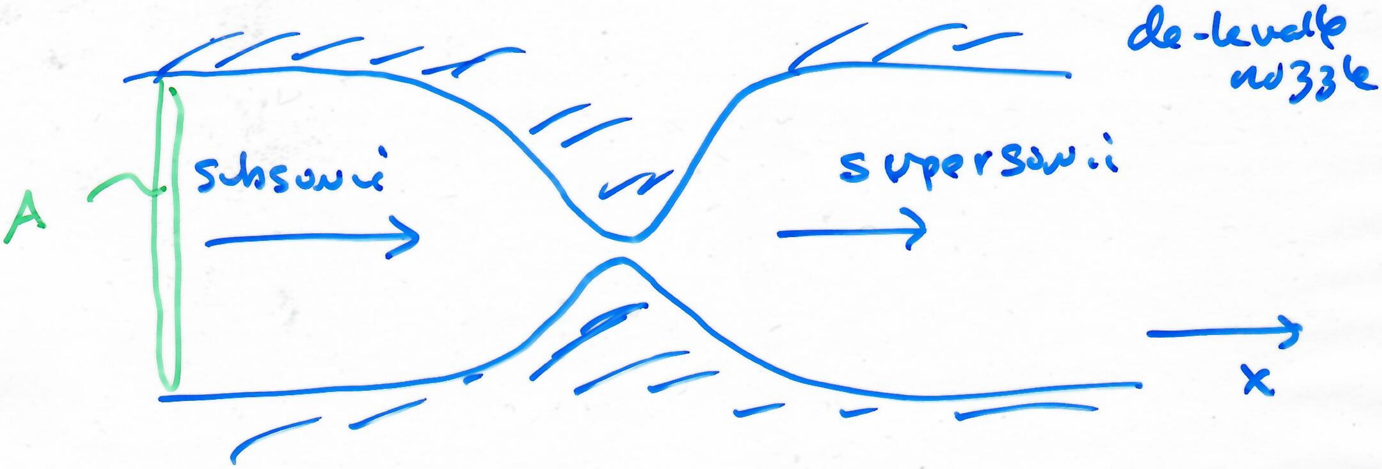
The de-Lavalle nozzle analogy to solar wind

Consider the following geometry



ignoring gravity, no mag. field, one-dimensional the momentum eq.

$$\rho u \frac{du}{dx} = -\frac{dP}{dx}$$

isothermal (without loss of generality in this case)

$$\frac{P}{\rho} = \text{const.} \Rightarrow P = P_0 \left(\rho/\rho_0\right) = c_s^2 \rho$$

$$c_s = \left(\frac{P_0}{\rho_0}\right)^{1/2}$$

$$\frac{dP}{dx} = c_s^2 \frac{d\rho}{dx}$$

$$\therefore \quad \rho u \frac{du}{dx} = -c_s^2 \frac{d\rho}{dx} \qquad (1)$$

also, cons. of mass requires

$$\rho u A = \text{const}$$

$$\Rightarrow \quad \frac{d\rho}{dx} u A + \rho \frac{du}{dx} A + \rho u \frac{dA}{dx} = 0$$

$$\frac{1}{\rho}\frac{d\rho}{dx} = -\frac{1}{u}\frac{du}{dx} - \frac{1}{A}\frac{dA}{dx} \qquad (2)$$

(2) → (1) gives

$$u \frac{du}{dx} = -c_s^2 \left(-\frac{1}{u}\frac{du}{dx} - \frac{1}{A}\frac{dA}{dx} \right)$$

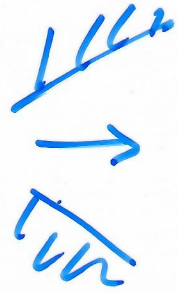
$$\Rightarrow \quad \frac{du}{dx}\left(u - \frac{c_s^2}{u} \right) = \frac{c_s^2}{A}\frac{dA}{dx}$$

$$\boxed{\frac{u^2 - c_s^2}{u}\frac{du}{dx} = \frac{c_s^2}{A}\frac{dA}{dx}}$$

$\frac{dA}{dx} < 0$

if $u < c_s \Rightarrow \frac{du}{dx} > 0 \Rightarrow u$ increases

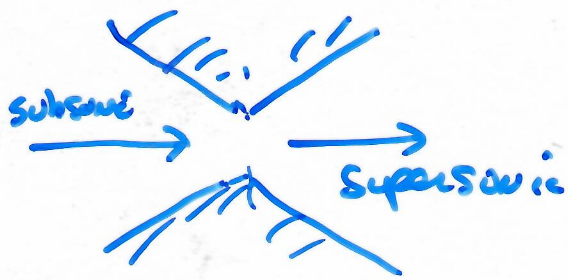
$u > c_s \Rightarrow \frac{du}{dx} < 0 \Rightarrow u$ decreases



$\frac{dA}{dx} > 0$

$u < c_s \Rightarrow \frac{du}{dx} < 0 \Rightarrow u$ decreases

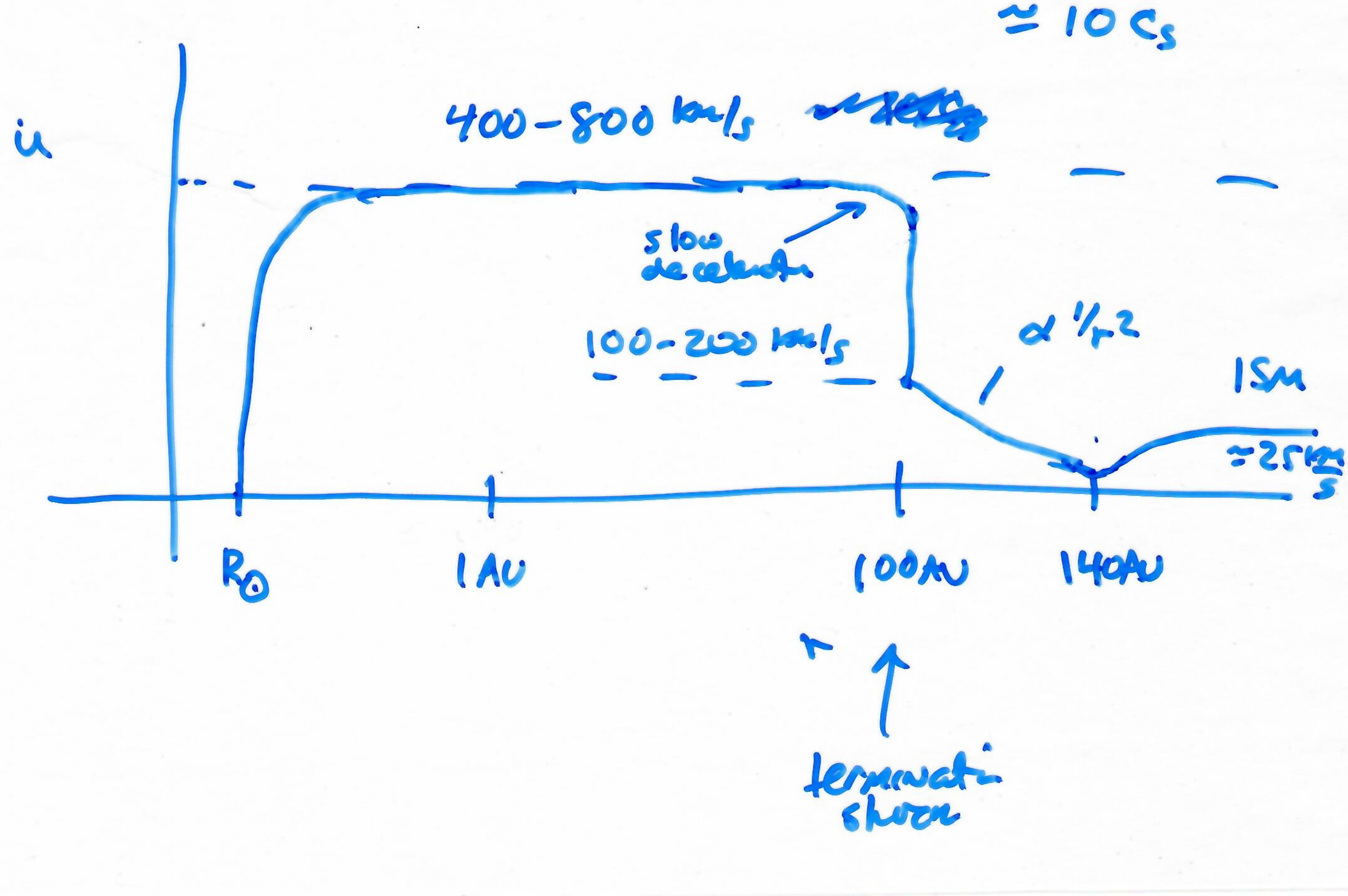
$u > c_s \Rightarrow \frac{du}{dx} > 0 \Rightarrow u$ increases



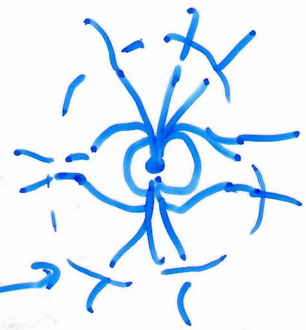
solar wind speed vs. r

V_{sw} 1AU $\simeq 10\, V_A$

$\simeq 10\, c_s$

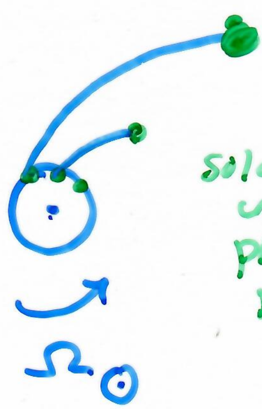


Parker spiral magnetic field



but, the Sun is rotating
what is the effect of this?

top view



← far from the Sun we have a spiral pattern

side view

due to no θ plasma motion

This is a kinematic problem (we are in the limit $\rho u^2 \gg B^2/8\pi$ ∴ field is weak and a passive tracer in the flow). We define the flow

$$\underset{\sim}{u} = \begin{cases} V_0 \hat{r} & (r > R_0) \\ + R_0 \Omega_0 \sin\theta \hat{\phi} & (r = R_0) \end{cases}$$

we solve $\frac{\partial \underset{\sim}{B}}{\partial t} = \nabla \times (\underset{\sim}{u} \times \underset{\sim}{B}) = 0$ because steady state

solve for $r > R_0$
use $r = R_0$ as B.C.

$$\nabla \times (\underset{\sim}{u} \times \underset{\sim}{B}) = 0$$

~~spher~~

$$\underset{\sim}{u} \times \underset{\sim}{B} = \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\phi} \\ V_0 & 0 & 0 \\ B_r & B_\theta & B_\phi \end{vmatrix}$$

$$= -V_0 B_\phi \hat{\theta} + V_0 B_\theta \hat{\phi}$$

* get $\nabla \times (\underset{\sim}{u} \times \underset{\sim}{B})$ in spherical coords.

r-component

$$\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta \, V_0 B_\theta) - \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(-V_0 B_\phi) = 0$$

$$\underbrace{V_0 \left(\frac{1}{r\sin\theta} \frac{\partial}{\partial\theta} (\sin\theta\, B_\theta) + \frac{1}{r\sin\theta} \frac{\partial B_\phi}{\partial\phi} \right)}_{} = 0$$

$$= -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 B_r)$$ because $\nabla \cdot \underset{\sim}{B} = 0$

$$\Rightarrow \quad -\frac{V_0}{r^2}\frac{\partial}{\partial r}(r^2 B_r) = 0$$

$$\Rightarrow \quad r^2 B_r = \text{constant}$$

$$B_r = B_r(R_0)\left(\frac{R_0}{r}\right)^2$$

$$\boxed{B_r = B_0 \left(\frac{R_0}{r}\right)^2}$$ B_0 is a signed quantity

θ- component

$$-\frac{1}{r}\frac{\partial}{\partial r}(r V_0 B_\theta) = 0$$

$$\Rightarrow \quad r B_\theta = \text{const.}$$

but $B_\theta = 0$ @ $r = R_0$

$$\therefore \quad \boxed{B_\theta = 0 \text{ everywhere}}$$

ϕ- component

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r V_0 B_\phi\right) = 0$$

$$\Rightarrow \quad r B_\phi = \text{constant}$$

$$= R_0 B_\phi(R_0)$$

an aside: consider the electric field near surface — $\underset{\sim}{E} = -\frac{1}{c}\underset{\sim}{u} \times \underset{\sim}{B}$

$$\underset{\sim}{E}(r = R_0) = -\frac{1}{c} B_r V_\phi \,\hat{\theta} \qquad (V_\phi = R_0 \Omega_0 \sin\theta)$$

$$\underset{\sim}{E}(r = R_0 + \epsilon) = \frac{1}{c} B_\phi V_0 \,\hat{\theta}$$

$\epsilon << R_0$
very small

these must be $=$ $\therefore$ @ $r = R_0$

$$-B_r(R_0) V_\phi(R_0) = B_\phi(R_0) V_0$$

$$\Rightarrow \boxed{B_\phi(R_0) = -B_0 \frac{R_0 \Omega_0 \sin\theta}{V_0}}$$

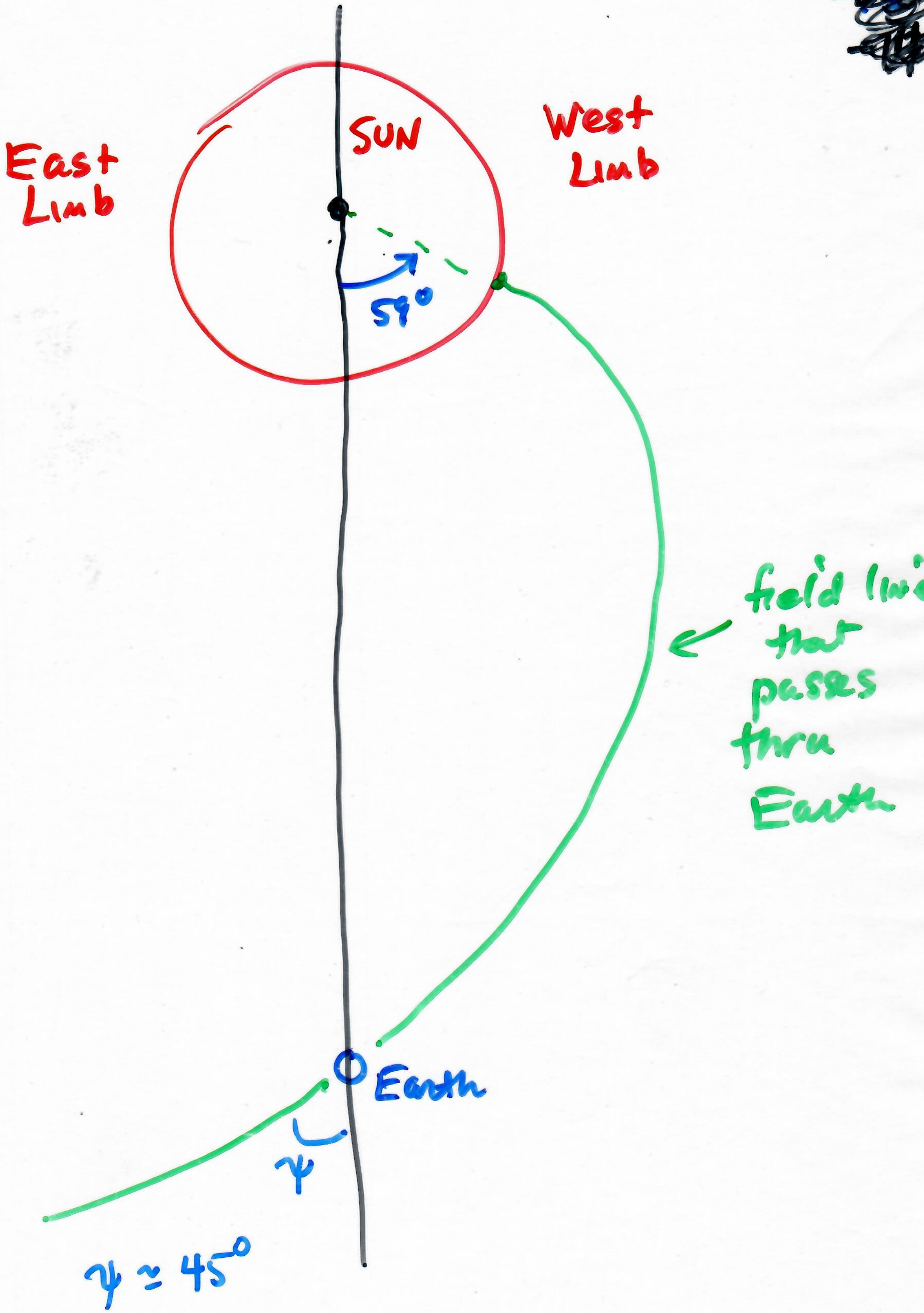
this gives

$$B_\phi(r) = \frac{R_0}{r}\left(-B_0 \frac{R_0 \Omega_0 \sin\theta}{V_0}\right)$$

$$\boxed{B_\phi(r) = -B_0 \frac{R_0}{r} \frac{R_0 \Omega_0 \sin\theta}{V_0}}$$

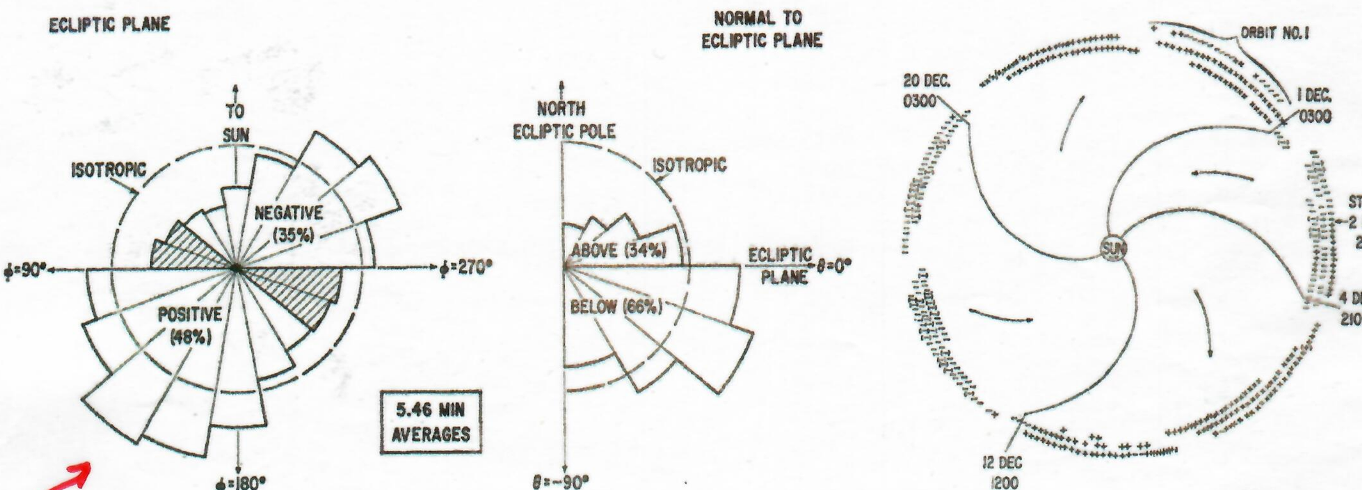
$$\boxed{\underset{\sim}{B} = B_0 \left(\frac{R_0}{r}\right)^2 \hat{r} - B_0 \frac{R_0}{r} \frac{R_0 \Omega_0 \sin\theta}{V_0} \hat{\phi}}$$

Parker-spiral
magnetic field
(a correct form)


SUN
East Limb
West Limb
59°
field line that passes thru Earth
Earth
ψ
ψ ≈ 45°

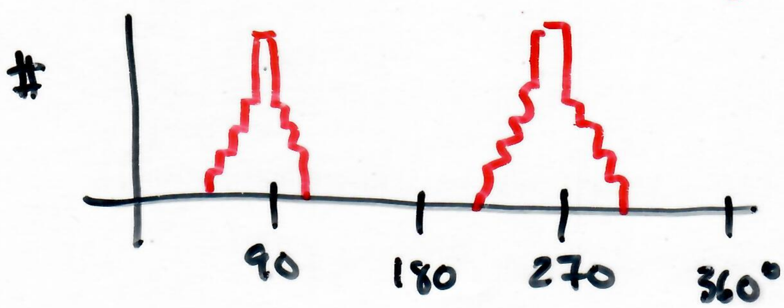
First observations of the "Parker Spiral"

Sector structure



at 1 AU the field is ~ 45° to the radial

Ness & Wilcox, 1965 (Science)



outer heliosphere the field is ~ 90° to the radial

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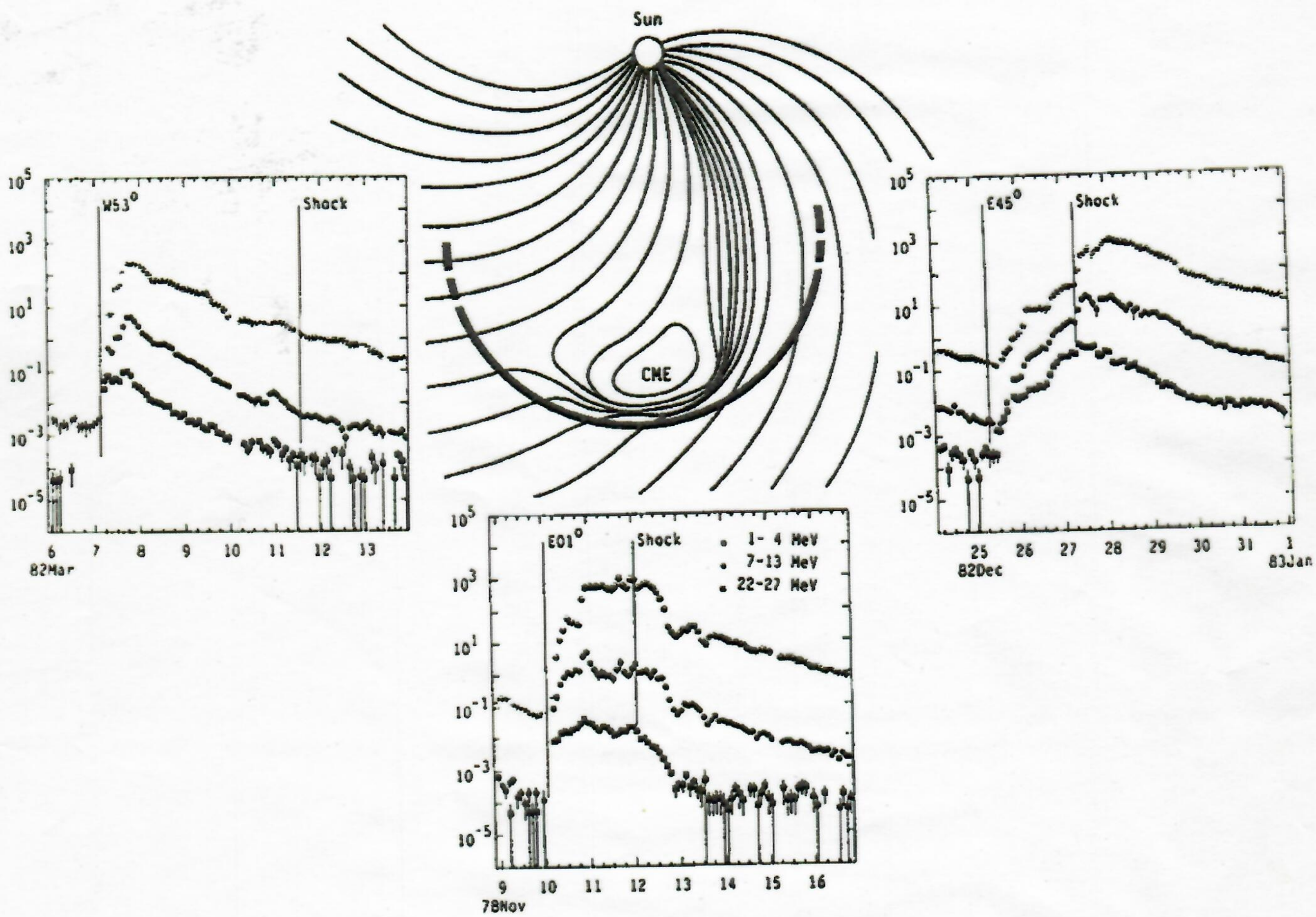

Figure 3.4. Typical intensity-time profiles are shown for 3 events viewed from different solar longitudes relative to the CME and shock.