PTYS558 – Plasma Physics with Astrophysical and Solar System Applications

Problem Set #5 – Due Monday, April 27

1. Derive the following dispersion relation for electromagnetic-wave propagation in an un-magnetized plasma

$$\omega^2 = \omega_{pe}^2 + k^2 c^2$$

where ω_{pe} is the electron plasma frequency and c is the speed of light. (Hint: this is problem 4.1 in Craven's Physics of Solar System Plasmas textbook). Be sure to justify any assumptions made in deriving this expression.

2. This problem is for single-fluid MHD waves.
Consider the transport of wave energy in a stellar atmosphere. Assume that the gas is perfectly (electrically) conducting and the plasma beta is low so that we can neglect thermal pressure (neglect gravitational forces also). Let z define the vertical axis. Take the background, unperturbed magnetic field (B_0) to be vertical, and there is no background flow (i.e. the unperturbed velocity is zero). On the boundary there is a transverse convective flow velocity given by:

$$\delta\mathbf{V}(x, y, 0, t) = V_g \cos\omega t(\hat{x}\cos kx \sin ky - \hat{y}\sin kx \cos ky)$$

   a. Assuming that the mass density is constant at the base (ρ_0), show that it must be constant everywhere.

   b. Linearize the ideal MHD equations, neglecting terms second order in perturbed quantities, to find a wave equation for the velocity, $\delta\mathbf{V}$, for z > 0.

   c. Assume a plane-wave solution (as we did in class) and solve this equation for $\delta\mathbf{V}$ subject to the boundary condition given above**.** Then determine $\delta\mathbf{B}$.

   d. Show that the Alfven waves are in *equipartition*. That is, show that the wave kinetic energy density ($\frac{1}{2}\rho\langle|\delta\mathbf{V}|^2\rangle$) and the magnetic energy density ($\langle|\delta\mathbf{B}|^2\rangle/(8\pi)$) of the waves are equal (note that the <> refers to a time average over one wave period).

(continued on the next page + one more problem)

e. The Poynting flux is given by.

$$\mathbf{F}_w = \frac{c}{4\pi}\delta\mathbf{E} \times \delta\mathbf{B}$$

Use this to determine the magnitude of the time-averaged energy flux carried by the Alfven waves. Note that you must include second-order terms, here, because the first-order terms vanish upon averaging.

f. Evaluate the magnitude of the Poynting flux for Alfven waves propagating out of the solar atmosphere using the typical parameters B_0 = 100G, $\rho_0 = 10^{-15}$ g/cm^3, V_g = 6x10^4 cm/s. How does this number compare with the total flux out of the solar corona (~5x10^6 erg cm^2 sec^{-1}, as discussed in section *9.2.5* of Foukal's book)?

3. Consider plasma waves moving along a mean magnetic field with strength B_0 pointing in the z direction. Assume the plasma consists of protons and electrons (these are plasma waves, not MHD waves). Assume the protons are immobile but maintain charge neutrality. Their number density is n_0. Assume the electrons are initially at rest. Assume the electrons are cold (ignore their thermal pressure).
   a. Write down the relevant equations that will determine how the plasma will respond to small perturbations in the system. Then, perturb the relevant quantities (magnetic field vector, electric field vector, electron velocity vector), and ignore terms that are second order in perturbed quantities. Write down the resulting equations.
   b. Assume plane-wave solutions for the perturbations, as we did in class, to write these equations in terms of the wave vector and frequency (note the waves are ALONG the mean magnetic field, so the wave vector points in the z direction).
   c. Decompose your equations into specific components (no more vectors!). You should have an equal number of equations and unknowns. List the unknowns. Now explain how you would proceed to determine the dispersion relation for these waves.
   d. (5 pts extra credit) Derive the dispersion relation for these waves.