# Parker's heliosphere model

Preliminary discussion on basic hydrodynamics

H.D. momentum eq.

viscosity ↙

$$\rho \frac{\partial \underset{\sim}{u}}{\partial t} + \rho \underset{\sim}{u} \cdot \nabla \underset{\sim}{u} = -\nabla P - \rho \underset{\sim}{g} - \mu \nabla^2 \underset{\sim}{u}$$

viscous H.D. momentum eq.
Navier Stokes

Can be written

$$\frac{\partial}{\partial t}(\rho \underset{\sim}{u}) + \nabla \cdot (\rho \underset{\sim}{u}\underset{\sim}{u} + P \underset{\approx}{I}) = -\rho \underset{\sim}{g} - \mu \nabla^2 \underset{\sim}{u}$$

defn $Re$ = ~~viscous~~ dynamic pressure gradient / viscous force per volume $= \left|\frac{2}{1}\right|$

$$= \frac{\rho u^2 / L}{\mu\, u / L^2}$$

$L$ = char. scale of problem

$$\boxed{Re = \frac{L \rho u}{\mu} = \frac{L u}{\nu}}$$

$\nu$ = static viscosity $\mu/\rho$

Also we should discuss Bernoulli's principle in this case, consider the H.D. energy eq. (isotropic pressure)

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho u^2 + \frac{3}{2}P\right) + \nabla\cdot\left[\left(\frac{5}{2}P + \frac{1}{2}\rho u^2\right)\underset{\sim}{u}\right] = -\rho\underset{\sim}{u}\cdot\underset{\sim}{g} \qquad (*)$$

includes work done by gravity

Consider <u>steady</u> flow. → $\partial/\partial t \to 0$

also, consider incompressible case ($\nabla\cdot\underset{\sim}{u} = 0$)

<u>Steady</u> & incompressible ⇒ $\rho$ = constant

$$\cancel{\frac{\partial \rho}{\partial t}}^{\,0} + \nabla\cdot(\rho\underset{\sim}{u}) = 0 \qquad \text{mass. cons.}$$

$$\rho\,\cancel{\nabla\cdot\underset{\sim}{u}}^{\,0} + \underbrace{\underset{\sim}{u}\cdot\nabla\rho}_{\Rightarrow\ \rho\ =\ \text{constant}} = 0$$

return to (*), steady state, we have

$$\nabla\cdot\left[\left(\frac{5}{2}P + \frac{1}{2}\rho u^2\right)\underset{\sim}{u}\right] = -\rho\underset{\sim}{u}\cdot\underset{\sim}{g}$$

$$\Rightarrow \nabla\cdot\left[\left(\frac{5}{2}P/\rho + \frac{1}{2}u^2\right)\rho\underset{\sim}{u}\right] = -\rho\underset{\sim}{u}\cdot\underset{\sim}{g}$$

$$\Rightarrow \rho\underset{\sim}{u}\cdot\nabla\left(\frac{5}{2}P/\rho + \frac{1}{2}u^2\right) = -\rho\underset{\sim}{u}\cdot\underset{\sim}{g}$$

also $\underline{g} = \frac{GM}{r^2}\hat{r}$ ← massive object (Sun)

$$= -\nabla \frac{GM}{r}$$

$$\Rightarrow \quad \rho \underset{\sim}{u} \cdot \left\{ \nabla \left( \frac{5}{2} p/\rho + \frac{1}{2} u^2 - \frac{GM}{r} \right) \right\} = 0$$

$$\Rightarrow \quad \boxed{\frac{5}{2}\frac{p}{\rho} + \frac{1}{2} u^2 - \frac{GM}{r} = \text{constant}}$$

this is for a compressible fluid!

Bernoulli principle

For an incompressible fluid, we have

$$\boxed{\frac{p}{\rho} + \frac{1}{2} u^2 - \frac{GM}{r} = \text{constant}}$$

note that near the surface of a planet, the last term is

$$\frac{GM}{r} = \frac{GM}{R_0 + z} \simeq \frac{GM}{R_0}\left(1 - \frac{z}{R_0}\right) \qquad z \ll R_0$$

$$\therefore \quad \frac{p}{\rho} + \frac{1}{2} u^2 - \underbrace{\frac{GM}{R_0}}_{\text{constant}} + \underbrace{\frac{GM}{R_0}\frac{1}{R_0}}_{g \,=\, \frac{GM}{R_0^2}} z = \text{const.}$$

Bernoulli in common form

$$\Rightarrow \quad \boxed{\frac{p}{\rho} + \frac{1}{2} u^2 + g z = \text{const}}$$

Finally, define vorticity

$$\underset{\sim}{\omega} = \nabla \times \underset{\sim}{u}$$

if $\nabla \cdot \underset{\sim}{u} = 0$ (incompressible), the H.D. mom. eq can be written

$$(**) \quad \frac{\partial \underset{\sim}{u}}{\partial t} + \underset{\sim}{u} \cdot \nabla \underset{\sim}{u} = -\nabla \left( P/\rho \right)$$

(inviscid case)
(no gravity)

$\rho$ = constant because it also assumed to be constant far from interaction w/ object

take $\nabla \times$ (curl) of both sides of (* *), use vector identities, it can be shown

⋮

$$\frac{D\underset{\sim}{\omega}}{Dt} = (\underset{\sim}{\omega} \cdot \nabla)\, \underset{\sim}{u}$$

if $\underset{\sim}{\omega} = 0$ far from interaction region (i.e. uniform flow), it must be zero everywhere.

$\therefore \quad \underset{\sim}{\omega} = 0$ (if $\underset{\sim}{\omega} = 0$ far away) this flow can be consider "irrotational"

$\nabla \cdot \underset{\sim}{u} = 0 \quad \leftarrow$ no compression

$\nabla \times \underset{\sim}{u} = 0 \quad \leftarrow$ no vorticity

also no viscosity $\rightarrow$ inviscid

note that if we write

$\underline{u} = \nabla \psi \qquad \psi$ = stream function

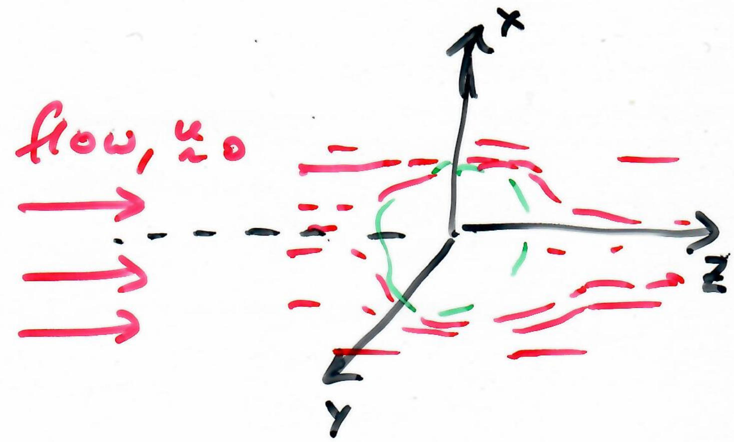
$\Rightarrow \quad \nabla^2 \psi = 0 \qquad$ Laplaces eq.

## ~~Return to~~ Return to Parker heliosphere model

$\underset{\sim}{u}$ is incompressible, vorticity free, inviscid

$\underset{\sim}{u} = \nabla \psi$

$\rightarrow \nabla^2 \psi = 0$




general solution

$$\psi = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \left[ a_n^m r^n + b_n^m r^{-(n+1)} \right] P_n^m(\cos\theta) e^{im\phi}$$

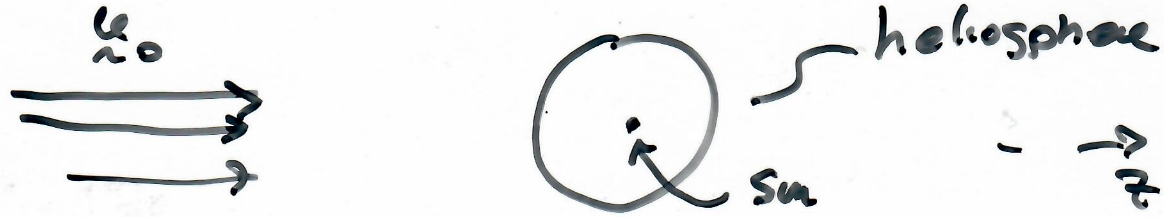
azimuthal symmetry $\Rightarrow m = 0$

Parker considered $n=0$ & $n=1$ terms.
we obtain

$$u_r = -\frac{b_0}{r^2} + \left(a_1 - \frac{2b_1}{r^3}\right)\cos\theta$$

$$u_\theta = -\left(a_1 + \frac{b_1}{r^3}\right)\sin\theta$$

far from the heliosphere




$$\underset{\sim}{u}_0 = u_0 \hat{z} = u_0 (\cos\theta\, \hat{r} - \sin\theta\, \hat{\theta})$$

as $r \to \infty$, we have

$$u_r = a_1 \cos\theta$$
$$u_\theta = -a_1 \sin\theta$$

if $a_1 = u_0$, we have uniform flow far away

$$\therefore \quad u_r = u_0 \cos\theta - \frac{b_0}{r^2} - \frac{2b_1}{r^3}\cos\theta$$

$$u_\theta = -u_0 \sin\theta - \frac{b_1}{r^3}\sin\theta$$

when near the Sun, we want radial flow
we get this if $b_1 = 0$

we set $V_{SW}$ = solar wind speed $= -\frac{b_0}{R_H^2} \Rightarrow b_0 = -R_H^2 V_{SW}$

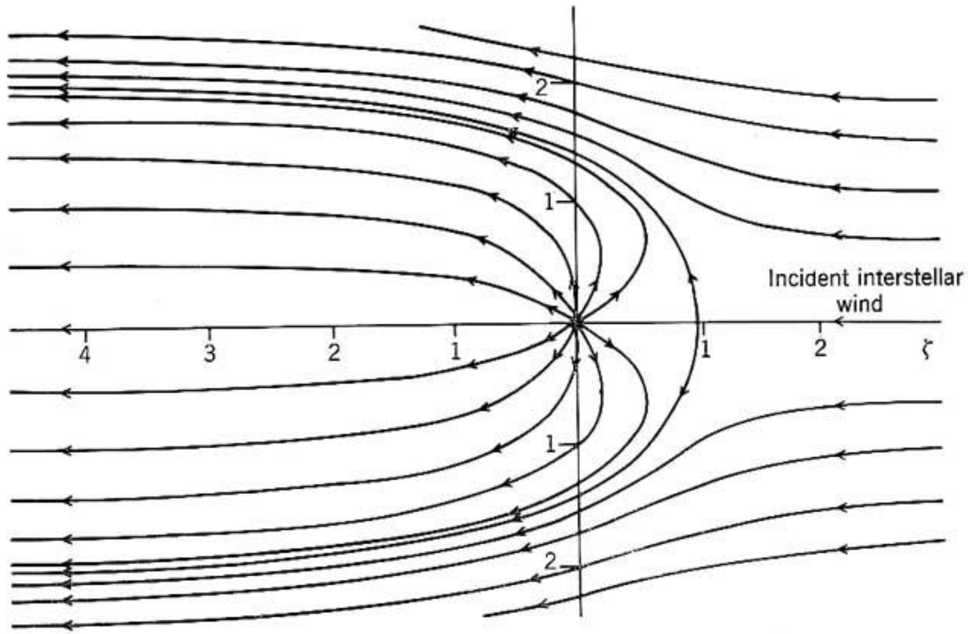
This gives the final solution

$$u_r = u_0 \cos\theta + V_{SW}\left(\frac{R_H}{r}\right)^2$$

$$u_\theta = -u_0 \sin\theta$$

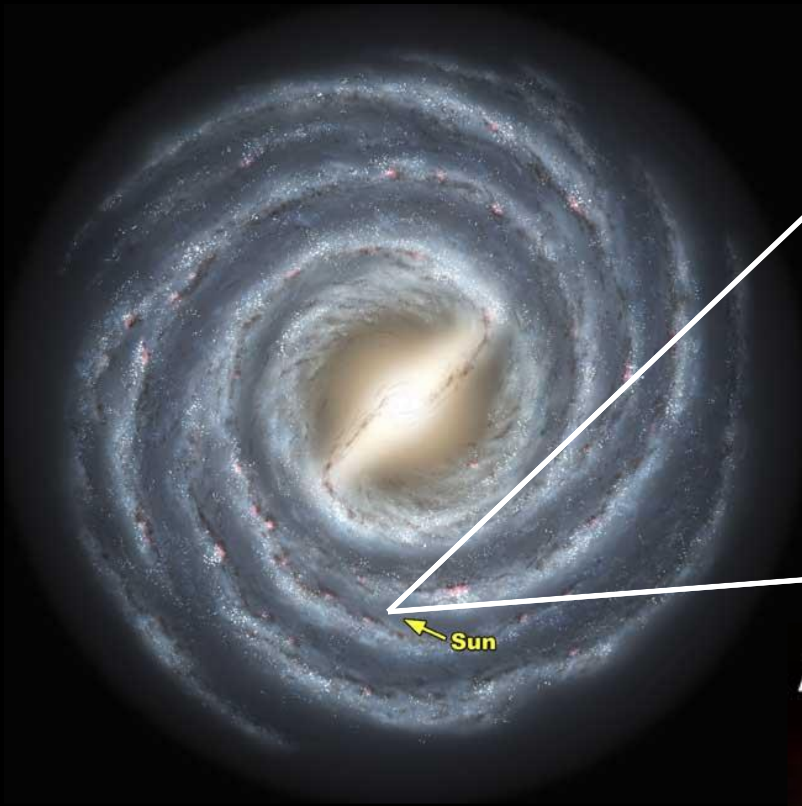
Parker's heliosphere solution

Parker's view of the heliosphere in 1961 – from an analytic formulation. He came up with a scale of **45-90** AU from knowledge of the ram pressure at 1 AU and the estimated interstellar pressure of (1-4) x $10^{-12}$ erg/cm$^2$.

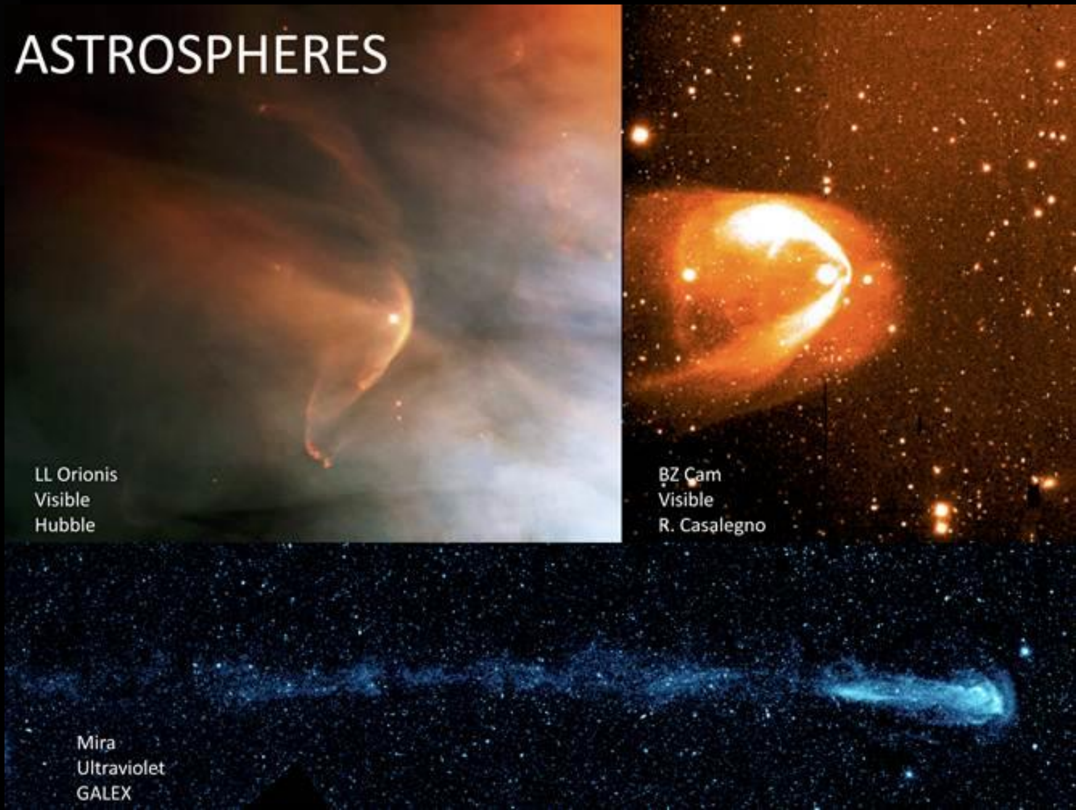



- The solar wind flows *supersonically* and nearly *radially* from the Sun
- Thus, its large-scale structure is determined mainly by the initial and boundary conditions at the Sun.
- The solar wind terminates at the termination shock – where

$$\rho V_w^2 = \rho_0 (r_0/r)^2 V_w^2 = P_{ism}.$$








As the Sun moves through its local environment, it carves out a region –the heliosphere – that is analogous to astrospheres seen surrounding other stars

# An instructive analog

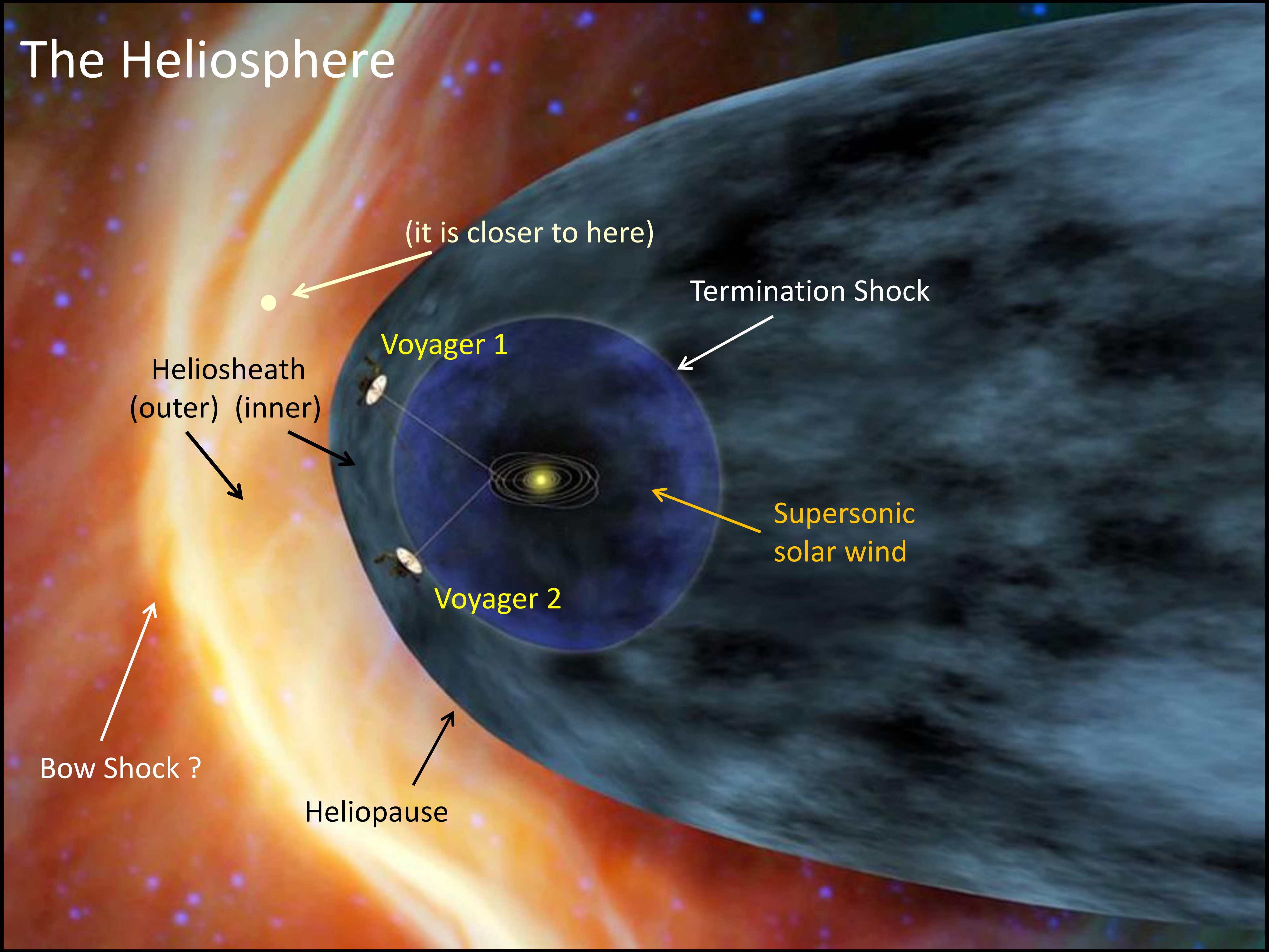
# Why we study the outer heliosphere

- The heliosphere represents the first of several boundaries shielding Earth from radiation coming outside our solar system – it is important to understand how it does this (especially if we *really* want to send humans to Mars or the moon, or elsewhere)

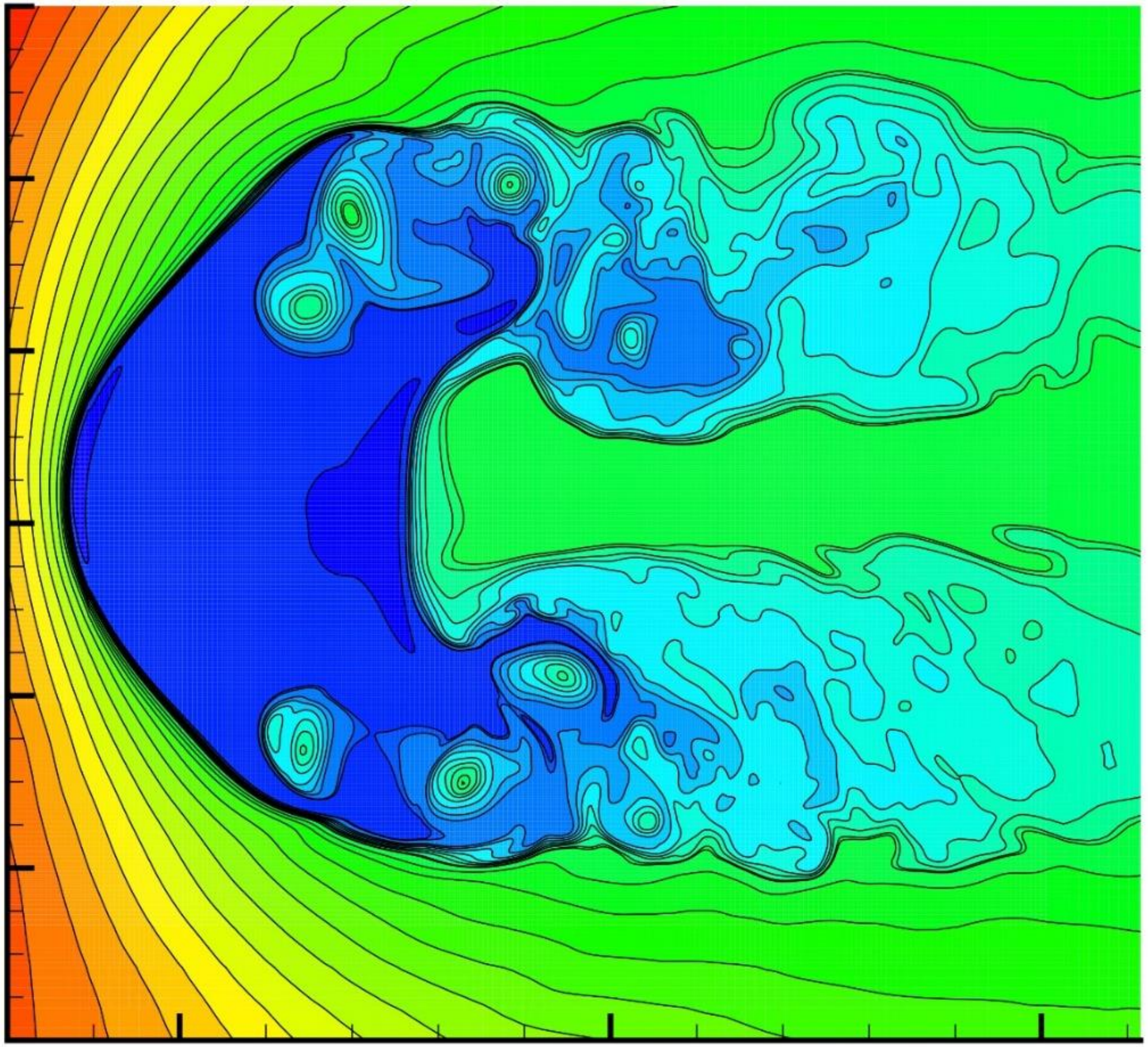
- NASA has several missions aimed at understanding the heliosphere (Voyager, IBEX, ACE, Ulysses, etc.) with rich data sets open for interpretation.

- It is a important laboratory for studying the physics of multi-component plasmas (interstellar neutral atoms, pickup ions, solar wind ions, cosmic rays), and particle acceleration.

- The source of many interesting and puzzling physics problems!


The Heliosphere
(it is closer to here)
Termination Shock
Voyager 1
Heliosheath
(outer) (inner)
Supersonic
solar wind
Voyager 2
Bow Shock ?
Heliopause

Another model suggests it is “Croissant” chaped (Opher et al., *Astrophys. J. Lett.*, 2015, and Opher et al., *Astrophys. J.*, 2017)



# How do we observe the Heliosphere?

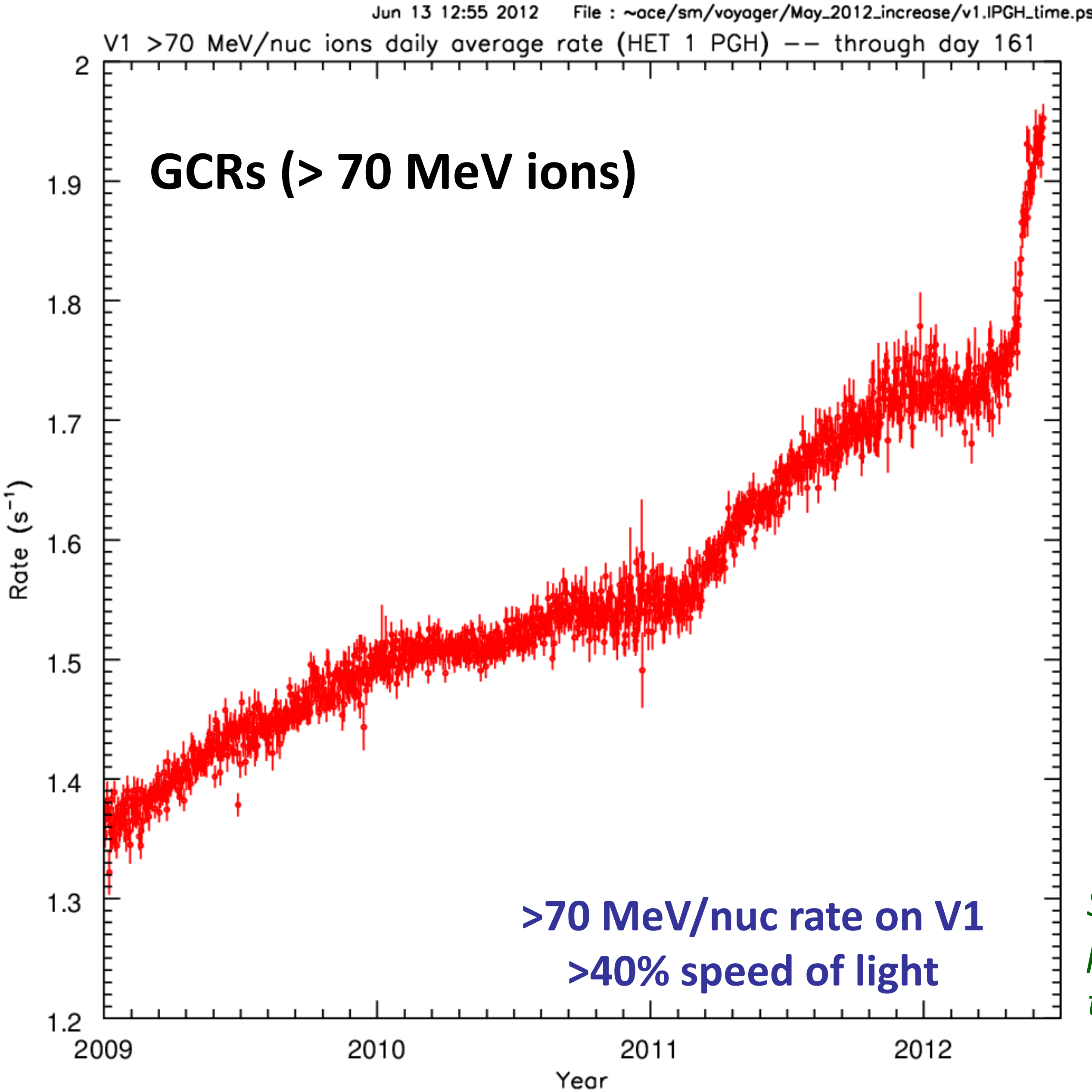
- *in situ* observations made possible by the two Voyager spacecraft have been essential to our present understanding.

- However, because of the turbulent, *essentially unpredictable* variations in the solar wind and large-scale structure at a given position and time, *remote* observations are also essential to provide a more-global view.
  - Cosmic rays (solar, galactic, anomalous). They provide us with valuable remote probes.
  - Radio-wave emissions.
  - Backscatter of photons from interstellar neutral atoms.
  - Also interstellar neutral atoms (e.g. IBEX, Cassini/INCA, Stereo, SoHO, and others before) and pickup ions (e.g. Ulysses, ACE).

**The interpretation of these remote probes relies on theory and modeling.**

# V1 crossing of the heliopause in 2012

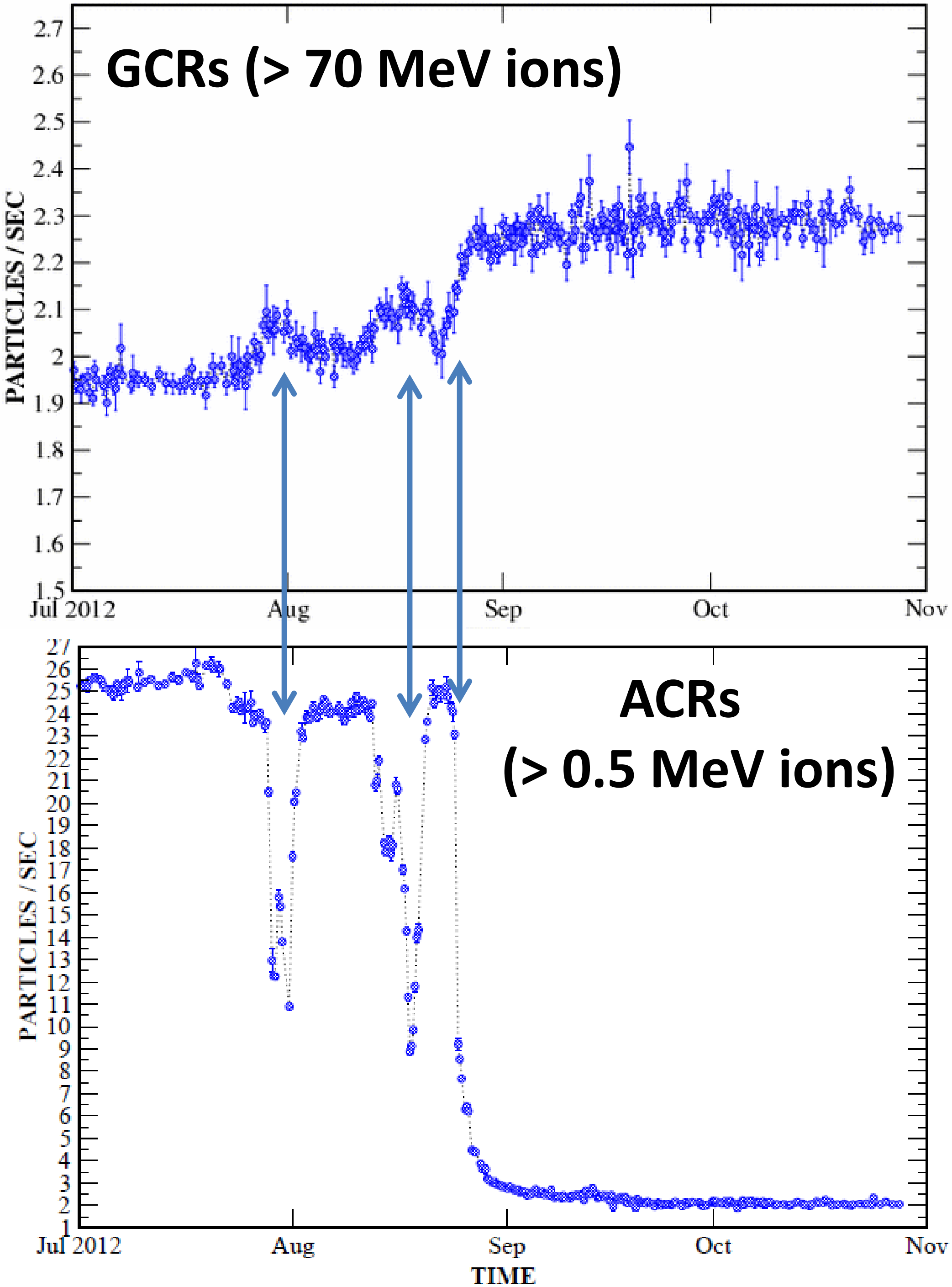
# Rate of Cosmic Rays Diffusing in from Galaxy




Rate increased 9% per year over last 3 years

In May 2012 the rate increased 5% in one week and 9% in a month

*Slide from E.C. Stone's presentation at Voyager team meeting, Dec. 2012*

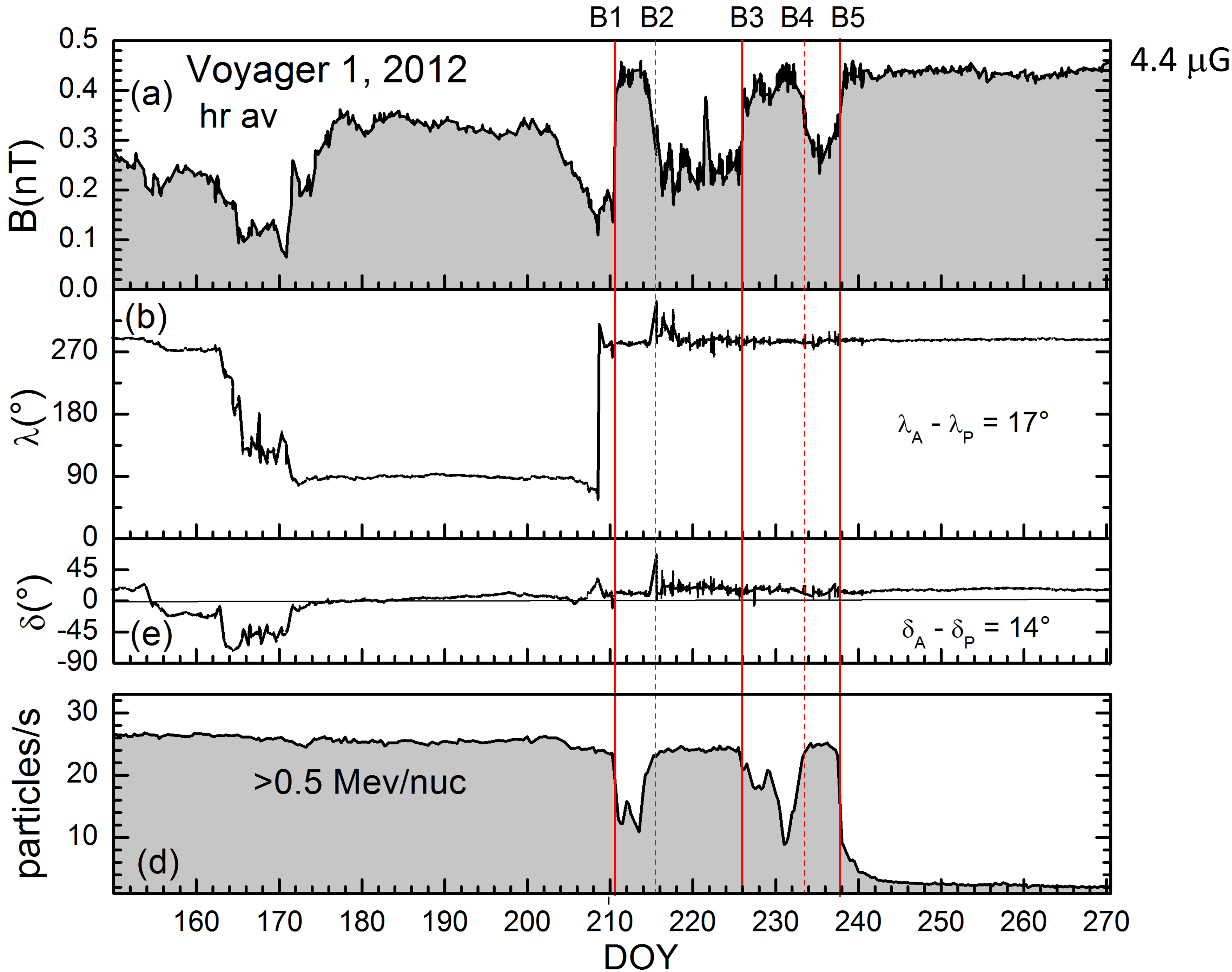



Cosmic rays from outside increased at the same time as ions from inside escaped

Is Voyager 1 in interstellar space,
or is there a new region inside that is connected to interstellar space outside?

voyager.gsfc.nasa.gov

*Slide from E.C. Stone's presentation at Voyager team meeting, Dec. 2012*




*Slide from Len Burlaga's presentation at Voyager team meeting, Dec. 2012*

## The science of the heliosphere received some attention in the popular media

**Ed Stone on the "Colbert report"**

http://www.cc.com/video-clips/g14s8s/the-colbert-report-ed-stone