PTYS558 – Plasma Physics with Astrophysical and Solar System Applications

Problem Set #4 – Due Wednesday, March 25

1. Determine the rate at which mass is being removed from the Sun by the solar wind as a function of the solar wind density and flow speed. Calculate the total amount of mass lost during the Sun's 4.6 Gy lifetime assuming the solar wind speed and density at 1AU have remained constant (4×10^7 cm/s, and 5 particles/cm^{-3}, respectively) during this time.

2. In class we considered the Parker-spiral magnetic field for the case of a non-radial solar wind speed. We assumed that the magnetic stresses were small at large distances from the Sun and ignored the terms involving the magnetic field. In this problem, we'll relax this assumption and include the magnetic stresses. Assume perfectly conducting solar wind plasma in steady state, azimuthally symmetric about the rotation axis of the Sun. For this problem, assume that the magnetic and rotation axes are aligned. Assume that the velocity and magnetic field are given by:

$$\mathbf{V} = V_r \hat{r} + V_\phi \hat{\phi}$$
$$\mathbf{B} = B_r \hat{r} + B_\phi \hat{\phi}$$

a. Show that

$$L = rV_\phi - \frac{rB_rB_\phi}{4\pi\rho V_r} \;=\; constant$$

What is the physical significant of *L*?

b. Obtain an equation for V_ϕ. Use this equation (which should have a singularity) to show that

$$L = R_A^2 \Omega_\odot \sin\theta$$

where R_A is the called the "Alfven radius".

c. Given that the total angular momentum flux density carried by the solar wind across a spherical surface at a heliocentric distance *r* is $\rho V_r L \sin\theta$ (where ρ is the solar wind mass density), show that the rate of solar angular momentum loss is related to the rate of solar mass loss by

$$\frac{dJ_\odot}{dt} = \frac{2}{3} R_A^2 \Omega_\odot \frac{dM_\odot}{dt}$$

d. Determine the rate at which the Sun's rotation is slowing down (i.e. the characteristic time scale of angular momentum loss). Take the Alfven radius = 8.5×10^{11} cm, the present solar mass = 1.99×10^{33} g, rotation frequency = 2.8×10^{-6} sec^{-1}, and total angular momentum = 1.9×10^{48} g-cm^2/s. (Other potentially useful parameters are: solar wind speed = 4×10^7 cm/s and solar wind number density at 1 AU = 5 cm^{-3}).