Lensing

Gravitational Lens in Abell 2218

HST · WFPC2

PF95-14 · ST ScI OPO · April 5, 1995 · W. Couch (UNSW), NASA
Strong lensing: distortion at large angular scales
Microlensing

microlensing: distortion at small angular scales ($\theta \approx 1$ mas)

Einstein Ring

If a lens is exactly (or very closely) aligned with a source there are magnified images of the source at (or near) an angle equal to the Einstein radius.
DISCUSSION

LENS-LIKE ACTION OF A STAR BY THE
DEVIATION OF LIGHT IN THE
GRAVITATIONAL FIELD

Some time ago, R. W. Mandl paid me a visit and asked me to publish the results of a little calculation, which I had made at his request. This note complies with his wish.

The light coming from a star $A$ traverses the gravitational field of another star $B$, whose radius is $R_o$. Let there be an observer at a distance $D$ from $B$ and at a distance $x$, small compared with $D$, from the extended central line $AB$. According to the general theory of relativity, let $\alpha_o$ be the deviation of the light ray passing the star $B$ at a distance $R_o$ from its center.

For the sake of simplicity, let us assume that $AB$ is large, compared with the distance $D$ of the observer from the deviating star $B$. We also neglect the eclipse (geometrical obscuration) by the star $B$, which indeed is negligible in all practically important cases. To permit this, $D$ has to be very large compared to the radius $R_o$ of the deviating star.

It follows from the law of deviation that an observer situated exactly on the extension of the central line $AB$ will perceive, instead of a point-like star $A$, a luminous circle of the angular radius $\beta$ around the center of $B$, where

$$ \beta = \sqrt{\frac{\alpha_o R_o}{D}}. $$

It should be noted that this angular diameter $\beta$ does not decrease like $1/D$, but like $1/\sqrt{D}$, as the distance $D$ increases.

Of course, there is no hope of observing this phenomenon directly. First, we shall scarcely ever approach closely enough to such a central line. Second, the angle $\beta$ will defy the resolving power of our instruments. For, $\alpha_o$, being of the order of magnitude of one second of arc, the angle $R_o/D$, under which the deviating star $B$ is seen, is much smaller. Therefore, the light coming from the luminous circle can not be distinguished by an observer as geometrically different from that coming from the star $B$, but simply will manifest itself as increased apparent brightness of $B$.

The same will happen, if the observer is situated at a small distance $x$ from the extended central line $AB$. But then the observer will see $A$ as two point-like light-sources, which are deviated from the true geometrical position of $A$ by the angle $\beta$, approximately.

The apparent brightness of $A$ will be increased by the lens-like action of the gravitational field of $B$ in the ratio $q$. This $q$ will be considerably larger than unity only if $x$ is so small that the observed positions of $A$ and $B$ coincide, within the resolving power of our instruments. Simple geometric considerations lead to the expression

$$ q = \frac{1 + \frac{x^2}{2l}}{\sqrt{1 + \frac{x^2}{4l^2}}}, $$

where

$$ l = \sqrt{\alpha_o DE_o}. $$

If we are interested mainly in the case $q > 1$, the formula

$$ q = \frac{1}{x} $$

is a sufficient approximation, since $\frac{x^2}{l^2}$ may be neglected.

Even in the most favorable cases the length $l$ is only a few light-seconds, and $x$ must be small compared with this, if an appreciable increase of the apparent brightness of $A$ is to be produced by the lens-like action of $B$.

Therefore, there is no great chance of observing this phenomenon, even if dazzling by the light of the much nearer star $B$ is disregarded. This apparent amplification of $q$ by the lens-like action of the star $B$ is a most curious effect, not so much for its becoming infinite, with $x$ vanishing, but since with increasing distance $D$ of the observer not only does it not decrease, but even increases proportionally to $\sqrt{D}$.

December 4, 1936

SCIENCE

Albert Einstein

Institute for Advanced Study,
Princeton, N. J.
\[ \alpha_{GR} = \frac{4GM_L}{c^2b} \]
lens equation derived from geometry …
(what is the mapping between beta and theta?)

Gaudi (2012)
\[ \theta_s = \theta_1 - 2R_s \frac{D_{LS}}{D_L D_S} \frac{1}{\theta_1} \]

(the “lens equation”)

(basic quad. eq.)

Recall that:

\[ \alpha_{GR} = \frac{4GM_L}{c^2 b} \]

\[ R_s = \frac{2GM_L}{c^2} \]

\[ u = y - y^{-1} \]

\[ y_{\pm} = \pm \frac{1}{2} \left( \sqrt{u^2 + 4} \pm u \right) \]
\[
\theta_s = \theta_1 - 2R_s \frac{D_{LS}}{D_L D_S} \frac{1}{\theta_1}
\]

(lens equation)

\[
\theta_\pm = \frac{1}{2} \left( \theta_s \pm \sqrt{\theta_s^2 + 4\theta_E^2} \right)
\]

\[
\alpha_{GR} = \frac{4GM_L}{c^2 b}
\]

\[
R_s = \frac{2GM_L}{c^2}
\]

“angular and linear Einstein Radii”

\[
\theta_E = \left[ \alpha_{GR} \frac{D_{LS}}{D_L D_S} \right]^{1/2}
\]

\[
R_E = \theta_E D_L
\]
For solar-mass lens halfway to galactic center:

\[ \theta_E \sim 1 \text{ mas} \]
\[ R_E \sim 4 \text{ AU} \]
theta (~ mas) usually too small to resolve spatially, observe total flux

$$\theta_{\pm} = \frac{1}{2} \left( \theta_s \pm \sqrt{\theta_s^2 + 4\theta_E^2} \right)$$

for $\theta_s \neq 0 \rightarrow$ two images!

see also Paczynski (1996)

image centroid moves in a non-linear path (think astrometry)
Microlensing (magnification)

Lensing conserves surface brightness, so think of lensing as a magnification of the surface. Flux = Area x surface brightness!

*magnification* is the ratio of image area to source area
Microlensing (magnification)

*magnification* is the ratio of image area to source area

\[ A \sim l \times w \]

\[
A_{\pm} = \left| \frac{y_{\pm}}{u} \frac{dy_{\pm}}{du} \right| = \frac{1}{2} \left[ \frac{u^2 + 2}{u\sqrt{u^2 + 4}} \pm 1 \right]
\]

(mag is time-dependent change in area)

(approximated by projected area above)

Gaudi (2012)
Animation is in the rest-frame of the lens (foreground star) 

from Scott Gaudi: [http://www.astronomy.ohio-state.edu/~gaudi/movies.html](http://www.astronomy.ohio-state.edu/~gaudi/movies.html)

\[
\theta_{\pm} = \frac{1}{2} \left( \theta_s \pm \sqrt{\theta_s^2 + 4\theta_E^2} \right) \]

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A_{\pm} = \left| \frac{y_{\pm}}{u} \frac{dy_{\pm}}{du} \right| = \frac{1}{2} \left[ \frac{u^2 + 2}{u\sqrt{u^2 + 4}} \pm 1 \right] \quad \text{(time-dependent)}
\]
single-lens observables

• time of peak-flux

• time of maximum magnification

• “duration”, Einstein-radius crossing time, $t_E$

Events near the galactic bulge: $t_E$ range ~ days to years

$$t_E \approx 24.8 \text{ days} \left( \frac{M}{0.5 \, M_\odot} \right)^{1/2} \left( \frac{\pi_{\text{rel}}}{125 \, \mu\text{as}} \right)^{1/2} \left( \frac{\mu_{\text{rel}}}{10.5 \, \text{mas year}^{-1}} \right)^{-1}$$

depends on both lens mass and distance
Microlensing Probability
(lensing optical depth)

\[ P = \frac{\text{Area covered by rings}}{\text{Area of sky}} \]

\[ \tau = \frac{1}{\Omega} \int_0^{D_s} n(D_l) \Omega D_l^2 \pi \theta_E^2 dD_l = \int_0^{D_s} n(D_l) D_l^2 \pi \theta_E^2 dD_l. \]

(For lensing < or = theta_E or A > ~ 1.34)
Microlensing Probability  
(lensing optical depth)

\[ P = \frac{\text{Area covered by rings}}{\text{Area of sky}} \]

\[ \tau = \frac{1}{\Omega} \int_0^{D_s} n(D_i) \Omega D_i^2 \pi \theta^2_E dD_i = \int_0^{D_s} n(D_i) D_i^2 \pi \theta^2_E dD_i. \]
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\[ n = \frac{\rho}{M} \]

\[ \pi \theta_E^2 \propto M \]

\[ \tau = \frac{4\pi GD_s^2}{c^2} \int_0^1 \rho(x)x(1-x)dx. \]

depends on mass density (not mass function) along line-of-sight.
\[
\alpha_{GR} = \frac{4GM_L}{c^2b}
\]

\[
\theta_E = \left[ \alpha_{GR} \frac{D_{LS}}{D_L D_S} \right]^{1/2}
\]

\[
R_E = \theta_E D_L
\]

\[
R_s = \frac{2GM_L}{c^2}
\]

\[
\theta_s = \theta_1 - 2R_s \frac{D_{LS}}{D_L D_S} \frac{1}{\theta_1}
\]

\[
\theta_{\pm} = \frac{1}{2} \left( \theta_s \pm \sqrt{\theta_s^2 + 4\theta_E^2} \right)
\]

(lens equation)
Microlensing Event Rate

\[ \Gamma = \frac{2}{\pi} \frac{\tau}{t_E} \; ; \; t_E = \frac{\theta_E}{\mu_{rel}} \]

does depend on mass function along line-of-sight
look to the Bulge!

Observe towards the Galactic bulge to increase stellar density
microlensing is rare ...

- lensing optical depth toward Galactic Bulge is $\sim 10^{-6}$

- median timescale of event $\sim 20$ days

- rate for lens event / star $\sim 10^{-5}$ per year!

- OGLE survey monitors $\sim 2.5 \times 10^8$ sources over 80 square degrees $\sim 1,500$ events / 8 month observing window.
Microlensing + planets
Binary Lens: adds three more parameters:

- mass ratio
- projected separation of binary at time of event
- angle between source-lens trajectory and binary axis at time of event
mapping between source and lens planes (beta ----> theta)

solution to lens magnification for multiple lens:

\[ A_j = \left. \frac{1}{\det J} \right|_{\theta = \theta_j} \]

caustic curves: \textit{source} positions where \( \det J = 0 \)

critical curves: \textit{image} positions

Gaudi (2012)
pretty “caustics”
pretty “caustics”

Caustics from http://www.mpa-garching.mpg.de/mpa/
"caustics" (typically 2 to 3 for one planet)

$q = 0.003$
Microlensing

Planetary perturbation example:

Magnified images along the Einstein ring pass near the planet and are magnified again ("the source crosses the planetary caustic")
planet outside and near ring:

For solar-mass lens halfway to galactic center:

\[ \theta_E \sim 1 \text{ mas} \]

\[ R_E \sim 4 \text{ AU} \]
effect of changing planet/source track angle:
effect of changing separation:

http://www.astronomy.ohio-state.edu/~gaudi/Movies/lcp_b.gif
What can you learn?

Planetary Perturbation

• Same as for the main event: measure time of maximum magnification, maximum magnification, and duration

• The duration is proportional to $q^{1/2}t_E$. With $t_E$ from the main event, you get $q = M_p/M_s$

• The time and magnitude of the perturbation give the separation and position angle of the planet
What can you learn?

**Planet-Star System**

- With the mass ratio \((q)\), you need to find some way to get the lens mass
- Finite source effects, and some assumptions about the source distance, give you a mass-distance relation for the lens
- Measure photometry and/or spectroscopy of the lens itself
- Measure the proper motion of the system
- Detect the microlens parallax
- Detect orbital motion of the planets
Powerful statistical probe of exoplanet population:

• very sensitive to planets > snow line

• magnification does not depend on planet mass (sensitive to low mass planets)

• sensitive to long-period planets (and free-floaters)

• probes galactic planet distribution

• detects multi-planet systems.
Microlensing Highlights:
OGLE-2005-BLG-390Lb

- $M_p = 5.5 \, M_{\text{earth}}$
- $a = 2.6 \, \text{AU}$
- $M_s = 0.20 \, M_{\text{sun}}$

(with big errors)

$\Rightarrow$ Super-Earths must be common

Beaulieu et al. 2006, Nature, 439, 437
Microlensing Highlights: OGLE-2006-BLG-109Lb

A Jupiter-Saturn analog system around a $M = 0.5 \, M_{\text{sun}}$ star

Orbital motion of the outer planet was detected

Gaudi et al. 2008, Science, 319, 927
Microlensing Highlights: Unbound Planets?

1.8^{+1.7}_{-0.8} \text{ planetary mass objects at } >10 \text{ AU per star}

Sumi et al. 2011, Nature, 473, 349
TRIPLE MICROLENS OGLE-2008-BLG-092L: BINARY STELLAR SYSTEM WITH A CIRCUMPRIMARY URANUS-TYPE PLANET

Poleski et al. (2014)
microlensing and WFIRST

WFIRST-AFTA will:

- Detect 2800 planets, with orbits from the habitable zone outward, and masses down to a few times the mass of the Moon.
- Be sensitive to analogs of all the solar system’s planets except Mercury.
- Measure the abundance of free-floating planets in the Galaxy with masses down to the mass of Mars
Finite Source Effects & Microlensing
Parallax Yield Lens System Mass

- **Finite source effects**
  Angular Einstein radius \( \theta_E = \theta_* t_E / t_* \)
  \( \theta_* = \) source star angular radius
  \( D_L \) and \( D_S \) are the lens and source distances

- **Microlensing Parallax**
  (Effect of Earth’s orbital motion)
  Einstein radius projected to Observer
  OR
- **One of above**
  Lens brightness & color (AO, HST)
  mass-distance relation \( \Rightarrow D_L \)

\[
M_L = \frac{c^2}{4G} \theta_E^2 \frac{D_S D_L}{D_S - D_L}
\]

\[
M_L = \frac{c^2}{4G} \tilde{r}_E^2 \frac{D_S - D_L}{D_S D_L}
\]

\[
M_L = \frac{c^2}{4G} \tilde{r}_E \theta_E
\]
Figure 3. Left: Keck image of OGLE-2005-BLG-169 in \(H\) band (\(~24'' \times 21''\)). Middle: a zoom on the target showing the lens on the upper left and the source. They are separated by \(~61\) mas. The extra star on the right was part of the measured blending in the microlensing light curve. Right: zoom on the target showing the flux contours.
microlensing and WFIRST

\begin{equation}
M_H = H_L - A_H - DM = H_L - A_H - 5 \log \frac{D_L}{10 \text{ pc}}
\end{equation}

\begin{table}
\centering
\caption{Model Parameters from the Microlensing Light Curve}
\begin{tabular}{llll}
\hline
Parameter & Units & Bennett2015 & Gould2006 \\
\hline
$\tau_E$ & days & $41.8 \pm 2.9$ & $43 \pm 4$ \\
$\theta_E$ & mas & $0.965 \pm 0.094$ & $1.00 \pm 0.22$ \\
$H_S$ & ... & $18.81 \pm 0.08$ & $18.83 \pm 0.09$ \\
$t_*$ & days & $0.0202 \pm 0.0017$ & $0.019 \pm 0.004$ \\
\hline
\end{tabular}
\end{table}

distance, and the relative lens–source proper motion, $\mu_{rel}$, where:

\begin{align}
\tau_E &= \frac{\theta_E}{\mu_{rel,geo}}, \\
\theta_E^2 &= \kappa M_L \pi_{rel}, \\
\pi_{rel} &= \text{AU} \left( \frac{1}{D_L} - \frac{1}{D_S} \right) \quad \text{and} \\
\kappa &= \frac{4G}{c^2 \text{AU}} = 8.144 \text{ mas } M_\odot^{-1}.
\end{align}