PTYS558 – Plasma Physics with Astrophysical and Solar System Applications

Problem Set #1 – Due Monday, Feb. 3

1. Verify the following vector identities

   a. $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$

   b. $\nabla(\vec{A} \cdot \vec{B}) = \vec{A} \times (\nabla \times \vec{B}) + \vec{B} \times (\nabla \times \vec{A}) + (\vec{A} \cdot \nabla)\vec{B} + (\vec{B} \cdot \nabla)\vec{A}$

   c. $\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$

   d. $\nabla \cdot (\nabla \times \vec{A}) = 0$

2. Obtain the full equations for x(t), y(t), and z(t) describing the trajectory of a charged particle, q, mass, m, injected with a velocity **v** = (v_{x0},v_{y0},v_{z0}) into a region with a spatially uniform magnetic field **B**=(0,0,B). Take the starting position to be (x_0,y_0,z_0). Show that the particle kinetic energy is conserved. Obtain expressions for the gyrofrequency and gyroradius.

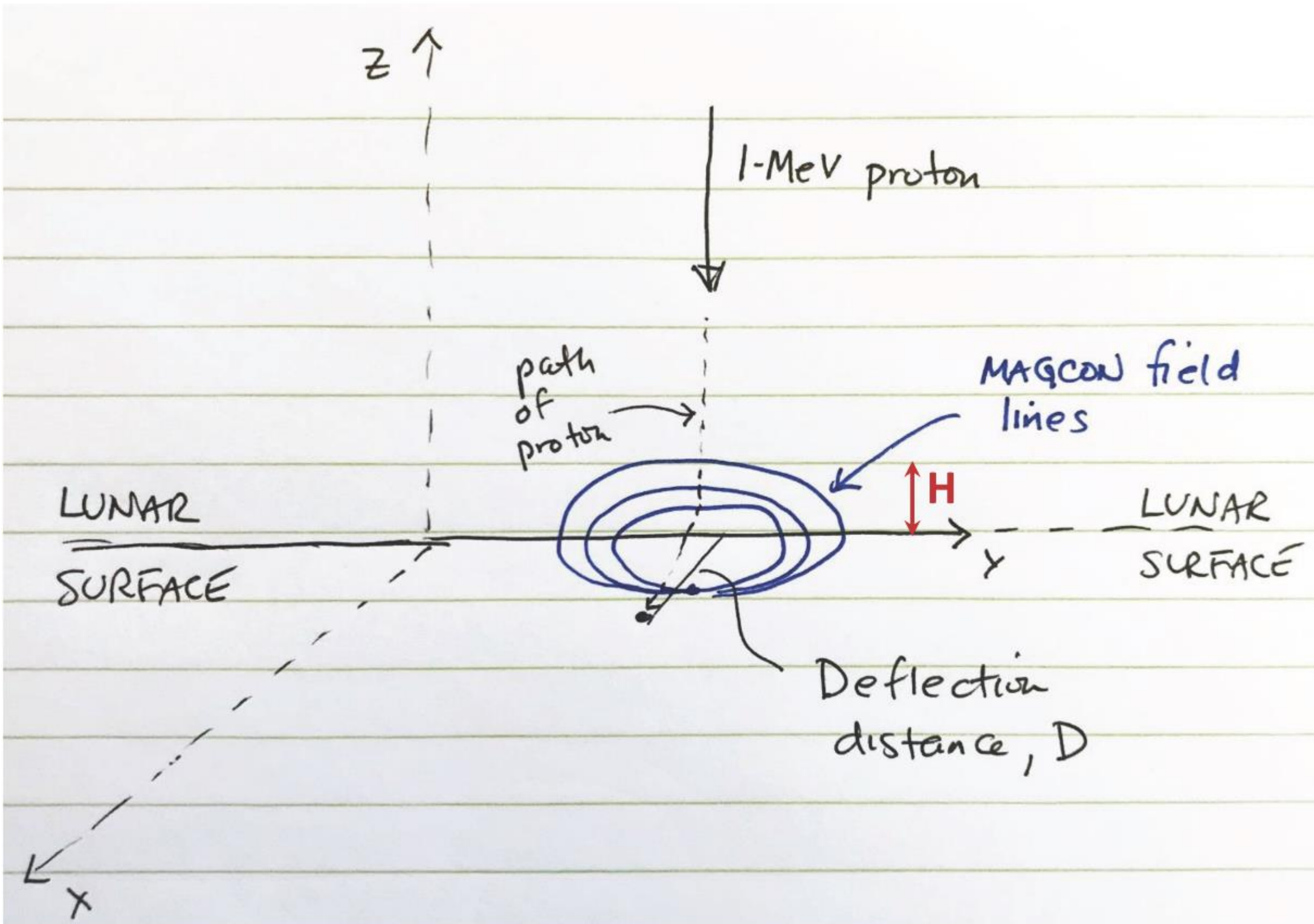
3. Consider a charged particle released in Earth's dipolar magnetic field above the equator with a pitch angle of 70 degrees. Determine the latitudes of its mirror points. Assume the dipolar magnetic field strength is given by:

   $$B(r) = M\,(1 + 3\sin^2\lambda)\ /\ r^3$$

   where M is the dipole moment of Earth's field, λ is latitude, and r is measured from the center of the Earth. To solve this problem you will also need the equation for a field line, which is $r = r_0 \cos^2\lambda$, where r_0 is the equatorial crossing distance of the field line.

4. Consider a 1-MeV proton moving directly normal to the surface of the Moon at a location where there exists a localized magnetic anomaly (a so-called "magcon"). The magnetic anomaly is like a dipole, laying on its side, so that the magnetic field lines are parallel to the surface of the Moon, as shown in the diagram at the top of the next page. For this problem, it is sufficient to treat the magnetic field (directly above the magcon) to be constant over the scale H, shown in the diagram. Take H to be 50 km and the magcon field strength to be $5\text{x}10^{-4}$ Gauss. Assume that at heights above H, the magcon field is negligible.

   a. Determine the initial speed of the proton and its gyroradius in the magcon field.
   b. Estimate the distance, D, shown in the diagram, that this proton is deflected by the magcon (this is the point along the x direction that the proton strikes the surface, and x=0 is where it would hit if there was no magcon).



5. (extra credit challenge)

The magnetic field of a monopole points in the radial direction with a magnitude:

$$B(r) = k / r^2$$

where k is a constant and r is the distance from the monopole.

Consider a particle with charge q, mass m, and velocity vector **v**, placed in this field. Show that the geometrical figure of its orbit is a spiral on the surface of a cone.