Units:

SI — Joule, Newton, meter, Kilogram, Second, etc. + Coulomb

c-g-s — centimeter, gram, second
dyne, erg, ...

ok for these, but equations are different in E&M and units are different.

$$|\underset{\sim}{F}|_{SI} = k_e \frac{q_1 q_2}{d^2} \qquad SI$$

$$|\underset{\sim}{F}|_{cgs} = \frac{q_1 q_2}{d^2} \qquad cgs$$

SI force between two charges of 1 Coulomb separated by 1 meter

$$9\times10^9 N = k_e \frac{(1C)(1C)}{(1m)^2}$$

$$\boxed{k_c = 9 \times 10^9 \frac{N \cdot m^2}{C^2}}$$

SI

for cgs, a similar question ...

$$\text{dyne} = \frac{(1\ \text{statC})(1\ \text{statC})}{(1\ \text{cm})^2}$$

$$\Rightarrow \boxed{1\ \text{stat C} = \text{dyne}^{1/2} \cdot \text{cm}}$$

note that, for c.g.s. case, imagine setting up two charges of a coulomb separated by 1 meter

$$9 \times 10^9\ N = \frac{(1\ C)^2}{(1\ m)^2}$$

$$9 \times 10^9 \cdot 1000 \cdot 100\ \text{dyne} = \frac{1\ C^2}{(100\ \text{cm})^2}$$

$$\Rightarrow 9 \times 10^{18} \underbrace{\text{dyne} \cdot \text{cm}^2}_{\text{stat Coulomb}} = 1\ C^2$$

$$\Rightarrow 1\ \text{stat C} = \frac{1}{(9 \times 10^{18})^{1/2}}\ C$$

ex. electron charge

$$\boxed{e = 1.6 \times 10^{-19} \text{ Coulombs} \qquad \underline{\text{SI}}}$$

is cgs

$$e = 1.6 \times 10^{-19} \, (9 \times 10^{18})^{1/2} \text{ stat C}$$

$$\boxed{e = 4.8 \times 10^{-10} \text{ stat C} \qquad \text{cgs}}$$

Maxwell's Equations

c.g.s.

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{E} = 4\pi\rho$$

$$\frac{\partial \mathbf{B}}{\partial t} = -c \, \nabla \times \mathbf{E}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}$$

$$\left(\mathbf{B} = \mu \mathbf{H} , \; \mathbf{E} = \frac{1}{\epsilon} \mathbf{D} \; ; \; \epsilon = \mu = 1 \right)$$

in free space

take an E&M wave moving through a vacuum

$$\Rightarrow \quad \underset{\sim}{J}, \rho = 0$$

$$\left.\begin{aligned} \frac{\partial \underset{\sim}{B}}{\partial t} &= -c\, \nabla \times \underset{\sim}{E} \\ \nabla \times \underset{\sim}{B} &= \frac{1}{c} \frac{\partial \underset{\sim}{E}}{\partial t} \end{aligned}\right] \Rightarrow \quad \frac{\partial^2 \underset{\sim}{B}}{\partial t^2} = c^2 \nabla^2 \underset{\sim}{B}$$

wave eq., wave moving w/ speed $\underline{c}$

do same thing for SI

$$\nabla \times \underset{\sim}{E} = -\frac{\partial \underset{\sim}{B}}{\partial t}$$

$$\nabla \times \underset{\sim}{H} = \frac{\partial \underset{\sim}{D}}{\partial t} + \underset{\sim}{J}$$

$$\nabla \cdot \underset{\sim}{D} = \rho$$

$$\nabla \cdot \underset{\sim}{B} = 0$$

take $\rho, \underset{\sim}{J} = 0$

do same as above

we get

$$\frac{\partial^2 \underset{\sim}{B}}{\partial t} = \frac{1}{\epsilon\mu} \nabla^2 \underset{\sim}{B}$$

$$\underset{\sim}{B} = \mu \underset{\sim}{H}, \quad \underset{\sim}{E} = \frac{1}{\epsilon} \underset{\sim}{D}$$

compare w/ above

$$\boxed{\epsilon\mu = 1/c^2}$$

note that $\underset{\sim}{E}$ & $\underset{\sim}{B}$ have the <u>same</u> units in c.g.s.

$$[\underset{\sim}{E}] = \frac{\text{stat volt}}{\text{cm}} \qquad \text{c.g.s.}$$

$$[\underset{\sim}{B}] = \text{Gauss} \qquad \text{c.g.s.}$$

$$\left.\begin{aligned} [\underset{\sim}{E}] &= \frac{\text{volts}}{\text{m}} \\ [\underset{\sim}{B}] &= \text{Tesla} \end{aligned}\right] \quad \text{SI}$$

Consider the Poynting flux

$$\underset{\sim}{S} = \underset{\sim}{E} \times \underset{\sim}{H}$$

$$[\underset{\sim}{S}] \rightarrow \frac{\text{energy}}{\text{area} \cdot \text{time}}$$

$$\nabla \cdot \underset{\sim}{S} = \nabla \cdot (\underset{\sim}{E} \times \underset{\sim}{H})$$

$$= \underset{\sim}{H} \cdot \nabla \times \underset{\sim}{E} - \underset{\sim}{E} \cdot \nabla \times \underset{\sim}{H}$$

$$\nabla \cdot (\underset{\sim}{A} \times \underset{\sim}{B}) = \underset{\sim}{B} \cdot \nabla \times \underset{\sim}{A} - \underset{\sim}{A} \cdot \nabla \times \underset{\sim}{B}$$

use Maxwell's ~~e~~ equations

⋮

$$\frac{\partial}{\partial t}\left(\frac{B^2}{2\mu} + \epsilon E^2\right) + \nabla\cdot \underset{\sim}{S} = 0$$

$$\frac{\partial}{\partial t}\left(\begin{matrix}\text{Energy}\\ \text{Density}\end{matrix}\right) + \nabla\cdot\left(\begin{matrix}\text{Energy}\\ \text{flux}\end{matrix}\right) = 0$$

$$\boxed{\begin{matrix}\text{Energy Density}\\ \text{in } \underset{\sim}{E} \text{ \& } \underset{\sim}{B} \text{ fields}\end{matrix} = \frac{B^2}{2\mu} + \frac{\epsilon E^2}{2} \quad \underline{\underline{\text{SI}}}}$$

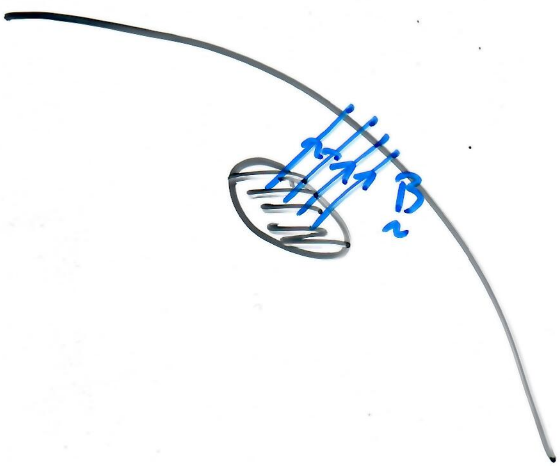
do same thing in c.g.s.

we find

$$\boxed{\begin{matrix}\text{Energy}\\ \text{Density}\\ \underset{\sim}{E} \text{ \& } \underset{\sim}{B} \text{ fields}\end{matrix} = \frac{E^2 + B^2}{8\pi} \quad \text{c.g.s.}}$$

in most relevant applications in heliophysics & astrophysics $E \ll B$!!

Consider a sunspot on the Sun



typical value $|\underset{\sim}{B}| \simeq 1000$ Gauss for a large sunspot

ϵ = energy density in sunspot field $= \dfrac{B^2}{8\pi} = \dfrac{(1000)^2}{8\pi} \dfrac{\text{erg}}{\text{cm}^3}$

$$= \frac{10^6}{8\pi} \frac{\text{erg}}{\text{cm}^3}$$

TOTAL energy in sunspot mag. field $= \epsilon \overset{\text{volume}}{V} \simeq \epsilon L^3$ (sunspot radius L)

large sunspot $L \simeq 3 \times 10^9$ cm

$$= \frac{10^6}{8\pi} \cdot 27 \times 10^{27} \text{ erg}$$

$$= \frac{27}{8\pi} \times 10^{33} \text{ erg} \sim 10^{33} \text{ erg.}$$