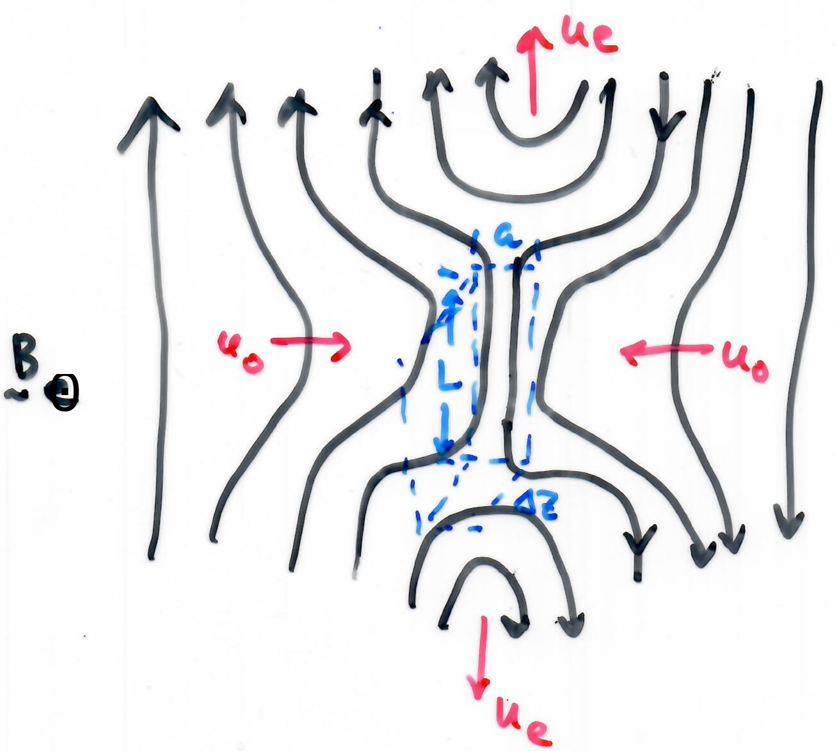
Sweet-Parker magnetic reconnection



steady-state magnetic reconnection

$\nabla \cdot \underset{\sim}{u} = 0$

$\Rightarrow$ incompressible

$\Rightarrow \rho =$ constant everywhere

- Conservation of particles

mass flux in = mass flux out

$$\rho u_0 L \Delta z = \rho u_e a \Delta z$$

$$\boxed{u_0 L = u_e a}$$ cons. of mass.

- Bernoulli's theorem – e.g. $\frac{1}{2}\rho u^2 + P =$ constant along streamline of flow.

$$\frac{1}{2}\rho u^2 + B^2/8\pi = \text{const. along streamline}$$

for our case

$$\cancelto{\text{small}}{\frac{1}{2}\rho u_0^2} + \frac{B_0^2}{8\pi} = \frac{1}{2}\rho u_e^2 + \cancelto{\text{small}}{\frac{B_e^2}{8\pi}}$$

$$\therefore \quad \boxed{u_e = \frac{B_0}{(4\pi\rho)^{1/2}}} = \text{Alfven speed.} = v_A$$

What is u_0? (same as asking what is reconnection rate?)

balance energy dissipation rate with incoming energy flux

$$\underbrace{(\eta J^2)}_{\text{energy diss.}} \underbrace{L\,\Delta z\,a}_{\text{volume}} = \underbrace{\frac{B^2}{8\pi} u_0}_{\text{energy flux}} \underbrace{L\,\Delta z}_{\text{area}}$$

$$J = \frac{c}{4\pi}\frac{B}{a} \simeq \frac{c}{4\pi}\nabla\times B$$

$$u_0 = \frac{2\eta c^2}{4\pi a}$$

recall $a = \frac{u_0}{u_e} L$

$$\Rightarrow u_0 = \frac{2\eta c^2}{4\pi\left[{}^{u_0}/_{u_e} L\right]}$$

$$\Rightarrow u_0^2 = \frac{2\eta c^2}{4\pi L} u_e = \frac{2\eta c^2}{4\pi L} v_A$$

$$\Rightarrow \quad u_0 = \left(\frac{2\eta c^2}{4\pi L} v_A\right)^{1/2}$$

recall R_M = magnetic Reynolds #

$$= \left(\frac{4\pi L v_A}{\eta c^2}\right)$$

$$= \left(\frac{2}{R_M} v_A^2\right)^{1/2}$$

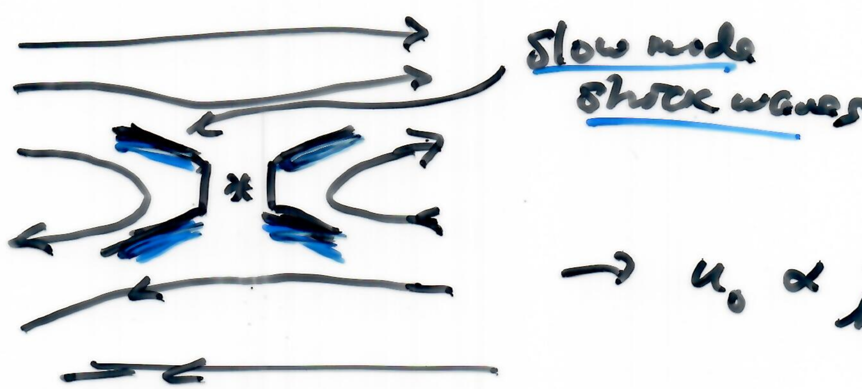
$$\boxed{u_0 = \left(\frac{2}{R_M}\right)^{1/2} v_A}$$

Reconnection rate in Sweet-Parker model

Solar atm. $R_M \sim 10^9$

$$\Rightarrow u_0 \ll v_A$$

* Petchek model reconnection



$$\rightarrow u_0 \propto \frac{1}{\ln(R_M)}$$

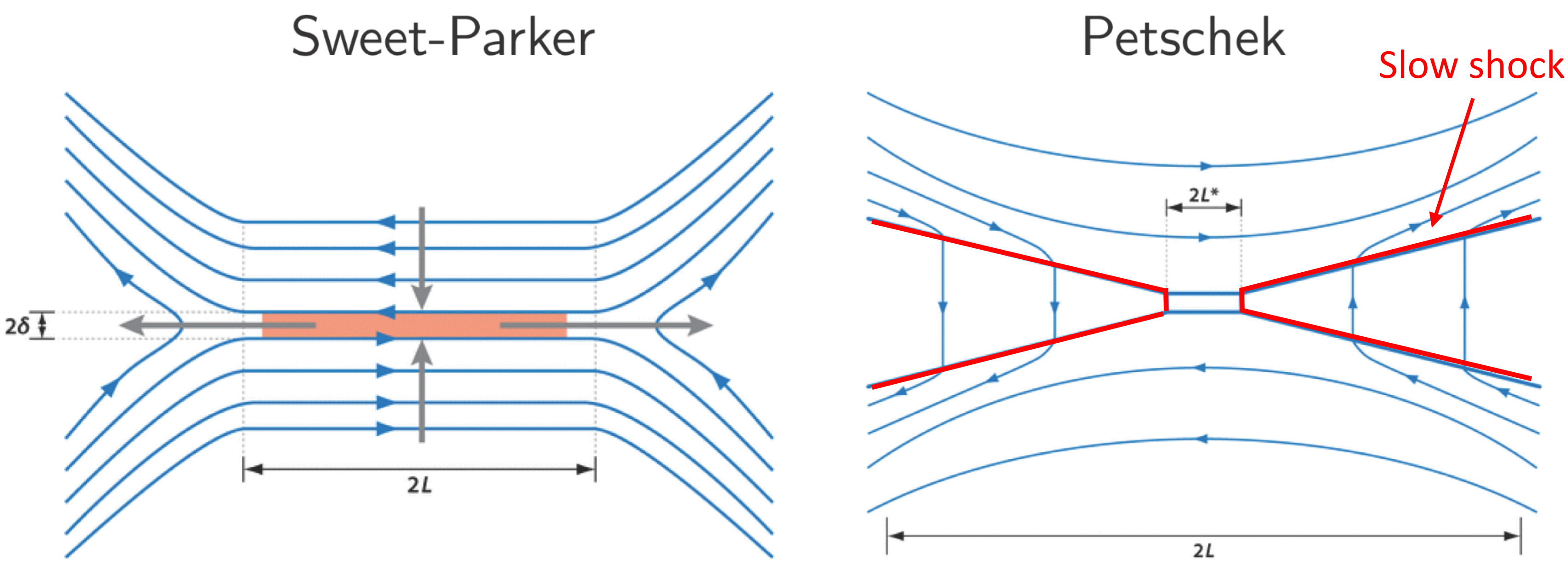
* more recent

HALL MHD → mostly solves problem

formation of magnetic "islands" → turbulence

- Sweet-Parker reconnection predicts a much slower reconnection rate than observed in solar flares and Earth's magnetosphere
  - Solar flares release energy in a time frame of a few to tens of minutes. Sweet-Parker predicts it would be months. This model cannot work for most astrophysical situations.

- Petchek (1964) proposed a different model, involving slow-mode shocks, allowing for a more-localized reconnection region – and X-point type geometry. The reconnection rate scales as $1/\ln(R_m)$ – fast reconnection.



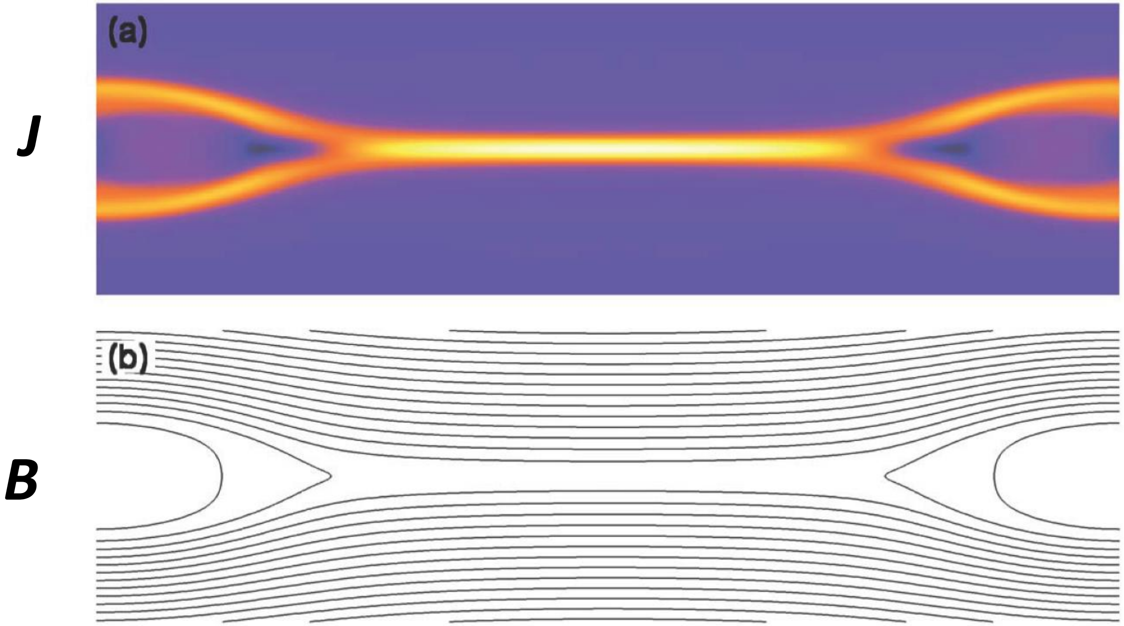
Zweibel & Yamada (2009)

- Petchek reconnection is not observed (in space or laboratory plasmas) to occur, thus this idea, is not the explanation for fast reconnection seen in space plasmas.
- So-called “Hall” reconnection does seem to agree with laboratory and space plasmas.
- What is “Hall” reconnection? To answer, this, consider the lecture on Feb 10 concerning the electric field in MHD. At the bottom of page 7 of those lecture notes, we wrote:

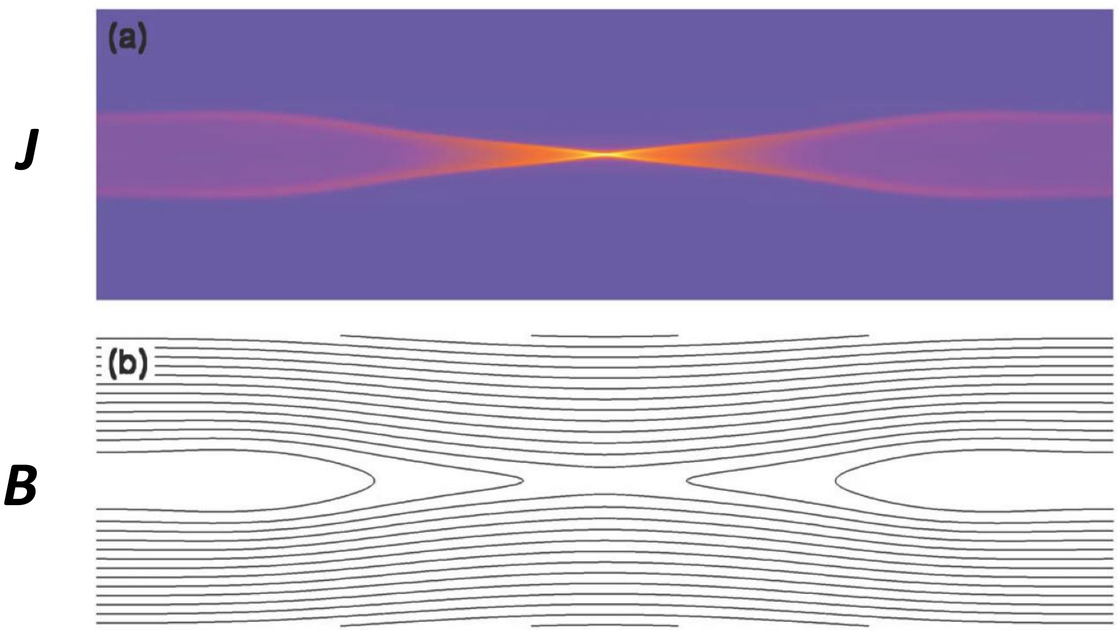
$$\vec{E} = -\frac{1}{c}\vec{U} \times \vec{B} \; + \; \frac{1}{nec}\vec{J} \times \vec{B} \; + \; \frac{1}{ne}\nabla P_e$$

Ideal MHD term **HALL term** **Polarization E field**

Numerical Simulations



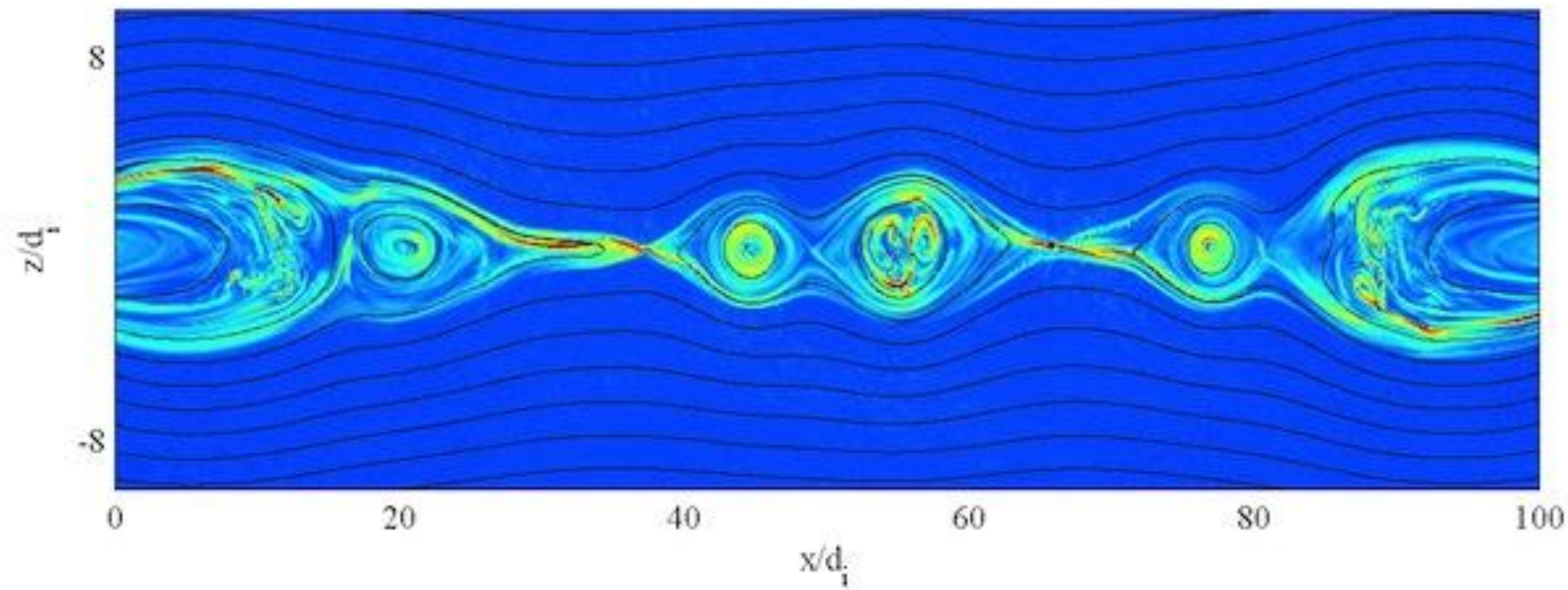
<u>Hall term OFF</u>: resistive MHD. Sweet-Parker-like



<u>Hall term ON</u>: fast reconnection. X-point geometry.

- The inclusion of the Hall term can lead to a “tearing-mode” instability for thin current sheets at small scales.
- This has been shown from fully kinetic numerical simulations to lead to the formation of “magnetic islands”, especially in the so-called “high-guide-field limit” (the guide field is the out of plane component of the magnetic field. High guide field is when this field dominates the in-plane magnetic field)

Current density, J, in a high-”guide-field” kinetic numerical simulation



Plasma Waves

$$\delta \underset{\sim}{v}(\underset{\sim}{x}, t) = \delta \underset{\sim}{v}(\underset{\sim}{k}, \omega)\, e^{i \underset{\sim}{k}\cdot \underset{\sim}{x} \mp i\omega t + \phi}$$

$\underset{\sim}{k}$ = wave vector $\qquad \lambda = \dfrac{2\pi}{|k|}$, wavelength

ω = wave frequency $\qquad T = \dfrac{2\pi}{\omega}$, wave period

ϕ = phase angle

$\delta \underset{\sim}{v}$ = amplitude + polarization

$\mp \rightarrow$ refers to direction of prop. (forward / backward)

plane waves

example

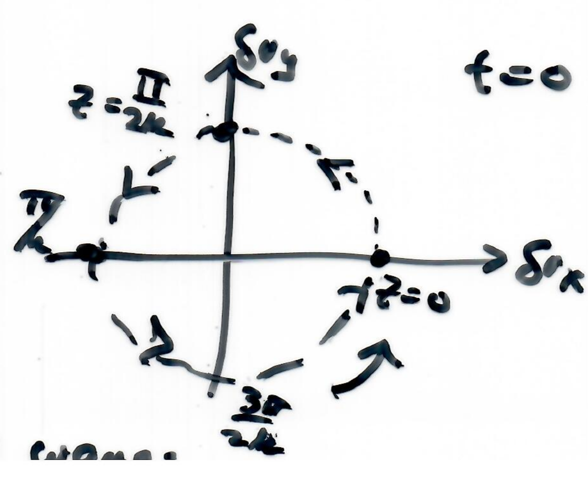
$$\delta \underset{\sim}{v}(\underset{\sim}{k}, \omega) = |\delta \underset{\sim}{v}|(\vec{x} - i\vec{y})$$

and propagation along z

circularly polarized plane wave

take $\underset{\sim}{k} = k\vec{z}$

$\phi = 0$



$\delta v_x = \delta v \cos(kz - \omega t)$

$\delta v_y = \delta v \sin(kz - \omega t)$

right-hand [redacted] polarized wave

This is a right-hand polarized, forward propagating plane wave

$\omega = \underset{\sim}{k} \cdot \underset{\sim}{v}_p$ $\qquad$ $\underset{\sim}{v}_p$ = phase velocity

$d\omega = d\underset{\sim}{k} \cdot \underset{\sim}{v}_g$ $\qquad$ $\underset{\sim}{v}_g$ = group velocity

Recall MHD equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underset{\sim}{u}) = 0$$

$$\rho \frac{\partial \underset{\sim}{u}}{\partial t} + \rho \underset{\sim}{u} \cdot \nabla \underset{\sim}{u} = -\nabla P + \frac{1}{4\pi}(\nabla \times \underset{\sim}{B}) \times \underset{\sim}{B} - \rho \underset{\sim}{g}$$

$$\left(\frac{\partial}{\partial t} + \underset{\sim}{u} \cdot \nabla\right)\left(P/\rho^{\gamma}\right) = 0$$

$$\frac{\partial \underset{\sim}{B}}{\partial t} = \nabla \times (\underset{\sim}{u} \times \underset{\sim}{B})$$

Perturb this by

$\rho = \rho_0 + \rho'$; $P = P_0 + P'$

$\underset{\sim}{u} = \underset{\sim}{u}'$; $\underset{\sim}{B} = \underset{\sim}{B}_0 + \underset{\sim}{B}'$

ρ', $\underset{\sim}{u}'$, P', $\underset{\sim}{B}'$ are small compared to unprimed substitute into MHD eq.'s

Continuity

$$\frac{\partial}{\partial t}(\rho_0+\rho') + \nabla\cdot\left[(\rho_0+\rho')\,\underset{\sim}{u}'\right] = 0$$

neglect second-order terms!

$$\Rightarrow \quad \boxed{\frac{\partial \rho'}{\partial t} + \rho_0 \nabla\cdot\underset{\sim}{u}' + \underset{\sim}{u}\cdot\nabla\rho_0 = 0} \qquad (1)$$

momentum eq.

$$(\rho_0+\rho')\frac{\partial}{\partial t}(\underset{\sim}{u}') + (\rho_0+\rho')\,\underset{\sim}{u}'\cdot\nabla\underset{\sim}{u}'$$

$$= -\nabla(P_0+P') + \frac{1}{4\pi}\left[\nabla\times(\underset{\sim}{B}_0+\underset{\sim}{B}')\right]\times(\underset{\sim}{B}_0+\underset{\sim}{B}')$$

$$-(\rho_0+\rho')\,\underset{\sim}{g}$$

h.s. equil.

$$\boxed{\nabla P_0 = -\rho_0\,\underset{\sim}{g}}$$

$$\boxed{\nabla\times\underset{\sim}{B}_0 = 0}$$ unif. back field

$$\boxed{\rho_0\frac{\partial\underset{\sim}{u}'}{\partial t} = -\nabla P' + \frac{1}{4\pi}(\nabla\times\underset{\sim}{B}_0)\times\underset{\sim}{B}' - \rho'\underset{\sim}{g}} \qquad (2)$$

energy eq. → more tedious, but leads

$$\boxed{\frac{\partial P'}{\partial t} + \underset{\sim}{u}'\cdot\nabla P_0 - c_s^2\left(\frac{\partial\rho'}{\partial t} + \underset{\sim}{u}'\cdot\nabla\rho_0\right) = 0} \qquad (3)$$

$$c_s = \left(\frac{\gamma P_0}{\rho_0}\right)^{1/2}$$

= sound speed

magnetic Induction eq.

$$\boxed{\frac{\partial \underset{\sim}{B}'}{\partial t} = \nabla \times (\underset{\sim}{u}' \times \underset{\sim}{B}_0)} \quad (4)$$

take $\partial/\partial t$ of (2), then substitute from (1), (3), (4) to give the <u>general wave equation</u>

$$\boxed{\begin{aligned}\frac{\partial^2 \underset{\sim}{u}'}{\partial t^2} = {} & c_s^2 \nabla(\nabla \cdot \underset{\sim}{u}') - (\gamma - 1)\, \underset{\sim}{g}\, (\nabla \cdot \underset{\sim}{u}') \\ & - g \nabla u_r' + \frac{1}{4\pi \rho_0}\left[\nabla \times \nabla \times (\underset{\sim}{u}' \times \underset{\sim}{B}_0)\right] \times \underset{\sim}{B}_0\end{aligned}}$$

~~gen~~

PTYS558 Spring 18 3/28/18

Recap

the general eq. for perturbed $\underset{\sim}{u}'$ is (wave eq.)

$$\boxed{\frac{\partial^2 \underset{\sim}{u}'}{\partial t^2} = c_s^2 \nabla (\nabla \cdot \underset{\sim}{u}') - (\gamma - 1) \underset{\sim}{g} (\nabla \cdot \underset{\sim}{u}') - g \nabla u_r' \; (g \nabla u_r') + \frac{1}{4\pi\rho_0} \left[\nabla \times \nabla \times (\underset{\sim}{u}' \times \underset{\sim}{B}_0) \right] \times \underset{\sim}{B}_0}$$

general wave eq.

assume plane-wave solution

$$\underset{\sim}{u}'(\underset{\sim}{x}, t) = \underset{\sim}{u}'(\underset{\sim}{k}, \omega)\, e^{i \underset{\sim}{k} \cdot \underset{\sim}{x} - i\omega t + \phi}$$

$$\nabla \rightarrow i \underset{\sim}{k}$$

$$\frac{\partial}{\partial t} \rightarrow -i\omega$$

$$-\omega^2 \frac{\partial^2 \underset{\sim}{u}}{\partial t^2} = -c_s^2 \underset{\sim}{k} (\underset{\sim}{k} \cdot \underset{\sim}{u}') - (\gamma - 1) \underset{\sim}{g} (\underset{\sim}{k} \cdot \underset{\sim}{u}')(-ik)$$

$$- i g \underset{\sim}{k} u_r' - \frac{1}{4\pi\rho_0} \left[\underset{\sim}{k} \times \underset{\sim}{k} \times (\underset{\sim}{u}' \times \underset{\sim}{B}_0) \right] \times \underset{\sim}{B}_0$$

general dispersion relation for MHD waves

$$\boxed{\omega^2 \underset{\sim}{u}' = c_s^2 \underset{\sim}{k} (\underset{\sim}{k} \cdot \underset{\sim}{u}') - i k (\gamma - 1) \underset{\sim}{g} (\underset{\sim}{k} \cdot \underset{\sim}{u}') + i g \underset{\sim}{k} u_r' + \frac{1}{4\pi\rho_0} \left[\underset{\sim}{k} \times \underset{\sim}{k} \times (\underset{\sim}{u}' \times \underset{\sim}{B}_0) \right] \times \underset{\sim}{B}_0}$$

Sound Waves

take $\mathbf{g} = 0$, $\mathbf{B}_0 = 0$

$$\frac{\omega^2}{c_s^2}\mathbf{u}' = \mathbf{k}(\mathbf{k}\cdot\mathbf{u}')$$

dot into $\mathbf{k}$

$$\left(\frac{\omega^2}{c_s^2} - k^2\right)(\mathbf{k}\cdot\mathbf{u}') = 0$$

get $\omega^2 = k^2 c_s^2$; $\omega = \pm k c_s$

if $\mathbf{k}\cdot\mathbf{u}' \neq 0$

these are sound waves.

$|\mathbf{v}_{phase}| = \pm c_s$

$v_g = \pm c_s$

owe their existence to compressibility of gas.

$\mathbf{k}\cdot\mathbf{u}' \neq 0 \Rightarrow \nabla\cdot\mathbf{u}' \neq 0$

compressible

Magnetic Waves

Intuitive picture

recall the ~~B~~ driving force is $\frac{1}{c}\underset{\sim}{J} \times \underset{\sim}{B}$

this had a pressure part & tension part.

Consider tension. From physics of waves on strings, it is found

$$v_p = \left(T/\rho^*\right)^{1/2}$$

T = tension of string,
ρ^* = mass per length of the string

for the analog to MHD

$$T \rightarrow B^2/4\pi$$

$$\rho^* \rightarrow \rho$$

$$v_p = \left(\frac{B^2/4\pi}{\rho}\right)^{1/2} = \left(\frac{B^2}{4\pi\rho}\right)^{1/2} = C_A \quad \text{Alfven speed.}$$

Consider pressure part

recall P/ρ^γ = const. $\quad C_s^2 = \frac{\gamma P}{\rho}$ $\quad$ analog sound pressure waves

it can be shown $\quad B/\rho$ = const. (normal to $\underline{B}$) derived from frozen flux thm.

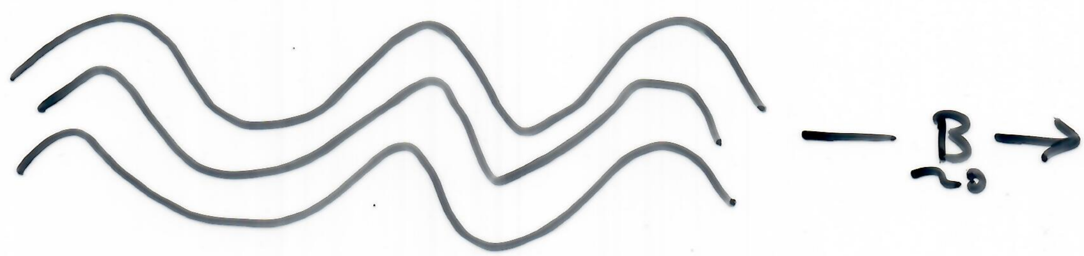
$\therefore \quad B^2/\rho^2 = \text{const.}$

$$\frac{B^2/8\pi}{\rho^2} = \text{const.} \qquad P_m$$

$$C_w^2 = \frac{\gamma P_m}{\rho} = \frac{\gamma\, B^2/8\pi}{\rho} = \frac{\gamma}{2}\frac{B^2}{4\pi\rho} = \frac{\gamma}{2} C_A^2$$

$\gamma = 2$, the pressure part of the $\underset{\sim}{J} \times \underset{\sim}{B}$ force is analogous to "sound" waves moving with speed C_A

tension



pressure