Statistical Properties of Exoplanets
Do these clumps, gaps and voids tell us anything?
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Things To Keep In Mind

• Selection effects
  ▪ Detection limits / bias / completeness
  ▪ Contamination: “false-positives”

• planet frequency

• Uncertainties (confidence intervals)
Where Do The Data Come From?

• **Radial Velocity** (HIRES & HARPS) surveys for the distributions of mass and orbital parameters, especially for Neptune and larger planets and those at larger separations that cannot be found by transit surveys.

• **Transits** (Kepler, etc.) provides the distributions of size and separation, especially for smaller planets with small orbits (<1 AU and <<1 AU)

• The *combination of radial velocity and transit data* for the mass-radius distribution of close-in planets.

• **Microlensing** provides the distributions of planet masses and separations, especially at intermediate/large (1 – 10 AU) separations.

• **Direct imaging** provides constraints on the frequency of massive planets at large (10s to 100s of AU) separations and at young ages.
Where are all the very massive planets?

What’s going on up here?
Planets versus Stars/Brown Dwarfs:

Raghavan et al. (2010) conclude that planets form just as often around single stars as binaries or multiple stars.

Shaded regions simply indicate the method by which the binary was found (visual, prop. motion acc., spectroscopic, multi-methods, etc.).
The Brown Dwarf Desert

There must be a fundamental difference in the formation of “planets” compared to stars

The Brown Dwarf Desert

There must be a fundamental difference in the formation of “planets” compared to stars

but wait!, There is water in the desert

Troup et al. 2016; Guillot et al. 2014, Bouchy et al. 2011, also Duchene & Kraus (2013)
but wait!, There is water in the desert

"we find a smaller and “wetter” BD desert with the BD desert only manifesting itself for orbital separations of a < 0.1 - 0.2 AU in this sample of candidate companions, much smaller than the 5AU proposed in previous studies.”

Troup et al. 2016

What is going on? Possible explanations …

P ~ 10 days for G stars
~ 30 days for F stars
smaller convective region, therefore
less angular momentum exchange
contours are fraction of main-sequence lifetime a planet will last before being eaten by star via momentum loss due to tidal interactions

\[ t = 12 \text{ Ma} \left( \frac{Q_*}{10^6} \right) \left( \frac{M_P}{1 \text{ M}_{\text{Jup}}} \right)^{-1} \left( \frac{M_*}{1 \text{ M}_{\odot}} \right)^{8/3} \left( \frac{R_*}{1 \text{ R}_{\odot}} \right)^{-5} \times \left( \frac{P_{\text{orb}}}{1 \text{ day}} \right)^{13/3} \left( 1 - \frac{P_{\text{orb}}}{P_{\text{spin}}} \right)^{-1} \]

see Guillot et al. 2014; Guillot, Lin, Morel in prep
What is going on? Possible explanations ...

Roche overflow, mass exchange ... and mass ratio distribution

\[
\frac{r_1}{a} = \frac{0.49q^{-2/3}}{0.6q^{-2/3} + \ln(1 + q^{-1/3})}
\]

P ~ 10 days for G stars
~ 30 days for F stars

mass exchange could explain the number of BDs orbiting evolved stars. especially for a > 0.2 AU

see Troup et al. and Duchene & Kraus (2013)
Results

- **Radial Velocity**: more than 50% (3σ lower limit) of Sun-like stars have at least one planet with $P < 10$ yr.

- **Transit**: more than 40% (3σ lower limit) of Sun-like stars have at least one planet with $P < 85$ d.

- **Transit**: 6.5% ± 3.6% of Sun-like stars have planets with radius between 0.8 and 1.25 $R_{\text{earth}}$ and $P < 85$ d.

- **RV and Transit**: At short periods, small planets are much more common and flat, multi-planet systems are the norm.

- **RV and Transit**: Smaller planets tend to have less hydrogen, but there is a wide diversity of planet sizes.

- **Microlensing**: 1.6 ± 0.8 planets per star between 0.5 and 10 AU

- **Direct Imaging**: Massive planets at 10s to 100s of AU are not common.
Radial Velocity

Fig. 1.—Minimum mass ($M \sin i$) and period ($P$) distribution of 182 extrasolar planets detected by radial velocity searches announced as of 2007 March. The dotted lines show velocity amplitudes of 3, 10, and 30 m s$^{-1}$ for a $1 M_\odot$ star. We take the orbital parameters from the updated Butler et al. 2006 catalog maintained by the California & Carnegie Planet Search.$^6$

Cumming et al. 2008, PASP, 120, 531
Giant Exoplanets
Radial Velocity (bias correction)

1 planet > 50 Mearth
P < 10 years

Mayor et al. 2011

Planet rate: 14% ± 2

~ 900 stars
After bias correction, the number of stars with at least 1 planet more massive than 50 Mearth with P < 10 years is 14.2%

call attention to aliasing at 1 day, N years

Mayor et al. 2011
Radial Velocity (bias correction)

1 planet > 50 Mearth
P < 10 years

\[ K_\star = 28.4 \text{ms}^{-1} \left( \frac{P}{1 \text{yr}} \right)^{-1/3} \left( \frac{M_p \sin(i)}{M_J} \right) \left( \frac{M_\star}{M_\odot} \right)^{-2/3} \]

~ 900 stars
After bias correction, the number of stars with at least 1 planet more massive than 50 Mearth with P < 10 years is 14.2 %

Mayor et al. 2011
Radial Velocity (bias correction)

Fig. 10. Observed mass histogram for the planets in the combined sample. Before any bias correction, we can already notice the importance of the sub-population of low-mass planets. We also remark a gap in the histogram between planets with masses above and below ~30 M$_\oplus$.

Fig. 12. Histograms of planetary masses, comparing the observed histogram (black line) and the equivalent histogram after correction for the detection bias (red line).

Mayor et al. 2011, arXiv:1109.2497
Radial Velocity (bias correction)

**Fig. 10.** Observed mass histogram for the planets in the combined sample. Before any bias correction, we can already notice the importance of the sub-population of low-mass planets. We also remark a gap in the histogram between planets with masses above and below \( \sim 30 \text{M}_{\oplus} \).

**Fig. 12.** Histograms of planetary masses, comparing the observed histogram (black line) and the equivalent histogram after correction for the detection bias (red line).

Mayor et al. 2011, arXiv:1109.2497
Radial Velocity

Rising function of Period, hot-Jupiters are rare, giant-planet migration is limited

Fig. 8. Histogram of the planet frequency for planets with masses \( m_2 \sin i > 50 \, M_\oplus \). The occurrence rate for gaseous giant planets is strongly increasing with the logarithm of the period \( \log P \).

Mayor et al. 2011
Radial Velocity

Gas giants only ($M_p \sin i > 0.3 \, M_{\text{jup}}$)

The biggest gas giants typically have smaller $e$.

eccentricity rises with $P$
Radial Velocity: Giant Planet – Metallicity Correlation

Key papers:
Santos et al. 2005 A&A, 437, 1127

PMC holds even for M dwarfs

typically uniform enhancement of elements heavier than H, He.
Radial Velocity:
Giant Planet – Host Star Mass Correlation

ca. 2008
Radial Velocity: Giant Planet – Putting it together

Gas giants only!

\[ f(M_*, [\text{Fe/H}]) = 0.07 \pm 0.01 \times (M_*/M_\odot)^{1.0\pm0.3} \times 10^{1.2\pm0.2[\text{Fe/H}]} . \]

Johnson et al. 2010, PASP, 122, 905
Giant Planet Formation:

- Two planet formation mechanisms: core-accretion and disk instability.
- Disk instability formation predicts no strong correlations with stellar properties.
- Core-accretion planet formation should scale with surface density of solids (thus metallicity and disk mass).
- On the contrary, there is evidence that disk instability forms planets less efficiently as metallicity increases (Meru & Bates 2010).
- The Planet-Metallicity/Mass Correlation strongly favors core-accretion planet formation (at least for the separations and masses included in the RV studies).
- Planet frequency rises with stellar mass and period may suggest a higher likelihood of directly imaging planets around massive stars.
Small/low-mass Exoplanets
(Super-Earth to Neptune-mass)
Radial Velocity (bias correction)

Fig. 10. Observed mass histogram for the planets in the combined sample. Before any bias correction, we can already notice the importance of the sub-population of low-mass planets. We also remark a gap in the histogram between planets with masses above and below \( \sim 30 M_{\oplus} \).

Fig. 12. Histograms of planetary masses, comparing the observed histogram (black line) and the equivalent histogram after correction for the detection bias (red line).

Mayor et al. 2011, arXiv:1109.2497
The majority of low-mass planets have $P < 100$ days

Mayor et al. 2011, arXiv:1109.2497
Extreme eccentricities are less common among low-mass planets compared to giants.

Mayor et al. 2011, arXiv:1109.2497
Transits

Only solar-type stars

R > 1.5 Rjup ... rare
P < 2 days ... rare
1 - 2 Rearth not complete (could be higher)

here planet occurrence, f, as the fraction of a defined population of stars (in Teff, log g, Kp) having planets within a domain of planet radius and period, including all orbital inclinations.

(Kp is Kep’s magnitude ~ 14 or so)

Transit

Solar-type stars

Independent search with consideration of completeness.

Figure 11. Distribution of planet occurrence for $R_P$ ranging from 1.0 to 8.0 $R_E$. We quote the fraction of Sun-like stars harboring a planet with $P = 5–50$ days for each $R_P$ bin. We observe a rapid rise in planet occurrence from 8.0 down to 2.8 $R_E$, as seen in H12. Below 2.8 $R_E$, the occurrence distribution is consistent with flat. This result rules out a power law increase in planet occurrence toward smaller radii. Adding up the two smallest radius bins, we find $15.1^{+1.8}_{-2.7}$% of Sun-like stars harbor a 1.0–2.0 $R_E$ planet within $\sim0.25$ AU. To compute occurrence as a function of $R_P$, we simply sum occurrence rates for all period bins shown in Figure 10. Errors due to counting statistics are computed by adding errors from each of the three period bins in quadrature. The gray portion of the histogram shows occurrence values before correcting for missed planets due to pipeline incompleteness. Our correction to account for missed planets is shown in red, and is determined by the injection and recovery of synthetic transits described in Section 5. We do not show occurrence values where the completeness is $<50%$.

Transit

Solar-type stars

With consideration of completeness and false-positive rate.

Figure 7. Average number of planets per size bin for main-sequence FGKM stars, determined here from the Q1–Q6 Kepler data and corrected for false positives and incompleteness.

Transit

Figure 11. Distribution of planet occurrence for \( R_P \) ranging from 1.0 to 8.0 \( R_E \). We quote the fraction of Sun-like stars harboring a planet with \( P = 5-50 \) days for each \( R_P \) bin. We observe a rapid rise in planet occurrence from 8.0 down to 2.8 \( R_E \), as seen in H12. Below 2.8 \( R_E \), the occurrence distribution is consistent with flat. This result rules out a power law increase in planet occurrence toward smaller radii. Adding up the two smallest radius bins, we find 15.1\( ^{+1.5}_{-2.7} \)% of Sun-like stars harbor a 1.0–2.0 \( R_E \) planet within \( \sim 0.25 \) AU. To compute occurrence as a function of \( R_P \), we simply sum occurrence rates for all period bins shown in Figure 10. Errors due to counting statistics are computed by adding errors from each of the three period bins in quadrature. The gray portion of the histogram shows occurrence values before correcting for missed planets due to pipeline incompleteness. Our correction to account for missed planets is shown in red, and is determined by the injection and recovery of synthetic transits described in Section 5. We do not show occurrence values where the completeness is \(<50\%\).
$5.7^{+2.2}_{-1.7}\%$ of Sun-like stars have a planet with $P=200-400$ d, $R_P = 1-2R_E$
See Mulders et al. 2015 (both papers)
Transit

Kepler M dwarfs

Figure 16. Planet occurrence rate as a function of planet radius for all candidates (black) and candidates with orbital periods shorter than <10 days (green) or between 10 and 50 days (purple). The error bars indicate the errors from binomial statistics and do not include errors from the stellar and planetary radius estimates.

small planets around M stars is 3.5 x more common than around FGK stars.

See Mulders et al. 2015 (both papers)
Radial Velocity vs. Transit

Mostly solar-type stars

Howard 2013, Science, 340, 572
Transit

Kepler M dwarfs

$0.15^{+0.13}_{-0.06}$ HZ planets per cool star

$>0.04$ HZ planets at 95% confidence

So, more than a billion HZ Earth-size planets around M dwarfs in the Galaxy???

($\sim 75\%$ of stars are M dwarfs)

Cool star HZ Per $\sim 17$ to $148$ days ($0.08$ to $0.4$ AU)

Radial Velocity:
No Small Planet – Host Star Metallicity Correlation

Fig. 16. Histograms of host star metallicities ([Fe/H]) for giant gaseous planets (black), for planets less massive than 30M⊕ (red), and for the global combined sample stars (blue). The latter histogram has been multiplied by 0.1 for visual comparison reason.

Mayor et al. 2011, arXiv:1109.2497
Transits:
No Small Planet – Host Star Metallicity Correlation

Terrestrial planets are common, ind. of metallicity (apparently can form around stars with 1/4 the Sun’s metallicity!)

Transit and Radial Velocity

The Rossiter-McLaughlin (RM) effect
Transit and Radial Velocity

The Rossiter-McLaughlin (RM) effect

HAT-P-6 (Albrecht et al. 2012)
Transit and Radial Velocity

The Rossiter-McLaughlin (RM) effect

Hot Jupiters only!

Transits

Short-period planets in multiple systems seem to be co-planar to within a few degrees.

\[ \frac{b_{out}}{b_{in}} = \frac{a_{out}}{a_{in}} \quad \text{(for coplanar,circ)} \]

\[ T_{dur} \simeq \left( (1 + r)^2 - b^2 \right)^{0.5} \frac{R_*}{V_{orb}} \]

\[ V_{orb} \propto P^{-1/3} \]

Ratio of normalized Tdur propto impact parameter ratio (in/out)

Fabrycky et al. 2014
Short-period multi-planet systems tend to reside just outside of mean motion resonances.

Microlensing

~ 17% (+6 -9) of stars host 0.3 -- 10 Mjup planets between 0.5 and 10AU.

~ 50 to 60% of stars host 5 to 30 Mearth planets between 0.5 and 10AU.

Microlensing

Figure 2 | Cool-planet mass function. a, The cool-planet mass function, $f$, for the orbital range $0.5-10$ AU as derived by microlensing. Red solid line, best fit for this study, based on combining the results from PLANET 2002–07 and previous microlensing estimates for slope (blue line; error, light-blue shaded area, s.d.) and normalization (blue point; error bars, s.d.). We find $dN/(dM d \log M) = 10^{-0.62 \pm 0.22} (M/M_{\text{Sat}})^{0.73 \pm 0.17}$, where $N$ is the average number of planets per star, $a$ the semi-major axis and $M$ the planet mass. The pivot point of the power-law mass function is at the mass of Saturn ($M_{\text{Sat}} = 95 M_\oplus$). The grey shaded area is the 68% confidence interval around the median (dash-dotted black line). For comparison, the constraint from Doppler measurements (green line and point; error, green shaded area, s.d.) is also displayed. Differences can arise because the Doppler technique focuses mostly on solar-like stars, whereas microlensing a priori probes all types of host stars. Moreover, microlensing planets are located further away from their stars and are cooler than Doppler planets. These two populations of planets may then follow a rather different mass function. b, PLANET 2002–07 sensitivity, $S$: the expected number of detections if all stars had exactly one planet, regardless of its orbit. c, PLANET 2002–07 detections, $k$. Thin black curves, distribution probabilities of the mass for the three detections contained in the PLANET sample; red line, the sum of these distributions.


“all stars”

“solar-like stars”
Direct Imaging (massive planets + wide orbits)

Gemini NICI planet finding campaign:

**Massive (B- and A-type) Stars**

fewer than 20% of $2 \, M_\odot$ stars can have giant planets greater than $4 \, M_{\text{Jup}}$ between 59 and 460 AU at 95% confidence, and fewer than 10% of these stars can have a planet more massive than $10 \, M_{\text{Jup}}$ between 38 and 650 AU. Overall, we find that large-separation giant planets are not common around B and A stars: fewer than 10% of B and A stars can have an analog to the HR 8799 b ($7 \, M_{\text{Jup}}$, 68 AU) planet at 95% confidence.


**Young Solar-Type Stars**

we restrict the frequency of 1–20 $M_{\text{Jup}}$ companions at semi-major axes from 10–150 AU to <18% at a 95.4% confidence level using DUSTY models and to <6% at a 95.4% using COND models. Our results strongly constrain the frequency of planets within semi-major axes of 50 AU as well. We restrict the frequency of 1–20 $M_{\text{Jup}}$ companions at semi-major axes from 10–50 AU to <21% at a 95.4% confidence level using DUSTY models and to <7% at a 95.4% using COND models.

Statistical Properties of Exoplanets

How do we combine planet frequencies from such different discovery techniques?

\[
\frac{d^2 N_{pl}}{d \log q d \log s} = ? + \frac{d^2 N_{pl}}{d \log \Delta m_p d \log s} = ? + \frac{d^2 N_{pl}}{d \log M_p d \log P} = ? + \frac{d^2 N_{pl}}{d \log R_p d \log P} = ? \quad \Rightarrow ?
\]
Illustration of combining results from multiple survey types (Clanton & Gaudi 2016)

\[
\frac{d^2N_{pl}}{d \log m_p \, d \log a} = \mathcal{A} \left( \frac{m_p}{M_{Sat}} \right)^\alpha \left( \frac{a}{2.5 \, \text{AU}} \right)^\beta.
\]

with an outer cutoff for the separation function: \( a_{\text{out}} \)

Goal: Find set of values \( \{ \alpha, \beta, \mathcal{A}, a_{\text{out}} \} \) that are consistent with the results of RV, microlensing and direct imaging surveys

see Clanton slides here: http://nexsci.caltech.edu/workshop/2016/Christian_Clanton_Sagan_Talk.pdf
Clanton & Gaudi (2016)
Clanton & Gaudi (2016)
Focusing on M dwarf hosts, and first comparing RV and micro:

Clanton & Gaudi (2016)
Assume planet population that matches freq. derived by microlensing surveys and ask how many would RV surveys discover.

\[
\frac{d^2 N_{pl}}{d \log q \ d \log s} = 0.23 \pm 0.1 \left( \frac{q}{5 \times 10^{-4}} \right)^{-0.68 \pm 0.20}
\]

Gould et al. 2010; Sumi et al. 2010

Must translate \((q,s)\) into \((K,\text{Per})\), requires a model of galactic distribution of stars, assumptions about orbits, etc.

Clanton & Gaudi (2016)
HARPS/South (Bonfils et al. 2013) RV survey should fine ~ 1 and they found 1

CPS/TRENDS survey (Montet et al. 2014) should find ~ 5 and found 4

So, *apparent* difference between RV and micro is really only driven by the steepness (in planet mass) of inferred planet freq. by microlensing and the biases of both survey types.

see Clanton & Gaudi (2014)
Now include direct imaging results and RV *trends* (still only thinking about M dwarfs)

Clanton & Gaudi (2016)
Goal: Find set of values \( \{ \alpha, \beta, A, a_{\text{out}} \} \) that are simultaneously consistent with the results of RV, microlensing and direct imaging surveys

For random sets of parameters, translate (mass, sep) \( \rightarrow \) survey observables and see if they match the measured freq. distribution.

- Hot-Start Models
- Cold-Start Models

68% Probability Contours
95% Probability Contours

\[
\frac{d^2 N_{pl}}{d \log m_p \, d \log a} = \mathcal{A} \left( \frac{m_p}{M_{\text{Sat}}} \right)^\alpha \left( \frac{a}{2.5 \text{ AU}} \right)^\beta
\]
Statistical Properties of Exoplanets

Next: Population Synthesis