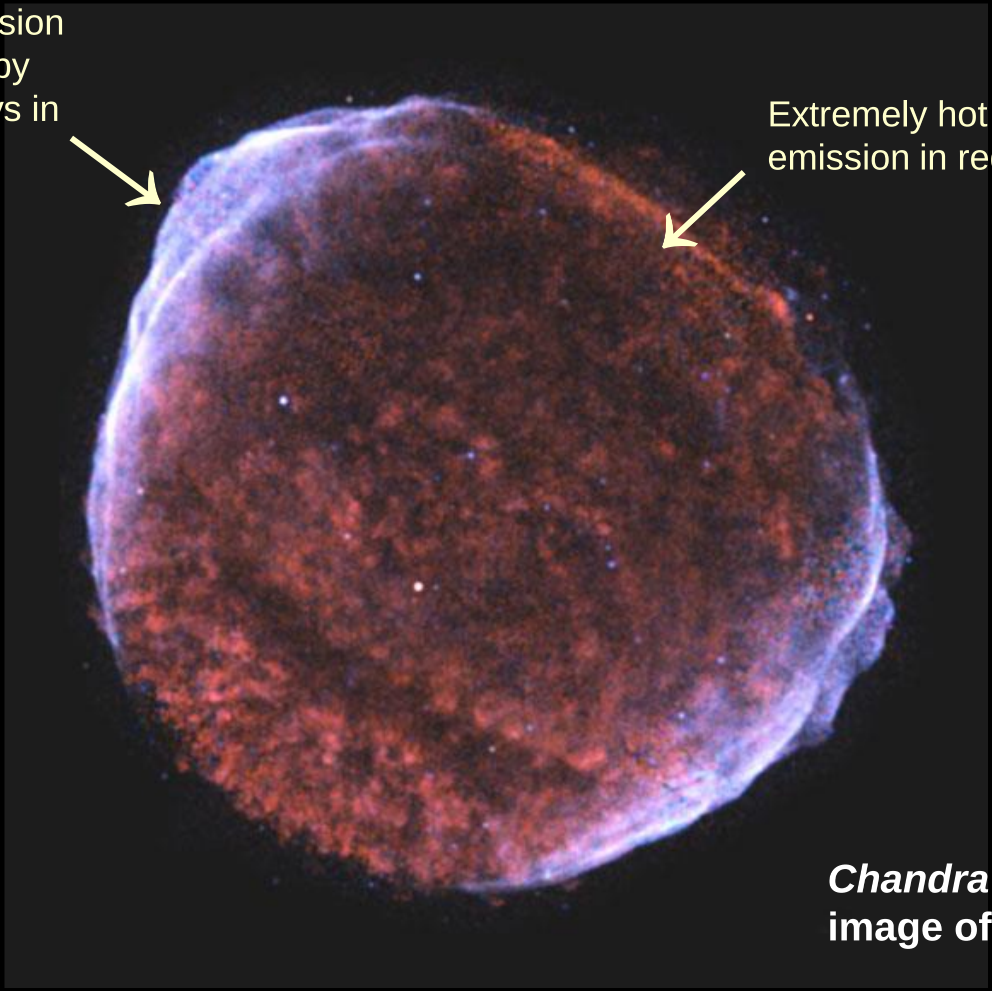
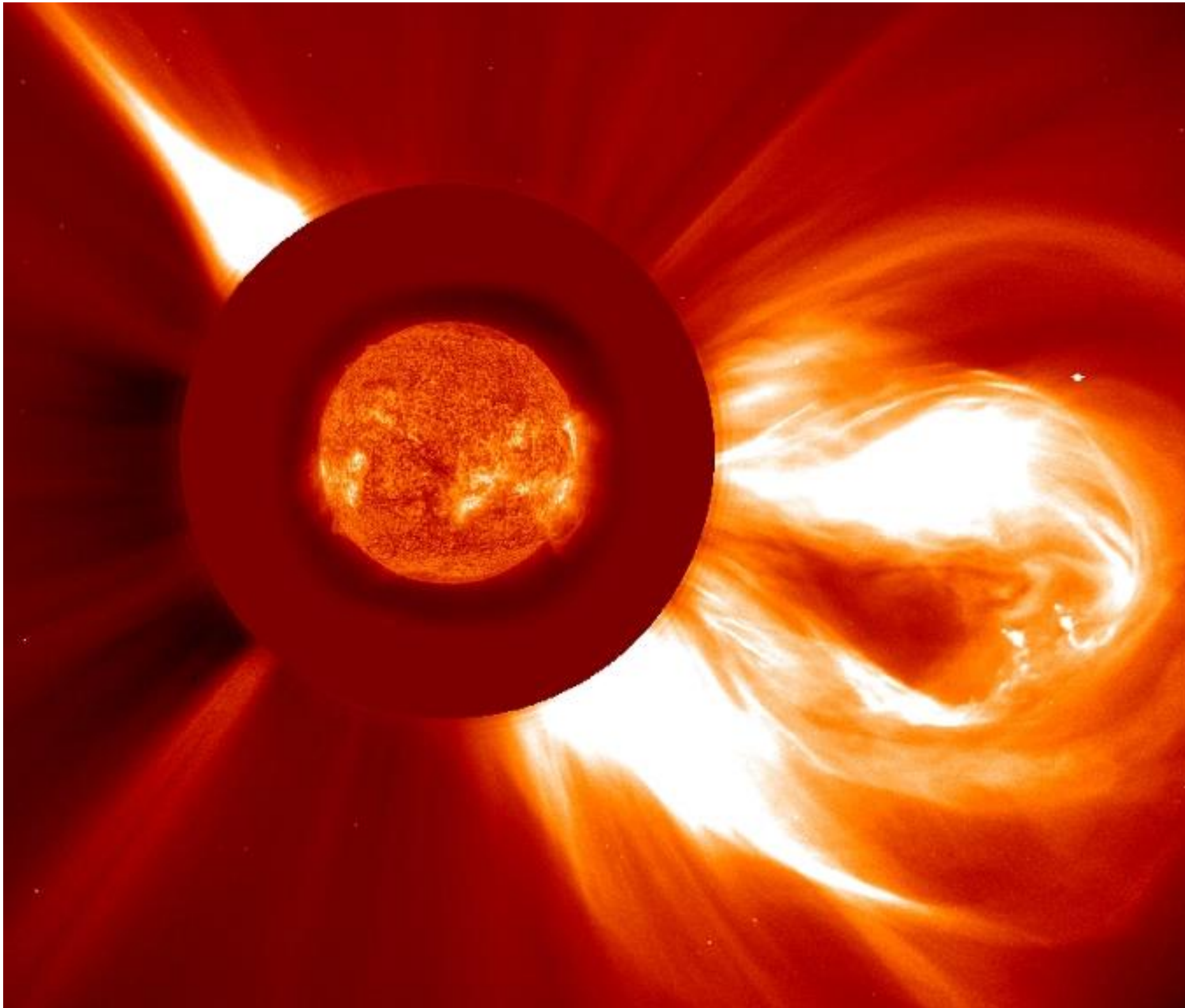
PTYS558 – April 1, 2020

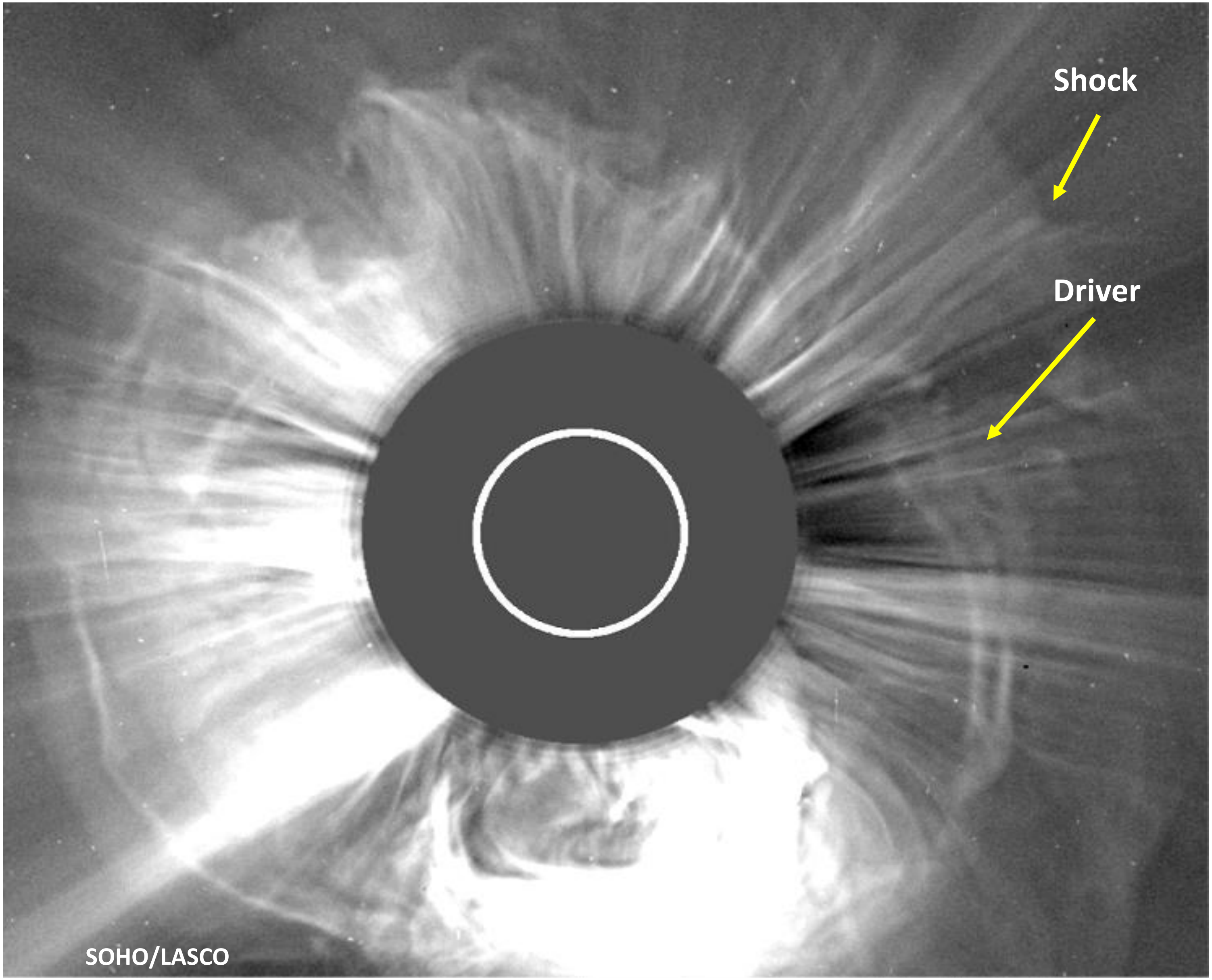
MHD Shocks

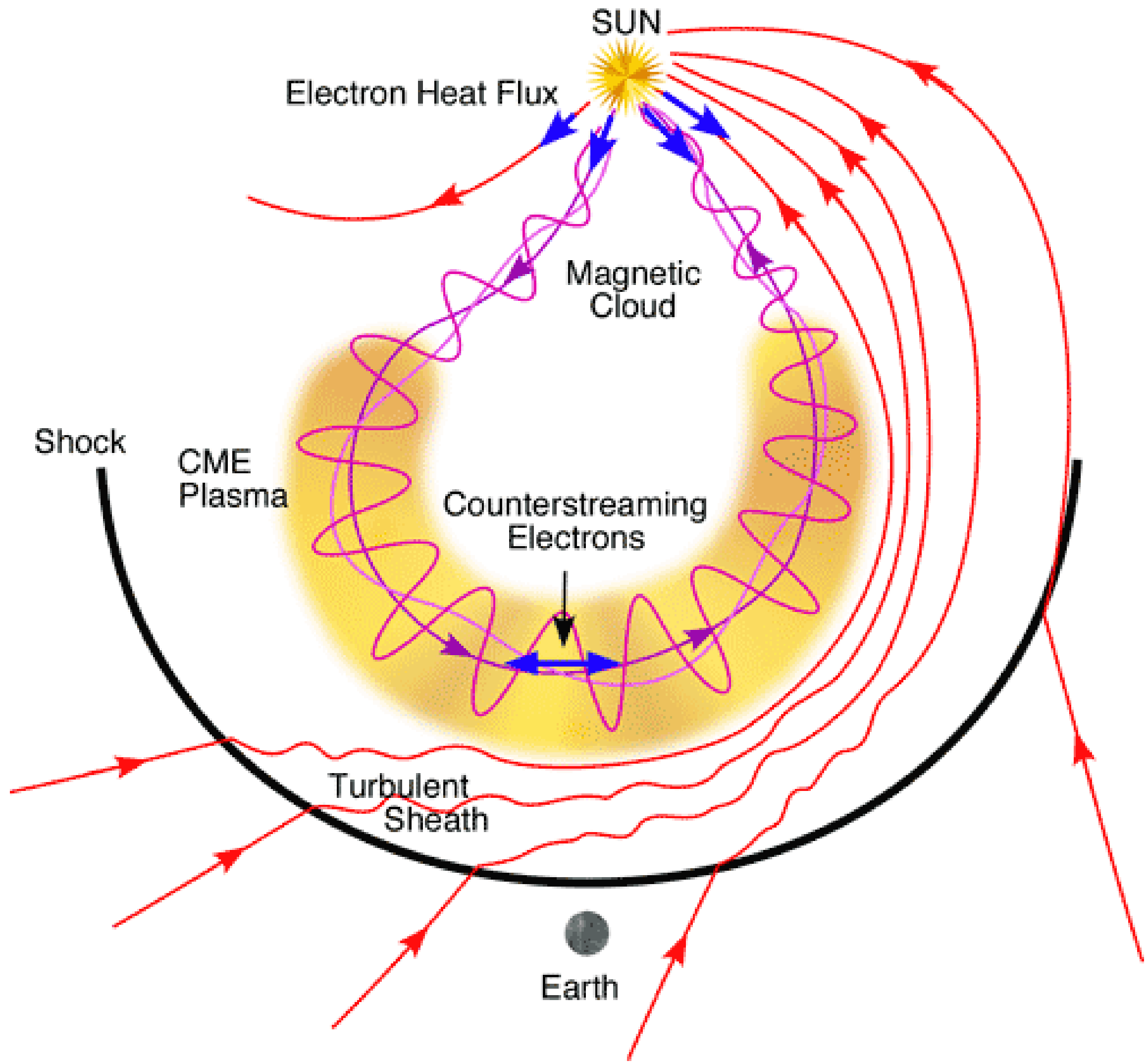


***Chandra* x-ray image of SN1006**

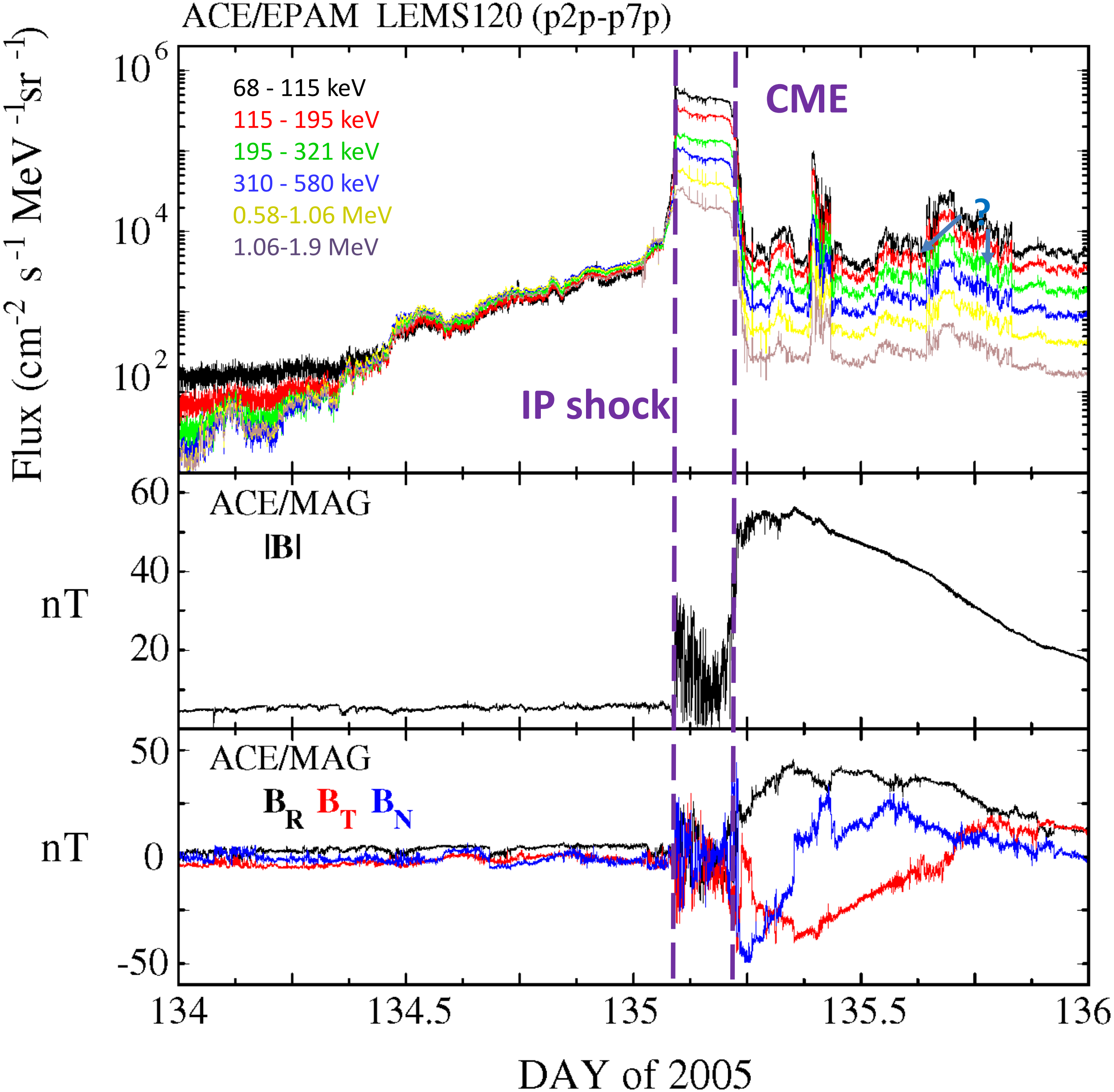
Bow shock in the Orion nebula (NASA/Hubble)




Shock
Driver
SOHO/LASCO

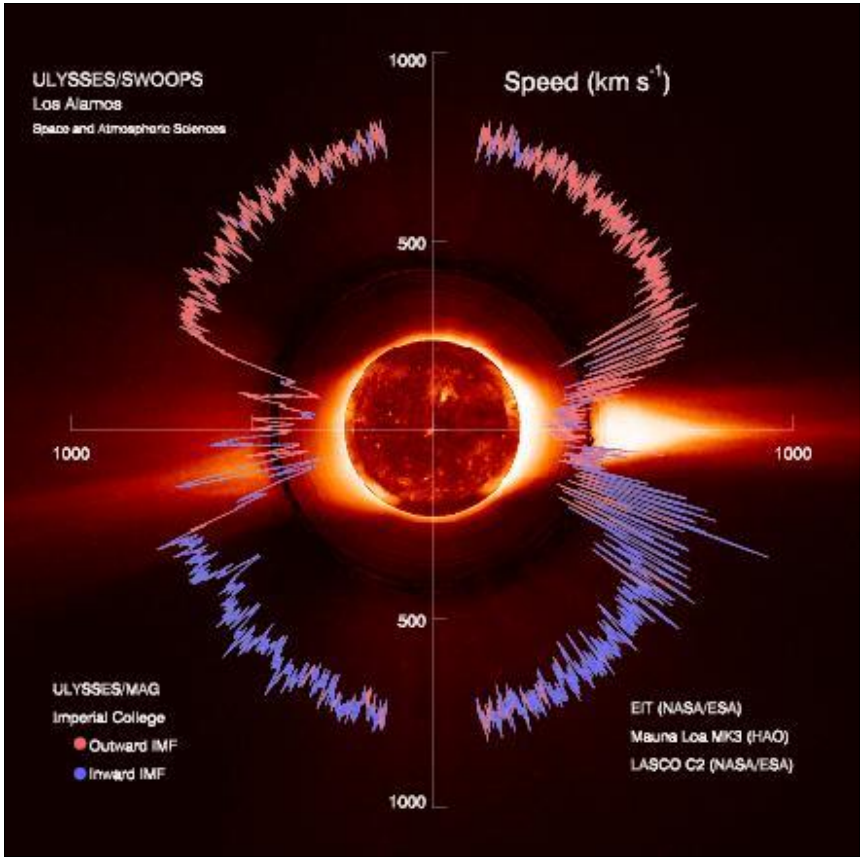


SUN
Electron Heat Flux
Magnetic
Cloud
Shock
CME
Plasma
Counterstreaming
Electrons
Turbulent
Sheath
Earth

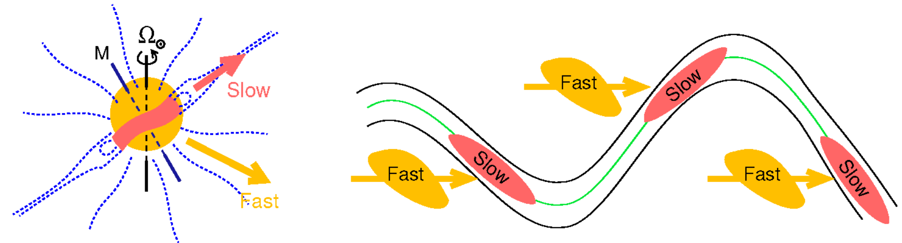


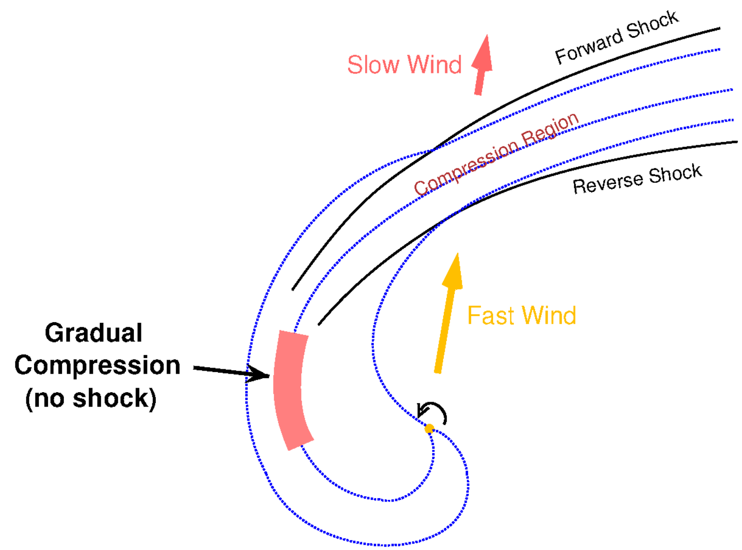
ACE/EPAM LEMS120 (p2p-p7p)
Flux (cm^{-2} s^{-1} MeV^{-1}sr^{-1})
68 - 115 keV
115 - 195 keV
195 - 321 keV
310 - 580 keV
0.58-1.06 MeV
1.06-1.9 MeV
CME
IP shock
?
ACE/MAG
|B|
nT
ACE/MAG
B_R B_T B_N
nT
DAY of 2005

Co-rotating Interaction Regions

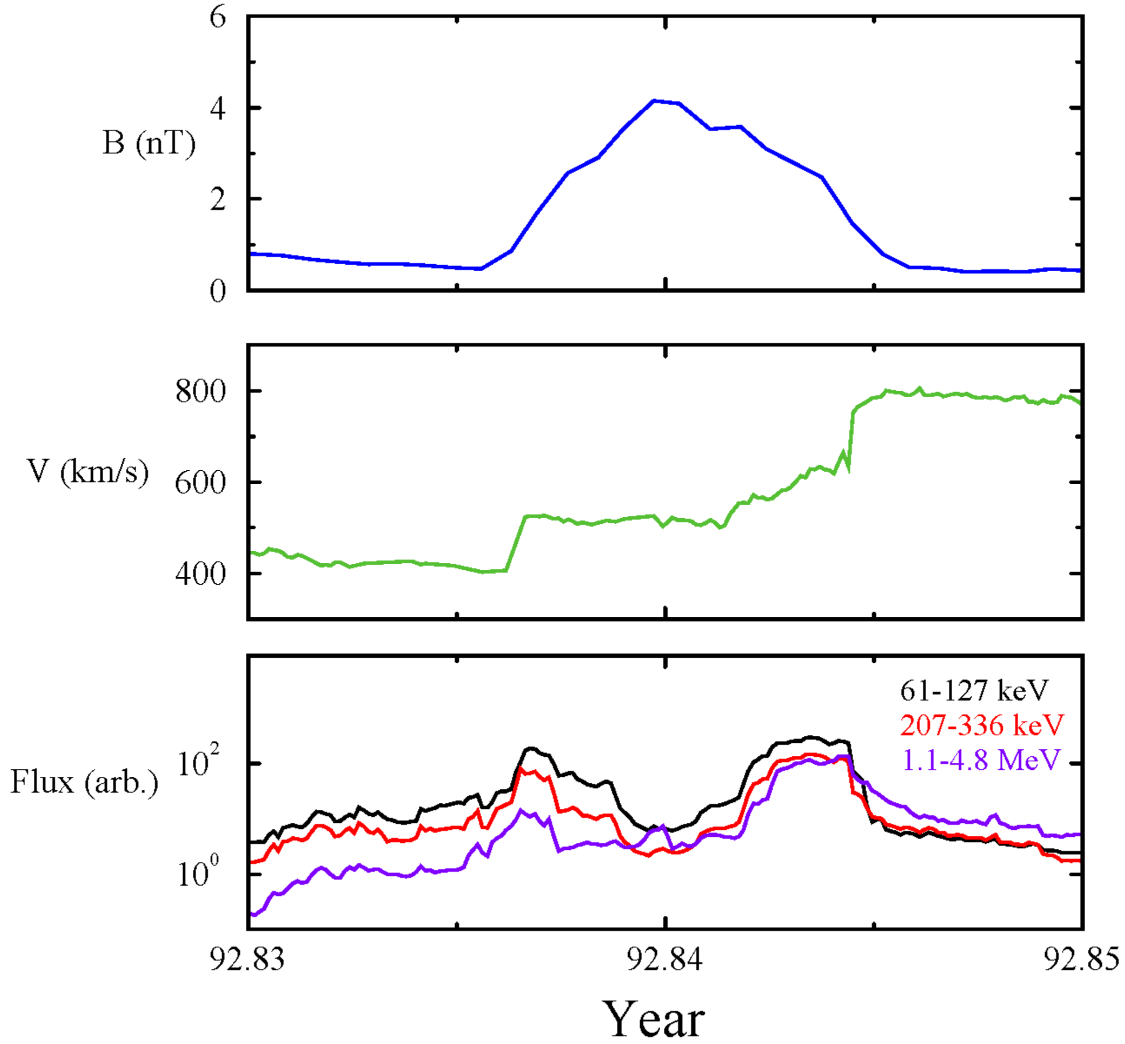


McComas et al., GRL, 1998





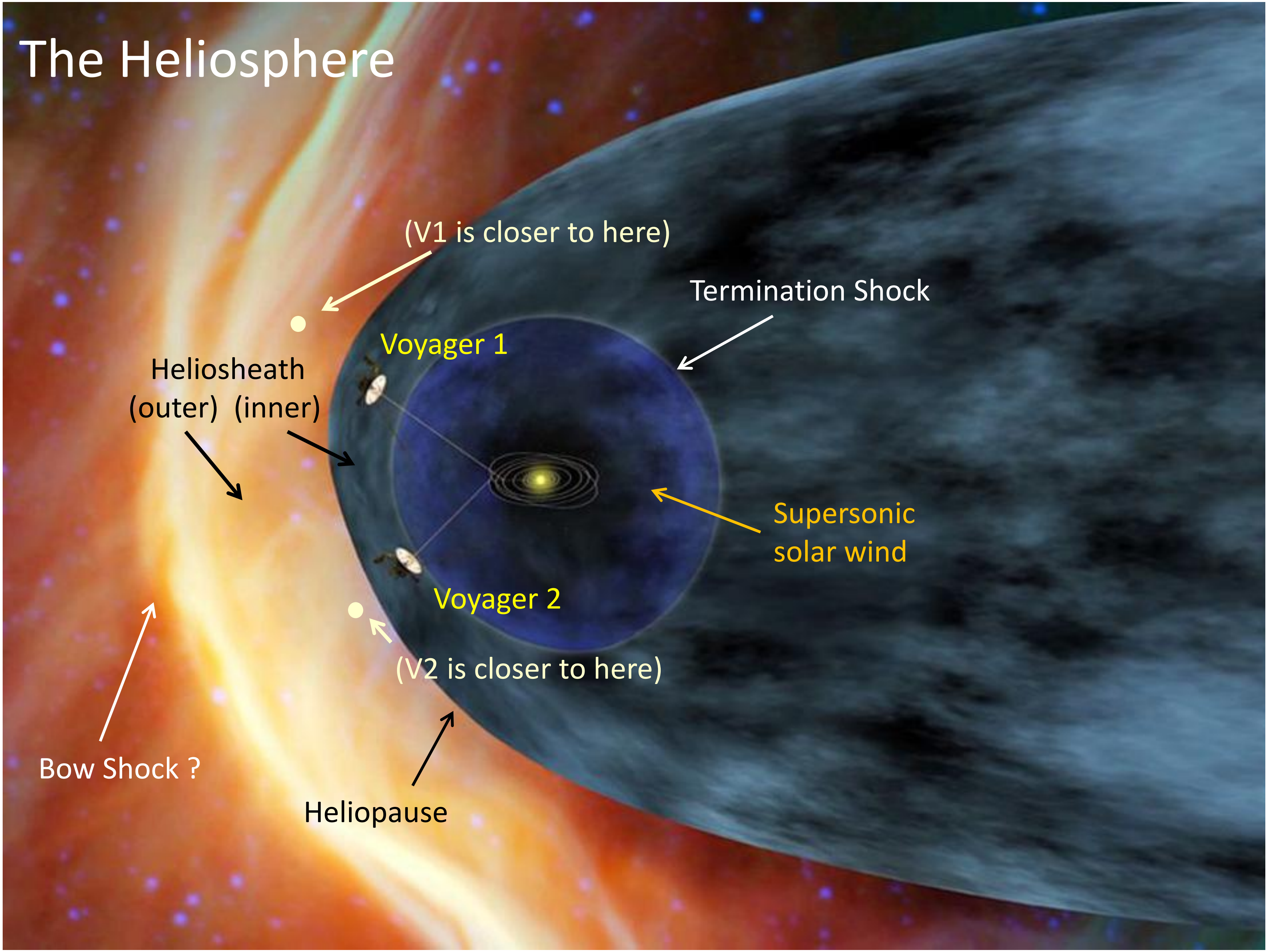
Ulysses data

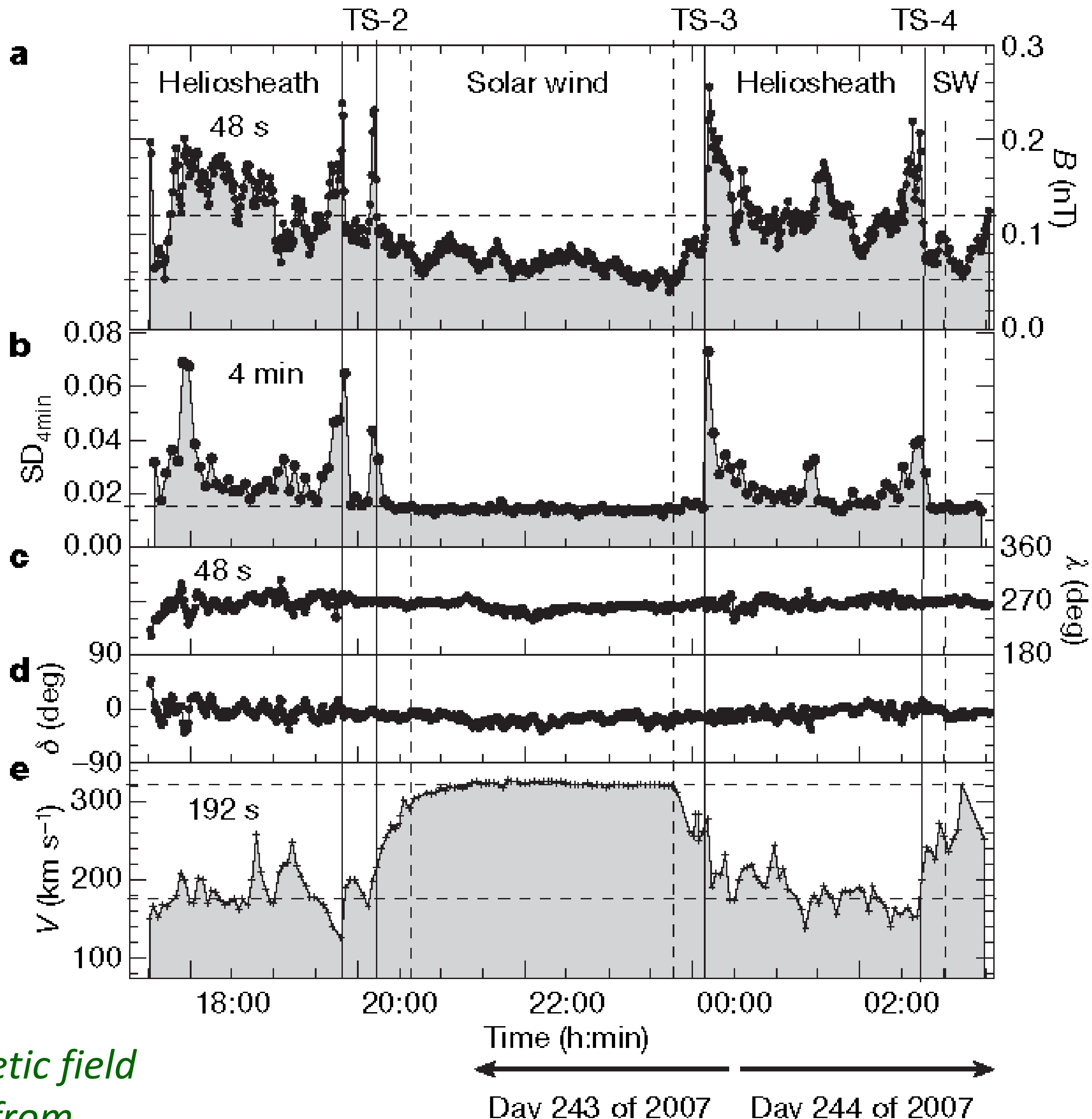


Compression of the magnetic field within CIR.

Slow, intermediate, and fast wind and both a Forward (F) and Reverse (R) shock.

Energetic Particles peaking at the F/R shocks, with a larger intensity at the <u>reverse</u> shock.


The Heliosphere
(V1 is closer to here)
Termination Shock
Voyager 1
Heliosheath
(outer) (inner)
Supersonic
solar wind
Voyager 2
(V2 is closer to here)
Bow Shock ?
Heliopause



Voyager 2 magnetic field and flow speed (from Burlaga et al., 2007)

Bikini Atoll atomic bomb test July 1, 1946

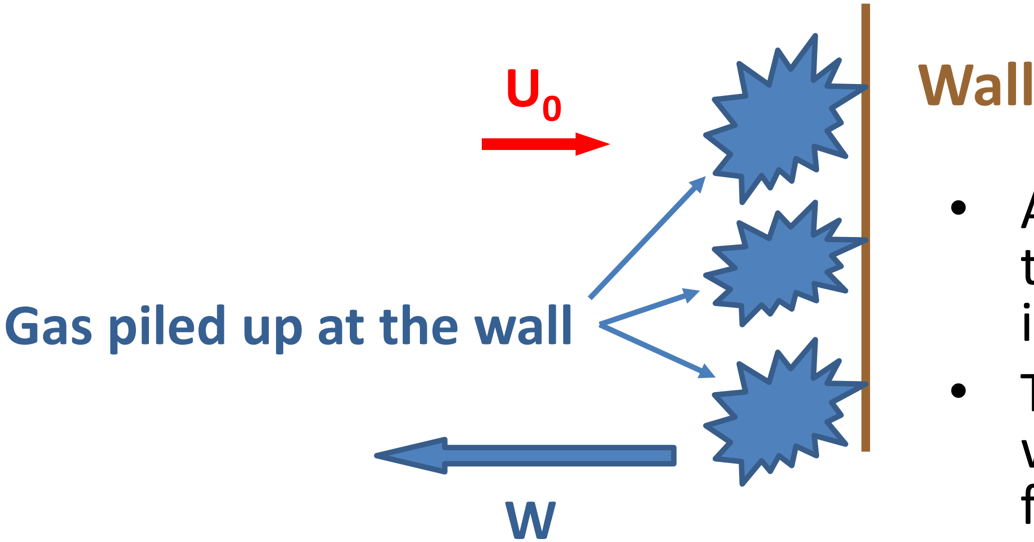
(NavSource Naval History)

How do shocks form?

- The “classic” explanation assumes that a wave steepens and eventually becomes a shock.
- For instance, imagine a sound wave moving through a gas. The pressure of the gas is at a maximum at the peak of the wave. But, the speed of the wave depends on the pressure. So, the peak of the wave is moving faster than other parts of the wave. Thus, as the wave steepens, it deforms as parts of the wave try to overtake other parts of the wave, eventually forcing a shock transition to occur.
- But, this explanation has a fundamental problem: the speed of the wave is based on linear theory! One must do the full non-linear problem to do this properly.
- Frank Shu has a very nice discussion of this in Chapter 15 of his book “The Physics of Astrophysics: Volume II: Gas Dynamics”.

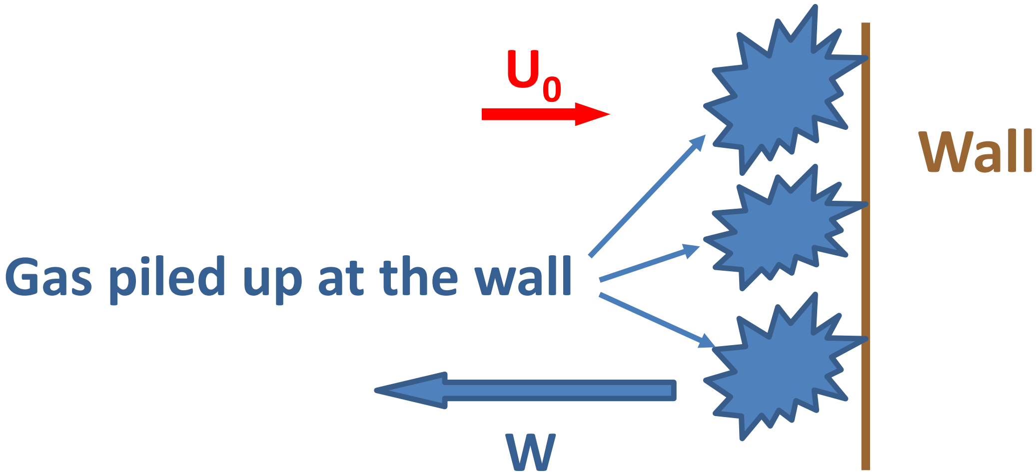
Consider a relatively simple “thought experiment”

- Lets take the limit of hydrodynamics (no magnetic field).
- Suppose we set up a very slow plasma flow, with speed U_0, into a rigid wall, as shown below.



- As gas piles up at the wall, the pressure and density increase there.
- This pressure leads to a wave, propagating away from the wall with a speed **W**, the sound speed associated with the pressure near the wall

- U_0+W is the speed of the gas upstream of the wave, measured in the wave frame of reference. It is greater than the sound speed of the upstream gas ($C_{s0}=(\gamma P_0/\rho_0)^{1/2}$). Thus, information cannot propagate upstream of the wave front!



- **Or, stated another way, consider the frame of reference moving with the upstream fluid (U_0). The wave front is moving faster than the sound speed in this frame! This is not consistent with ordinary linear wave theory.**
- **In this case, "piston-driven" shock case, a HD shock forms very rapidly.**

Shocks are NOT adiabatic!

- Consider the situation we just described. Lets look at the conservation equations.

$$n_0(U_0 + W) = nW$$

Conservation of number. n_0 is the density in the inflowing gas, n the near-wall side

$$n_0(U_0 + W)^2 + P_0 = nW^2 + P$$

Conservation of momentum

$$\frac{P}{n^\gamma} = \frac{P_0}{n_0^\gamma}$$

Lets suppose its adiabatic

- Using the top equation, the second equation becomes:

$$n_0(U_0 + W)^2 + P_0 = n_0(U_0 + W)W + P$$

$$= n_0(U_0 + W)W + P_0\left(\frac{U_0 + W}{W}\right)^\gamma$$

- The last term on the right follows from the third equation above

Shocks are NOT adiabatic!

- Re-arranging, gives:

$$n_0 U_0 (U_0 + 2W) = P_0 \left[\left(\frac{U_0}{W} + 1 \right)^{\gamma} - 1 \right]$$

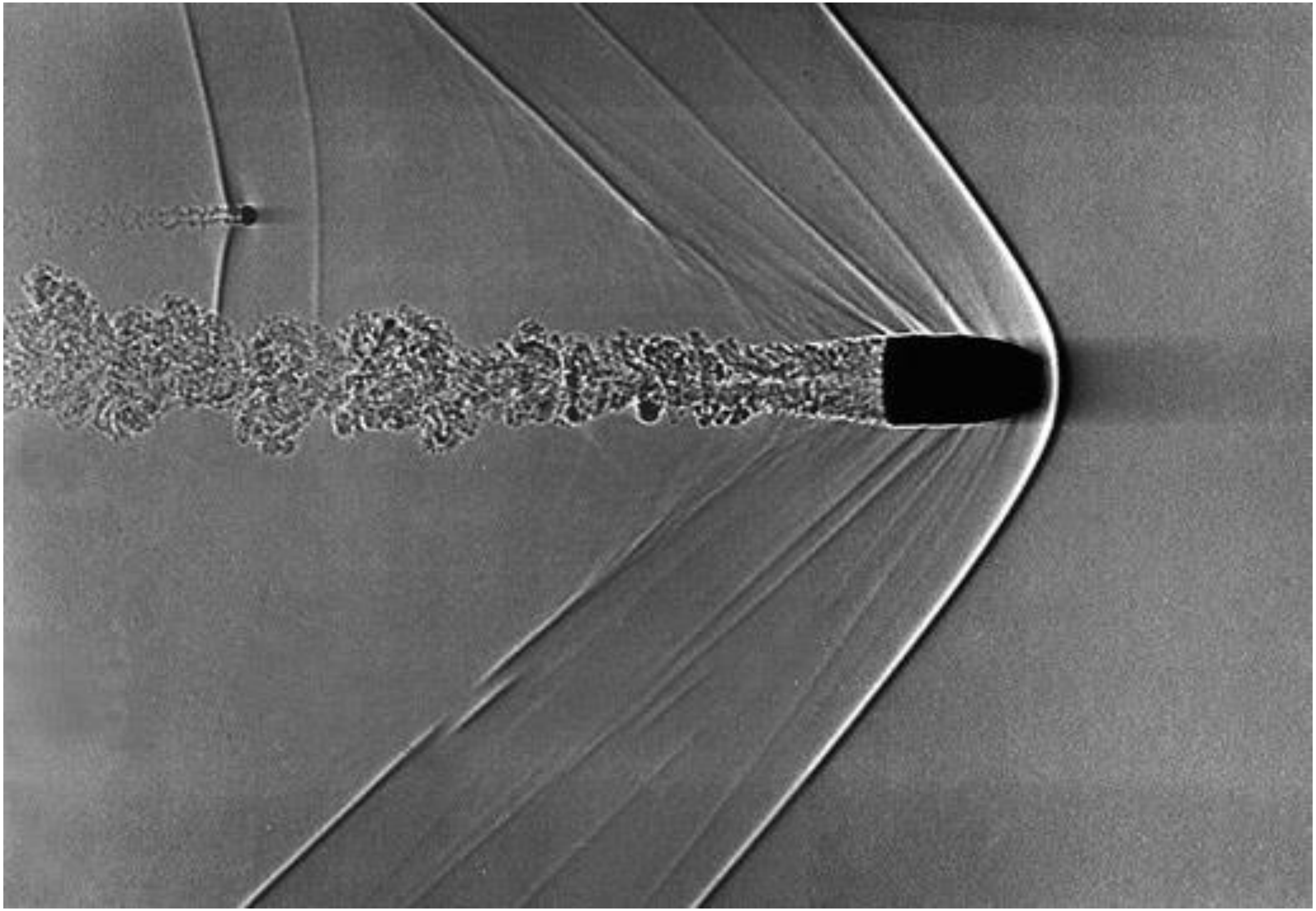
- Which leads to:

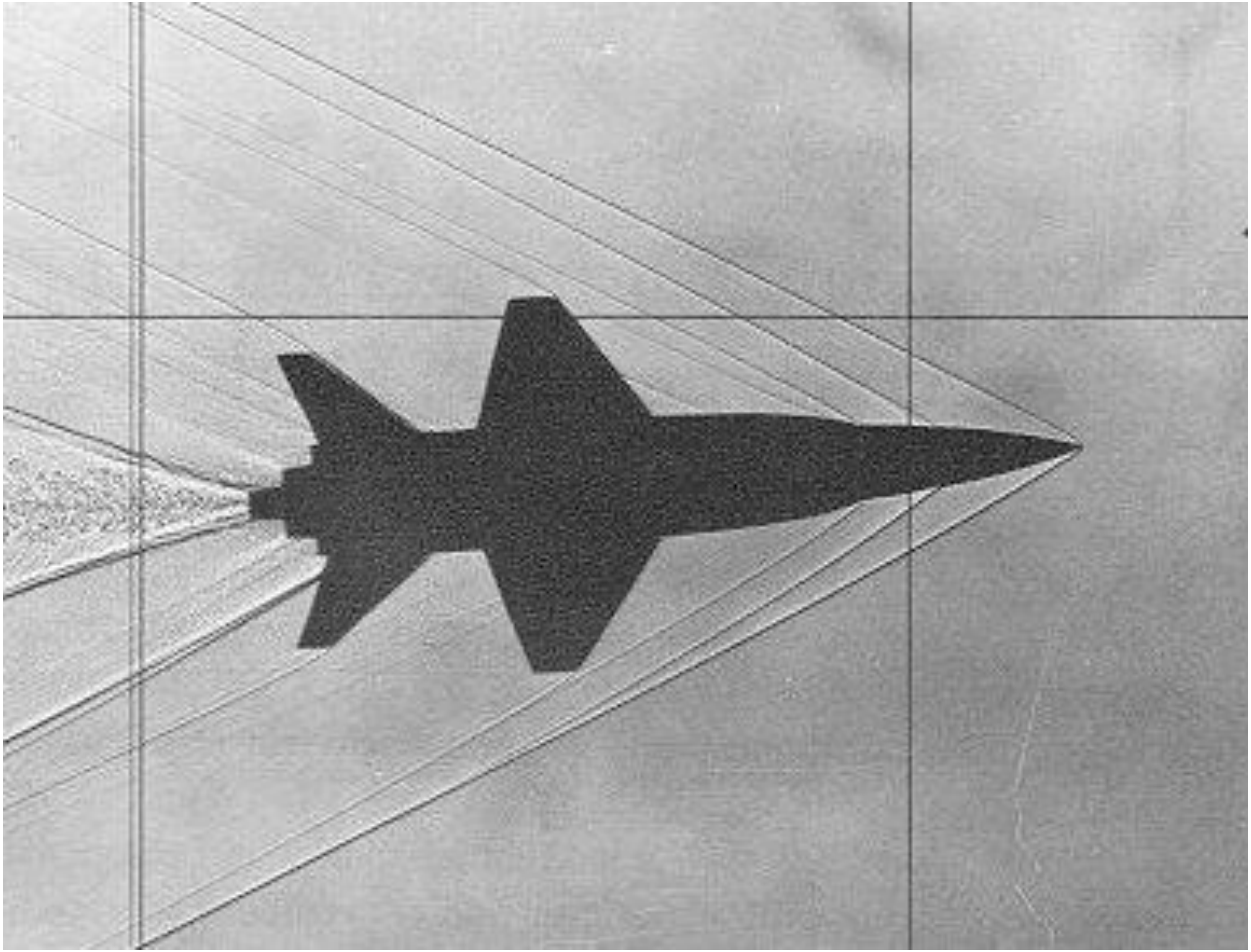
$$1 + \frac{2W}{U_0} = \frac{P_0}{n_0 U_0^2} \left[\left(\frac{U_0}{W} + 1 \right)^{\gamma} - 1 \right]$$

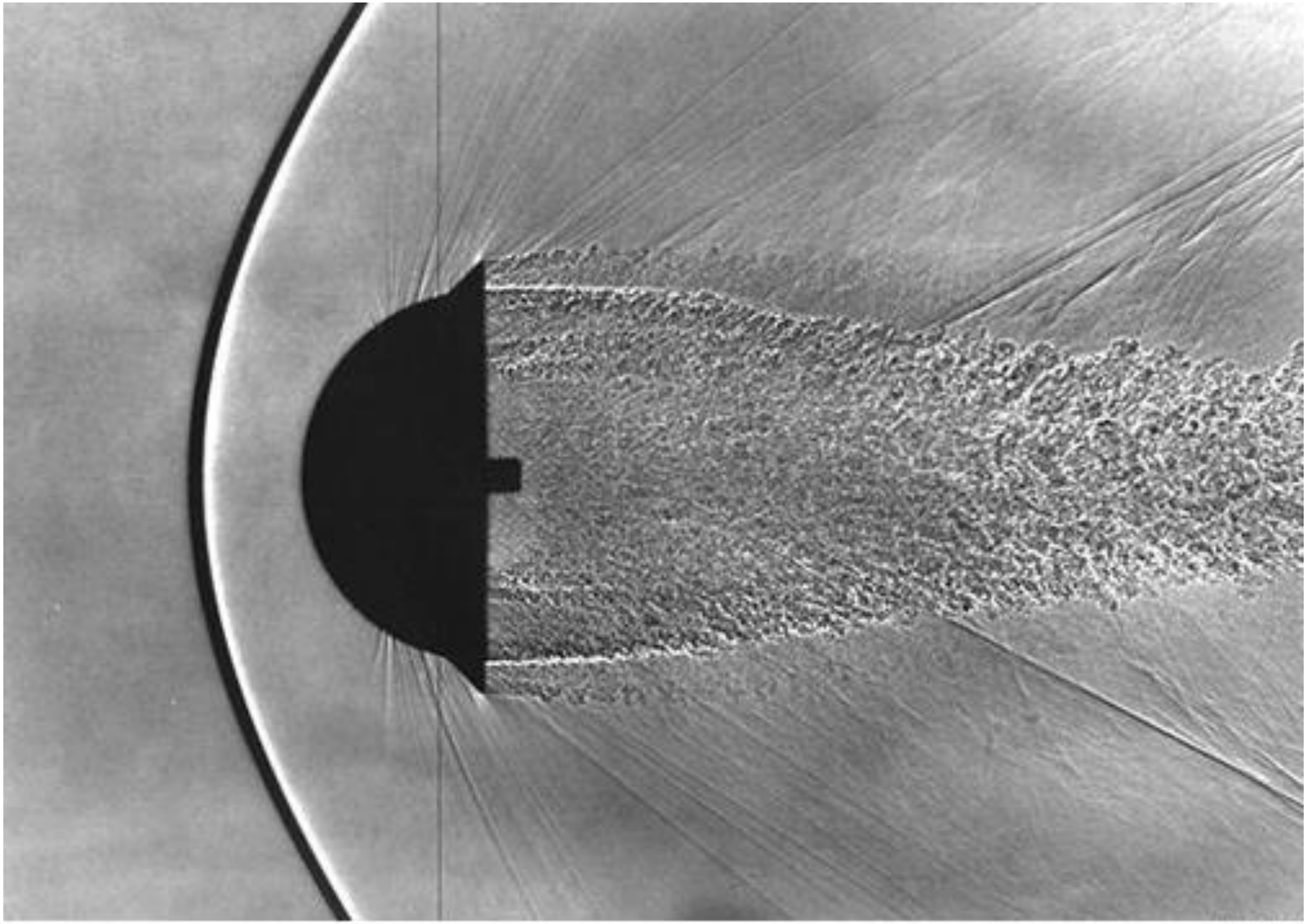
- Note that since $W>>U_0$, the RHS is nearly zero, while the LHS is large. This is not consistent!
- Cannot conserve energy across this wave using adiabatic approximation for the gas. The entropy must increase across the wave.
- Basically, if $W>>U_0$, to conserve mass and momentum, the gas density and pressure cannot change (much) across the wave, but this would imply the downstream sound speed is not much different from the upstream sound speed; but this is essentially the case in which no gas piles up at the wall. We cannot satisfy this with the adiabatic approximation.

Some other considerations

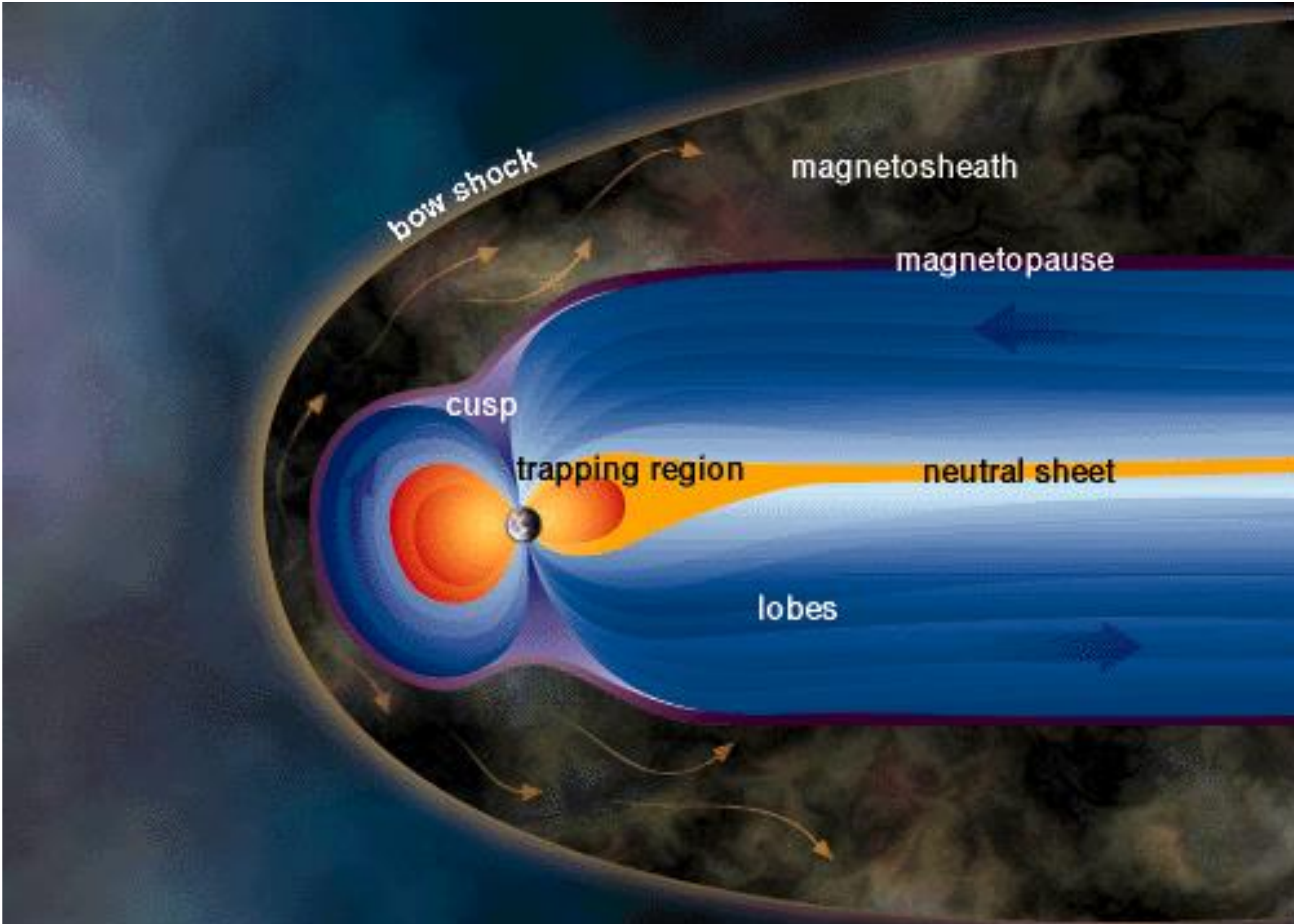
- Note that In the INERTIAL frame, i.e. that at rest with respect to the wall, there is no plasma speed anywhere which exceeds the local sound speed!
- I often hear researchers say that a gas must be moving faster than the local sound speed to have a shock. But, this is stated incorrectly!
- Example: a CME moving away from the Sun, which behaves like the wall. The CME does not have to be moving faster than the sound speed to get a shock. Yet, you often hear people say this and it is commonly written in the literature.







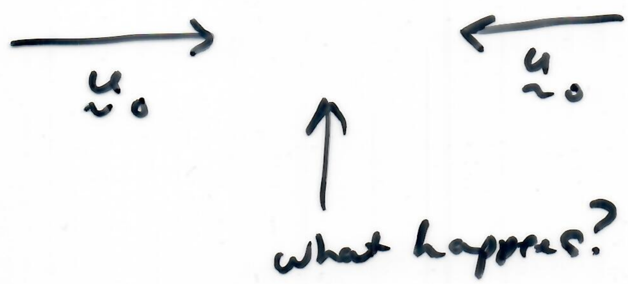
(NASA)


bow shock
magnetosheath
magnetopause
cusp
trapping region
neutral sheet
lobes

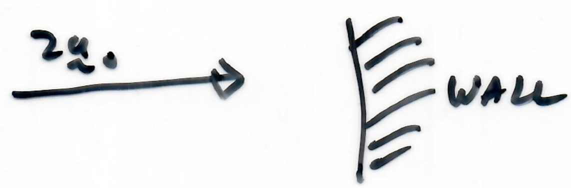
MHD Shocks

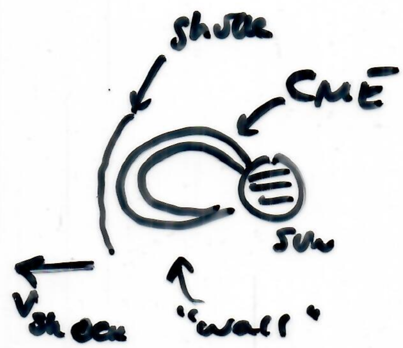
how do shocks form?

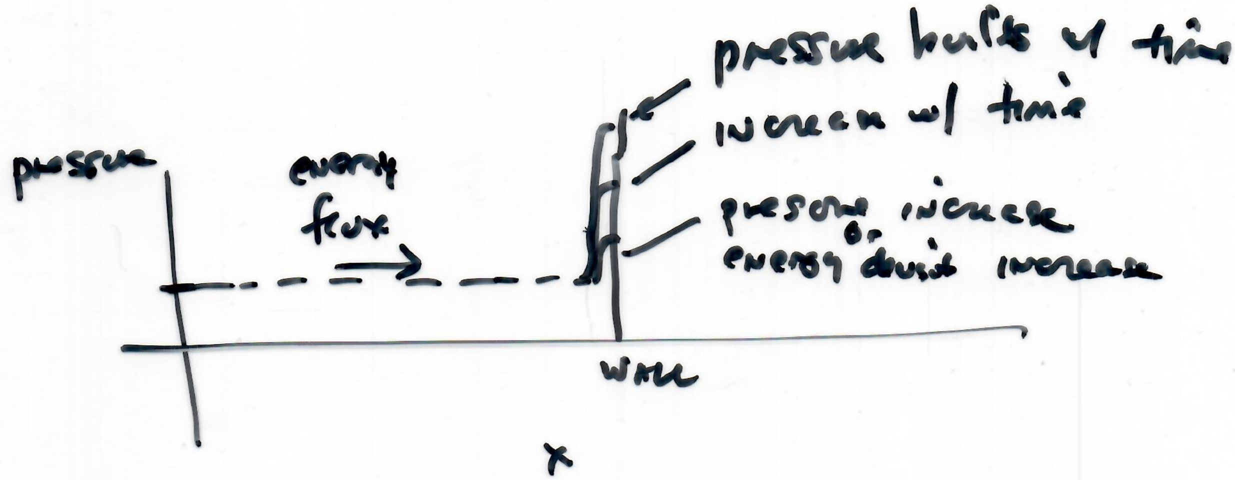
Consider two opposing flows



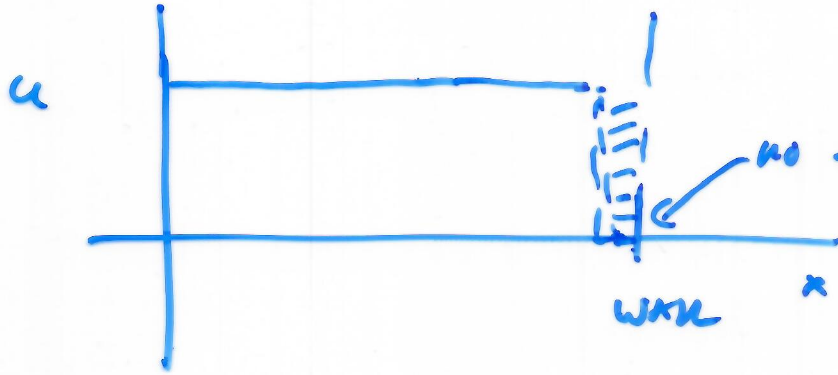
this is analogous



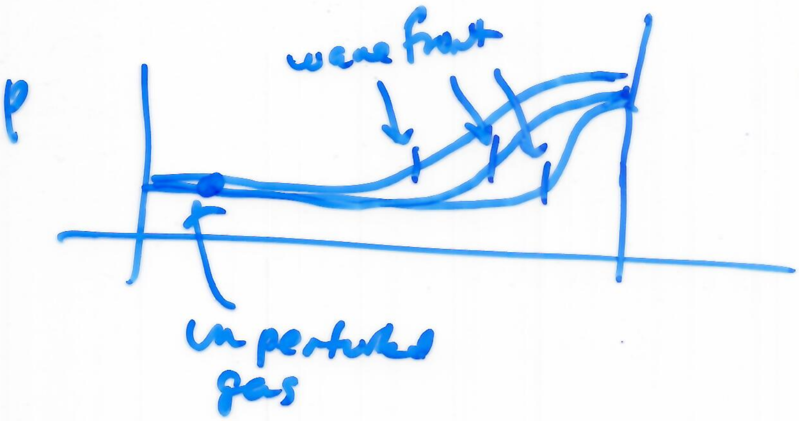




this pressure buildup will expand back into gas

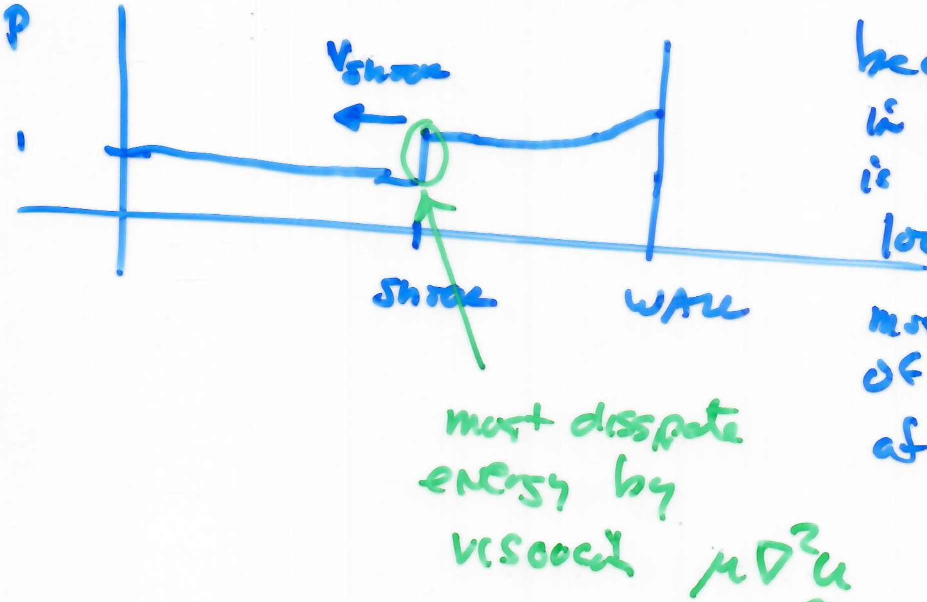


in wall frame high pressure, no flow
plasma expands with speed = sound speed
of this gas $(\gamma P/\rho)^{1/2}$



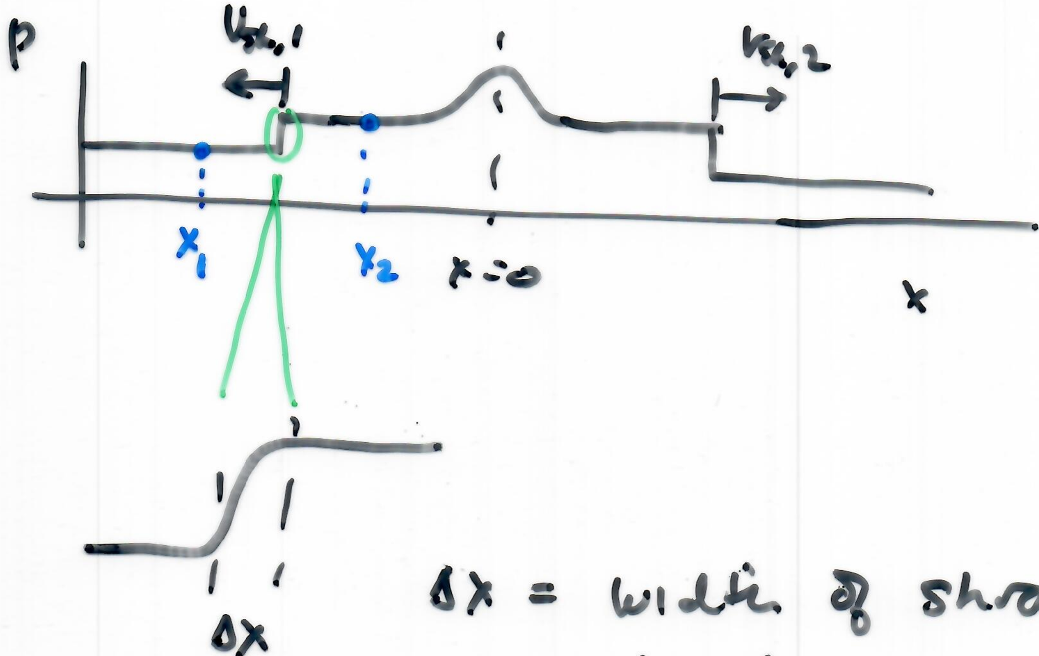
wave front moves with speed $(\gamma P/\rho)^{1/2}$ > sound speed in unperturbed gas

the result



because the wave in unperturbed gas is moving faster than local sound speed

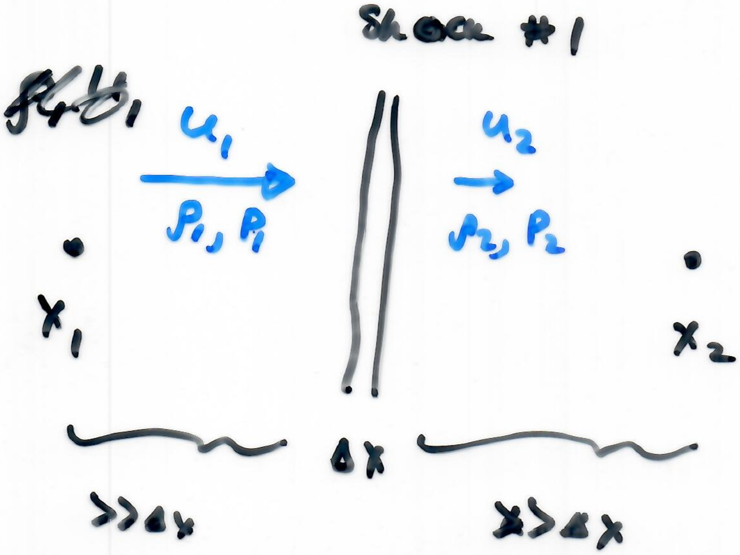
must have dissipation of energy in front of the wave for our original case – 2 shocks moving opposite direction



Δx = width of shock
depends on μ, coef. of viscosity

x_1 & x_2 are at distances from the shock >> Δx

can write down conservation equations at point 1 & 2. (steady state in frame of shock)



Look @ HD eq.'s at end point

$$\rho_1 u_1 = \rho_2 u_2$$

$$\rho_1 u_1^2 + P_1 = \rho_2 u_2^2 + P_2 \qquad \text{(no mag. field)}$$

$$\frac{1}{2}\rho_1 u_1^3 + \frac{\gamma}{\gamma-1} P_1 u_1 = \frac{1}{2}\rho_2 u_2^3 + \frac{\gamma}{\gamma-1} P_2 u_2$$

note: we need complete energy eq.
Shocks are __NOT__ adiabatic
entropy INCREASES across a shock

Consider dimensionless variables

$$r = \rho_2/\rho_1$$

$$u = u_2/u_1$$

$$p = P_2/P_1$$

$$M^2 = \frac{u_1^2}{c_{s1}^2} = \frac{u_1^2}{\gamma P_1/\rho_1} = \frac{\rho_1 u_1^2}{\gamma P_1} = \text{(sonic) Mach \#}$$

Continuity eq:

$$1 = ru$$

Momentum eq:

$$1 + \underbrace{\frac{p_1}{\rho_1 u_1^2}}_{\frac{1}{\gamma M^2}} = ru^2 + \underbrace{\frac{p_2}{\rho_1 u_1^2}}_{\frac{p}{\gamma M^2}}$$

$$1 + \frac{1}{\gamma M^2} = ru^2 + \frac{p}{\gamma M^2}$$

Energy eq.

$$1 + \frac{2p_1}{\rho_1 u_1^2}\frac{\gamma}{\gamma-1} = ru^3 + \frac{2p_2 u_2}{\rho_1 u_1^3}\frac{\gamma}{\gamma-1}$$

$$1 + \frac{2}{M^2(\gamma-1)} = ru^3 + \frac{2pu}{M^2(\gamma-1)}$$

use top equation to eliminate ru in bottom two eq's

$$u + \frac{p}{\gamma M^2} = 1 + \frac{1}{\gamma M^2}$$

eliminated r

$$u^2 + \frac{2pu}{M^2(\gamma-1)} = 1 + \frac{2}{M^2(\gamma-1)}$$

eliminate p by using top eq. substituted into bottom leaving

$$u^2 + \frac{2u}{M^2(\gamma-1)}\left[\left(1 + \frac{1}{\gamma M^2} - u\right)\gamma M^2\right] = 1 + \frac{2}{M^2(\gamma-1)}$$

$$u^2\left(1 - \frac{2\gamma}{\gamma-1}\right) + u\left(\frac{2\gamma}{\gamma-1}\left(1 + \frac{1}{\gamma M^2}\right)\right) = 1 + \frac{2}{M^2(\gamma-1)}$$

$$-u^2\,\frac{\gamma+1}{\gamma-1} + u\,\frac{2\gamma}{\gamma-1}\left(1 + \frac{1}{\gamma M^2}\right) = 1 + \frac{2}{M^2(\gamma-1)}$$

$$\vdots$$

$$u^2 - u\,\frac{2(\gamma M^2+1)}{(\gamma+1)M^2} + \frac{2(\gamma M^2+1)}{(\gamma+1)M^2} - 1 = 0$$

$$\underbrace{u^2 - 1}_{(u+1)(u-1)} - \frac{2(\gamma M^2+1)}{(\gamma+1)M^2}(u-1) = 0$$

$$(u-1)\left[u + 1 - \frac{2(\gamma M^2+1)}{(\gamma+1)M^2}\right] = 0$$

$u = 1 \Rightarrow$ trivial non-shock sol'n

the other is

$$u = \frac{2(\gamma M^2 + 1)}{(\gamma+1)M^2} - 1$$

$$= \frac{2\gamma M^2 + 2 - \gamma M^2 - M^2}{(\gamma+1)M^2}$$

$$= \frac{\gamma M^2 + 2 - M^2}{(\gamma+1)M^2} = \frac{M^2(\gamma-1)+2}{M^2(\gamma+1)}$$

$$\boxed{u = \frac{\gamma - 1 + 2/M^2}{\gamma+1}}$$ flow decrease

$$\boxed{r = \frac{1}{u} = \frac{\gamma+1}{\gamma - 1 + 2/M^2}}$$ density increase

monatomic gas $\gamma = 5/3$

$$\Rightarrow \boxed{u = \frac{1 + 3/M^2}{4} \; ; \; r = \frac{4}{1 + 3/M^2}}$$

Strong shock limit $M \to \infty$

$u \to 1/4$

$r \to 4$

pressure jump

$$p = \left(1 + \frac{1}{\gamma M^2} - u\right)\gamma M^2$$

$$= \gamma M^2 + 1 - u\gamma M^2$$

$$= \gamma M^2 + 1 - \frac{\gamma - 1 + 2/M^2}{\gamma + 1} \cdot \gamma M^2$$

$\vdots$

$\propto M^2 \Rightarrow p$ increases w/ M^2

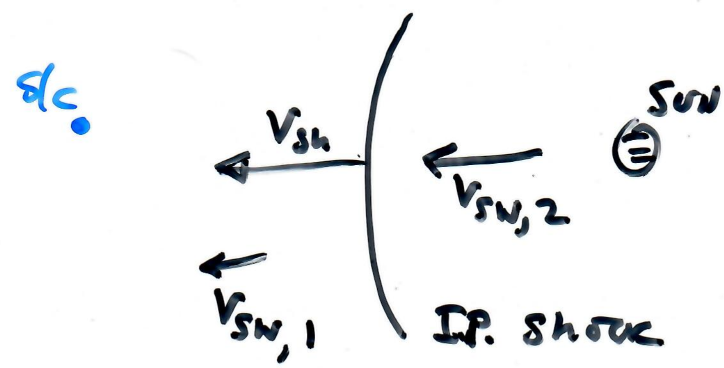
downstream temp. ~~[illegible]~~ $P_2 = n_2 k T_2$

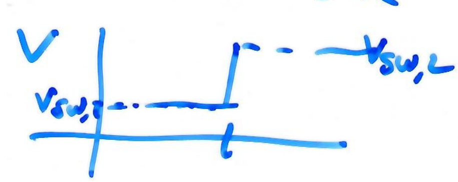
$\Rightarrow T_2 = \dfrac{P_2}{n_2 k}$ k = Boltzmann's const

10/24/18

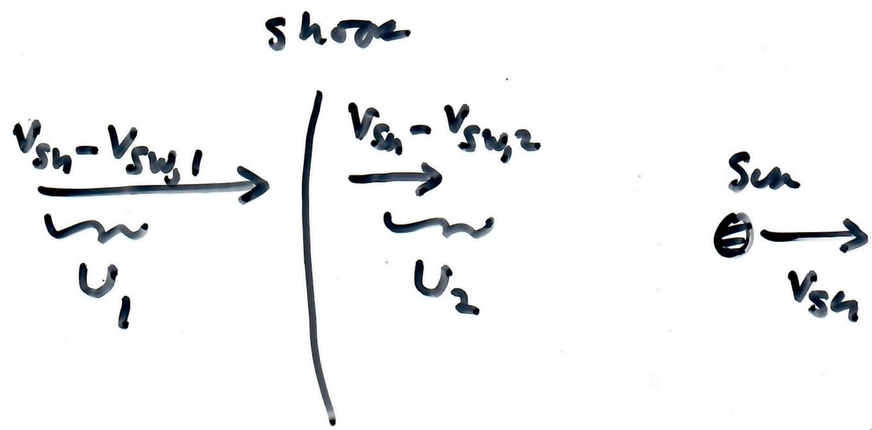
Example of using shock-jump conditions

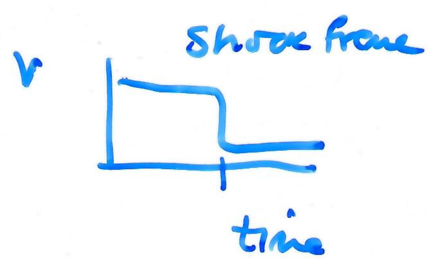
interplanetary shock





In the frame of the shock

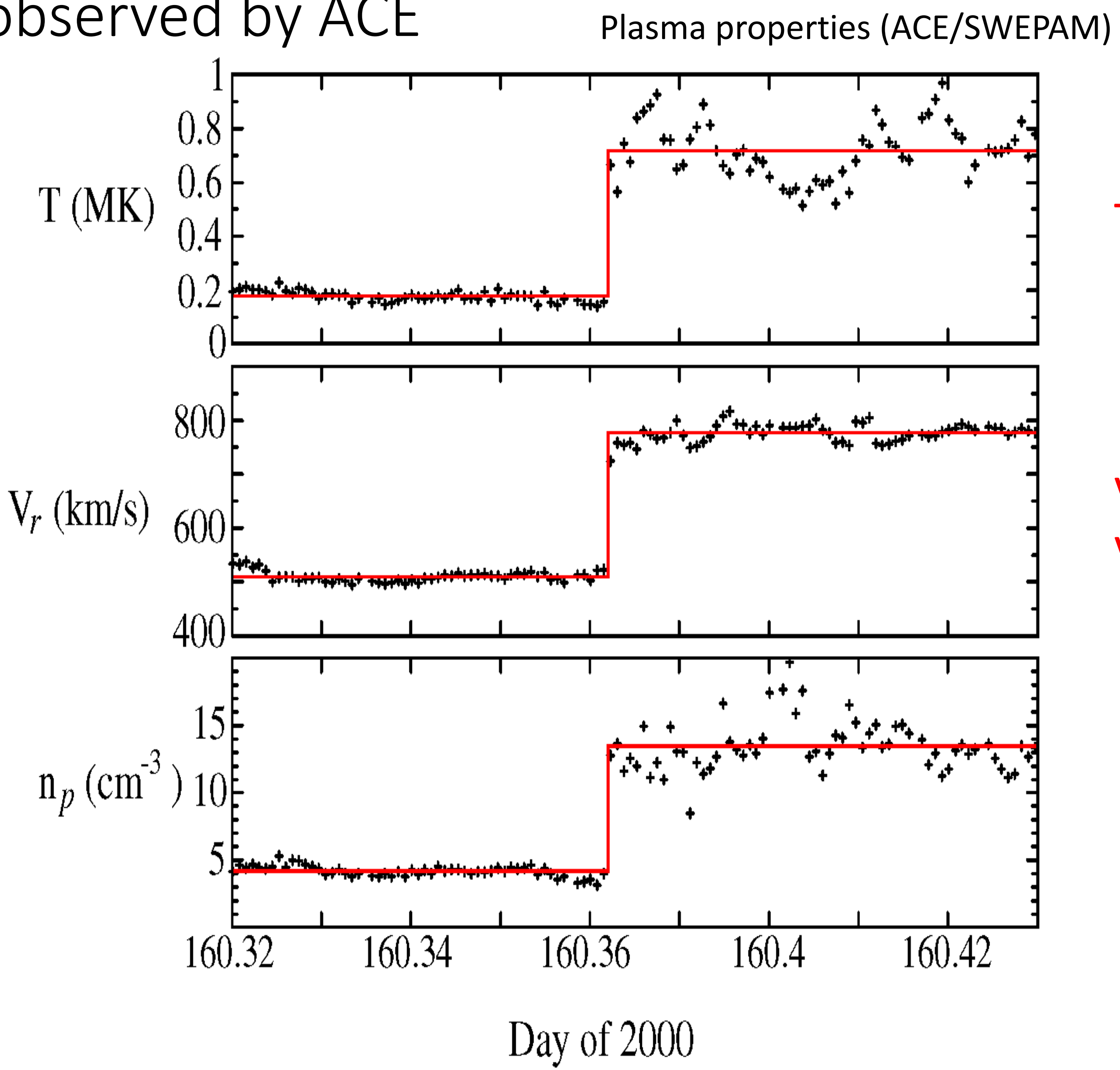




$$\frac{U_1}{U_2} = \frac{n_2}{n_1} \quad \text{cons. of mass}$$

$$\Rightarrow \quad \frac{V_{sh} - V_{sw,1}}{V_{sh} - V_{sw,2}} = \frac{n_2}{n_1}$$

IP shock observed by ACE



$T_2/T_1 = 4$

$V_1 = 510$ km/s

$V_2 = 777$ km/s

$n_2/n_1 = 3.2$

leads to

$$V_{sh} = \frac{(n_2/n_1)\, V_{sw,2} - V_{sw,1}}{n_2/n_1 - 1}$$

$$= \frac{(3.2)(777) - 510}{3.2 - 1} \ \frac{km}{s}$$

$$\boxed{V_{sh} = 898 \ km/s}$$

What is Mach number?

$$M = \frac{U_1}{c_s} \qquad c_s = \text{sound speed}$$

$$= \left(\frac{\gamma P}{\rho}\right)^{1/2} = \left(\frac{\gamma kT}{m}\right)^{1/2}$$

$m = m_p = 1.67 \times 10^{-24}$ g

$\gamma = 5/3$

$k = 1.38 \times 10^{-16}$ erg/K

$T = 200{,}000$ K

$\Rightarrow c_s = 52.5$ km/s

$$U_1 = V_{sh} - V_{sw,1}$$

$$= (898 - 510) \ \frac{km}{s}$$

$$U_1 = 388 \ km/s$$

$$\therefore \boxed{M = \frac{388}{52.5} = 7.4}$$

What do jump conditions give for density compression?

$$\frac{n_2}{n_1} = \frac{(\gamma+1)M^2}{(\gamma-1)M^2+2} = \frac{(8/3)(7.4)^2}{(2/3)(7.4)^2+2}$$

$$= 3.8$$

not 3.2!

because we assumed pure hydro – not MHD!
the mag. field is needed
and we do find agreement
w/ jump conditions + mag. field