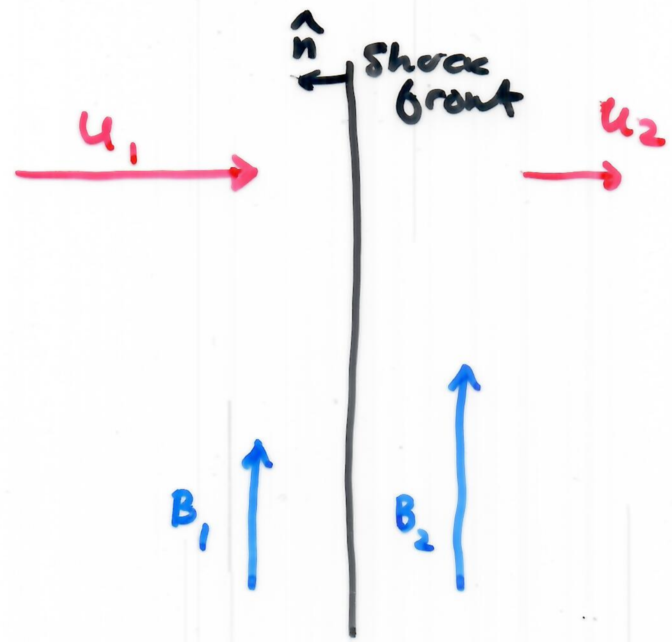
PTYS558: April 6, 2020

- MHD Shocks Part 2
- Collisionless Shock Microstructure

Effect of mag. field on shock jump conditions

Consider a "perpendicular" shock



$\hat{n}$ = unit normal to shock front

$\underset{\sim}{B} \perp \hat{n}$

$\Rightarrow$ "perpendicular" shock

The jump conditions are derived from

$$\rho_1 u_1 = \rho_2 u_2 \qquad \text{continuity}$$

$$\rho_1 u_1^2 + P_1 + \frac{B_1^2}{8\pi} = \rho_2 u_2^2 + P_2 + \frac{B_2^2}{8\pi} \qquad \text{momentum cons.}$$

$$\frac{1}{2}\rho_1 u_1^3 + \frac{\gamma}{\gamma-1} P_1 u_1 + \frac{B_1^2}{4\pi} u_1 = \frac{1}{2}\rho_2 u_2^3 + \frac{\gamma}{\gamma-1} P_2 u_2 + \frac{B_2^2}{4\pi} u_2 \qquad \text{energy cons.}$$

$$u_1 B_1 = u_2 B_2 \qquad \text{magnetic induction equation}$$

The complete MHD energy equation (monatomic gas, isotropic pressure) is often written

$$\boxed{\frac{\partial}{\partial t}\left(\frac{1}{2}\rho u^2 + \frac{1}{\gamma-1} + \frac{B^2}{8\pi}\right) + \nabla\cdot\left[\left(\frac{1}{2}\rho u^2 + \frac{\gamma}{\gamma-1}p\right)\underset{\sim}{u} + \frac{c}{4\pi}\underset{\sim}{E}\times\underset{\sim}{B} + \underset{\sim}{Q}\right] = 0}$$

can include work done by gravity, e.g. here typical to define dimensionless quantities

$$r = \rho_2/\rho_1 \qquad \text{and} \quad \left[\begin{array}{l} M_S = \dfrac{u_1}{c_s} \; ; \; c_s^2 = \dfrac{\gamma P_1}{\rho_1} \\[2ex] M_A = \dfrac{u_1}{c_A} \; ; \; c_A^2 = \dfrac{B_1^2}{4\pi\rho_1} \end{array}\right.$$

$$u = u_2/u_1$$

$$p = P_2/P_1$$

$$b = B_2/B_1$$

this leads to

$$ru = 1$$

$$ru^2 + \frac{1}{\gamma M_s^2}\, p + \frac{1}{2M_A^2}\, b^2 = 1 + \frac{1}{\gamma M_s^2} + \frac{1}{2M_A^2}$$

$$\frac{1}{2} r u^3 + \frac{1}{\gamma-1}\frac{1}{M_s^2}\, pu + \frac{1}{M_A^2}\, b^2 u = \frac{1}{2} + \frac{1}{(\gamma-1)M_s^2} + \frac{1}{M_A^2}$$

$$bu = 1$$

4 eq.'s, 4 unknowns

solve, tedious but straightforward. Be sure to factor out the trivial solution ($u = 1$)

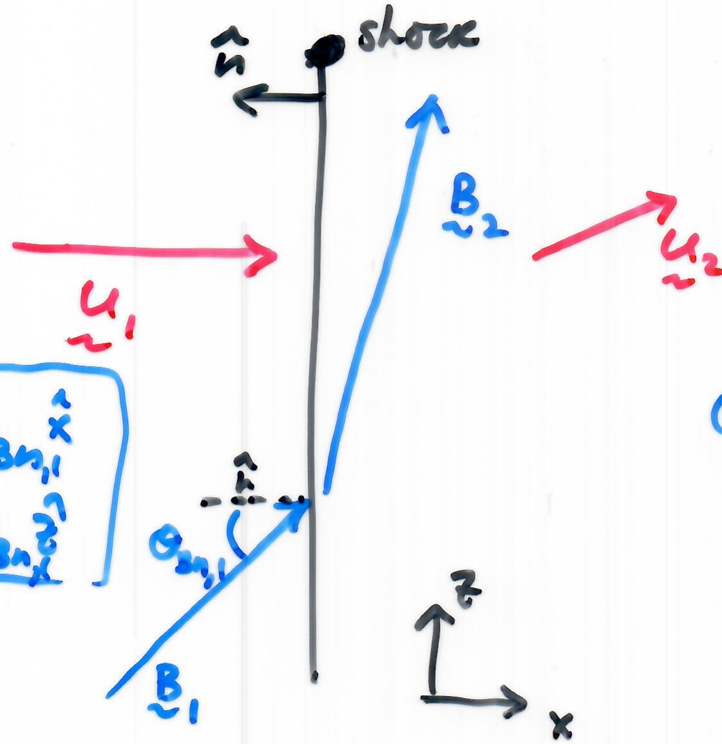
$$\boxed{u^2 - u\left[\frac{\gamma-1}{\gamma+1} + \frac{1}{\gamma+1}\left(\frac{2}{M_s^2} + \frac{2-\gamma}{M_A^2}\right)\right] + \frac{\gamma-2}{(\gamma+1)M_A^2} = 0}$$

jump condition for u

main thing to note: $\underset{\sim}{B}$ remains ~~parallel~~ perp. to $\underset{\sim}{u}$ throughout

also, if $M_A \to \infty$ we get the same result as H.D. case we did before

Oblique shocks



deflected by change in $\underset{\sim}{B}$ ($\sim \frac{1}{M_A^2}$ effect)

$$\boxed{\underset{\sim}{B}_1 = B_1 \cos\theta_{Bn,1}\,\hat{x} + B_1 \sin\theta_{Bn,1}\,\hat{z}}$$

$\theta_{Bn,1}$ = angle between $\underset{\sim}{B}_1$ & $\hat{n}$

"shock normal angle"

things of note: $\nabla \cdot \underset{\sim}{B} = 0 \Rightarrow \frac{dB_x}{dx} = 0 \Rightarrow B_x = \text{const.}$

$$\boxed{B_{x1} = B_{x2}}$$

$\nabla \times (\underset{\sim}{u} \times \underset{\sim}{B}) = 0$ in steady state $\therefore \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ u & 0 & 0 \\ B_x & 0 & B_z \end{vmatrix} = \text{const.}$

$$u B_z = \text{const.}$$

$$\boxed{u_1 B_{z1} = u_2 B_{z2}}$$

$$\therefore \frac{B_2}{B_1} = \left(\frac{B_{2x}^2 + B_{2z}^2}{B_{1x}^2 + B_{1z}^2}\right)^{1/2} = \left(\frac{\cos^2\theta_{Bn,1} + r^2 \sin^2\theta_{Bn,1}}{\cos^2\theta_{Bn,1} + \sin^2\theta_{Bn,1}}\right)^{1/2}$$

$$= \left(\cos^2\theta_{Bn,1} + r^2 \sin^2\theta_{Bn,1}\right)^{1/2}$$

$\theta_{Bn,1} = 0$ ∥ shock, no change in B

$\theta_{Bn,1} = 90°$ ⊥ shock

B changes here

COMPUTER MODELING OF TEST PARTICLE ACCELERATION AT OBLIQUE SHOCKS

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Abstract. We review the basic techniques and results of numerical codes used to model the acceleration of charged particles at oblique, fast-mode, collisionless shocks. The emphasis is upon models in which accelerated particles (ions) are treated as test particles, and particle dynamics is calculated by numerically integrating along exact phase-space orbits. We first review the case where ions are sufficiently energetic so that the shock can be approximated by a planar discontinuity, and where the electromagnetic fields on both sides of the shock are defined at the outset of each computer run. When the fields are uniform and static, particles are accelerated by the scatter-free drift acceleration process at a single shock encounter. We review the characteristics of scatter-free drift acceleration by considering how an incident particle distribution is modified by interacting with a shock. Next we discuss drift acceleration when magnetic fluctuations are introduced on both sides of the shock, and compare these results with those obtained under scatter-free conditions. We describe the modeling of multiple shock encounters, discuss specific applications, and compare the model predictions with theory. Finally, we review some recent numerical simulations that illustrate the importance of shock structure to both the ion injection process and to the acceleration of ions to high energies at quasi-perpendicular shocks.

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Space Science Reviews **48** (1988) 195–262.

Shock microstructure

Basic equations, continuity for both electrons & protons, momentum conservation, Maxwell's eq's etc.

assume steady state, cold plasma ($P_e = P_p = 0$)

and, at least to start, $m_e = 0$ → no electron inertia (we have to put it in later)

$\mathbf{B} = B\hat{z} = B(x)\hat{z}$ → $\perp$ shock

Continuity

$$\nabla \cdot (n_p \underset{\sim}{u}_p) = 0 \qquad \nabla \to \frac{d}{dx}$$

$$\nabla \cdot (n_e \underset{\sim}{u}_e) = 0$$

$$\Rightarrow \quad \frac{d}{dx}(n_p u_{xp}) = 0$$

$$\frac{d}{dx}(n_e u_{xe}) = 0$$

assume $\boxed{n_e = n_p = n}$ → quasi-neutrality

$\therefore$ $\boxed{u_{xp} = u_{xe} = u_x}$ electrons & protons move with same speed in x direction

The other eq.'s are

protons

$$\begin{cases} m_p u_x \dfrac{du_x}{dx} = eE_x + \dfrac{e}{c} u_{yp} B & \text{x- comp. of momentum} \\[2ex] m_p u_x \dfrac{du_{yp}}{dx} = eE_y - \dfrac{e}{c} u_x B & \text{y- comp. of momentum} \end{cases}$$

electrons

$$\begin{cases} 0 = -eE_x - \dfrac{e}{c} u_{ye} B & \text{x- comp. mom. eq.} \\[2ex] 0 = -eE_y + \dfrac{e}{c} u_x B & \text{y- comp. mom. eq.} \end{cases}$$

also have

$$\underset{\sim}{J} = -\frac{c}{4\pi}\frac{dB}{dx}\hat{y} = ne(u_{yp} - u_{ye})\hat{y}$$

$\therefore$

$$\boxed{\frac{c}{4\pi}\frac{dB}{dx} = ne(u_{ye} - u_{yp})}$$

also, $\dfrac{\partial \underset{\sim}{B}}{\partial t} = -c\nabla\times\underset{\sim}{E} = 0 \rightarrow E_y = \text{constant} = -\frac{1}{c}u_x B$

$$\boxed{u_x B = u_1 B_1}$$

Add y-comp. of mom. eq. for protons & electrons

$$\Rightarrow \quad m_p u_x \frac{du_{yp}}{dx} = 0$$

$$\Rightarrow \quad \boxed{u_{yp} = \text{const.} = 0}$$

Therefore, we have from the current

$$\frac{c}{4\pi} \frac{dB}{dx} = n e u_{ye}$$

$$\Rightarrow \quad \boxed{u_{ye} = \frac{c}{4\pi n e} \frac{dB}{dx}}$$ electron drift speed

Insert this into x-component of electron momentum eq. we find

$$\boxed{E_x = -\frac{B}{4\pi n e} \frac{dB}{dx}}$$ ← "cross-shock" electric field

Shock microstructure (cont.)

recall from last lecture, the cross-shock E_x is

$$E_x = -\frac{B}{4\pi n e}\frac{dB}{dx}$$

subst. into ion-momentum eq.

$$m_p u_x \frac{du_x}{dx} = -\frac{B}{4\pi n}\frac{dB}{dx}$$

$$\Rightarrow \quad \frac{du_x}{dx} = -\frac{B}{4\pi n m_p u_x}\frac{dB}{dx}$$

but, ion-continuity eq. gives (steady state)

$$\frac{d}{dx}(n u_x) = 0$$

$$\Rightarrow n u_x = \text{const.} = n_1 u_1$$

$$\therefore \quad \frac{du_x}{dx} = -\frac{d}{dx}\left(\frac{\frac{1}{2} B^2}{4\pi m_p n_1 u_1}\right)$$

$$\Rightarrow \quad u_x = \frac{-B^2}{8\pi m_p n_1 u_1} + C$$

far from shock $u_x = u_1$ (upstream)
$B = B_1$

$$\Rightarrow \quad c = u_1 + \frac{B_1^2}{8\pi m_p n_1 u_1}$$

$$\boxed{u_x = u_1 + \frac{B_1^2 - B^2}{8\pi m_p n_1 u_1}} \qquad (A)$$

to get an eq. for just B, recall from last class

$$u_{ye} = \frac{c}{4\pi ne}\frac{dB}{dx}$$

$$\Rightarrow \quad \frac{dB}{dx} = \frac{4\pi e}{c} n\, u_{ey}$$

$$\Rightarrow \quad u_x \frac{dB}{dx} = \frac{4\pi e}{c}(n u_x)\, u_{ey} = \frac{4\pi e}{c} n_1 u_1 u_{ey}$$

$$\Rightarrow \quad \frac{d}{dx}\left(u_x \frac{dB}{dx}\right) = \frac{4\pi e}{c} n_1 u_1 \frac{du_{ey}}{dx}$$

$$u_x \frac{d}{dx}\left(u_x \frac{dB}{dx}\right) = \frac{4\pi e}{c} n_1 u_1 \left[u_x \frac{du_{ey}}{dx}\right] \qquad (*)$$

now we bring in the electron momentum

the electron mom. eq. for finite elec. mass is (y-component only)

$$n m_e u_x \frac{du_{ey}}{dx} = -ne E_y + \frac{ne}{c} u_x B$$

$$= -ne\left(-\frac{1}{c} u_1 B_1\right) + \frac{ne}{c} u_x B$$

$$\Rightarrow \quad u_x \frac{du_{ey}}{dx} = \frac{e}{m_e c}\left(u_x B + u_1 B_1\right)$$

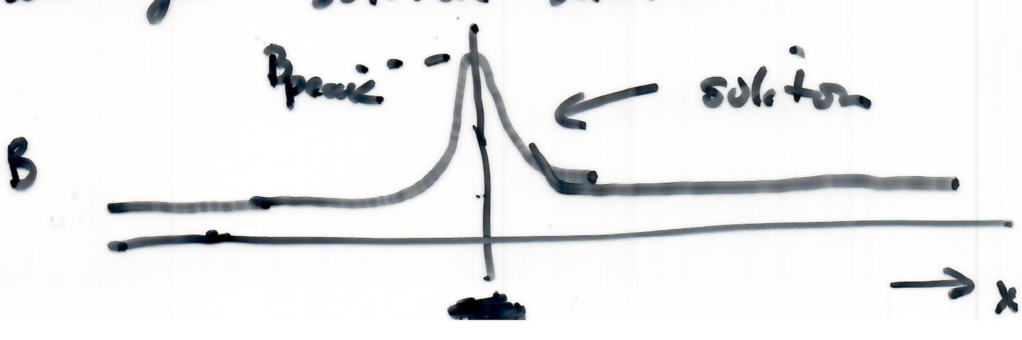
(insert into A)

$$u_x \frac{d}{dx}\left(u_x \frac{dB}{dx}\right) = \frac{\omega_e^2}{c^2} u_1 \left(u_x B + u_1 B_1\right)$$

to get a single eq. for B, substitute (A) on prev. page into this eq.

Complicated! but solvable on computer

we get soliton solution



B_{peak} depends on Mach # $= \frac{u_1}{c_A}$

If we add dissipation, we get a shock.

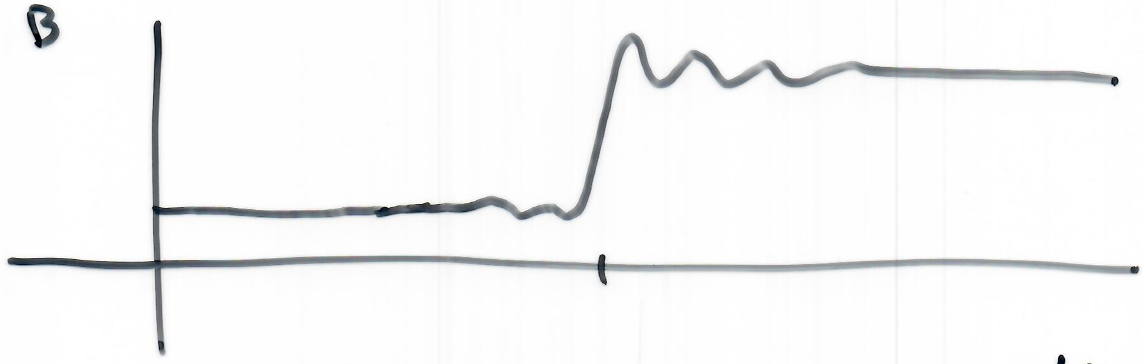
a way to do this is to add a force to both the elec. & proton momentum equations

$-\nu u_{ey}$ ⟵ add to prot. mom. eq.

$+\nu u_{ey}$ ⟵ add to elec. mom. eq.

opposite signs ⇒ preserves total cons. of momentum

then we get a shock



But, only for a limited range in $M_A = \frac{u_1}{c_{A1}}$

(like $M_A < 2$ or so)

Last comment about shock microstructure

recall that the cross-shock elec. field

$$E_x = -\frac{B}{4\pi ne}\frac{dB}{dx}$$

but, we assumed $n_e = n_i = n \Rightarrow$ charge density $= 0$

which, by Poisson's law, $\Rightarrow E_x$ should be constant (zero)

what is the charge separation?

$$\nabla \cdot \underset{\sim}{E} = 4\pi(n_i e - n_e e)$$

assume $n_i = n$

$n_e = n + \delta n$ $\qquad n_i - n_e = \delta n$

this gives a constraint on δn

$$\nabla \cdot \underset{\sim}{E} = 4\pi e\,\delta n$$

$$\Rightarrow \frac{E_x/\Delta x}{4\pi e} = \delta n \qquad \Delta x = \text{width of shock}$$

$$\Rightarrow \frac{(B/4\pi ne)\frac{B}{\Delta x}\frac{1}{\Delta x}}{4\pi e} = \delta n$$

$\Delta x = c/\omega_e$

$$\omega_e^2 = \frac{4\pi n e^2}{m_e}$$

$$\Rightarrow \quad \frac{\delta n}{n} = \frac{B^2}{(4\pi)^2 n^2 e^2 \Delta x^2} = \frac{B^2}{(4\pi n e)^2 \dfrac{c^2}{\underbrace{\left[\frac{4\pi n e^2}{m_e}\right]}_{\omega_e^2}}}$$

$$= \frac{B^2}{4\pi n c^2 m_e}$$

$$= \frac{m_i}{m_e} \frac{B^2}{4\pi n m_i} \frac{1}{c^2}$$

$$\boxed{\frac{\delta n}{n} = \frac{m_i}{m_e} \left(\frac{v_A}{c}\right)^2}$$

we need $\frac{\delta n}{n} \ll 1$ for our approach to work.

for typical parameters in solar wind

$$\left.\begin{aligned} v_A &\simeq 45 \text{ km/s} \\ c &\simeq 300{,}000 \text{ km/s} \\ m_i &\simeq 2000\, m_e \end{aligned}\right\} \quad \frac{\delta n}{n} \sim 4.5 \times 10^{-5}$$

Structure of a collisionless shock

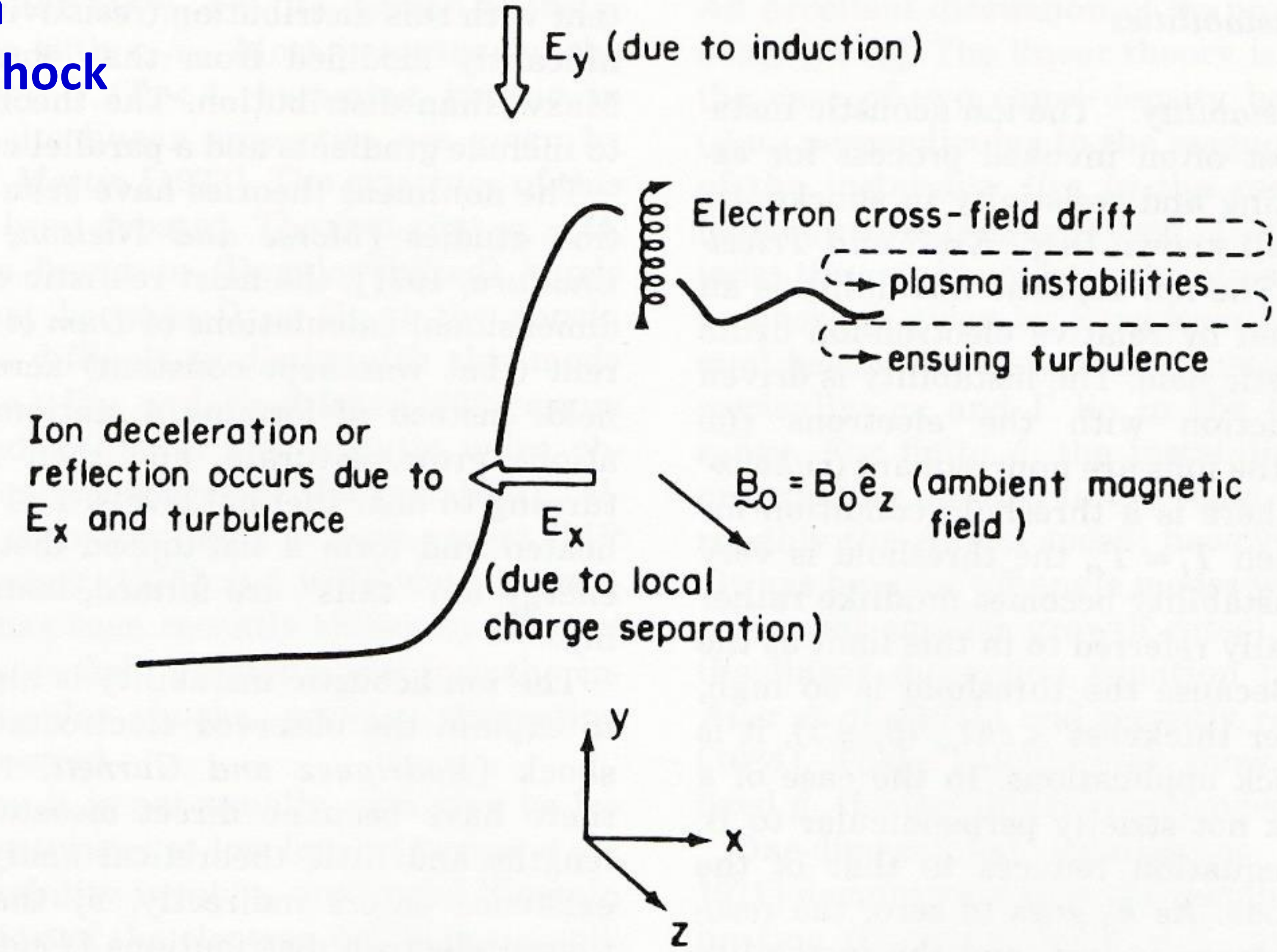
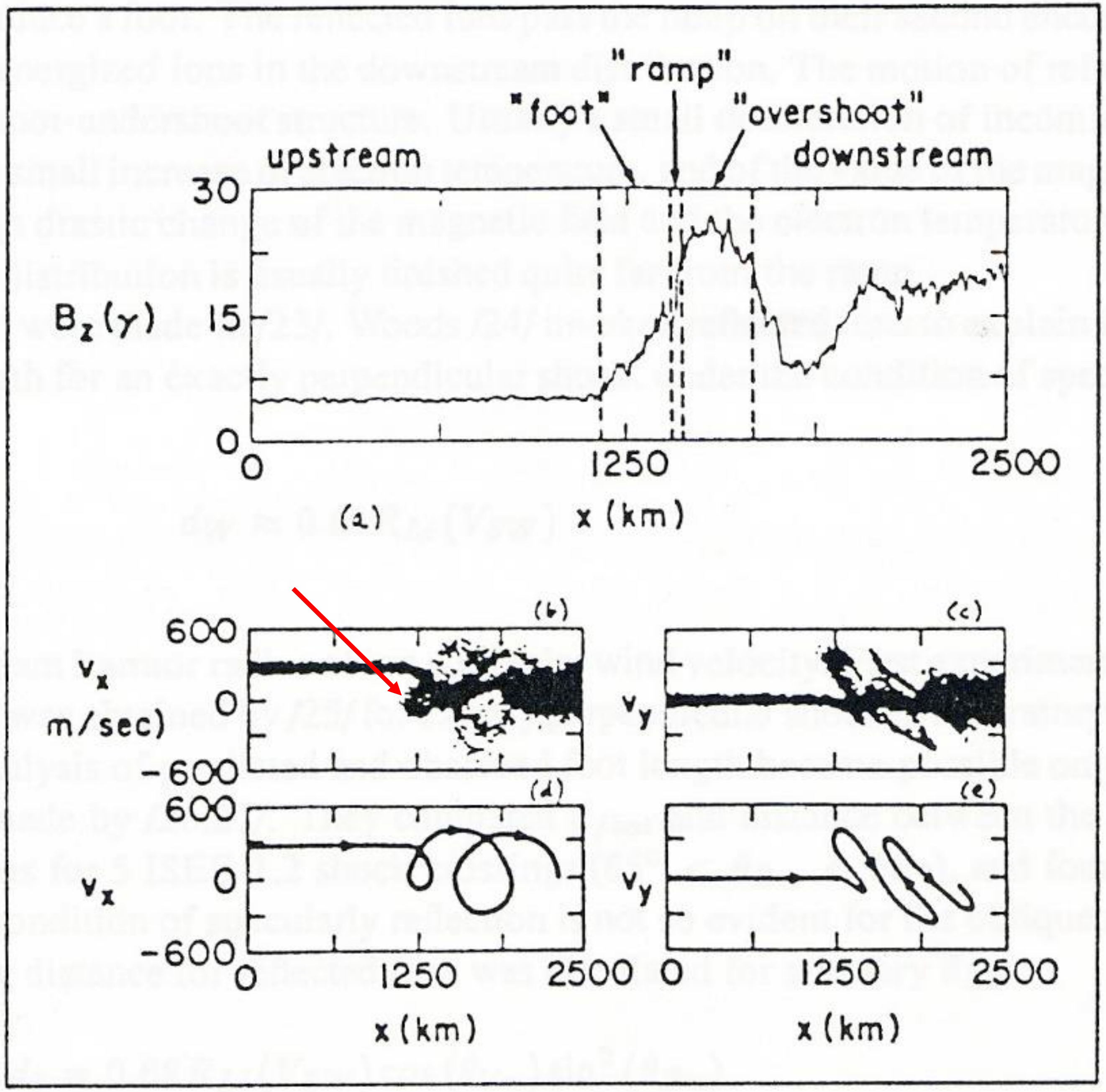


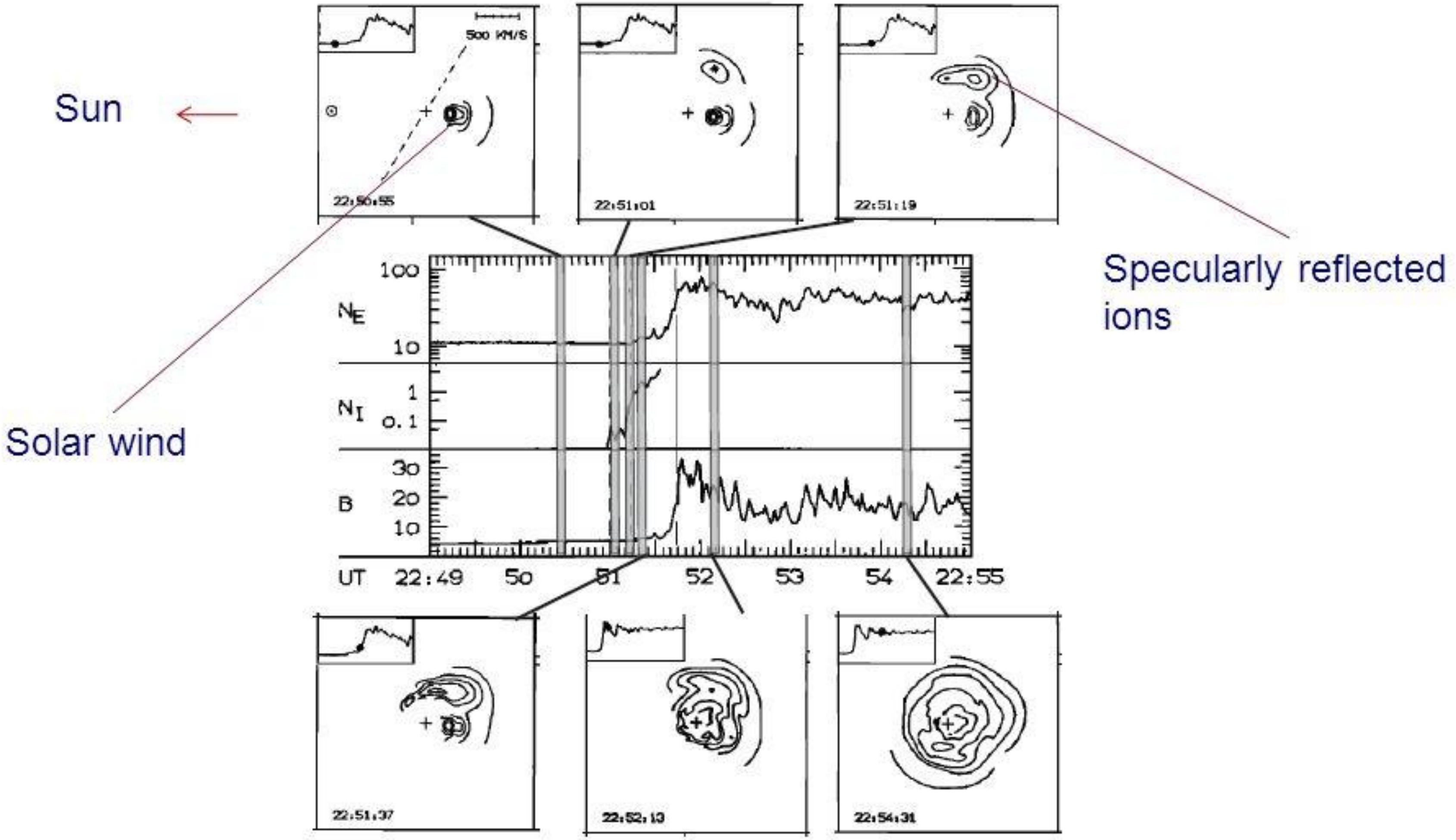
Fig. 1. Geometry of a perpendicular shock showing the field structure and sources of free energy [*Wu*, 1982].



“Specularly reflected ions” seen in kinetic simlations of shocks

This is a non-resistive type of energy dissipation

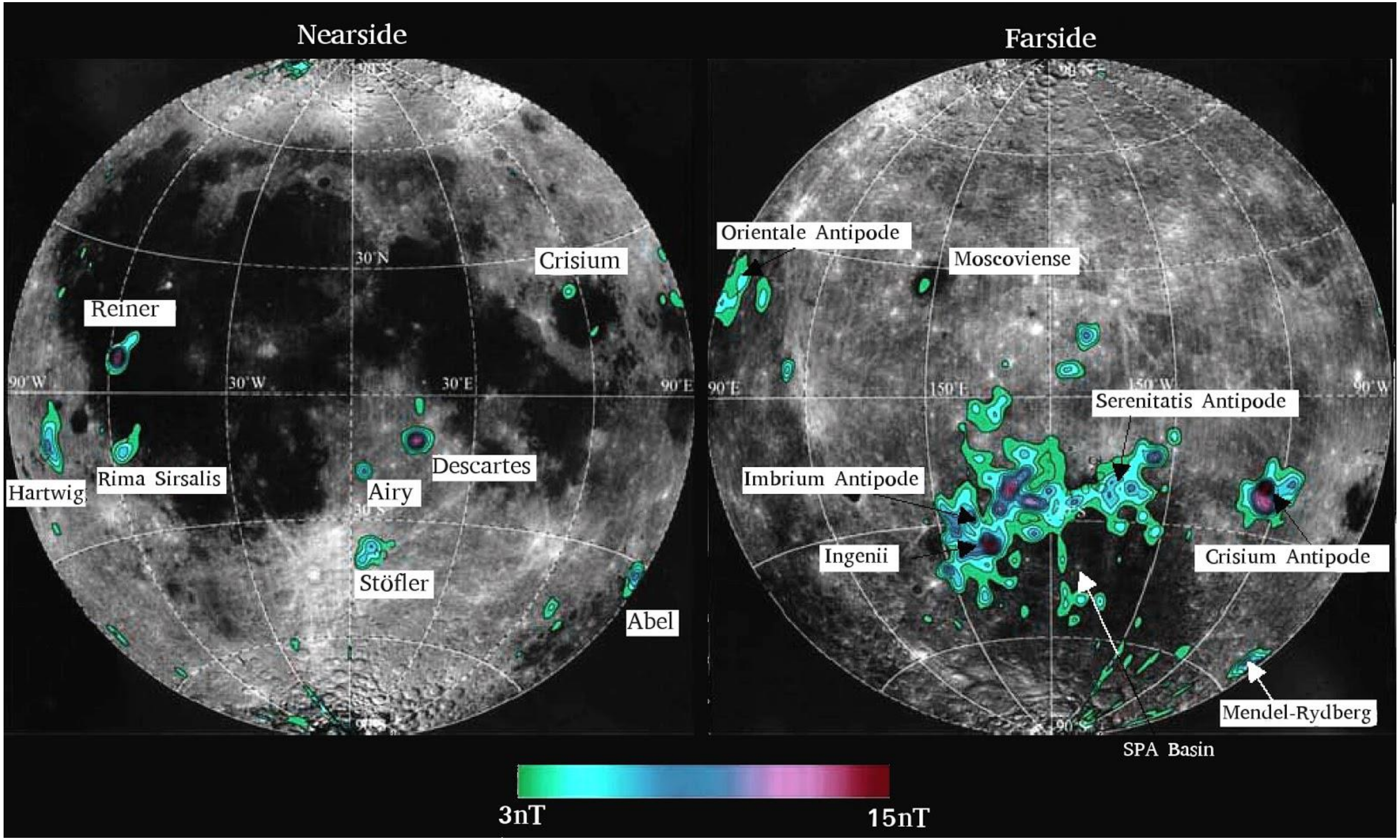
Specularly reflected ions in the foot of the quasi-perpendicular bow shock – in situ observations



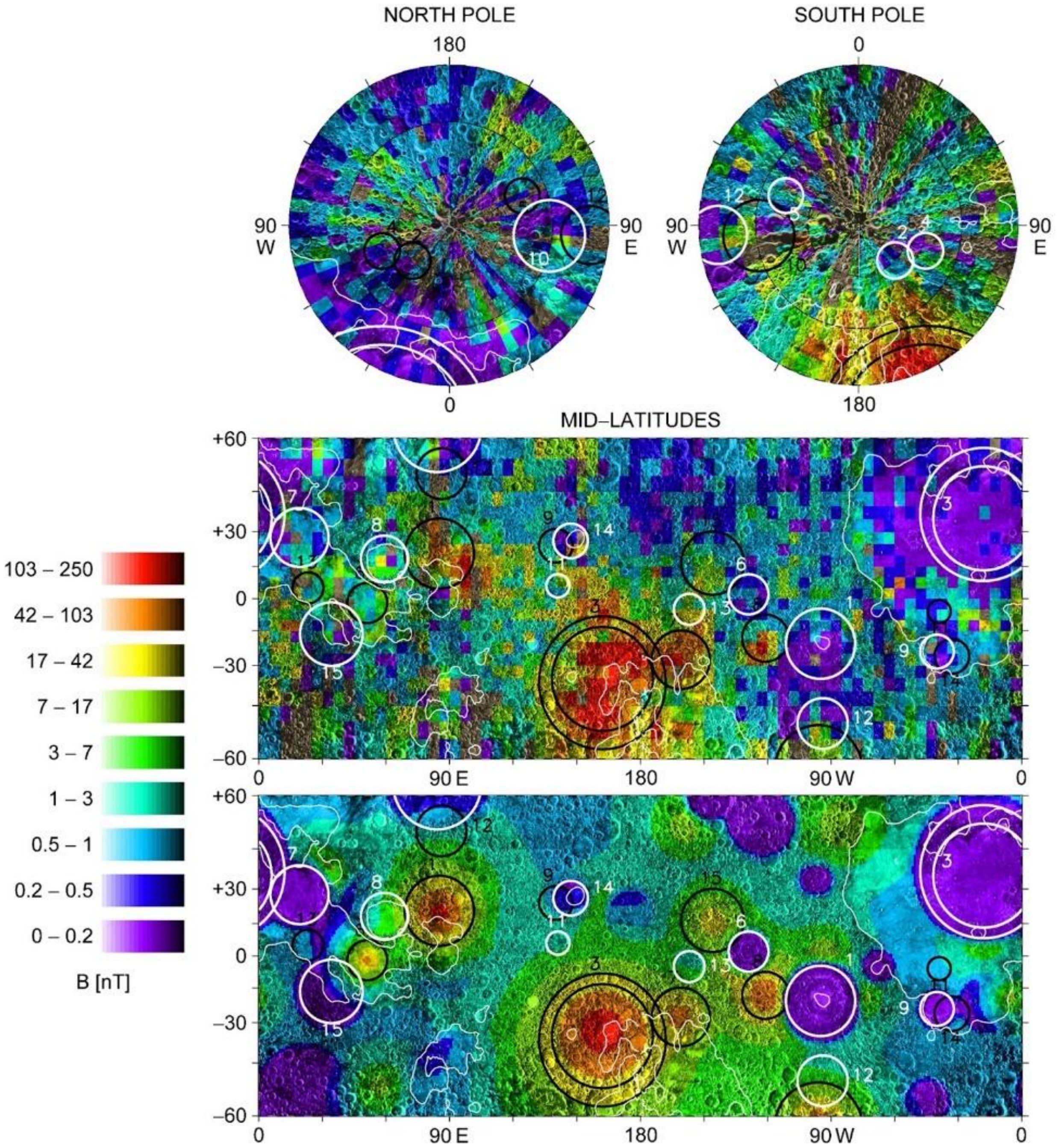
Ion velocity space distributions for an inbound bow shock crossing. Phase space density is shown in the ecliptic plane with sunward flow to the left.

Sckopke et al. 1983

Lunar magnetic anomalies



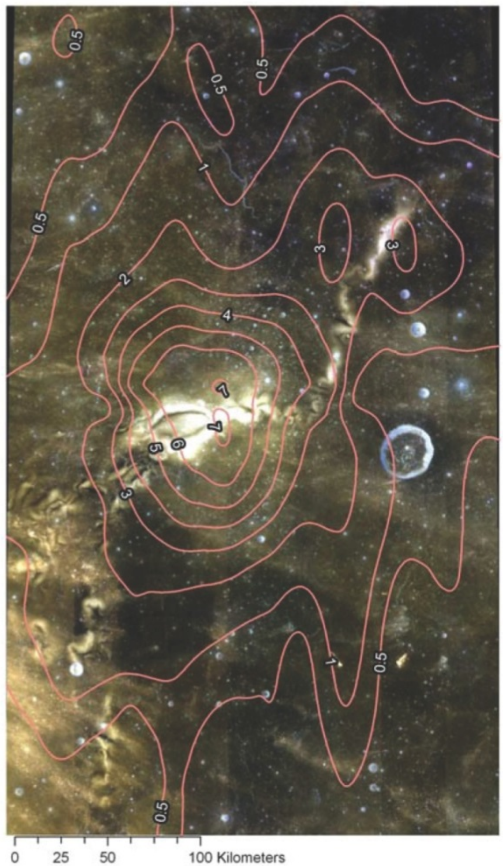
Lunar magnetic anomalies

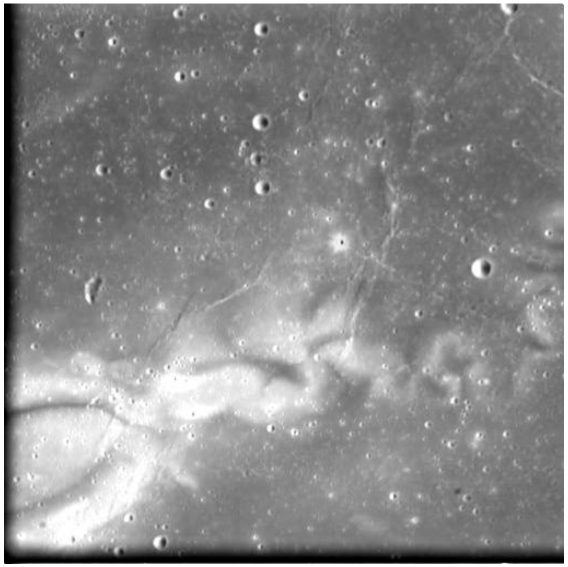


Lunar "Swirls"

- A certain class of surface albedo feature (called "swirls") has also been identified on the moon that are roughly spatially co-located with magnetic anomalies.

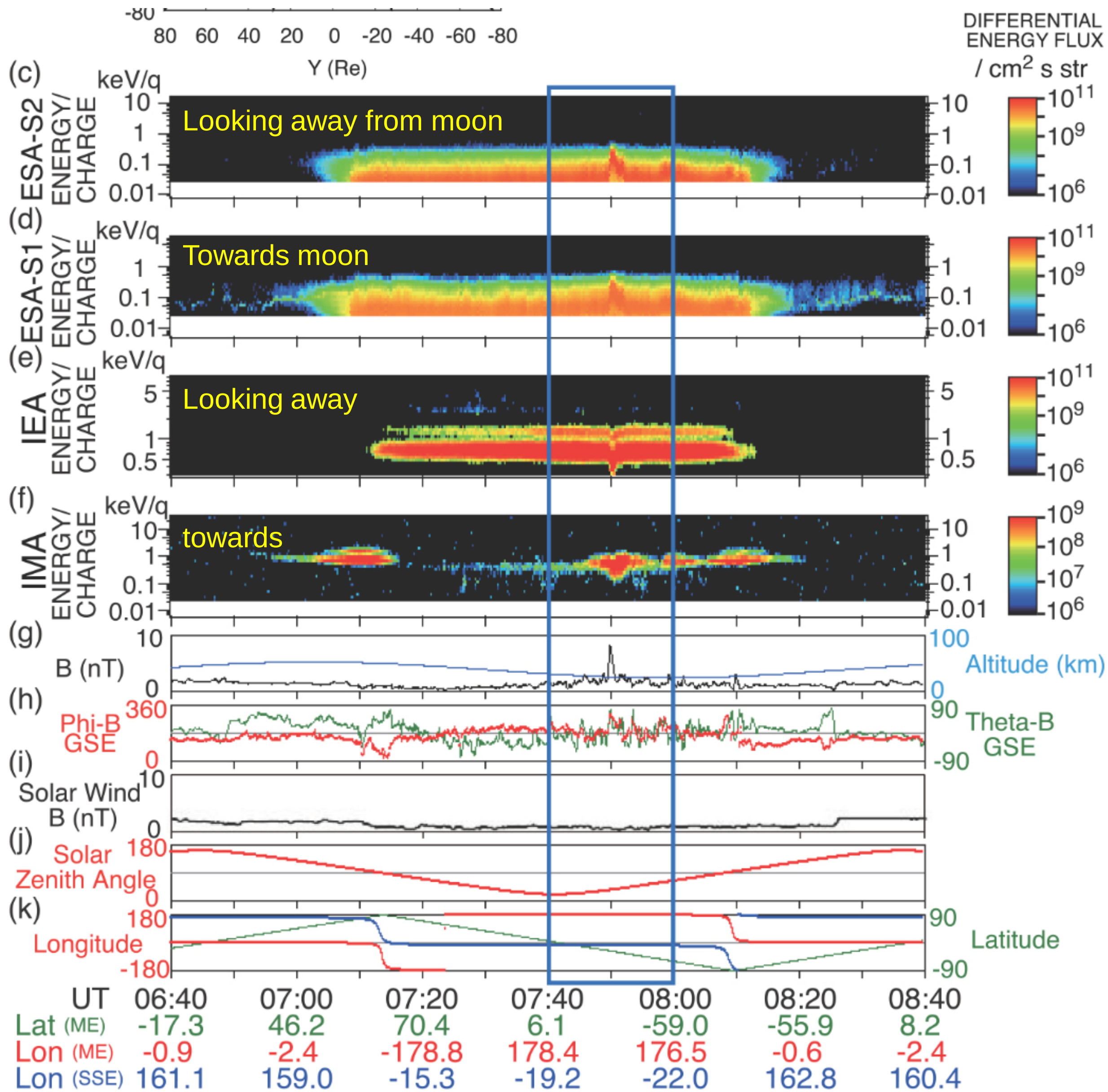
- Swirls are probably the result of variations in "space-weathering" of the lunar surface near magcons, but the process itself is not well understood





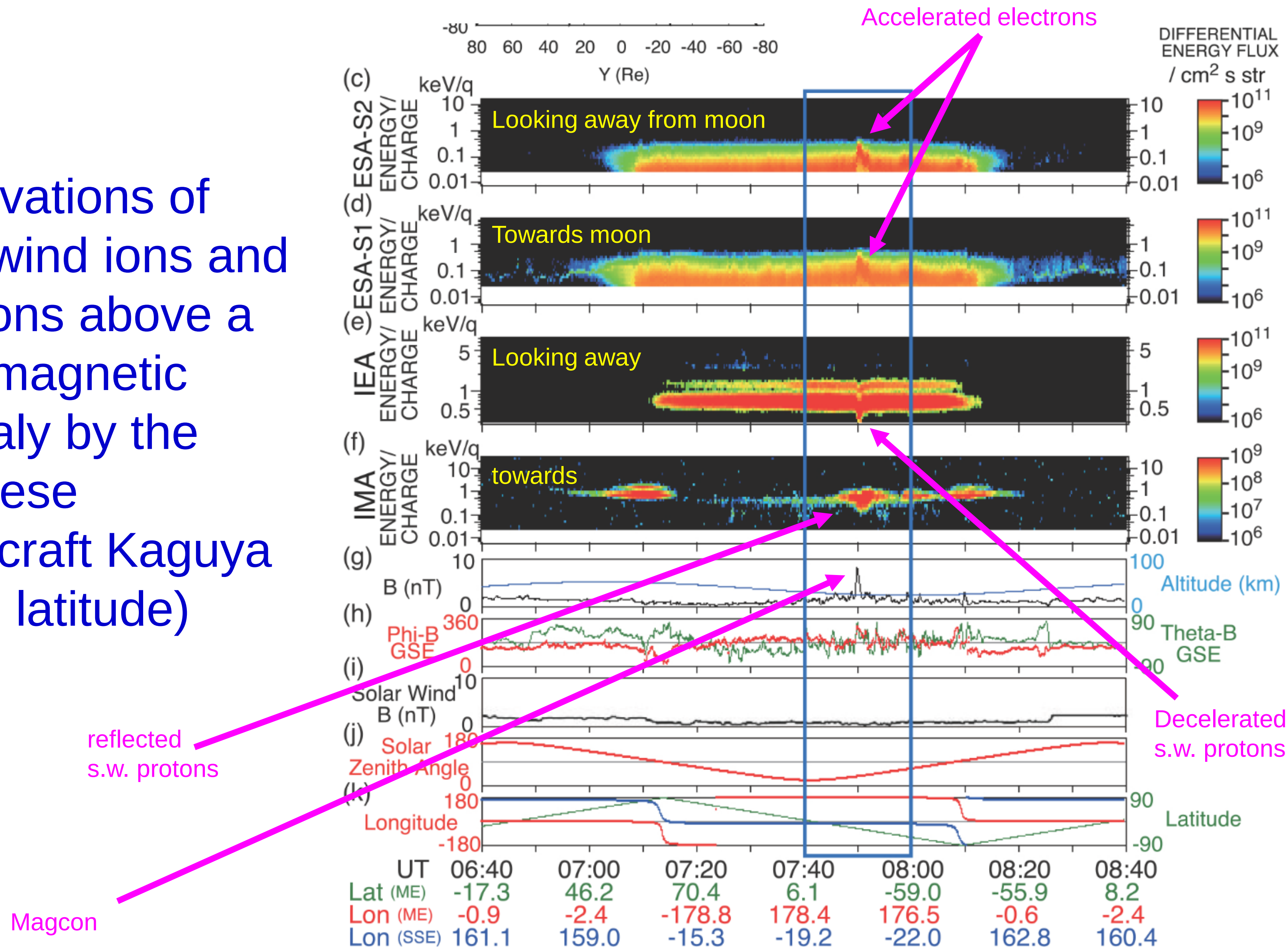
The Reiner-Gamma albedo feature, near the lunar crater Reiner

Observations of solar-wind ions and electrons above a lunar magnetic anomaly by the Japanese spacecraft Kaguya (25km latitude)



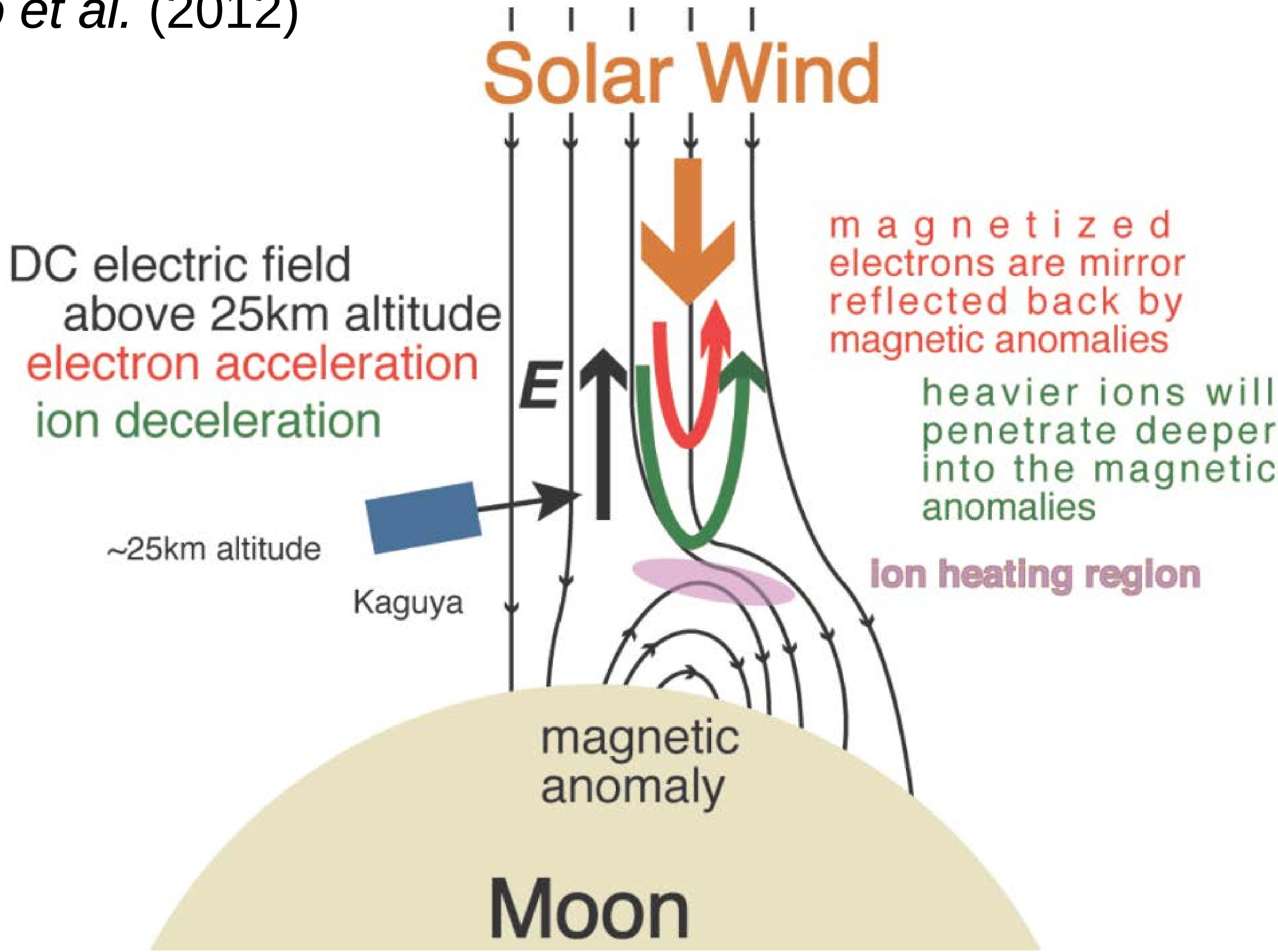
Saito et al., 2012

Observations of solar-wind ions and electrons above a lunar magnetic anomaly by the Japanese spacecraft Kaguya (25km latitude)



Saito et al., 2012

A picture put forth by
Saito et al. (2012)



Results from the simulations

Reflected protons