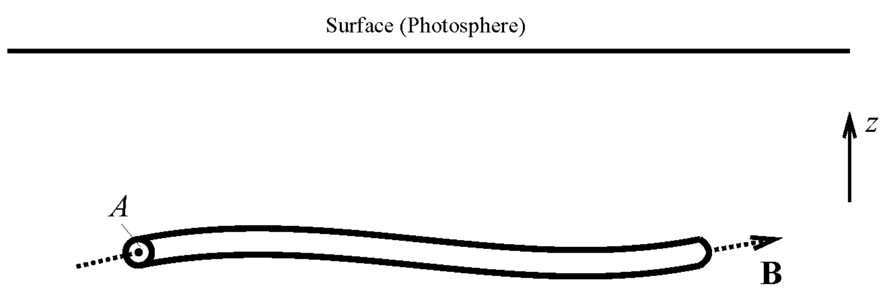
PTYS558 – Plasma Physics with Astrophysical and Solar System Applications

Problem Set #2 – Due Wednesday, Feb. 19

1. Consider a thin “tube” of magnetic flux isolated within the solar interior with cross-sectional area, *A*, as shown below. Consider the flux tube to be in steady state.



   a. Suppose that the gas inside this flux tube is in thermal equilibrium with the gas outside of it (locally, i.e., at a particular value of *z*). Show that there is a buoyant force acting on the flux tube that will cause it to rise.
   b. Show that along the direction of the flux tube (the same as the direction of the magnetic field), the gas inside satisfies the equation of hydrostatic equilibrium.
   c. Show that the magnetic field strength as a function of vertical height (*z*) can be written (don’t forget to include the gravity force in the fluid momentum equation!).

$$B(z) = B(z_0)exp\left(-\frac{1}{2}\int_{z_0}^{z}\frac{mg(z')}{kT(z')}dz'\right)$$

   Where *m* is the mean mass of the gas particles, *g* is the gravitational acceleration, *k* is Boltzmann’s constant and *T* is the local gas temperature, and z_0 is the initial height of the flux tube.

   d. (extra credit challenge #1) Show that the relationship between the force due to the tension, F_t, in the magnetic field is related to the buoyant force (per unit length), F_b, through the following relationship.

$$\frac{dF_t}{dz} = -F_b$$

2. Suppose that $\mathcal{E} = (1/2)mv^2 + q\,\phi\,(\mathbf{x})$ is an exact constant of the motion for an individual charged particle. Show that $f(\mathcal{E})$ is a solution to the time-independent Vlasov equation.

3. Consider perfectly (electrically) conducting plasma flowing from the Sun with a constant speed V_0. It carries with it a magnetic field. The flow and field have the analytic forms:

$$\mathbf{V} = V_0\ \hat{r}$$
$$\mathbf{B}(r,\theta) = \frac{a}{r^2}\ \hat{r} + \frac{b}{r}\sin\theta\ \hat{\phi}$$

where a and b are constants. This is an idealized solar wind and Parker-spiral magnetic field (that we will discuss in class soon).

a. Determine the Lorentz force (per volume) acting on the plasma.

b. Determine the electric field vector and resulting charge distribution.

c. Determine the electric force per unit volume resulting from this charge distribution and show that it can be neglected in the single-fluid MHD momentum equation.

4. (extra credit challenge #2)

Consider the energy equation of hydrodynamics (same as we derived, but without the work done by any external forces) assuming a variation of the energy flux in the x direction only and ignoring the work done on the fluid by any external forces, given by:

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho u^2 + \frac{1}{\gamma - 1}p\right) + \frac{\partial}{\partial x}\left(\left[\frac{1}{2}\rho u^2 + \frac{\gamma}{\gamma - 1}p\right]u + Q_x\right) = 0$$

where γ is the ratio of specific heats. Using the continuity and momentum equations (also assuming 1D flow with scalar pressure), show that

$$\frac{\rho^\gamma}{\gamma - 1}\frac{D}{Dt}\left(\frac{P}{\rho^\gamma}\right) = -\frac{\partial Q_x}{\partial x}$$

How would this result change if this were a plasma and the work done on the fluid by the electric field were included?