

## ANALYTIC LIGHT CURVES FOR PLANETARY TRANSIT SEARCHES

KAISEY MANDEL<sup>1</sup> AND ERIC AGOL<sup>1,2</sup>

*Received 2002 September 20; accepted 2002 October 7; published 2002 November 8*

### ABSTRACT

We present exact analytic formulae for the eclipse of a star described by quadratic or nonlinear limb darkening. In the limit that the planet radius is less than a tenth of the stellar radius, we show that the exact light curve can be well approximated by assuming the region of the star blocked by the planet has constant surface brightness. We apply these results to the *Hubble Space Telescope* observations of HD 209458, showing that the ratio of the planetary to stellar radii is  $0.1207 \pm 0.0003$ . These formulae give a fast and accurate means of computing light curves using limb-darkening coefficients from model atmospheres that should aid in the detection, simulation, and parameter fitting of planetary transits.

*Subject headings:* binaries: eclipsing — eclipses — occultations — planetary systems

### 1. INTRODUCTION

The eclipse of the star HD 209458 by an orbiting planet was recently used to measure the size and mass of the planet, which had been found with velocity measurements (Charbonneau et al. 2000; Henry et al. 2000). With this landmark discovery, the planetary transit tool was added to the planet finder’s toolbox, already yielding several planetary candidates (Udalski et al. 2002a, 2002b; Dreizler et al. 2002). Several large surveys that aim to find planets using the transit signature are now being carried out or planned and will soon yield large numbers of light curves requiring fast computation of eclipse models to find the transit needles within the haystack of variability (Borucki et al. 2001; Howell et al. 2000; Mallen-Ornélas et al. 2002; Koch et al. 1998; Deeg et al. 2000; Street et al. 2002). Light-curve fits to transit events may be used to characterize the planet and star, yielding important constraints on planet formation (Cody & Sasselov 2002; Hubbard et al. 2001; Seager & Mallen-Ornélas 2002). The recent activity in this new field of astronomy motivates a return to the equations describing the transit light curve, the subject of this Letter.

The limb darkening of main-sequence stars is typically represented by functions of  $\mu = \cos \theta$ , where  $\theta$  is the angle between the normal to the stellar surface and the line of sight to the observer (Fig. 1a). Claret (2000) has found that the most accurate limb-darkening functions are the quadratic law in  $\mu$  and the “nonlinear” law, which is a Taylor series to fourth order in  $\mu^{1/2}$ ; the latter conserves flux to better than 0.05%. The data require an accurate description of limb darkening as demonstrated by *Hubble Space Telescope* (*HST*) observations of HD 209458 of such high quality that a quadratic limb-darkening law was needed to fit the transit light curve rather than the usual linear limb-darkening law (Brown et al. 2001).

In this Letter, we compute analytic functions for transit light curves for the quadratic and nonlinear limb-darkening laws and make available our codes to the community (§ 7). For treatment of subtler effects during planetary transits, see Seager, Whitney, & Sasselov (2000), Seager & Sasselov (2000), Hubbard et al. (2001), and Hui & Seager (2002). In § 2, we review the light curve of a uniform spherical source. In § 3, we derive the light curve for eclipses of nonlinear limb-darkened stars. In § 4, we give a simpler form in the limit of a quadratic limb-darkening law. In § 5, we give an approximation for the light curve in the case  $p \lesssim 0.1$ , which is very fast to compute and is fairly accurate. In § 6, we apply the results to some example cases, and in § 7 we conclude.

### 2. UNIFORM SOURCE

We model the transit as an eclipse of a spherical star by an opaque, dark sphere. In what follows,  $d$  is the center-to-center distance between the star and the planet,  $r_p$  is the radius of the planet,  $r_*$  is the stellar radius,  $z = d/r_*$  is the normalized separation of the centers, and  $p = r_p/r_*$  is the size ratio (Fig. 1b). The flux relative to the unobscured flux is  $F$ .

For a uniform source, the ratio of obscured to unobscured flux is  $F^e(p, z) = 1 - \lambda^e(p, z)$ , where

$$\lambda^e(p, z) = \begin{cases} 0, & 1 + p < z, \\ \frac{1}{\pi} \left[ p^2 \kappa_0 + \kappa_1 - \sqrt{4z^2 - (1 + z^2 - p^2)^2} \right], & |1 - p| < z \leq 1 + p, \\ p^2, & z \leq 1 - p, \\ 1, & z \leq p - 1, \end{cases} \quad (1)$$

and  $\kappa_1 = \cos^{-1} [(1 - p^2 + z^2)/2z]$ ,  $\kappa_0 = \cos^{-1} [(p^2 + z^2 - 1)/2pz]$ . We next consider the effects of limb darkening.

<sup>1</sup> California Institute of Technology, Mail Code 130-33, Pasadena, CA 91125; kmandel@tapir.caltech.edu, agol@tapir.caltech.edu.

<sup>2</sup> Chandra Fellow.

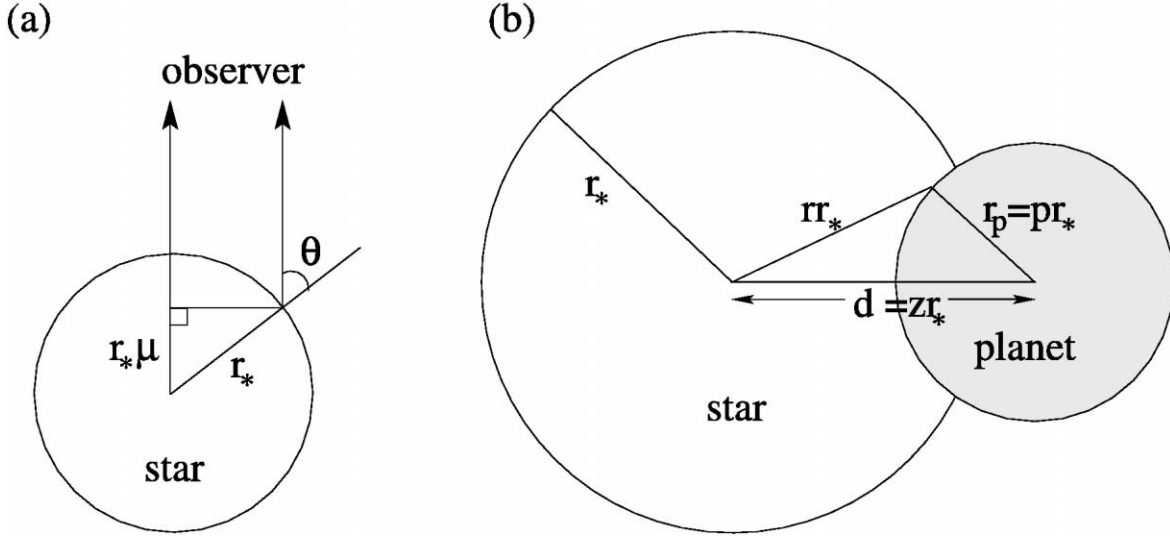


FIG. 1.—(a) Geometry of limb darkening. The star is seen edge-on, with the observer off the top of the page. The star has radius  $r_*$ , and  $\theta$  is defined as the angle between the observer and the normal to the stellar surface, while  $\mu = \cos \theta$ . (b) Transit geometry from the perspective of the observer.

### 3. NONLINEAR LIMB DARKENING

Limb darkening causes a star to be more centrally peaked in brightness compared to a uniform source. This leads to more significant dimming during eclipse and creates curvature in the trough. Thus, including limb darkening is important for computing accurate eclipse light curves. Claret (2000) proposed a nonlinear limb-darkening law that fits well a wide range of stellar models and observational bands,  $I(r) = 1 - \sum_{n=1}^4 c_n (1 - \mu^{n/2})$ , where  $\mu = \cos \theta = (1 - r^2)^{1/2}$ ,  $0 \leq r \leq 1$  is the normalized radial coordinate on the disk of the star and  $I(r)$  is the specific intensity as a function of  $r$  or  $\mu$  with  $I(0) = 1$ . Figure 1a shows the geometry of lensing and the definition of  $\mu$ . The light curve in the limb-darkened case is given by

$$F(p, z) = \left[ \int_0^1 dr 2r I(r) \right]^{-1} \int_0^1 dr I(r) \frac{d[F^e(p/r, z/r)r^2]}{dr}, \quad (2)$$

where  $F^e(p, z)$  is the light curve of a uniform source defined in § 2.

In what follows,  $c_0 \equiv 1 - c_1 - c_2 - c_3 - c_4$ . For convenience, we define  $a \equiv (z - p)^2$ ,  $b \equiv (z + p)^2$ , and  $\Omega = \sum_{n=0}^4 c_n (n + 4)^{-1}$ . We partition the parameter space in  $z$  and  $p$  into the regions and cases listed in Table 1. Next we describe each of these cases in turn.

In case 1, the star is unobscured, so  $F = 1$ . In case 2, the planet disk lies on the limb of the star but does not cover the center of the stellar disk. We define

$$N = \frac{(1-a)^{(n+6)/4}}{(b-a)^{1/2}} B\left(\frac{n+8}{4}, \frac{1}{2}\right) \left[ \frac{z^2-p^2}{a} F_1\left(\frac{1}{2}, 1, \frac{1}{2}, \frac{n+10}{4}; \frac{a-1}{a}, \frac{1-a}{b-a}\right) - {}_2F_1\left(\frac{1}{2}, \frac{1}{2}; \frac{n+10}{4}; \frac{1-a}{b-a}\right) \right]. \quad (3)$$

In the above equations,  $B(a, b)$  is the beta function,  $F_1(a, b_1, b_2, c; x, y)$  is Appell's hypergeometric function of two variables, and  ${}_2F_1(a, b; c; x)$  is the Gauss hypergeometric function. The relative flux is  $F = 1 - (2\pi\Omega)^{-1} \sum_{n=0}^4 N c_n (n+4)^{-1}$ . This case covers the ingress/egress where the light curve is steepest. For cases 3 and 4, the planet's disk lies entirely inside the stellar disk but does not cover the stellar center. We define

$$M = (1-a)^{(n+4)/4} \left[ \frac{z^2-p^2}{a} F_1\left(\frac{1}{2}, -\frac{n+4}{4}, 1, 1; \frac{b-a}{1-a}, \frac{a-b}{a}\right) - {}_2F_1\left(-\frac{n+4}{4}, \frac{1}{2}; 1; \frac{b-a}{1-a}\right) \right] \quad (4)$$

and  $L = p^2 (1 - p^2/2 - z^2)$ . Then the relative flux is given by  $F = 1 - (4\Omega)^{-1} [c_0 p^2 + 2 \sum_{n=1}^3 M c_n (n+4)^{-1} + c_4 L]$ . This case requires the planet to be less than half of the size of the star. In case 5, the edge of the planet touches the center of the stellar disk and the planet lies entirely within the stellar disk. The relative flux is  $F = \frac{1}{2} + (2\Omega)^{-1} \sum_{n=0}^4 c_n (n+4)^{-1} {}_2F_1[\frac{1}{2}, -(n+4)/4, 1; 4p^2]$ . For case 6, the planet's diameter equals the star's radius and the edge of the planet's disk touches both the stellar center and the limb of the star. The relative flux is

$$F = \frac{1}{2} + \frac{1}{2\sqrt{\pi}\Omega} \sum_{n=0}^4 \frac{c_n}{n+4} \Gamma\left(\frac{3}{2} + \frac{n}{4}\right) \Gamma\left(2 + \frac{n}{4}\right). \quad (5)$$

TABLE 1  
LIMB-DARKENED OCCULTATION

Case	$p$	$z$	$\lambda^d(z)$	$\eta^d(z)$
1 .....	$(0, \infty)$	$[1 + p, \infty)$	0	0
	0	$[0, \infty)$	0	0
2 .....	$(0, \infty)$	$(\frac{1}{2} +  p - \frac{1}{2} , 1 + p)$	$\lambda_1$	$\eta_1$
3 .....	$(0, \frac{1}{2})$	$(p, 1 - p)$	$\lambda_2$	$\eta_2$
4 .....	$(0, \frac{1}{2})$	$1 - p$	$\lambda_5$	$\eta_2$
5 .....	$(0, \frac{1}{2})$	$p$	$\lambda_4$	$\eta_2$
6 .....	$\frac{1}{2}$	$\frac{1}{2}$	$1/3 - 4/9\pi$	$3/32$
7 .....	$(\frac{1}{2}, \infty)$	$p$	$\lambda_3$	$\eta_1$
8 .....	$(\frac{1}{2}, \infty)$	$[ 1 - p , p)$	$\lambda_1$	$\eta_1$
9 .....	$(0, 1)$	$(0, \frac{1}{2} -  p - \frac{1}{2} )$	$\lambda_2$	$\eta_2$
10 .....	$(0, 1)$	0	$\lambda_6$	$\eta_2$
11 .....	$(1, \infty)$	$[0, p - 1)$	1	1

In case 7, the edge of the planet’s disk touches the stellar center, but the planet is not entirely contained inside the area of the stellar disk. The relative flux is

$$F = \frac{1}{2} + \frac{1}{4p\pi\Omega} \sum_{n=0}^4 \frac{c_n}{n+4} B\left(\frac{1}{2}, \frac{n+8}{4}\right) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}; \frac{5}{2} + \frac{n}{4}; \frac{1}{4p^2}\right). \tag{6}$$

In case 8, the planet covers the center and limb of the stellar disk. The relative flux is  $F = -(2\pi\Omega)^{-1} \sum_{n=0}^4 c_n N(n+4)^{-1}$ , except at  $p = 1, z = 0$ , when  $F = 0$ . This and the previous case apply when the planet is larger than half the size of the star. For case 9, the planet’s disk lies entirely inside the stellar disk, and the planet covers the stellar center. The relative flux is  $F = (4\Omega)^{-1} [c_0(1 - p^2) + c_4(\frac{1}{2} - L) - 2 \sum_{n=1}^3 c_n(n+4)^{-1}M]$ . This is the bottom of the transit trough for nearly edge-on inclinations if  $p \ll 1$ . In case 10, the planet is concentric with the disk of the star, at the precise bottom of the transit trough. In this case,  $F = \Omega^{-1} \sum_{n=0}^4 c_n(1 - p^2)^{(n+4)/4}(n+4)^{-1}$ . This formula applies only for edge-on orbits when there is a central transit. Finally, in case 11 the planet completely eclipses the star, so that  $F = 0$ . In this case, the “planet” is likely a star. In the event that  $c_1 = c_3 = 0$ , these light curves can be simplified as we describe in § 4.

4. QUADRATIC LIMB DARKENING

In this section, we describe the limb darkening with a function that is quadratic in  $\mu, I(r) = 1 - \gamma_1(1 - \mu) - \gamma_2(1 - \mu)^2$ , where  $\gamma_1 + \gamma_2 < 1$ . The nonlinear law in § 3 reduces to this case when  $c_1 = c_3 = 0, c_2 = \gamma_1 + 2\gamma_2$ , and  $c_4 = -\gamma_2$ . In this limit, the hypergeometric functions reduce to elliptic integrals, which are much faster to compute, so in this section we provide these simpler formulae.

For a quadratic limb-darkening law, the light curve is  $F = 1 - (4\Omega)^{-1} \{(1 - c_2)\lambda^e + c_2 [\lambda^d + \frac{2}{3}\Theta(p - z)] - c_4\eta^d\}$ , where  $\lambda^e$  is defined in equation (1), while  $\lambda^d$  and  $\eta^d$  are given in Table 1.

In Table 1, the various functions are

$$\begin{aligned} \lambda_1 &= \frac{1}{9\pi\sqrt{pz}} \left\{ [(1 - b)(2b + a - 3) - 3q(b - 2)]K(k) + 4pz(z^2 + 7p^2 - 4)E(k) - 3\frac{q}{a} \Pi\left(\frac{a-1}{a}, k\right) \right\}, \\ \lambda_2 &= \frac{2}{9\pi\sqrt{1-a}} \left[ (1 - 5z^2 + p^2 + q^2)K(k^{-1}) + (1 - a)(z^2 + 7p^2 - 4)E(k^{-1}) - 3\frac{q}{a} \Pi\left(\frac{a-b}{a}, k^{-1}\right) \right], \\ \lambda_3 &= \frac{1}{3} + \frac{16p}{9\pi} (2p^2 - 1)E\left(\frac{1}{2k}\right) - \frac{(1 - 4p^2)(3 - 8p^2)}{9\pi p} K\left(\frac{1}{2k}\right), \\ \lambda_4 &= \frac{1}{3} + \frac{2}{9\pi} [4(2p^2 - 1)E(2k) + (1 - 4p^2)K(2k)], \\ \lambda_5 &= \frac{2}{3\pi} \cos^{-1}(1 - 2p) - \frac{4}{9\pi} (3 + 2p - 8p^2), \\ \lambda_6 &= -\frac{2}{3} (1 - p^2)^{3/2}, \\ \eta_1 &= (2\pi)^{-1} \left[ \kappa_1 + 2\eta_2\kappa_0 - \frac{1}{4} (1 + 5p^2 + z^2)\sqrt{(1-a)(b-1)} \right], \\ \eta_2 &= \frac{p^2}{2} (p^2 + 2z^2), \end{aligned} \tag{7}$$

where  $k = [(1 - a)/(4zp)]^{1/2}$  and  $q = p^2 - z^2$ . Here  $\Pi(n, k)$  is the complete elliptic integral of the third kind with the sign convention of Gradshteyn & Ryzhik (1994). For linear limb darkening,  $\gamma_2 = 0$ , Merrill (1950) presents an equivalent analytic expression in terms of an ‘‘eclipse function,’’  $\alpha$ . The expressions here require fewer evaluations of the elliptic integrals, which decreases computation time, and include quadratic limb darkening. Our expression for eclipse with quadratic limb darkening decreases computation time by more than an order of magnitude compared to evaluating the expressions in § 3 or numerical integration of the unocculted flux.

#### 5. SMALL PLANETS

For a small planet,  $p \lesssim 0.1$ , the interior of the light curve,  $z < 1 - p$ , can be approximated by assuming the surface brightness of the star is constant under the disk of the planet, so that  $F = 1 - p^2 I^*(z)/(4\Omega)$  and  $I^*(z) = (4zp)^{-1} \int_{z-p}^{z+p} I(r)2r dr$ . If one knows the limb-darkening coefficients of the star in question (from, say, spectral information), and if the semimajor axis is much larger than the size of the star so that the orbit can be approximated by a straight line, then the shape of the eclipse for  $p \lesssim 0.1$  is simply determined by the smallest impact parameter,  $z_0 = a_p r_*^{-1} \cos i$ , where  $a_p$  is the semimajor axis and  $i$  is the inclination angle. For example, at the midpoint of the eclipse,  $z = z_0$ , while at the one-fourth and three-fourth phases of the eclipse,  $z = z_{1/4} = (1 + 3z_0^2)^{1/2}/2$ . Taking the ratio of the depth of the eclipse at these points yields  $R = [1 - F(z_0)]/[1 - F(z_{1/4})] = I(z_0)/I(z_{1/4})$ . This then determines an equation for  $z_0$ , and the resulting  $z_0$  can then be used to determine  $p = \{4\Omega[1 - F(z_0)]/I(z_0)\}^{1/2}$ . When  $1 - p < z < 1 + p$  and  $p \lesssim 0.1$ , an approximation to the light curve is

$$F = 1 - \frac{I^*(z)}{4\Omega} \left[ p^2 \cos^{-1} \left( \frac{z-1}{p} \right) - (z-1) \sqrt{p^2 - (z-1)^2} \right], \quad (8)$$

where  $I^*(z) = (1 - a)^{-1} \int_{z-p}^1 I(r)2r dr$ , which is accurate to better than 2% of  $1 - F(0)$  for  $p = 0.1$  and  $\sum_{n=1}^4 c_n \leq 1$  (see Fig. 2). This is a very fast means of computing transit light curves with reasonable accuracy and may be used for any limb-darkening function, generalizing the approach of Deeg, Garrido, & Claret (2001).

#### 6. DISCUSSION

Figure 2 shows five light curves, the first of which has  $c_n = 0$ ,  $\{n = 1, 4\}$ , while the other four have  $c_n = 1$ ,  $c_m = 0$ ,  $\{m \neq n\}$  for  $p = 0.1$ ; these may be thought of as a basis set for any nonlinear limb darkening. Note that the higher order functions have flux that is concentrated more strongly toward the center of the star and thus have a more gradual ingress and a deeper minimum as more flux is blocked at the center than the edge. All of the curves cross near  $z \sim 0.7$ , which means that accurate observations are required near minimum and egress/ingress to constrain the coefficients of the various basis functions. If the inclination is large enough that  $z \gtrsim 0.7$  for the entire transit, then it may be difficult to constrain the  $c_n$ 's.

As Claret (2000) claims that the nonlinear limb-darkening law is the most accurate, we have compared the transit with  $p = 0.1$  for the nonlinear and quadratic laws. Of the entire grid of models computed by Claret (2000), which covers  $2000 \text{ K} < T_{\text{eff}} < 50,000 \text{ K}$ ,  $0 < \log g < 5$ ,  $-5 < [\text{M}/\text{H}] < 1$ , and filters  $u, b, v, y, U, B, V, R, I, J, H$ , and  $K$ , the largest difference between the quadratic and nonlinear models for  $p = 0.1$  is 3% of the maximum of  $1 - F$ . Thus, the quadratic law should be sufficient for main-sequence stars when an accuracy of less than 3% is required; indeed, the average difference for the entire grid of models is about 1% of the maximum value of  $1 - F$  for each light curve. In absolute terms, this is about  $10^{-4}(p/0.1)^2$  of the total flux, an accuracy that can be achieved from space.

So far we have only presented the light curve as a function of  $z$  and  $p$ . To determine  $z$  as a function of time requires the planetary orbital parameters, which for zero eccentricity is given as  $z = a_p r_*^{-1} [(\sin \omega t)^2 + (\cos i \cos \omega t)^2]^{1/2}$ , where  $\omega$  is the orbital frequency, while  $t$  is the time measured from the center of the transit. Contribution of flux from the planetary companion or other companions may be added to the light curve, reducing the transit depth.

To illustrate the utility of our formulae, we have fitted the nonlinear limb-darkened light curve to the *HST* Space Telescope Imaging Spectrograph (STIS) data of HD 209458 (Brown et al. 2001). The best-fit parameters in this case are  $p = 0.12070 \pm 0.00027$ ,  $i = 86.591 \pm 0.055$ ,  $a_p/r_* = 8.779 \pm 0.032$ ,  $c_1 = 0.701$ ,  $c_2 = 0.149$ ,  $c_3 = 0.277$ , and  $c_4 = -0.297$ , with a reduced  $\chi^2 = 1.046$ . The  $c_n$ 's are poorly constrained given the small differences in the basis functions relative to the observed errors and the large impact parameter for this system. The errors on the other parameters are marginalized over the  $c_n$ 's. Limiting the limb darkening to quadratic, we find  $\gamma_1 = 0.296 \pm 0.025$  and  $\gamma_2 = 0.34 \pm 0.04$ , consistent with the values derived by Brown et al. (2001) and with stellar atmosphere predictions. The value for  $p$  in the quadratic case is consistent with the nonlinear case, indicating that the fit is independent of the assumed limb-darkening law. For a stellar mass of  $1.1 \pm 0.1 M_\odot$  (Mazeh et al. 2000) and period of  $T = 3.5248$  days (Brown et al. 2001), we find  $r_* = 1.145 \pm 0.035 R_\odot$  and  $r_p = 1.376 \pm 0.043 R_{\text{Jup}}$ .

We apply the small-planet approximation described in § 5 to HD 209458, assuming quadratic limb darkening. Cody & Sasselov (2002) determined the effective temperature and surface gravity for this star, which imply  $\gamma_1 = 0.292$  and  $\gamma_2 = 0.35$  for the *I*-band flux (close to the effective wavelength of the *HST* STIS data) from the models of Claret (2000). From the light curve, one finds  $F(z_0) = 0.9835$  and  $F(z_{1/4}) = 0.9847$ . Solving the equations from § 5 gives  $z_0 = 0.546$  and  $p = 0.12$ , very similar to the parameters derived by a fit to the entire light curve. This technique may be used for finding initial parameters for light-curve fitting.

#### 7. CONCLUSIONS

We have derived analytic expressions for an eclipse including quadratic limb darkening and nonlinear limb darkening. The nonlinear law (§ 3) provides an accurate fit to realistic stellar limb darkening, while the quadratic fit (§ 4) provides a fast means of obtaining a relatively accurate light curve. For an extremely fast and fairly accurate approximation for any limb-darkening law,

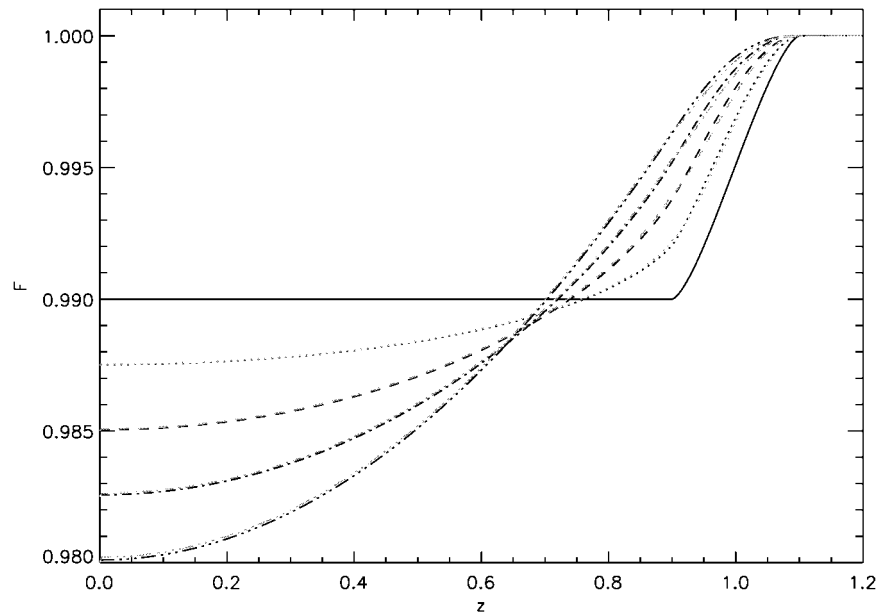


FIG. 2.—Transit light curves for  $p = 0.1$  and  $c_1 = c_2 = c_3 = c_4 = 0$  (solid line), and all coefficients equal zero but  $c_1 = 1$  (dotted line),  $c_2 = 1$  (dashed line),  $c_3 = 1$  (dash-dotted line), or  $c_4 = 1$  (dash-triple-dotted line). The thinner lines (nearly indistinguishable) show the approximation of § 5.

the equations in § 5 may be used to derive light curves. If the limb-darkening law is known from the spectral type of the star, then one can use the formulae in § 5 to analytically estimate both the minimum impact parameter (in units of stellar radius) and the ratio of the planetary radius to the stellar radius. We have written a code that takes the properties of a host star, finds the limb-darkening coefficients in the tables of Claret (2000), and computes light curves for the parameters of a given planetary transit. This code will be useful for simulating planetary transit searches (Gaudi 2000; Defaÿ, Deleuil, & Barge 2001; Jenkins, Caldwell, & Borucki 2002; Remund et al. 2002; Jenkins 2002; Pepper & Gould 2002), searching for planetary transit signals in light curves collected by a given search, and fitting and measuring the errors of the parameters of detected planetary transit events. Planetary searches suffer from two important backgrounds: grazing eclipsing binaries and triple systems in which two stars eclipse while the flux from the third reduces the depth of the eclipse. Using the appropriate limb-darkening coefficients for each star's spectral type will help to distinguish these contaminants from true planetary transits, which can be accomplished using the formulae presented here. These routines are available from the authors.<sup>3</sup>

We thank Sara Seager and Leon Koopmans for useful discussions. Support for E. A. was provided by the National Aeronautics and Space Administration through *Chandra* Postdoctoral Fellowship award PF0-10013 issued by the *Chandra* X-Ray Observatory Center, which is operated by the Smithsonian Astrophysical Observatory for and on behalf of the National Aeronautics Space Administration under contract NAS8-39073. K. M. was supported by a Caltech Summer Undergraduate Research Fellowship.

<sup>3</sup> See <http://www.astro.washington.edu/agol>.

#### REFERENCES

- Borucki, W. J., et al. 2001, *PASP*, 113, 439  
 Brown, T. M., Charbonneau, D., Gilliland, R. L., Noyes, R. W., & Burrows, A. 2001, *ApJ*, 552, 699  
 Charbonneau, D., Brown, T., Latham, D., & Mayor, M. 2000, *ApJ*, 529, L45  
 Claret, A. 2000, *A&A*, 363, 1081  
 Cody, A. M., & Sasselov, D. D. 2002, *ApJ*, 569, 451  
 Deeg, H. J., Garrido, R., & Claret, A. 2001, *NewA*, 6, 51  
 Deeg, H. J., et al. 2000, in *ASP Conf. Ser. 219, Disks, Planetessimals, and Planets*, ed. F. Garzón, C. Eiroa, D. de Winter, & T. J. Mahoney (San Francisco: ASP), 578  
 Defaÿ, C., Deleuil, M., & Barge, P. 2001, *A&A*, 365, 330  
 Dreizler, S., Rauch, T., Hauschildt, P., Schuh, S. L., Kley, W., & Werner, K. 2002, *A&A*, 391, L17  
 Gaudi, B. S. 2000, *ApJ*, 539, L59  
 Gradshteyn, I. S., & Ryzhik, I. M. 1994, *Table of Integrals, Series, and Products* (New York: Academic)  
 Henry, G. W., Marcy, G. W., Butler, R. P., & Vogt, S. S. 2000, *ApJ*, 529, L41  
 Howell, S. B., Everett, M., Davis, D. R., Weidenschilling, S. J., McGruder, C. H., III, & Gelderman, R. 2000, *AAS DPS Meeting*, 32, 32.03  
 Hubbard, W. B., Fortney, J. J., Lunine, J. I., Burrows, A., Sudarsky, D., & Pinto, P. 2001, *ApJ*, 560, 413  
 Hui, L., & Seager, S. 2002, *ApJ*, 572, 540  
 Jenkins, J. M. 2002, *ApJ*, 575, 493  
 Jenkins, J. M., Caldwell, D. A., & Borucki, W. J. 2002, *ApJ*, 564, 495  
 Koch, D., Borucki, W., Webster, L., Dunham, E., Jenkins, J., Marriot, J., & Reitsema, H. 1998, *Proc. SPIE*, 3356, 599  
 Mallen-Ornelas, G., Seager, S., Yee, H. K. C., Minniti, D., Gladders, M. D., Mallen-Fullerton, G., & Brown, T. M. 2002, *ApJ*, submitted (astro-ph/0203218)  
 Mazeh, T., et al. 2000, *ApJ*, 532, L55  
 Merrill, J. E. 1950, *Contrib. Princeton Univ. Obs.* 23  
 Pepper, J., & Gould, A. 2002, *ApJ*, submitted (astro-ph/0208042)  
 Remund, Q. P., Jordan, S. P., Updike, T. F., Jenkins, J. M., & Borucki, W. J. 2002, *Proc. SPIE*, 4495, 182  
 Seager, S., & Mallen-Ornelas, G. 2002, *ApJ*, submitted (astro-ph/0206228)  
 Seager, S., & Sasselov, D. D. 2000, *ApJ*, 537, 916  
 Seager, S., Whitney, B. A., & Sasselov, D. D. 2000, *ApJ*, 540, 504  
 Street, R. A., et al. 2002, preprint (astro-ph/0208154)  
 Udalski, A., et al. 2002a, *Acta Astron.*, 52, 1  
 ———. 2002b, *Acta Astron.*, 52, 115