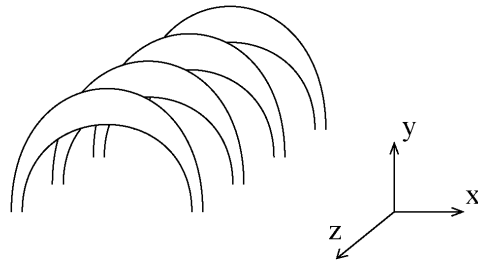


**Problem Set #3 – Due Wednesday, March 4**

1. **The force-free magnetic arcade:** Consider a magnetic field with arched magnetic field lines, like a tunnel. Such features are commonly seen on the Sun, particularly after solar flares. These field lines can be regarded as extending from the Sun, going up to some height, and then returning again to the Sun, as in the sketch below. The main point of this problem is to show that magnetic field lines tend to rise when sheared at the base, and this shearing also adds to the magnetic energy.



Ignore gravity, assume that there are no bulk motions, the arcade is in equilibrium, and there are no variations in the  $z$  direction. Assume a magnetic field of the form

$$\mathbf{B} = B_x(x, y)\hat{x} + B_y(x, y)\hat{y} + B_z(x, y)\hat{z}$$

assume that the field is force free .

- a. Show that the gas pressure is constant everywhere.
- b. For a force-free field, assume  $\text{curl}(\mathbf{B}) = \alpha \mathbf{B}$ , where  $\alpha = \text{constant}$ . Show that

$$B_z = \alpha A + \text{constant}$$

Where  $A$  is the vector potential as we discussed in class. From this point on take the added constant to be zero.

- c. Now consider the following form for the vector potential for the arcade

$$A = -\cos(kx)e^{-qy}$$

where  $k$  and  $q$  are positive constants. Determine the resulting components of the magnetic field (i.e. determine  $B_x$ ,  $B_y$ , and  $B_z$ )

- d. Show that  $q^2 = k^2 - \alpha^2$
- e. Determine the magnetic energy density ( $B^2/8\pi$ ). What happens to the total magnetic energy when  $\alpha$  (the shear) is increased (for  $y>0$ )?
- f. Determine the equations for field lines projected onto the  $x$ - $z$  plane (i.e., what is  $z(x)$  for a field line?). What happens to the field lines when  $\alpha$  is increased from zero, while  $k$  is held fixed?
- g. Determine the equations for field lines projected onto the  $x$ - $y$  plane (i.e., what is  $y(x)$  for a field line?).
- h. Now consider a field line which starts at some  $x=x_0$  and  $y=0$ . Consider the projection of the field lines onto the  $x$ - $y$  plane. Keeping  $k$  and  $x_0$  fixed, what happens to this field line as  $\alpha$  is increased? Draw a little sketch showing the field line for small and large  $\alpha$ .