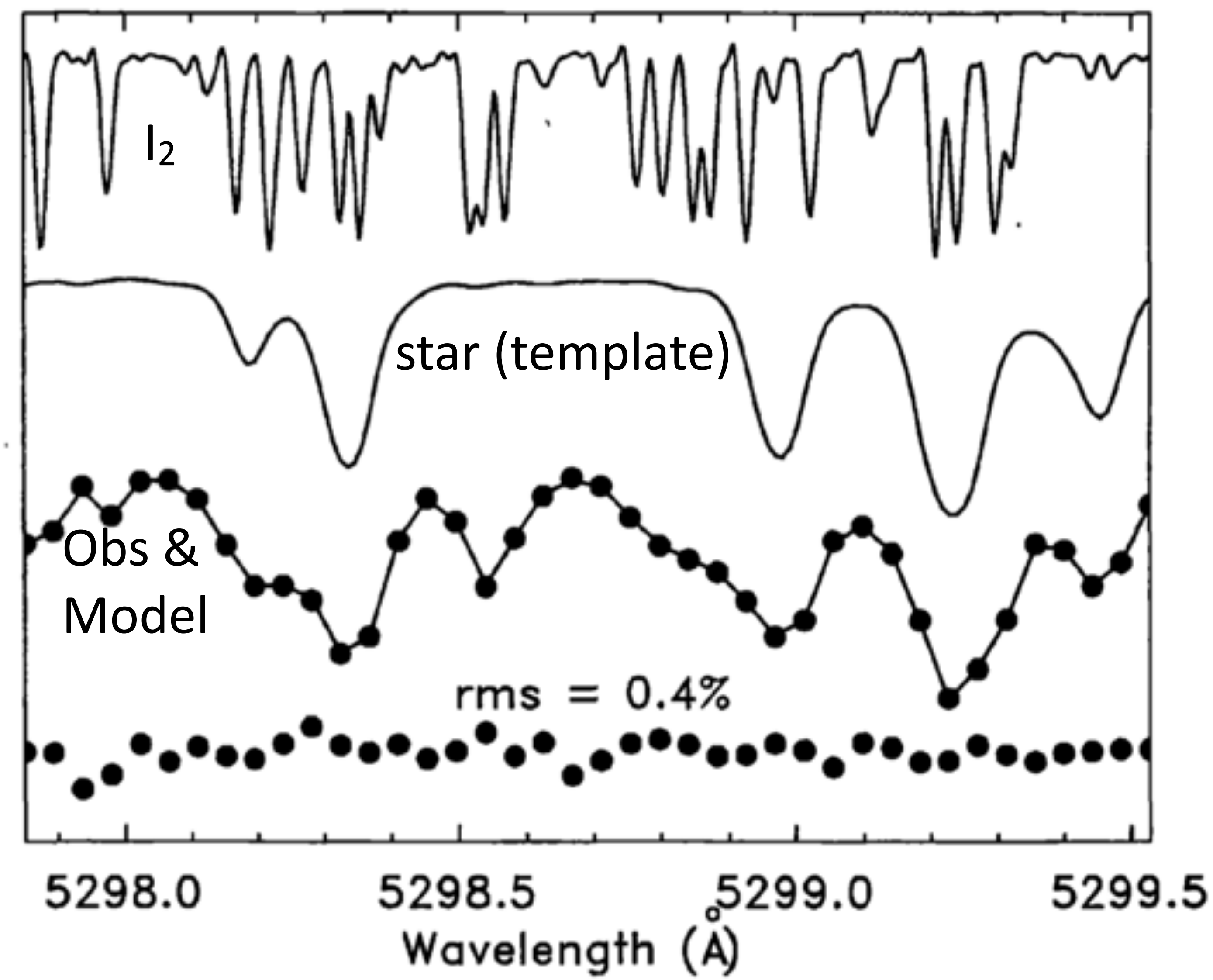


Gas cell technique:
It is a little more complicated
(involves forward modeling of spectrum)

$$I_{obs}(\lambda) = k[T_{I2}(\lambda) I_s(\lambda + \Delta\lambda)] * \mathrm{PSF},$$

instrumental profile





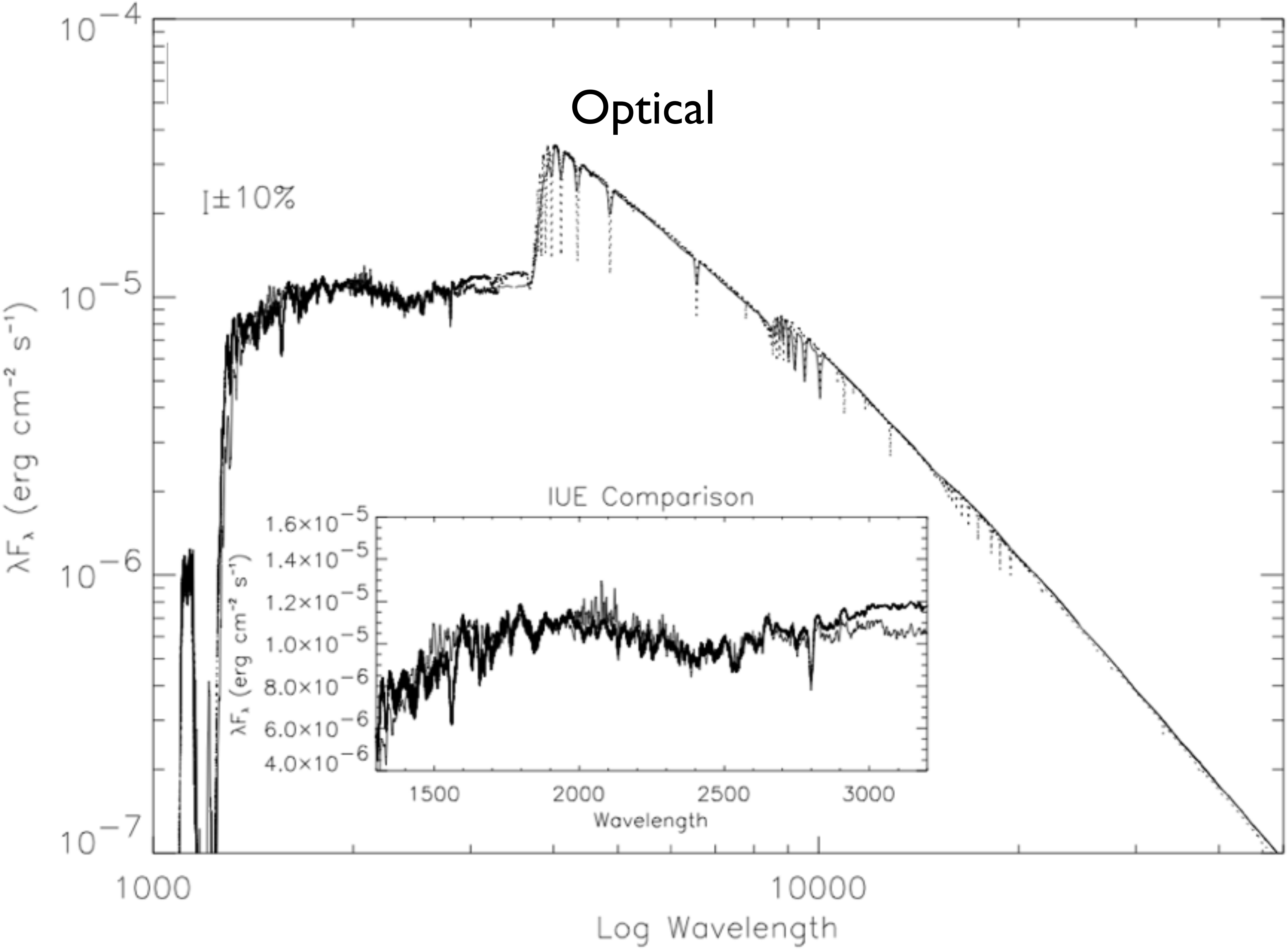
Butler et al. 1996, PASP, 108, 500

Calibrated instruments:
HIRES@ 10m Keck telescope in Hawaii

Spectrograph not particularly stabilized.

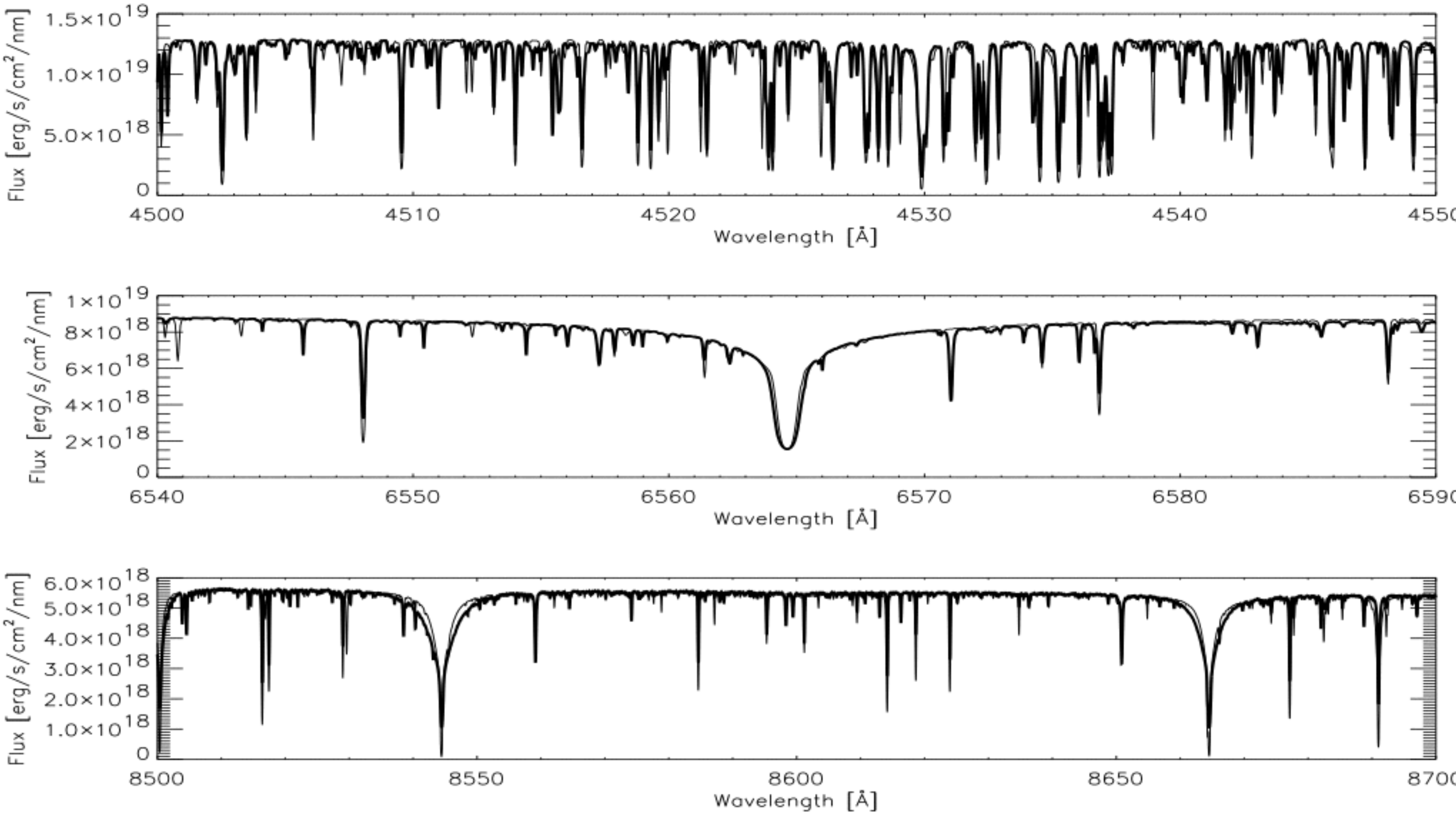
Which stars to observe?

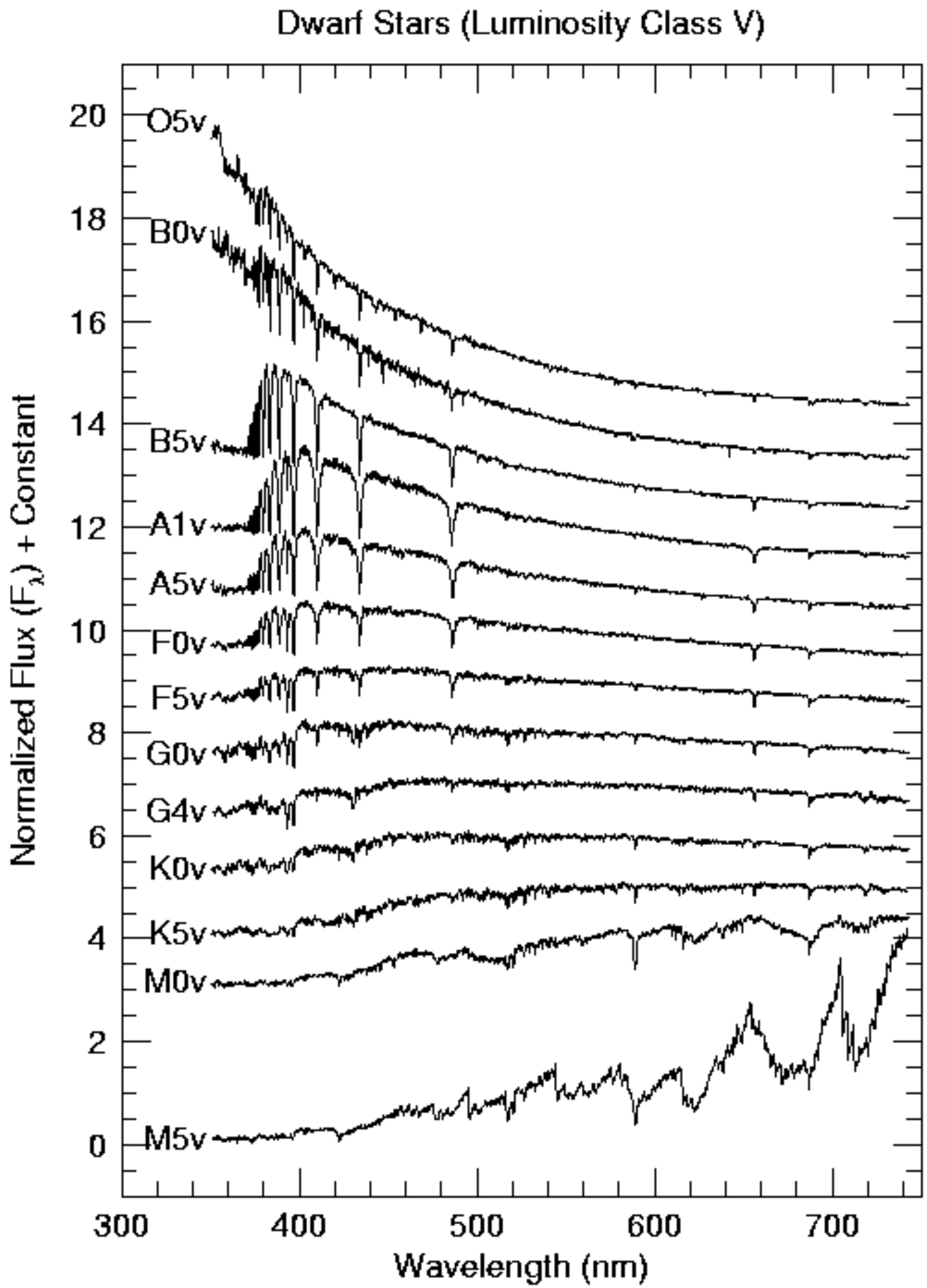
Vega

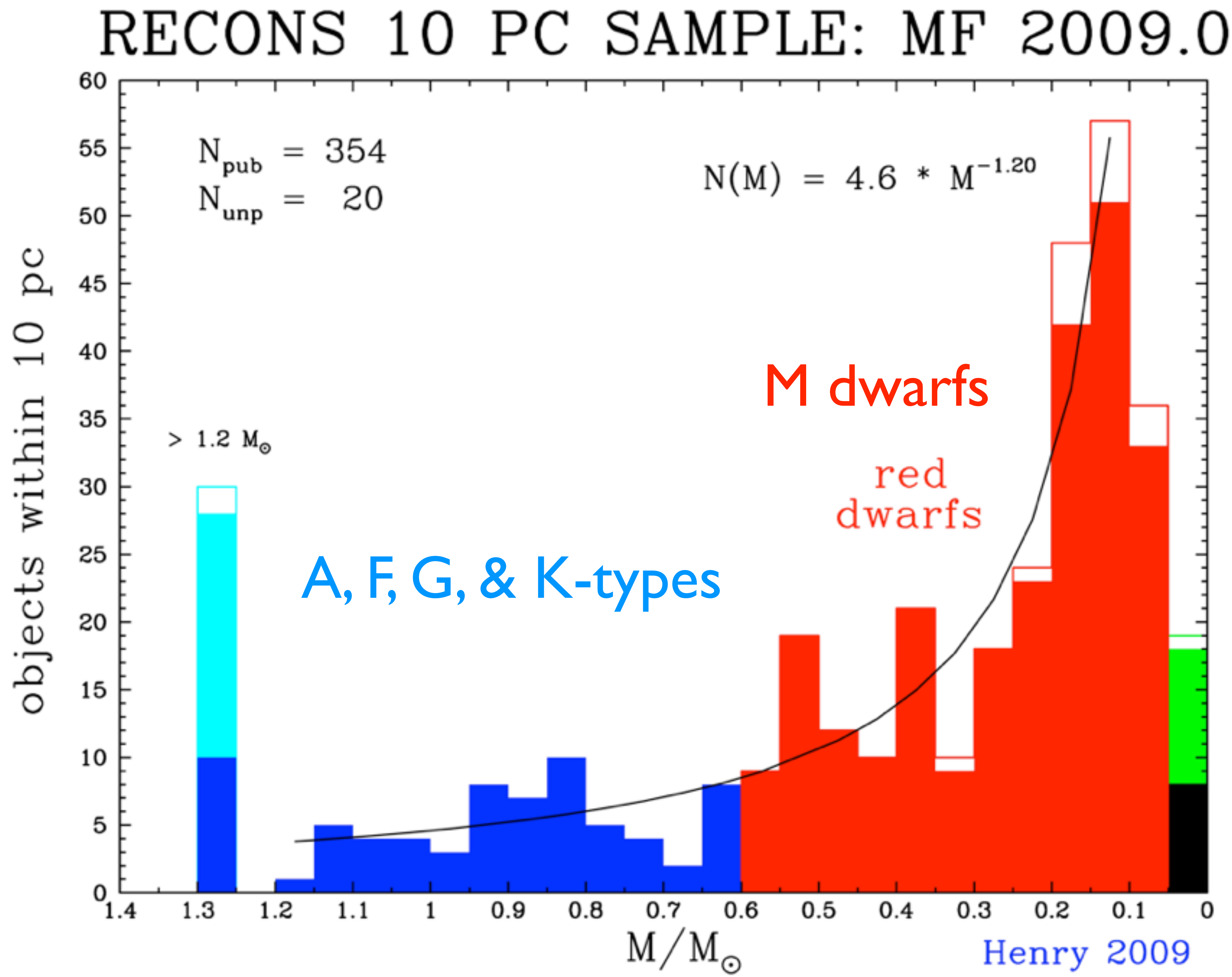


Optical
I±10%
IUE Comparison
λFλ (erg cm⁻² s⁻¹)
Log Wavelength
Wavelength
1000
10000
1500
2000
2500
3000

Solar-type (The Sun) stars:

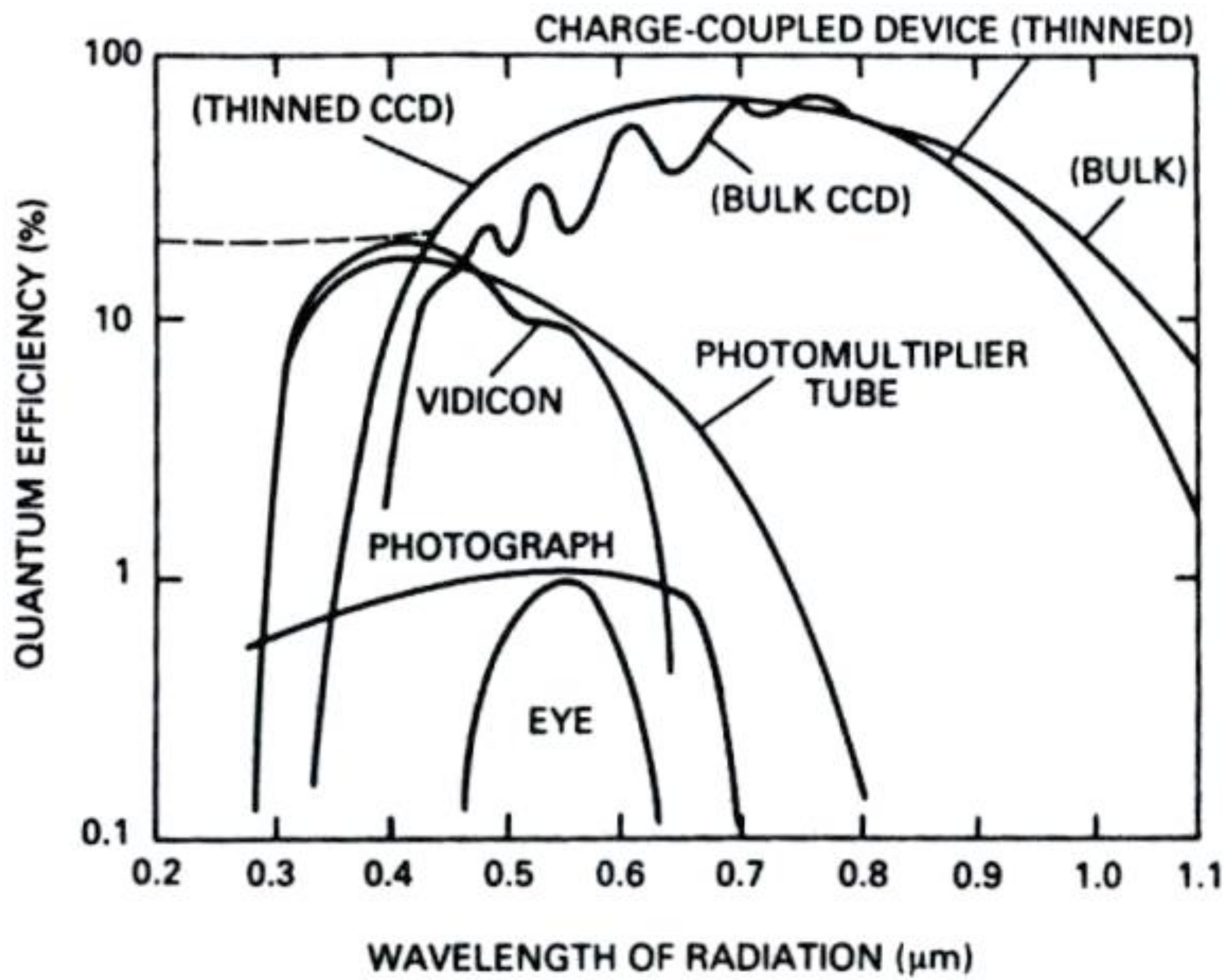



Dwarf Stars (Luminosity Class V)
O5v
B0v
B5v
A1v
A5v
F0v
F5v
G0v
G4v
K0v
K5v
M0v
M5v
Normalized Flux (F_λ) + Constant
Wavelength (nm)
0
2
4
6
8
10
12
14
16
18
20
300
400
500
600
700

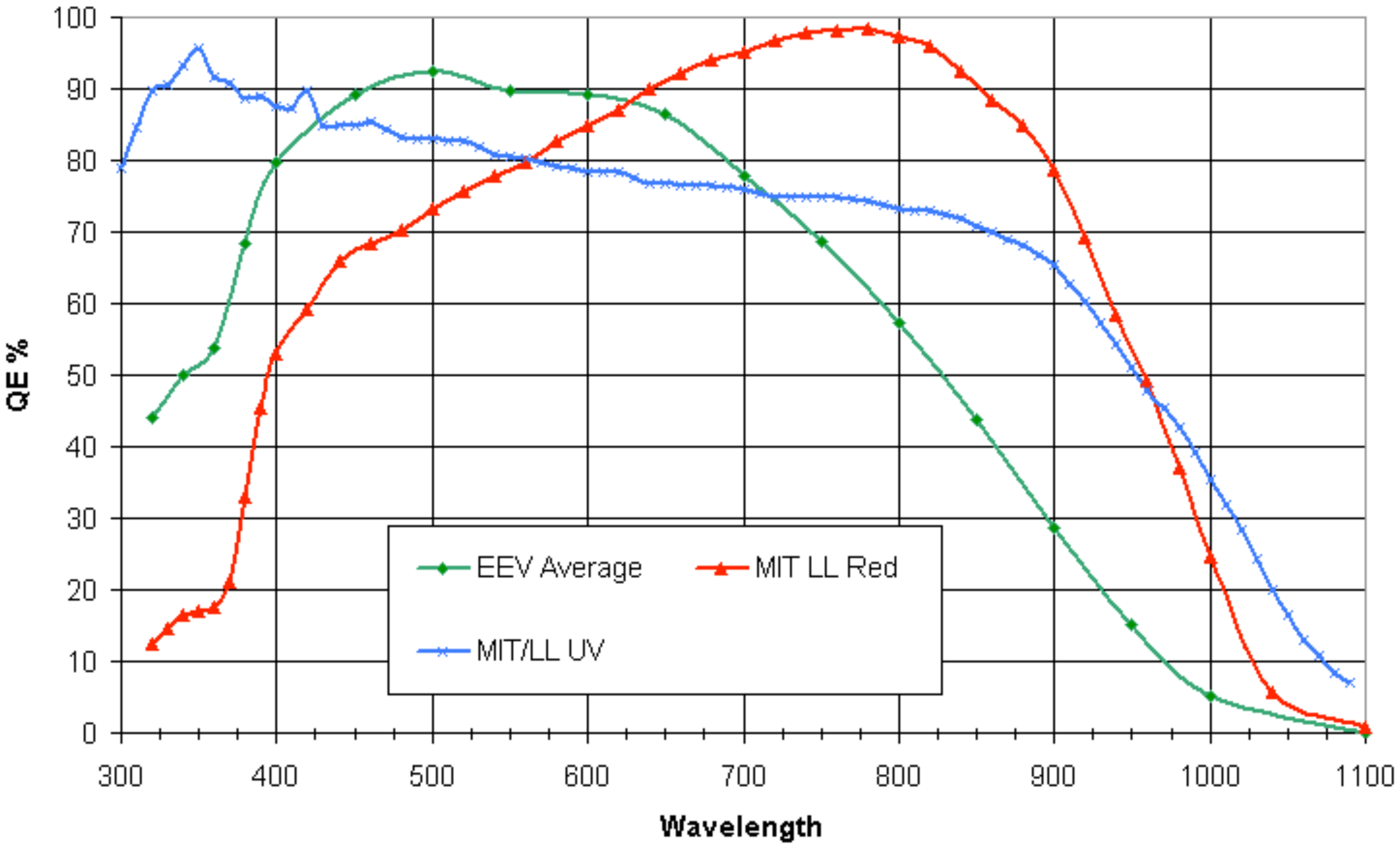


RECONS 10 PC SAMPLE: MF 2009.0
N_{pub} = 354
N_{unp} = 20
$N(M) = 4.6 * M^{-1.20}$
> 1.2 $M_\odot$
M dwarfs
red dwarfs
A, F, G, & K-types
objects within 10 pc
$M/M_\odot$
Henry 2009

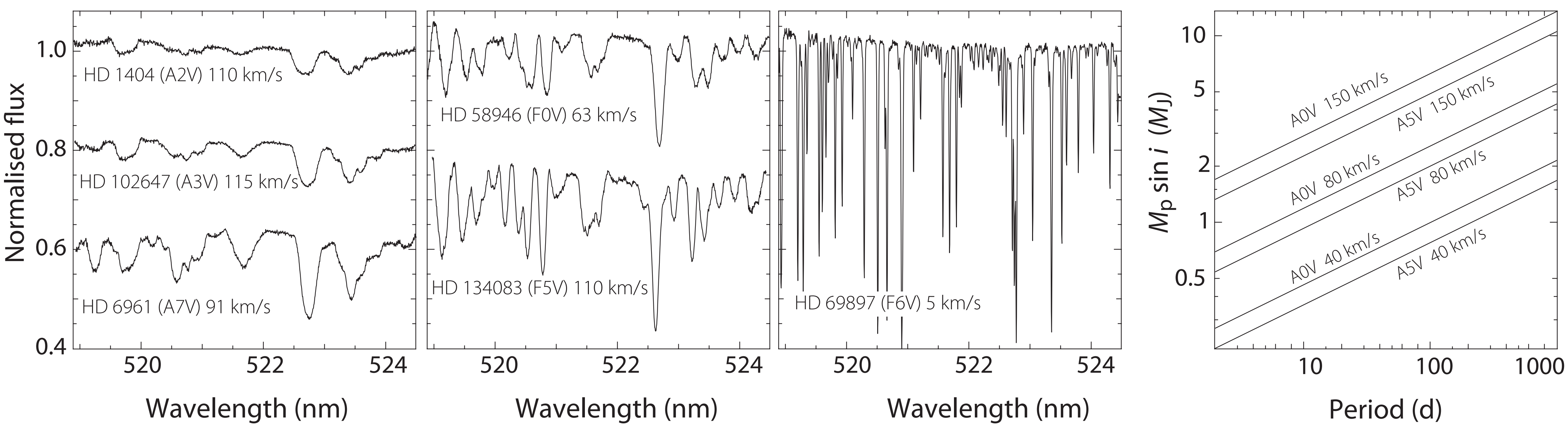
quantum-efficiency



quantum-efficiency

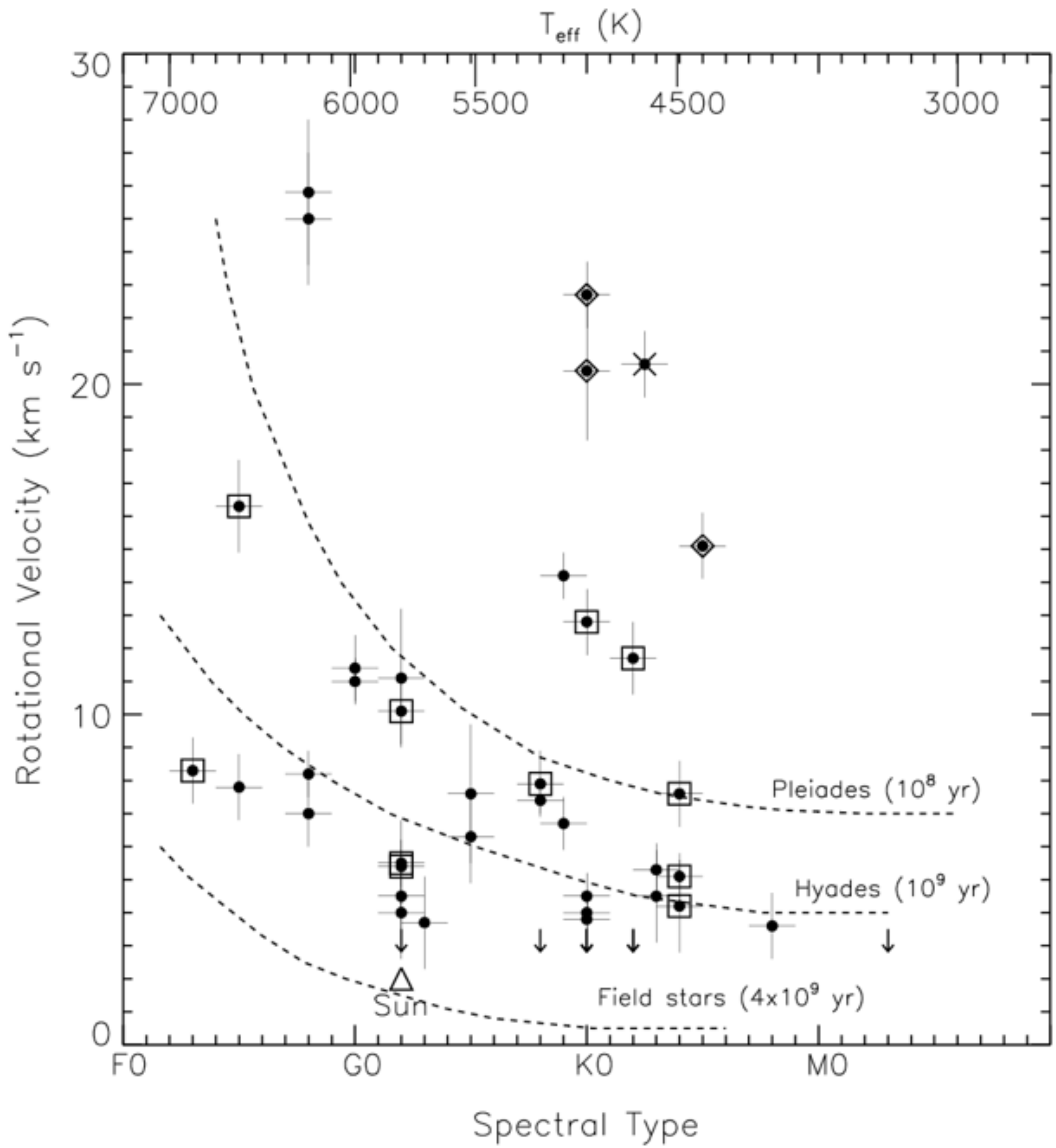


Rotational Broadening:



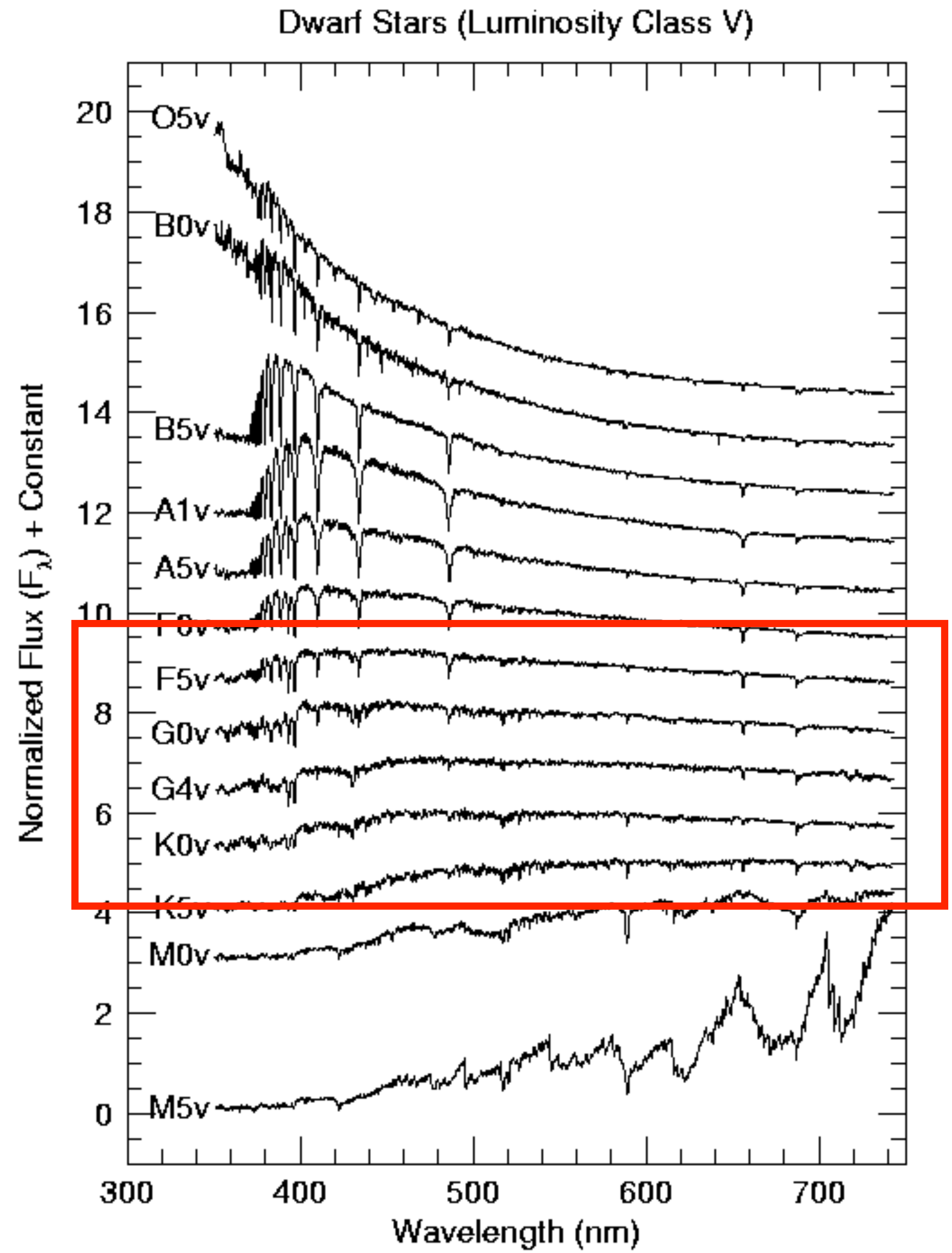
- young stars have larger rotational velocities, and are more active.

Rotational Broadening (vs SpT):

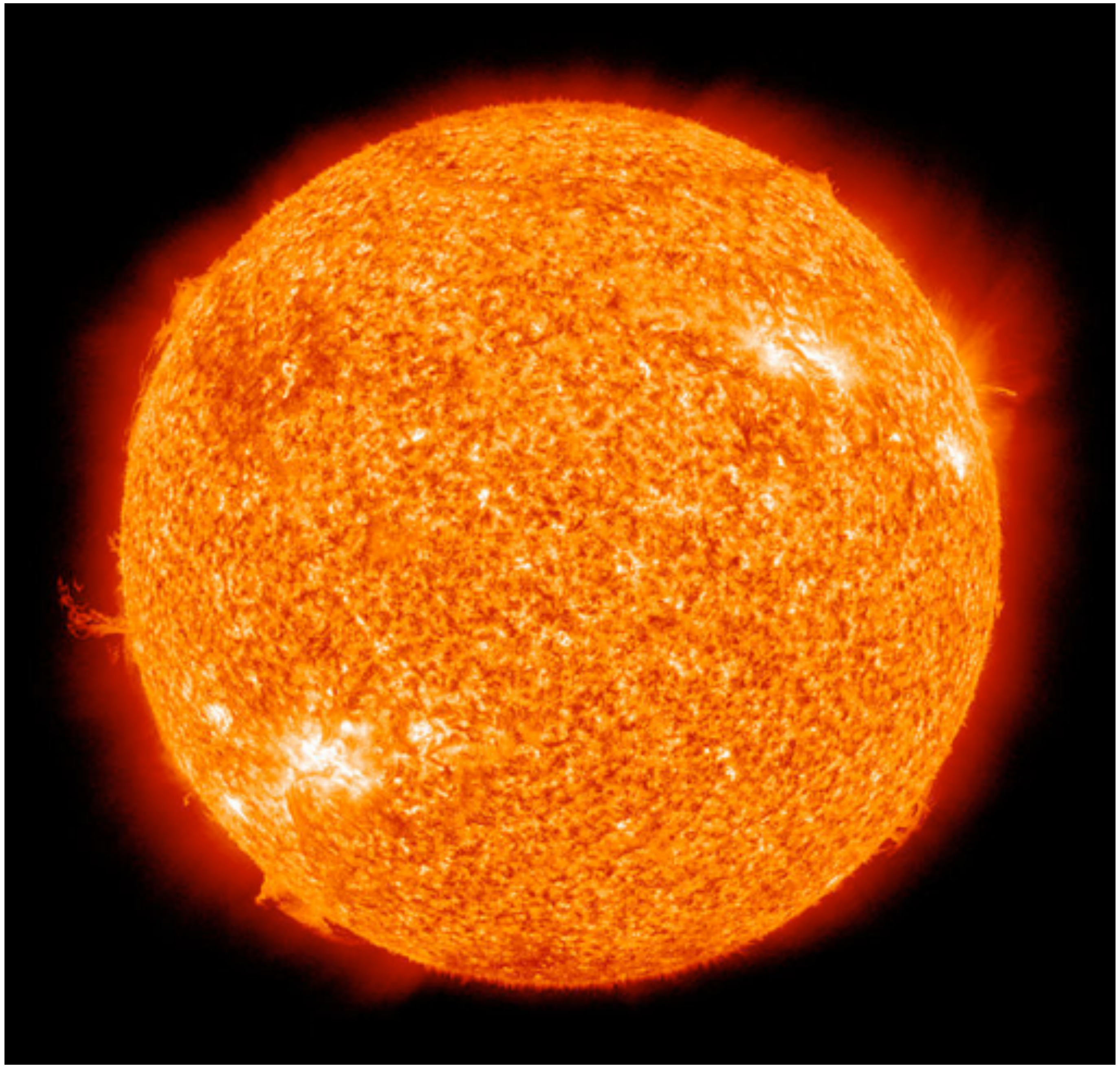


Affer et al. (2008)

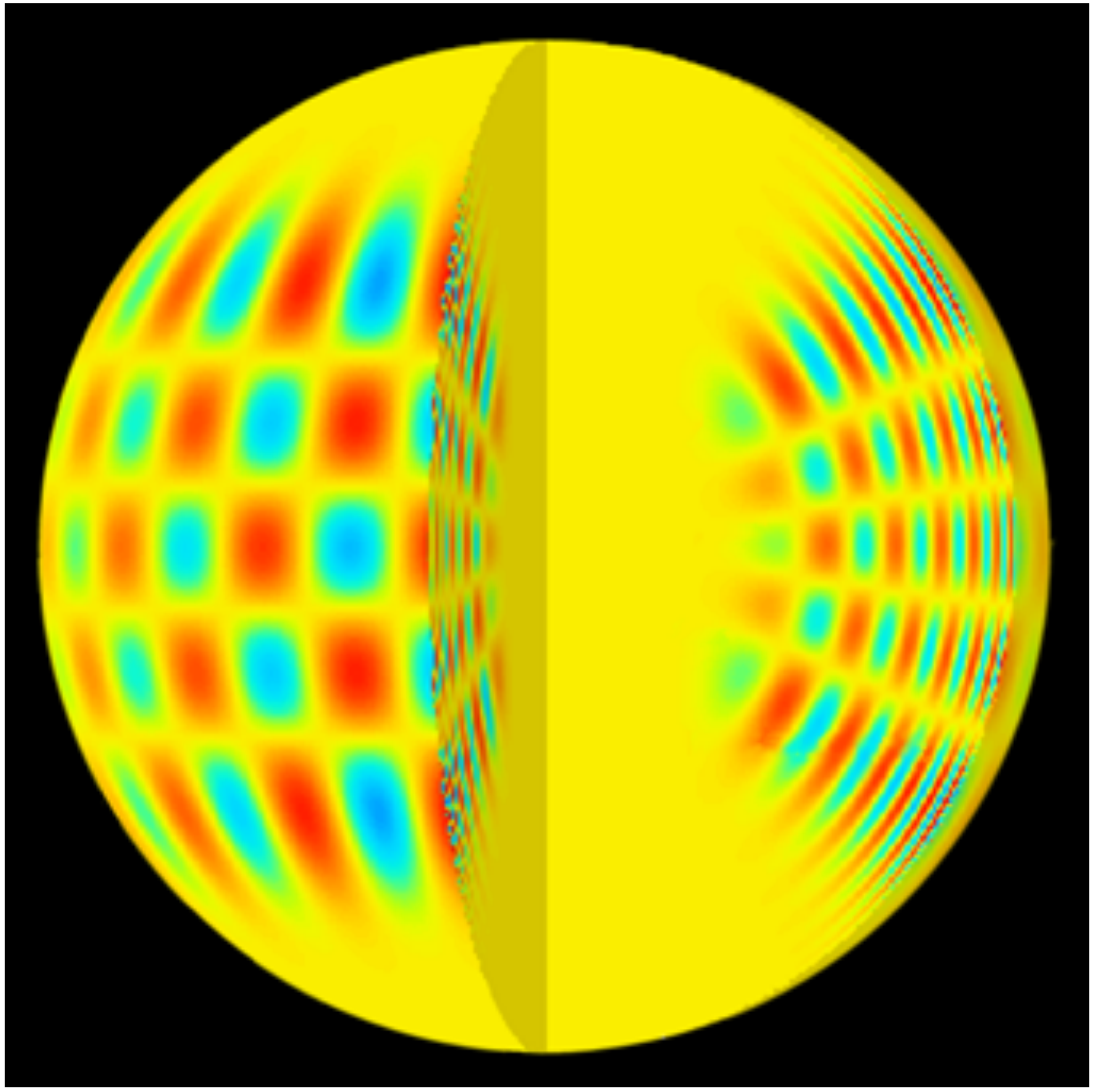
- spectral “richness”
- near peak of QE
- low V*sin*(i)
- metal-rich
- old FGK-type = sweet-spot



Problem: areas on the surface with different velocities and brightnesses + changes with time



P-modes
(acoustic pressure waves)

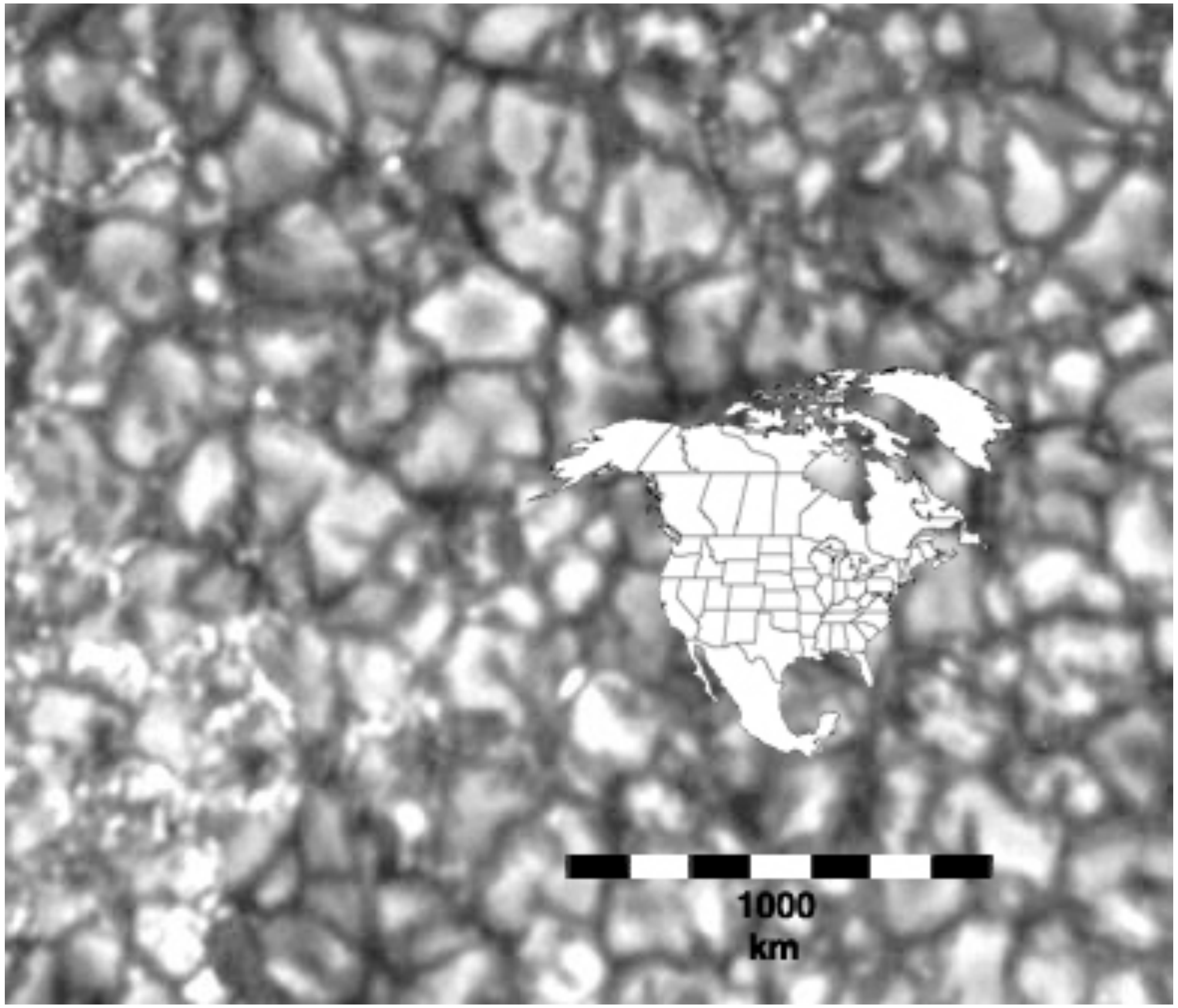


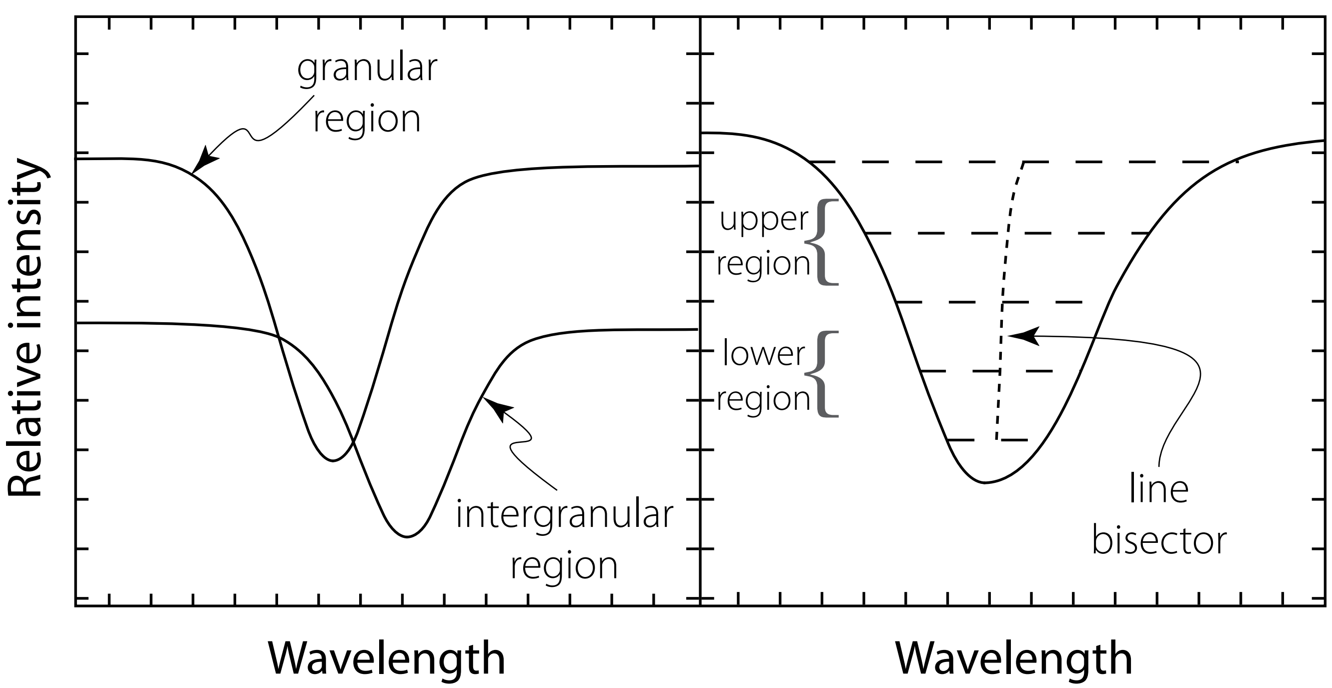
The sun oscillates with a characteristic timescale of five minutes.

The timescale scales as sqrt(density), so lower-mass stars have longer timescales, and higher-mass stars have shorter timescales.

Amplitude is approximately 1 m/s.

Granulation





Convective cells on the surface. The bright center is the upwelling of hot gas, and the dark edges are the downward motion of cooler gas.

Length ~ 1000 km
Timescale ~ 10 minutes
Velocity ~ 1 km/s
Number ~ 10^6

Spots

A few words about Fourier Transforms

Fourier Transform $$f(\sigma) = \int_{-\infty}^{\infty} F(x) e^{2\pi i x \sigma} dx$$

$$e^{i\theta} = cos(\theta) + isin(\theta)$$

Inverse Fourier Transform $$F(x) = \int_{-\infty}^{\infty} f(\sigma) e^{-2\pi i x \sigma} dx$$

$$f(\sigma) = \int_{-\infty}^{\infty} F_{\mathrm{R}}(x) \cos 2\pi x \sigma dx + i \int_{-\infty}^{\infty} F_{\mathrm{I}}(x) \cos 2\pi x \sigma dx$$

$$+ i \int_{-\infty}^{\infty} F_{\mathrm{R}}(x) \sin 2\pi x \sigma dx - \int_{-\infty}^{\infty} F_{\mathrm{I}}(x) \sin 2\pi x \sigma dx$$

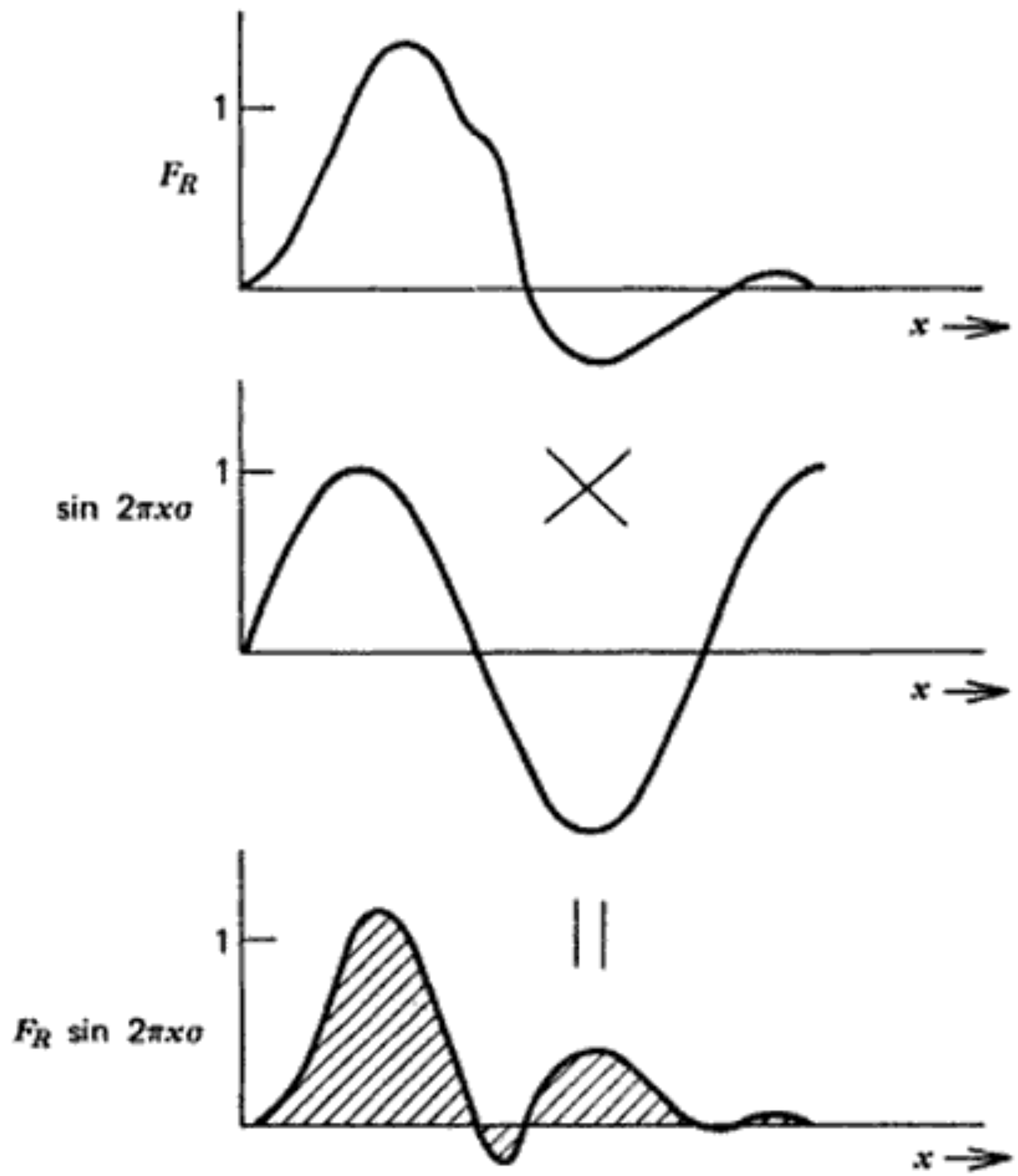
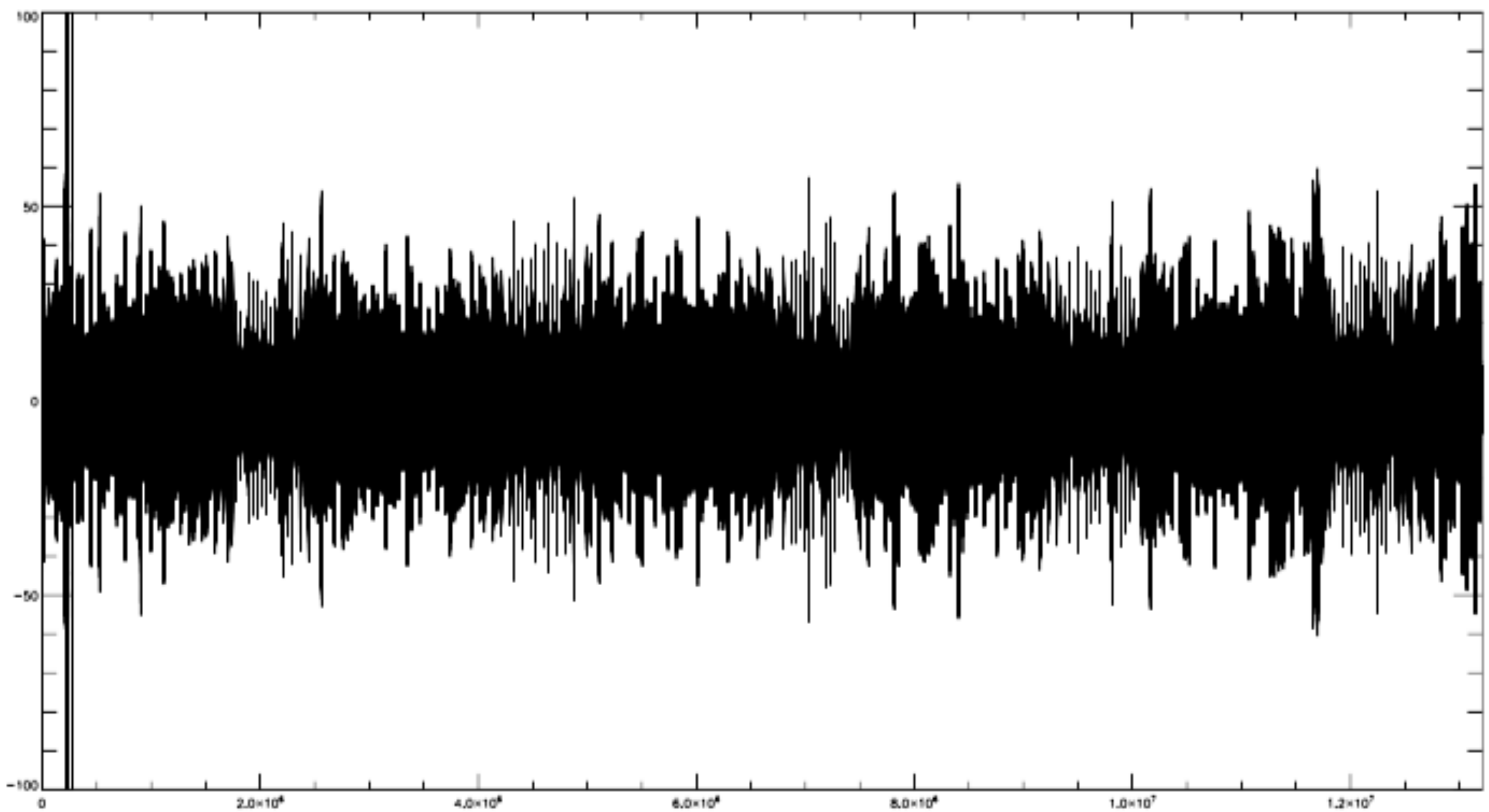


Fig. 2.1. The arbitrary real function $F_R(x)$ is multiplied by $\sin 2\pi x\sigma$ to give the bottom curve. The area is the value of the transform $f(\sigma)$ for one value of σ.

see Gray, 3rd ed. chapter 2

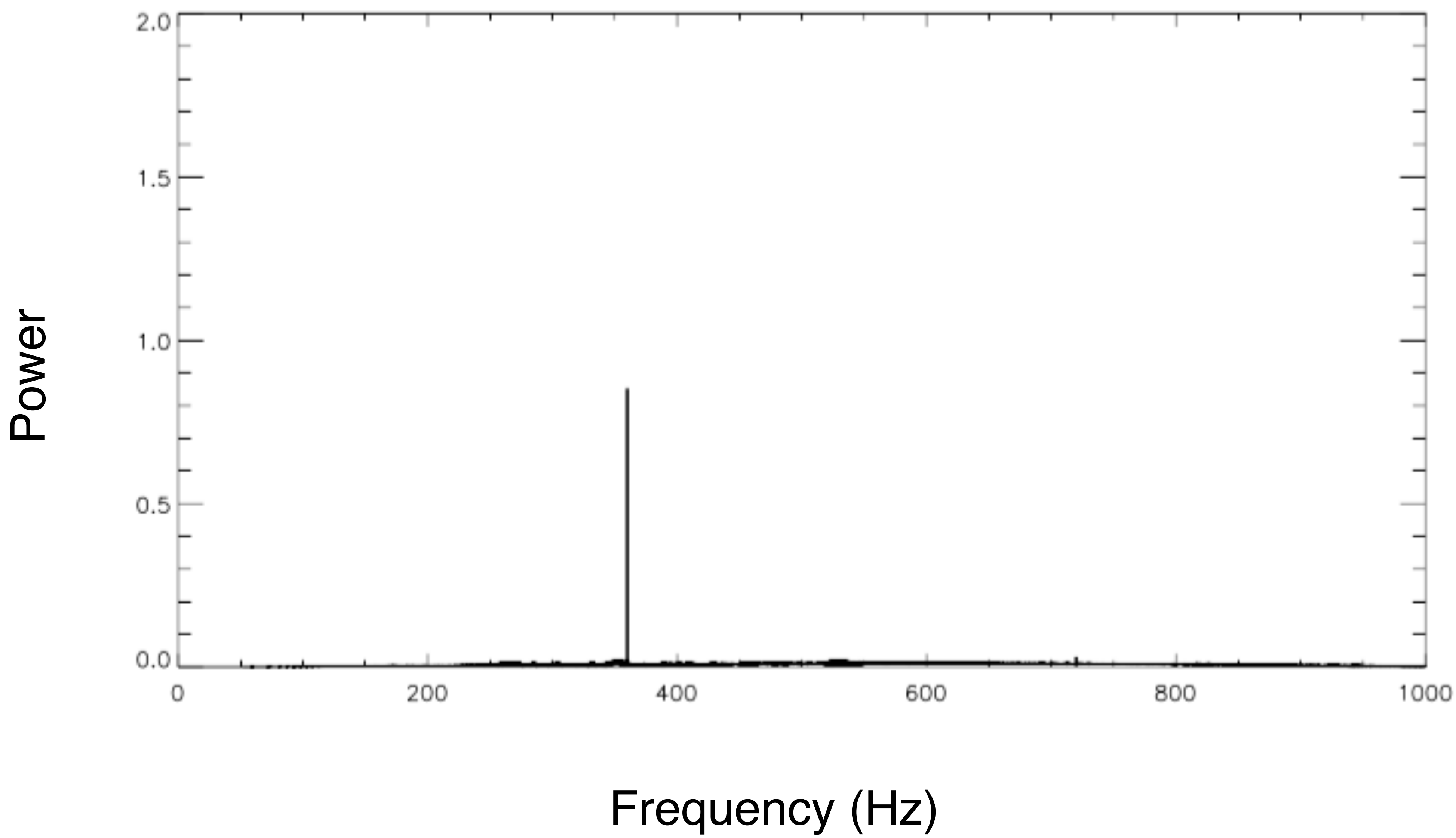
The professor's watch



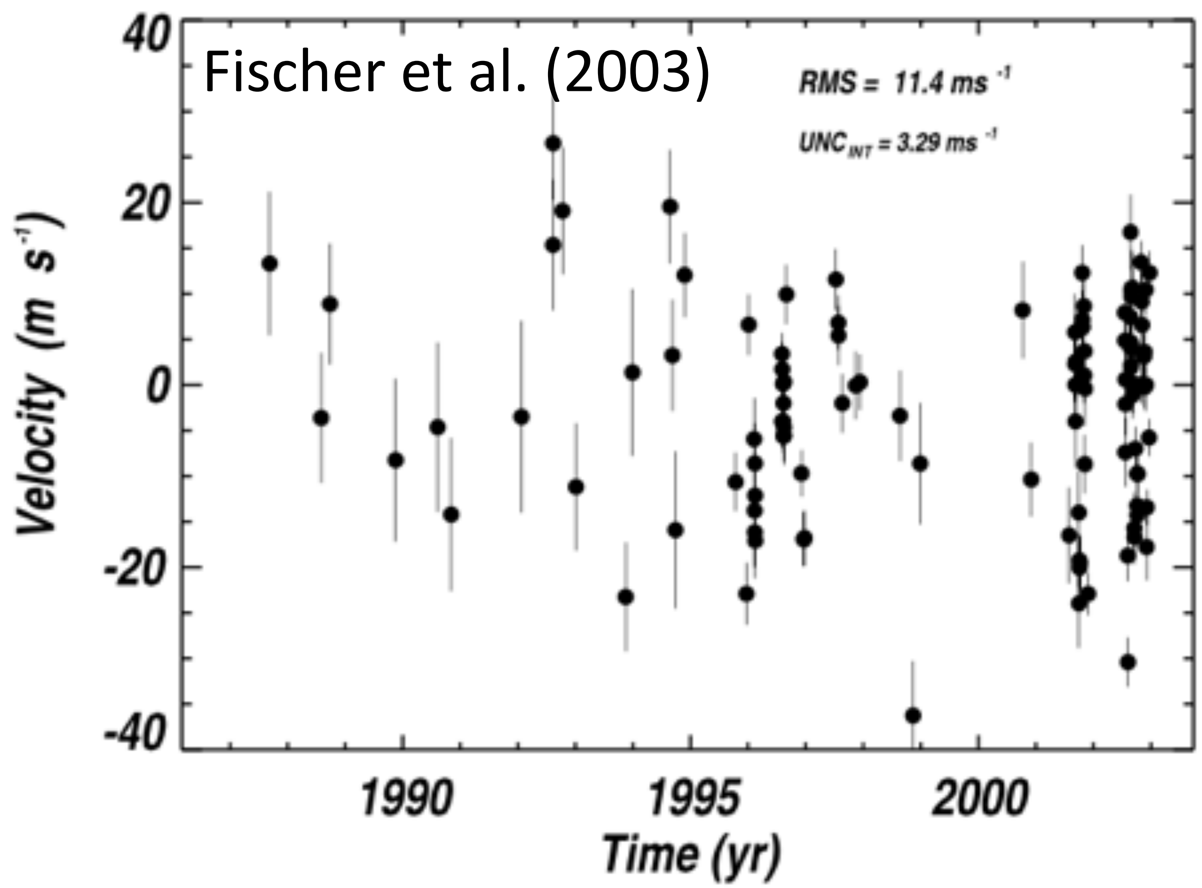
Amplitude

Time

The professor's watch



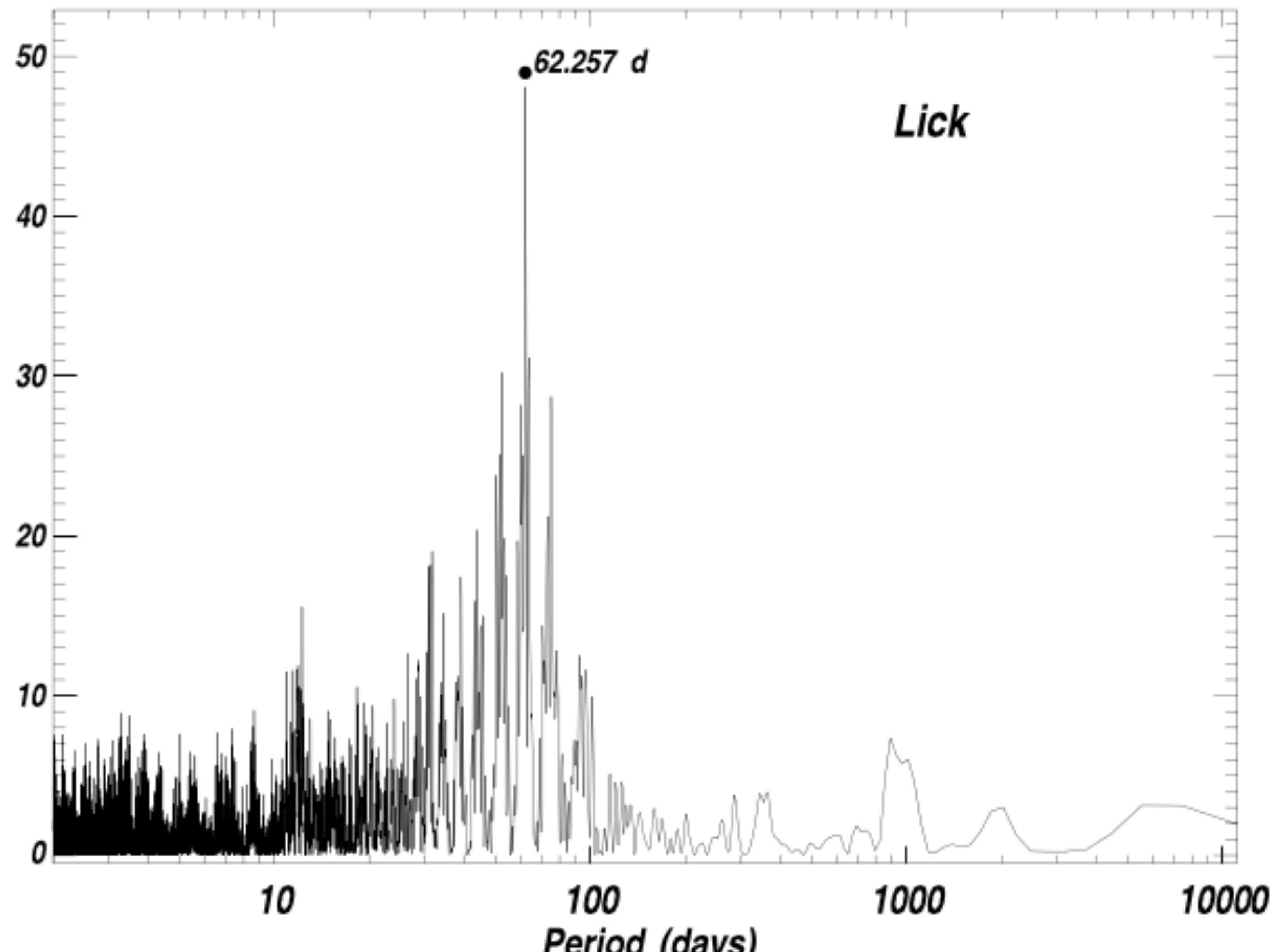
Sub-saturn mass planet HD3651

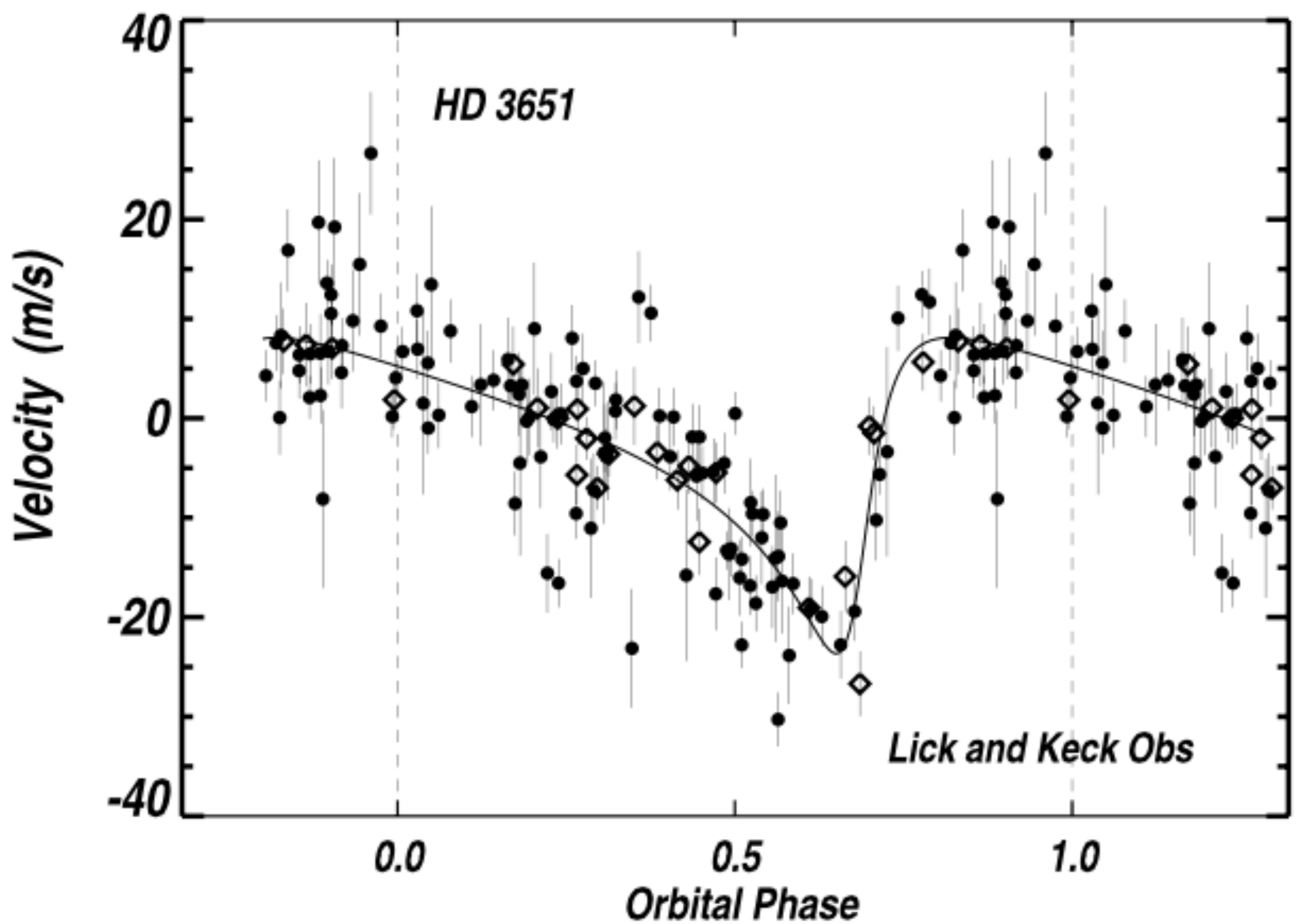


"Power Spectrum"
(or Lomb-Scargle
periodogram, Lomb 1976)

$$
\begin{aligned}
P_v(\omega) &= \frac{1}{N}|\mathrm{FT}_v(\omega)|^2 \\
&= \frac{1}{N_0}\left|\sum_{j=1}^{N_0} v(t_j)\exp(-\mathrm{i}\omega t_j)\right|^2 \\
&= \frac{1}{N_0}\left[\left(\sum_j v_j\cos(\omega t_j)\right)^2 + \left(\sum_j v_j\sin(\omega t_j)\right)^2\right]
\end{aligned}
$$

$$
\begin{aligned}
P_v(\omega) &= \frac{1}{N}|\mathrm{FT}_v(\omega)|^2 \\
&= \frac{1}{N_0}\left|\sum_{j=1}^{N_0} v(t_j)\exp(-\mathrm{i}\omega t_j)\right|^2 \\
&= \frac{1}{N_0}\left[\left(\sum_j v_j\cos(\omega t_j)\right)^2 + \left(\sum_j v_j\sin(\omega t_j)\right)^2\right]
\end{aligned}
$$



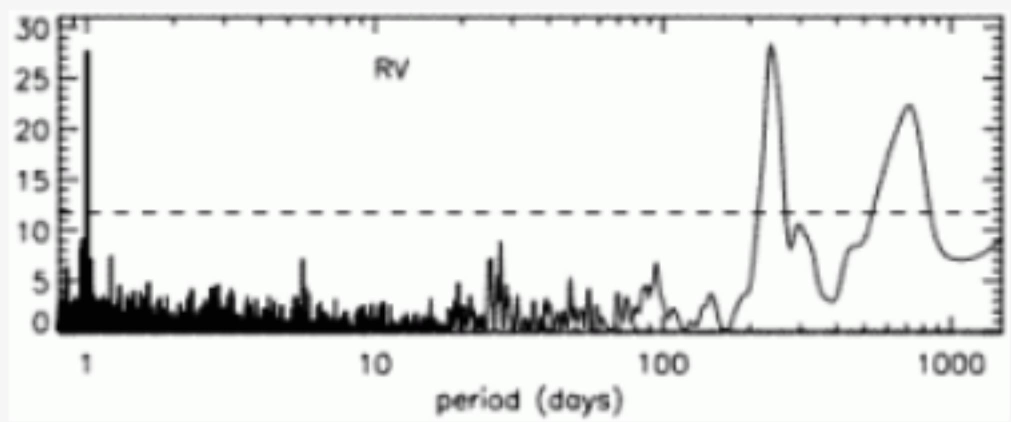

HD 3651
Lick and Keck Obs
Velocity (m/s)
Orbital Phase
40
20
0
-20
-40
0.0
0.5
1.0

Multi-planets Example:

Example from HARPS southern survey,
some planets orbiting active star BD-08 2823
(Hebrard et al. 2010)

https://exoplanetmusings.wordpress.com/2012/12/24/the-real-ones/

Multi-planets Example:

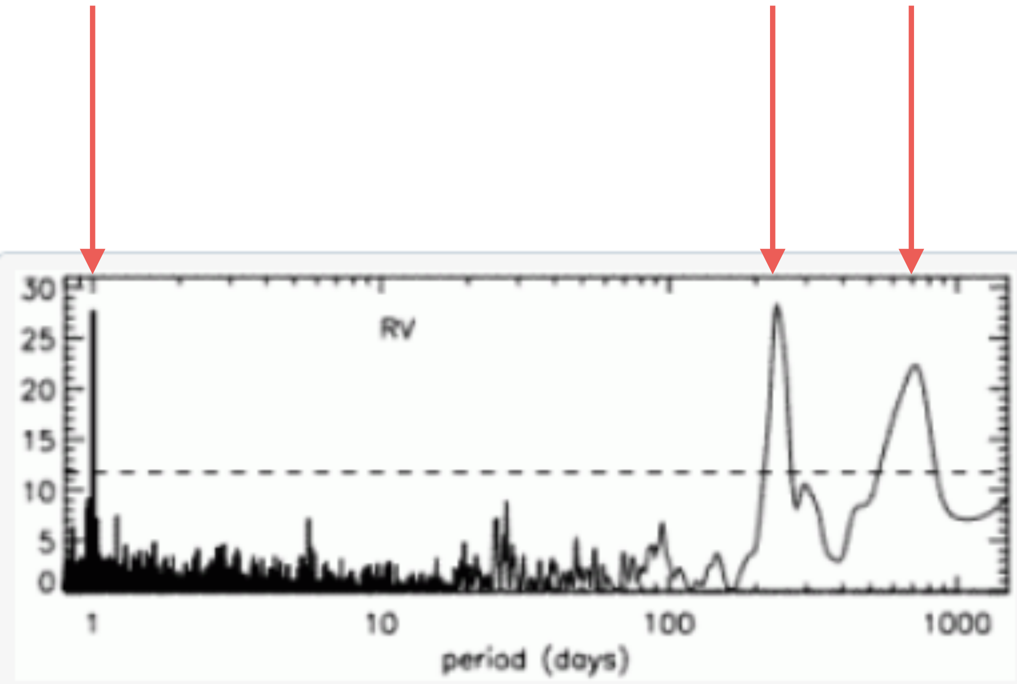


BD-08 2823 RV Data Periodogram

How many planets do you see?

https://exoplanetmusings.wordpress.com/2012/12/24/the-real-ones/

Multi-planets Example:

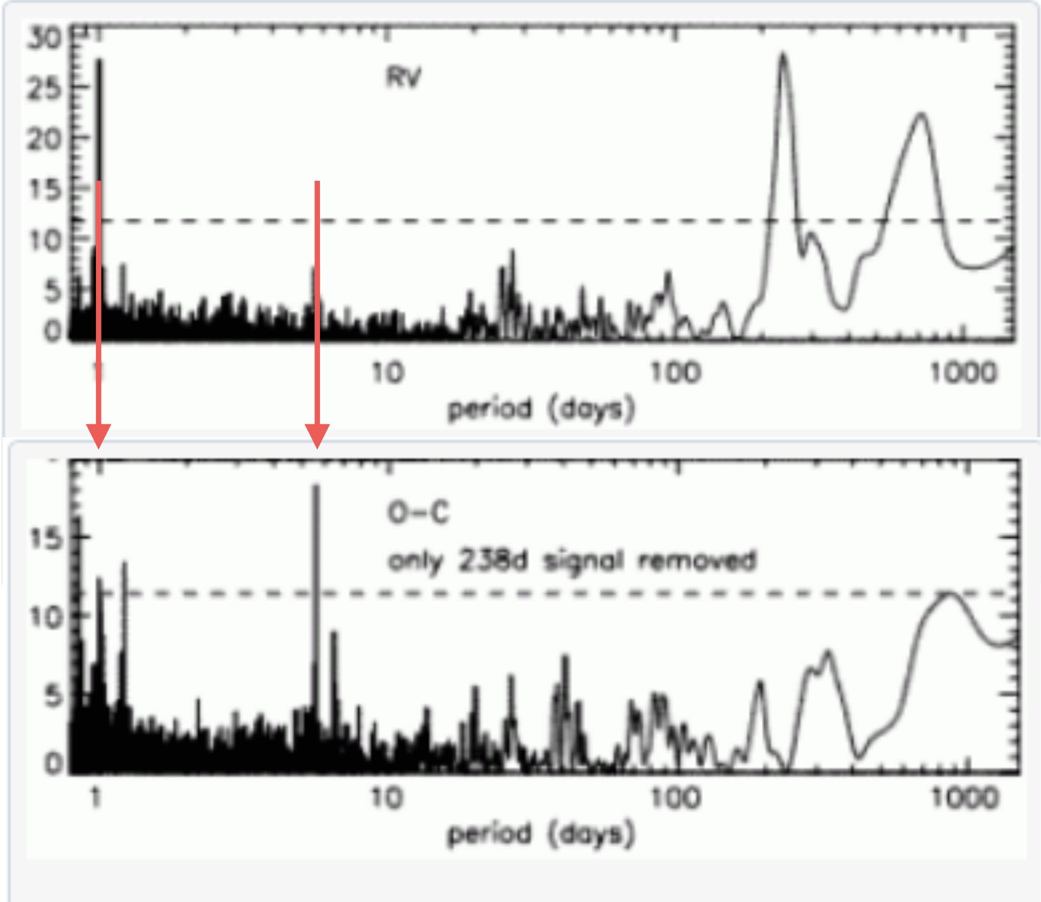


BD-08 2823 RV Data Periodogram

How many planets do you see?

https://exoplanetmusings.wordpress.com/2012/12/24/the-real-ones/

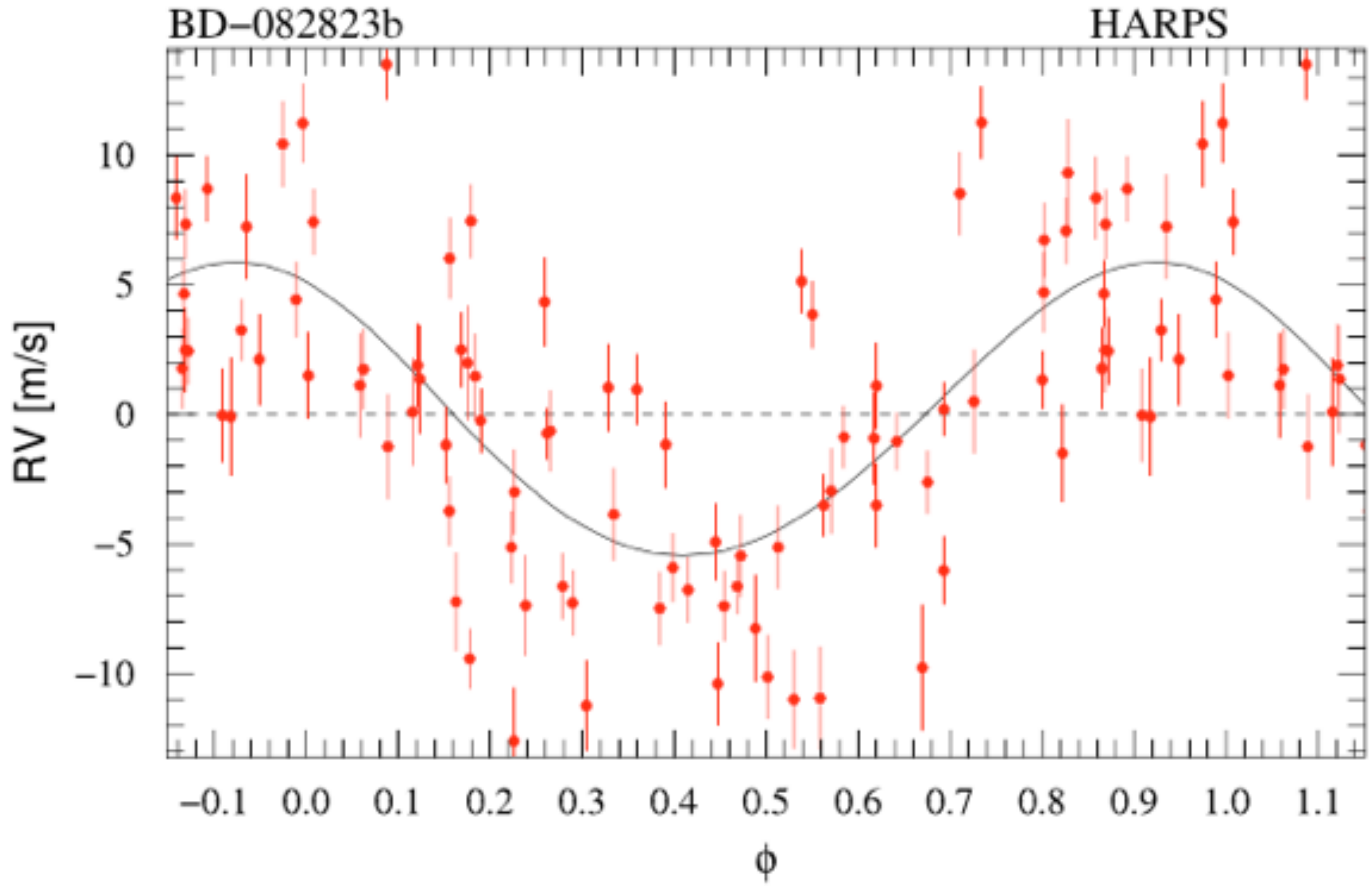
Multi-planets Example:

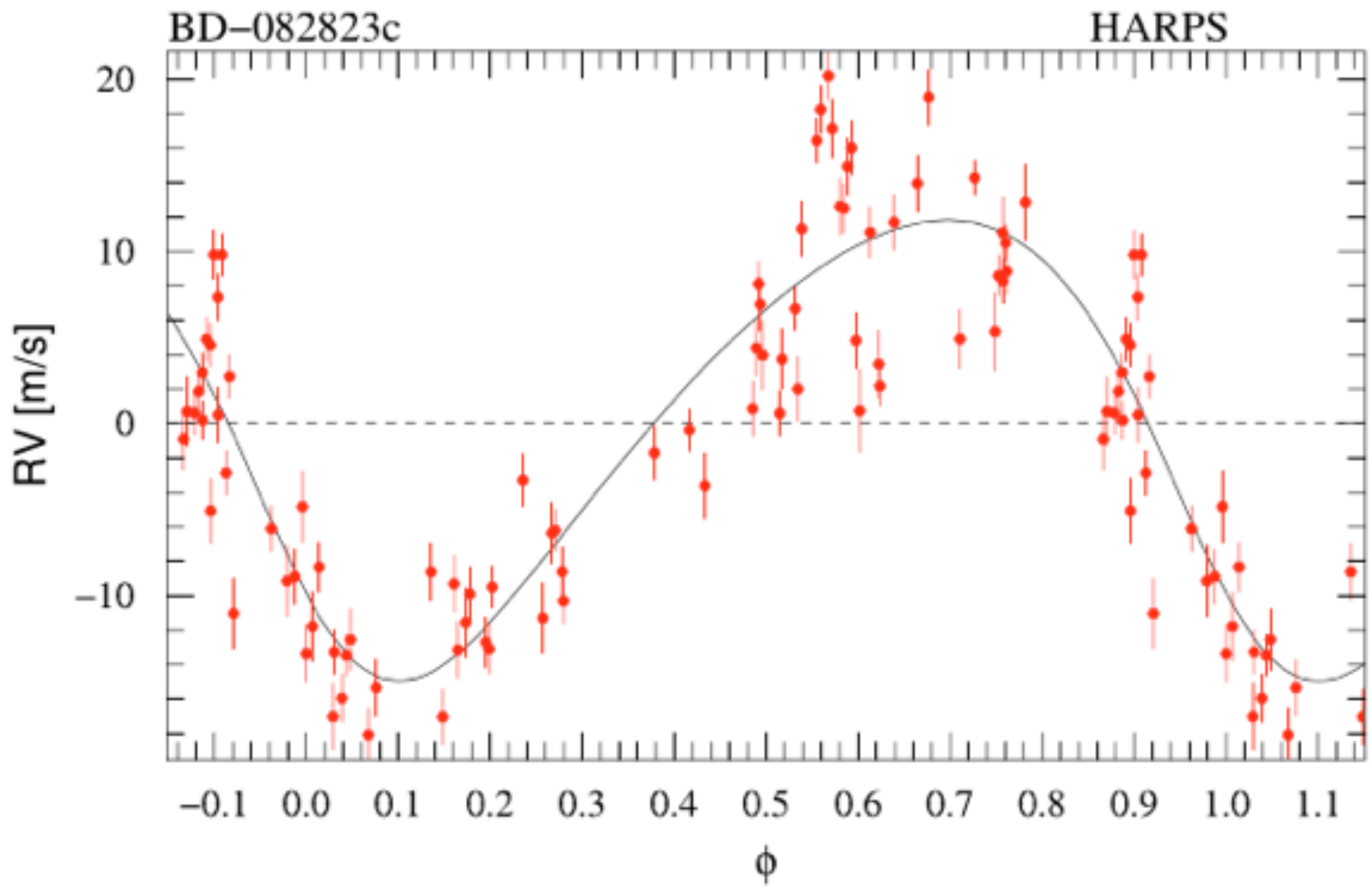


BD-08 2823 Periodogram of Residuals

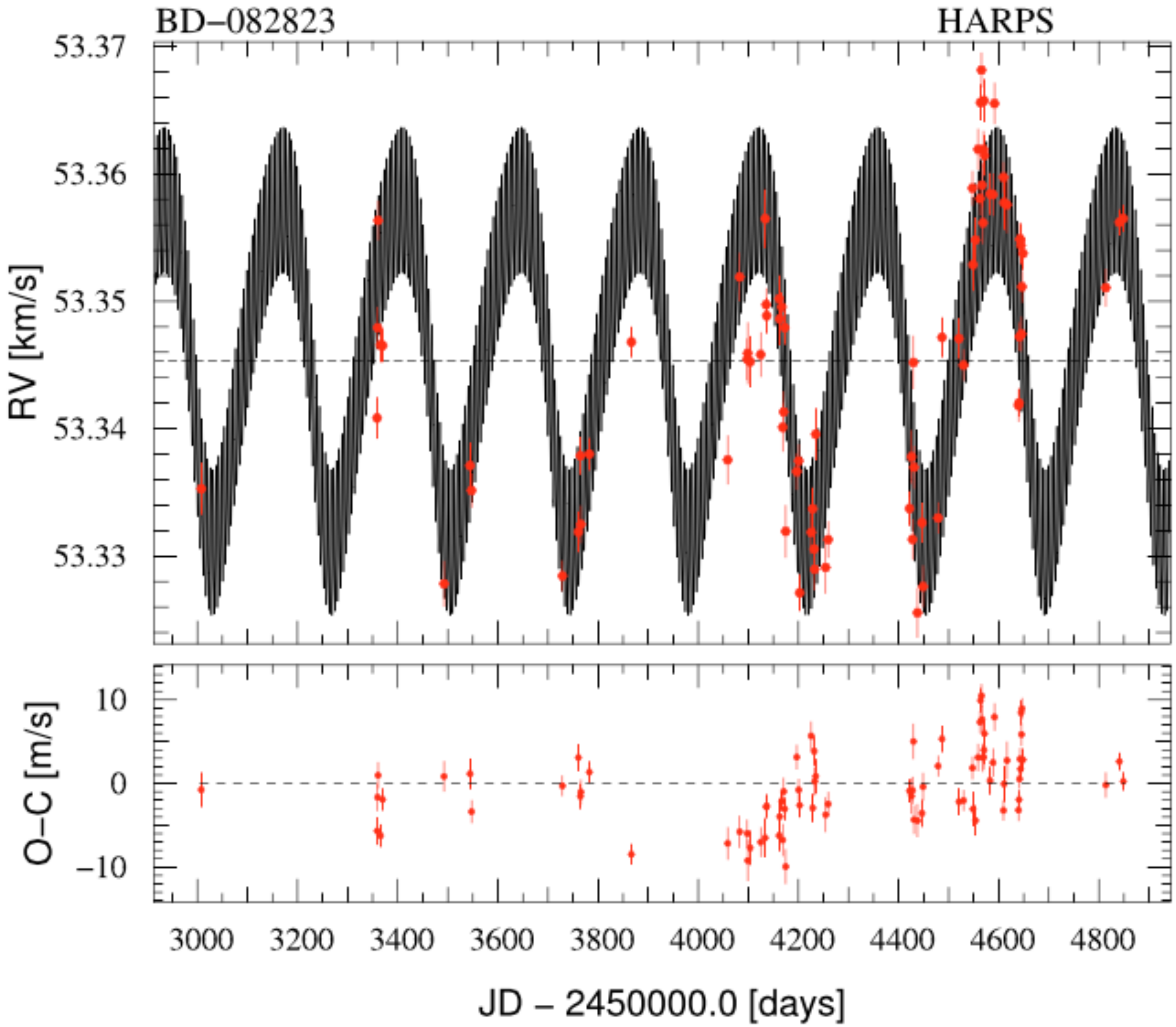
With 238 day signal removed

https://exoplanetmusings.wordpress.com/2012/12/24/the-real-ones/

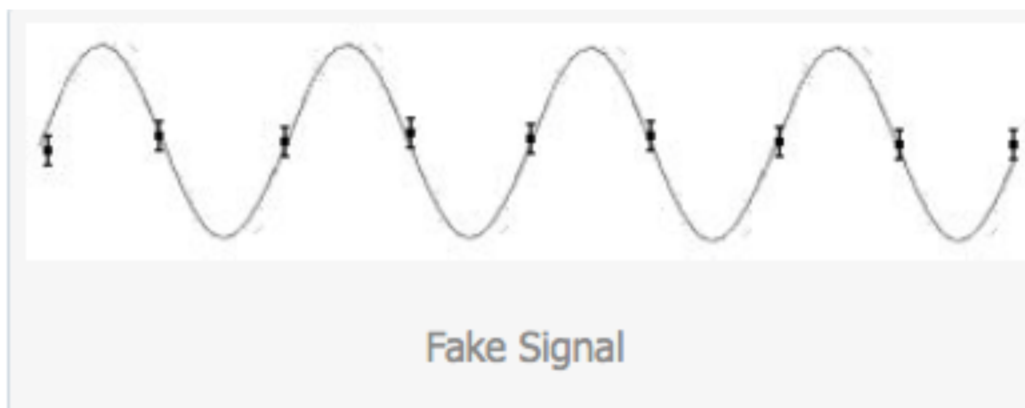

BD−082823b
HARPS
RV [m/s]
10
5
0
−5
−10
−0.1
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
ϕ


BD−082823c
HARPS
RV [m/s]
20
10
0
−10
−0.1
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
ϕ

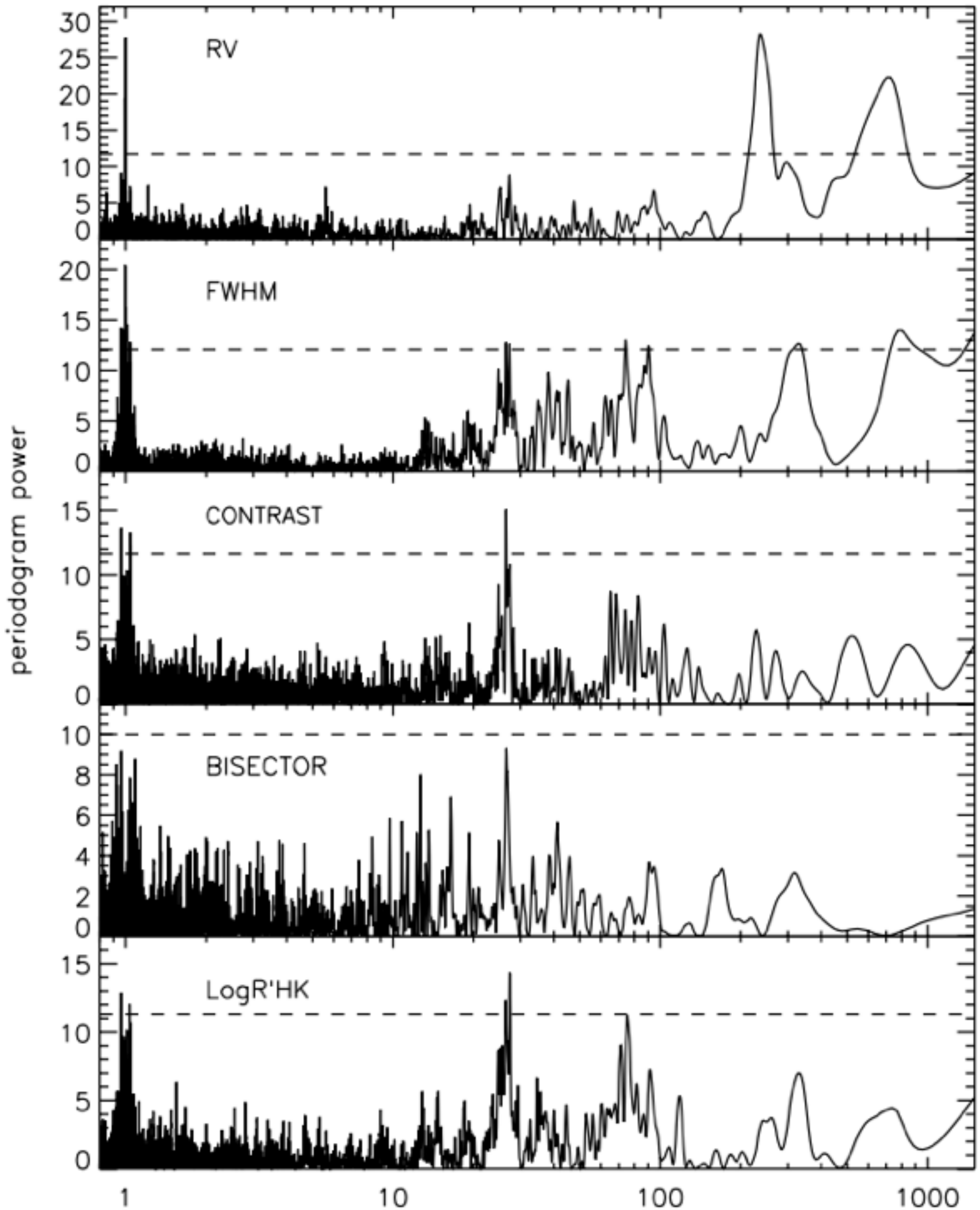
Combined Fits to the data



What about the planet at 1 day?

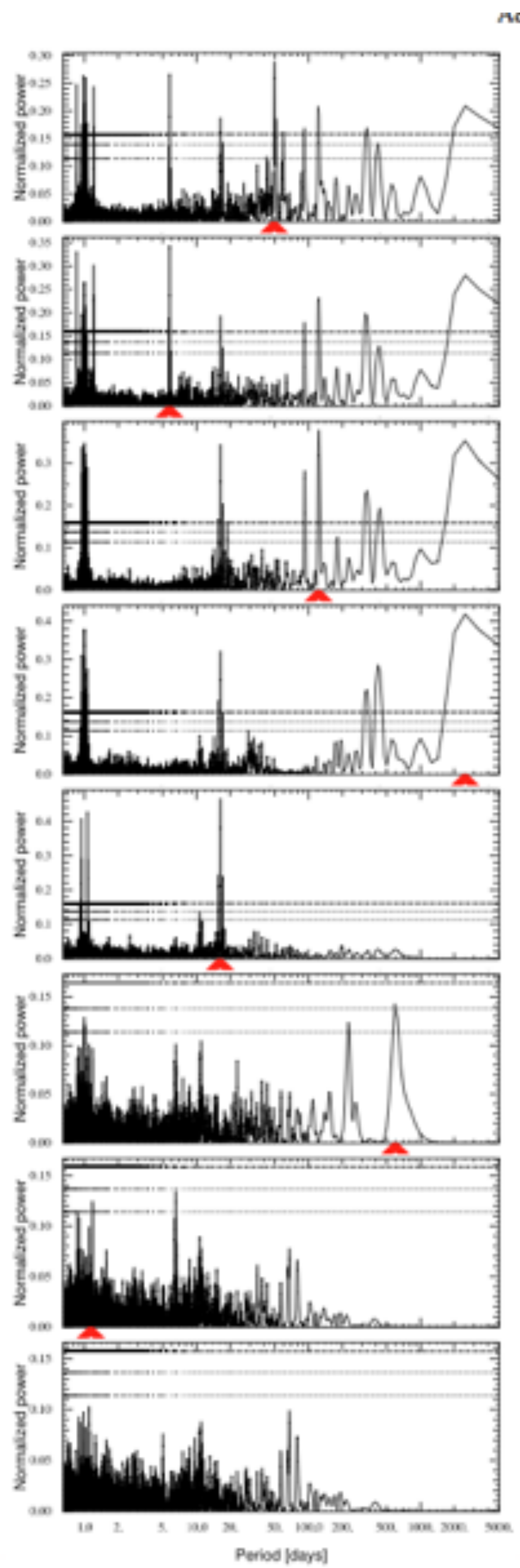


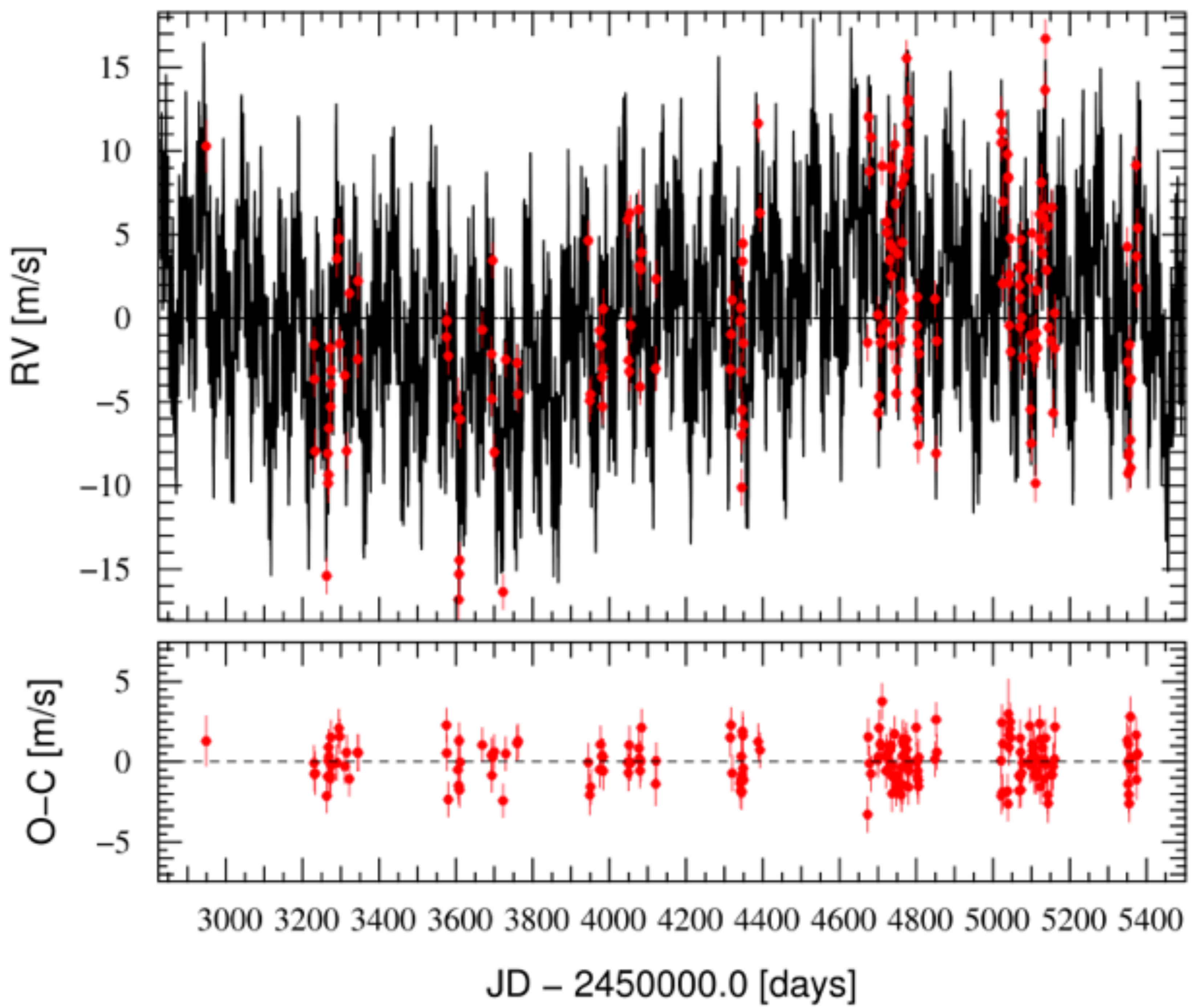
https://exoplanetmusings.wordpress.com/2012/12/24/the-real-ones/



This is an active star

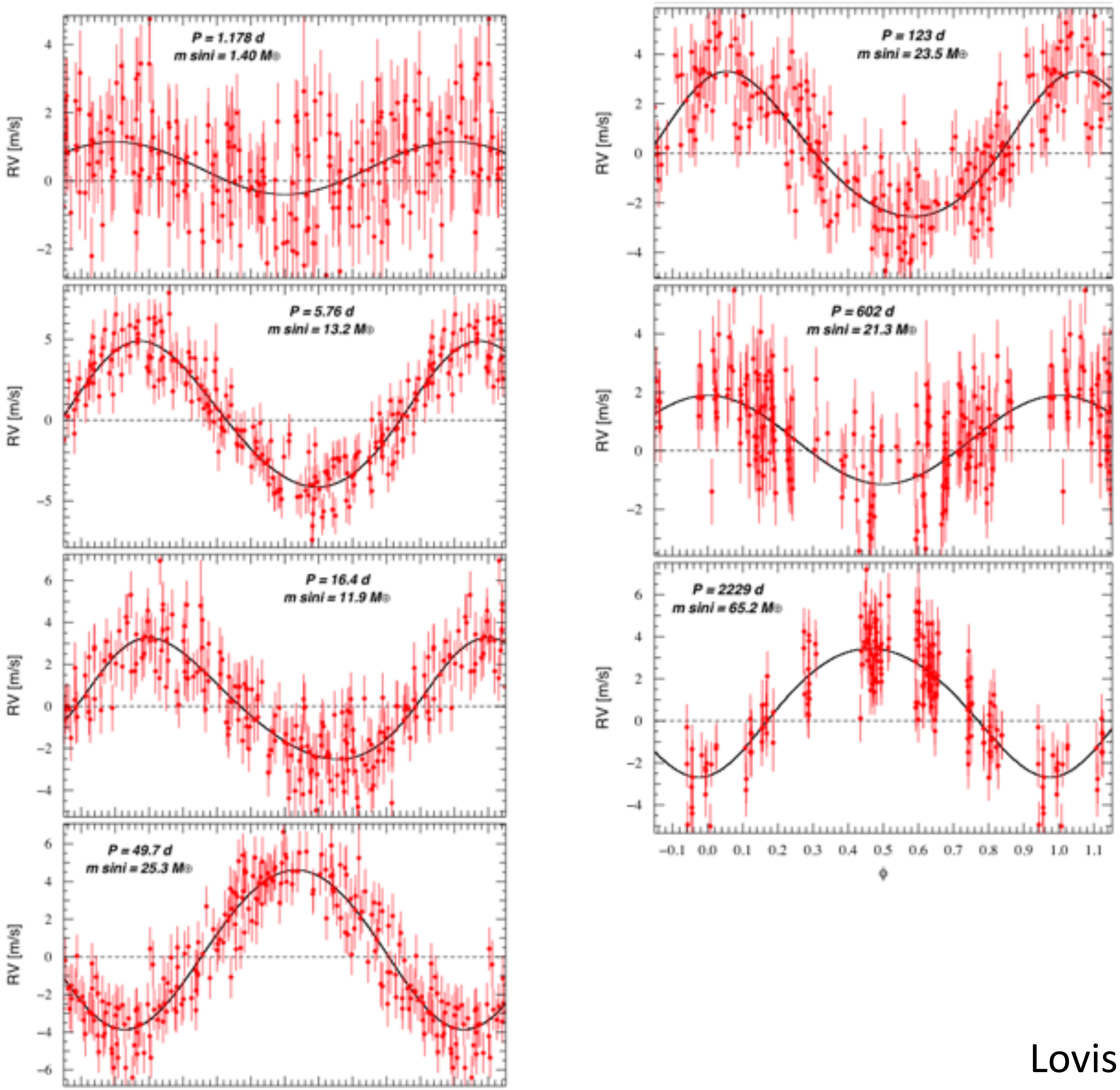
Seven planets around a single star (HD10180)





Lovis et al. 2011, A&A, 528, 112

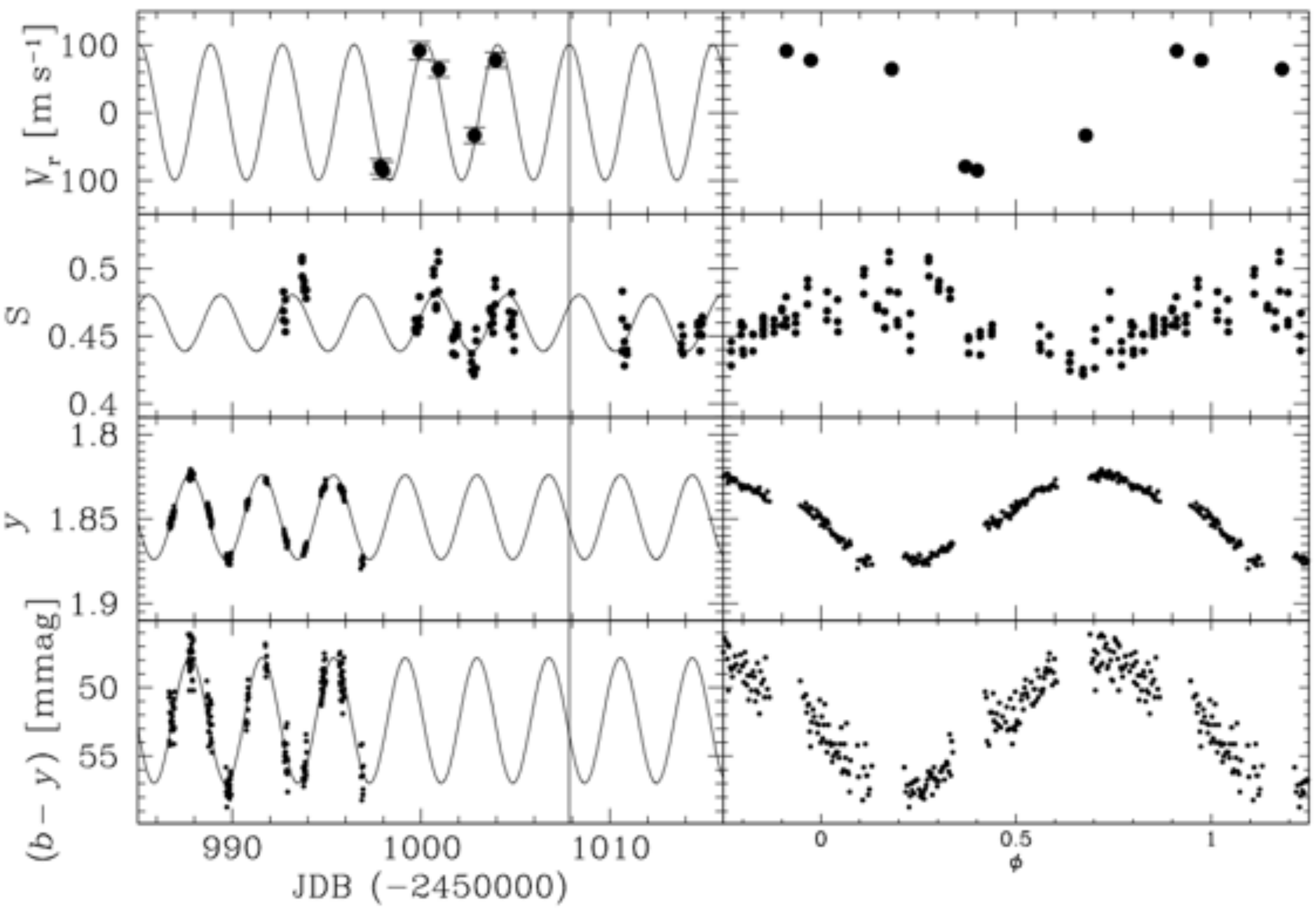
Seven planets around a single star (HD10180)

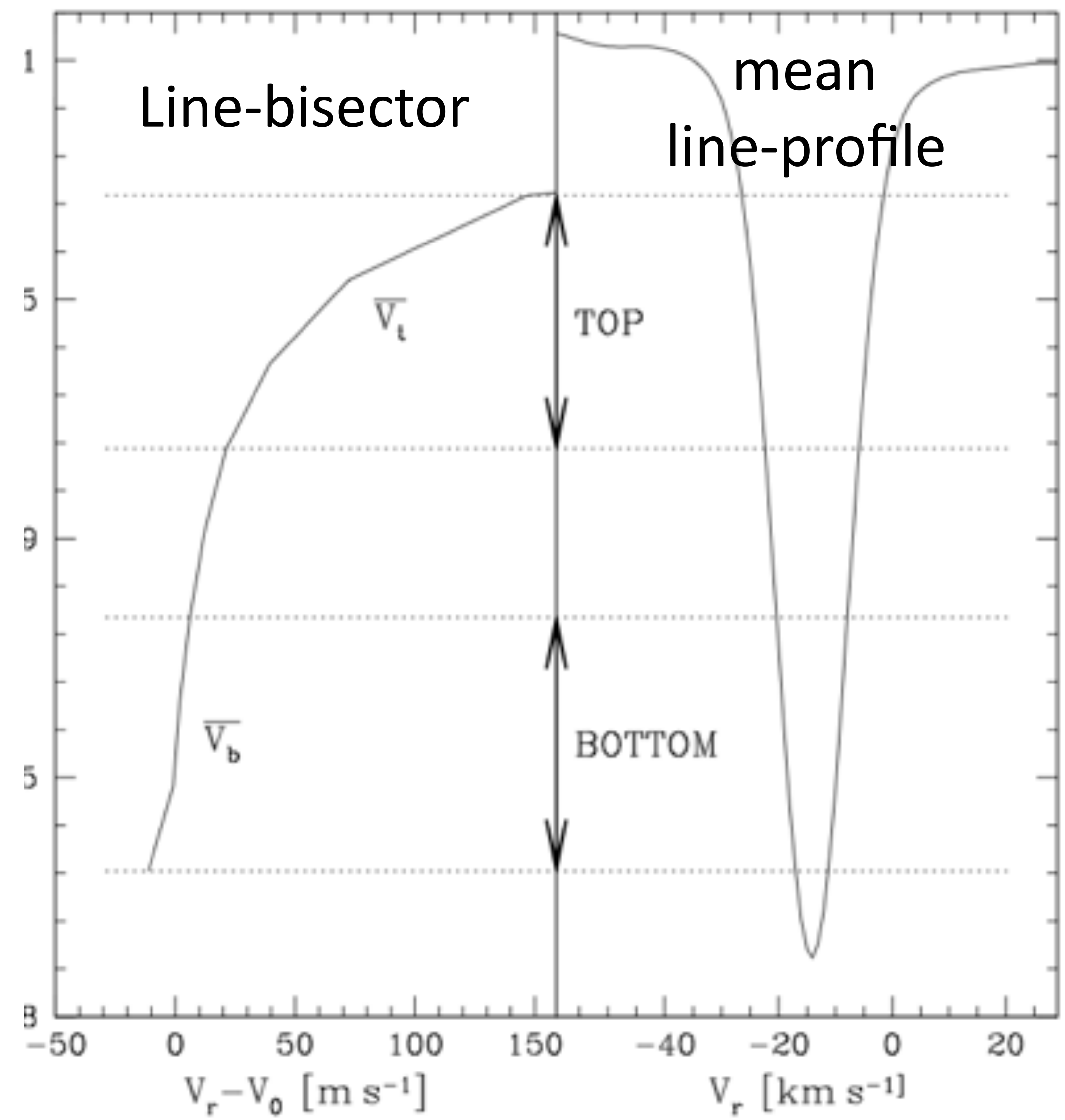


Lovis et al. 2011, A&A, 528, 112

Activity and RVs

HD166435: spots masquerading as a planet!

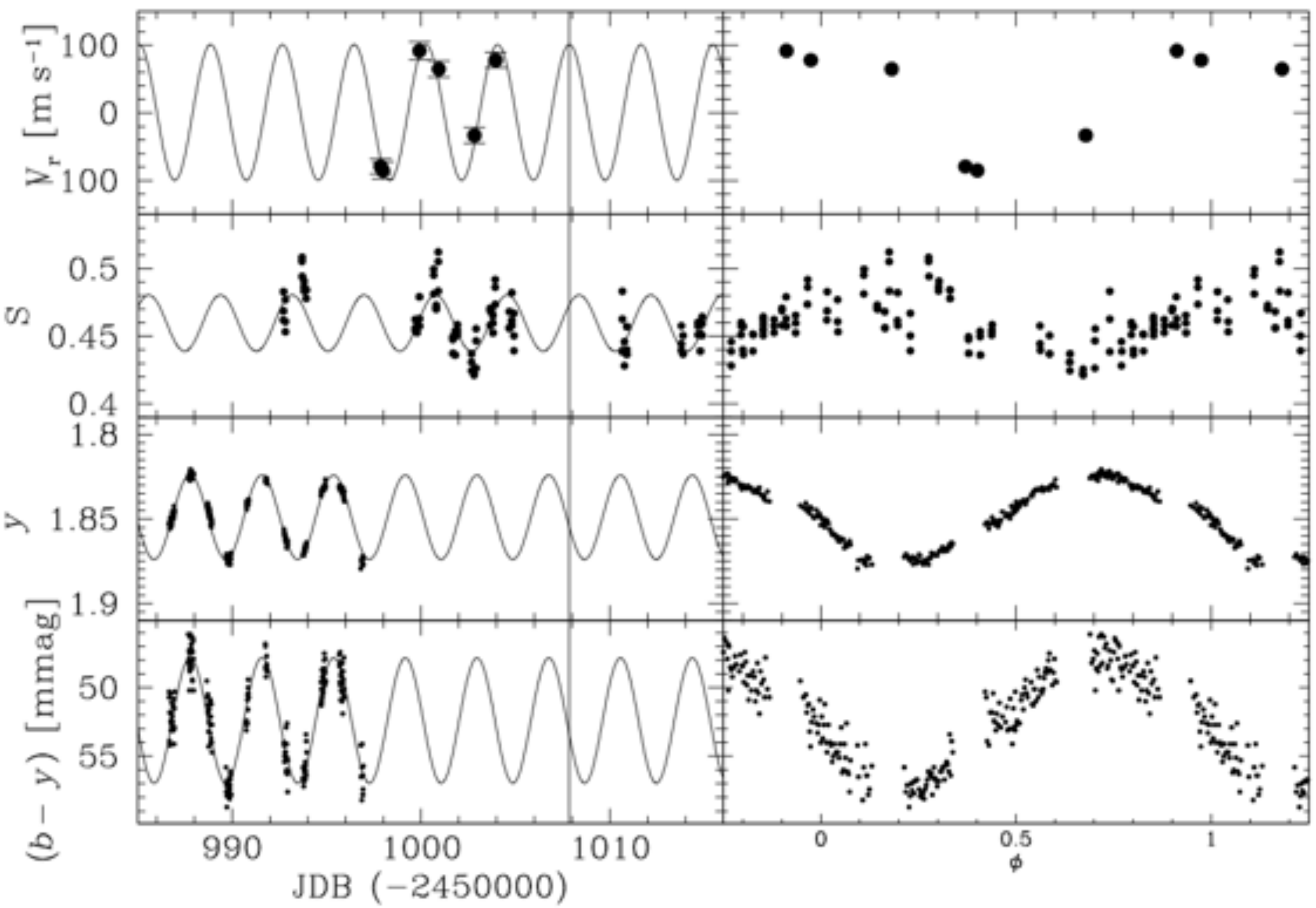


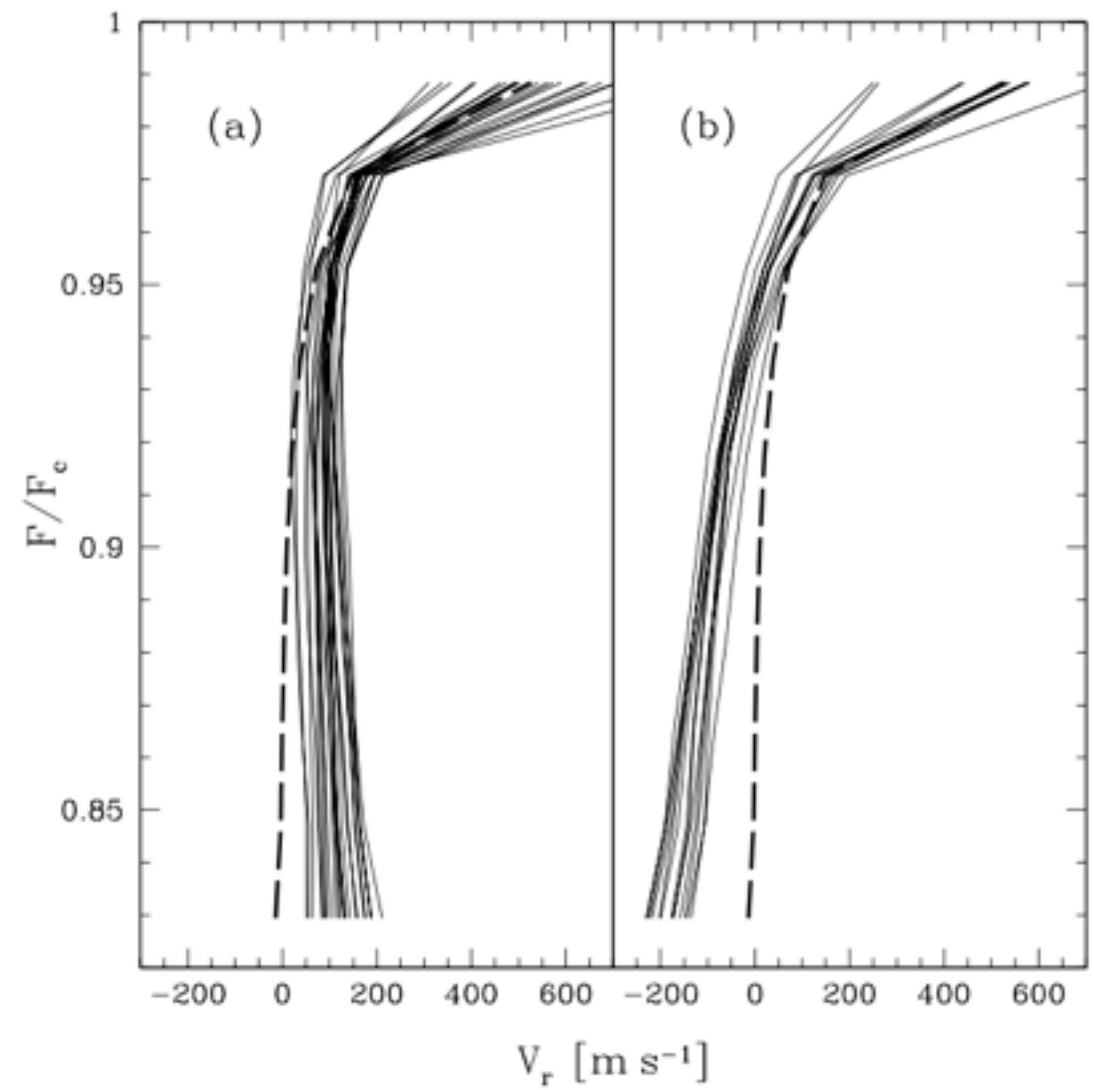


Queloz et al. 2001, A&A, 379, 279

Activity and RVs

HD166435: spots masquerading as a planet!





Queloz et al. 2001, A&A, 379, 279