More preliminaries

My notation:

$\underset{\sim}{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$

vectors $\underset{\sim}{A}$ also $\vec{A}$ A_i $\underset{\sim}{A} = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$

I use this

tensors $\underset{\approx}{A}$ $A_{ij} = \begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix}$

vector algebra

dot product $\underset{\sim}{A} \cdot \underset{\sim}{B}$ = scalar

cross product $\underset{\sim}{A} \times \underset{\sim}{B}$ = vector

dyadic product $\underset{\sim}{A}\,\underset{\sim}{B}$ = tensor

$\underset{\sim}{A} \cdot \underset{\sim}{B} = A_i B_i$ i index repeated ⇒ sum

$= A_1 B_1 + A_2 B_2 + A_3 B_3$

$= A_x B_x + A_y B_y + A_z B_z$

$$\underset{\sim}{A} \times \underset{\sim}{B} = \epsilon_{ijk} A_j B_k$$

ϵ_{ijk} = Levi-Civita symbol

$\epsilon_{ijk} = 0$ if $i=j$, or $i=k$, or $j=k$

$\epsilon_{123} = 1 \qquad \epsilon_{213} = -1$

$\epsilon_{132} = -1 \qquad \epsilon_{231} = 1$

$\epsilon_{321} = -1 \qquad \epsilon_{312} = 1$

$$(\underset{\sim}{A} \times \underset{\sim}{B})_x = \epsilon_{xjk} A_j B_k \qquad \text{sum over } j \,\&\, k$$

$$= \overset{0}{\cancel{\epsilon_{xxx} A_x}} B_x + \overset{0}{\cancel{\epsilon_{xxy} A_x}} B_y + \overset{0}{\cancel{\epsilon_{xxz} A_x}} B_z$$
$$+ \overset{0}{\cancel{\epsilon_{xyx} A_y}} B_x + \overset{0}{\cancel{\epsilon_{xyy} A_y}} B_y + \overset{1}{\epsilon_{xyz}} A_y B_z$$
$$+ \overset{0}{\cancel{\epsilon_{xzx} A_z}} B_x + \overset{-1}{\epsilon_{xzy}} A_z B_y + \overset{0}{\cancel{\epsilon_{xzz} A_z}} B_z$$

$$= A_y B_z - A_z B_y$$

$$(\underset{\sim}{A} \times \underset{\sim}{B})_x = \left\| \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \right\|_x$$

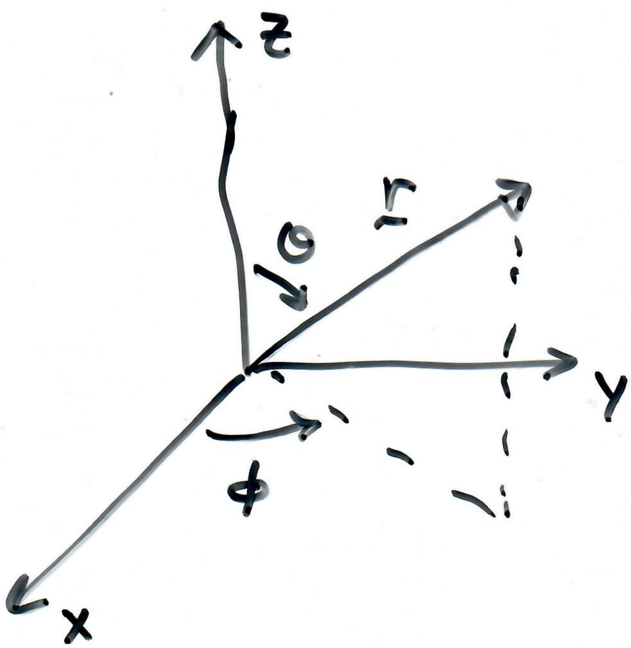
$$= \hat{x} \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} = \hat{x}(A_y B_z - A_z B_y)$$

$$\epsilon_{ijk}\,\epsilon_{imn} = \delta_{jm}\,\delta_{kn} - \delta_{jn}\,\delta_{km}$$

where $\delta_{ij} = 0 \quad i \neq j$ Kronecker delta

$\qquad = 1 \quad i = j$

Final note about my notation: geometry



θ = polar angle

ϕ = azimuth angle

plasma = any material for which the degree of ionization is sufficient for the dynamics to effected by electric & magnetic forces

if we define

n_i = number density of ions

n_e = " " " electrons

n_n = " " " neutral atoms

we have Saha's equation

$$\frac{n_i n_e}{n_n} = \frac{2 g_i}{g_n} \frac{(2\pi m_e kT)^{3/2}}{h^3} e^{-\frac{\chi_n}{kT}}$$

where $\chi_n \simeq 13.6\ eV$

where $1 eV \sim 10^4\ {}^\circ K$

typically have a plasma at about $kT \sim .1\chi_n$

for most materials $10^4 K \rightarrow$ plasma

Basic Equations

$$\nabla \cdot \underset{\sim}{B} = 0$$

$$\nabla \times \underset{\sim}{B} = \frac{4\pi}{c} \underset{\sim}{J} + \frac{1}{c} \frac{\partial \underset{\sim}{E}}{\partial t}$$

$$\nabla \cdot \underset{\sim}{E} = 4\pi \rho$$

$$\nabla \times \underset{\sim}{E} = -\frac{1}{c} \frac{\partial \underset{\sim}{B}}{\partial t}$$

$\underset{\sim}{J}$ = current density

$$= \sum_l q_l n_l \underset{\sim}{v}_l$$

$$\rho = \sum_l q_l n_l = \text{charge density}$$

Consider Ampere's law

$$\nabla \times \underset{\sim}{B} = \frac{4\pi}{c} \underset{\sim}{J} + \frac{1}{c} \frac{\partial \underset{\sim}{E}}{\partial t}$$

dimensional analysis

$$\underset{①}{\frac{B}{L}} :: \underset{②}{\frac{4\pi}{c} J} :: \underset{③}{\frac{1}{c} \frac{E}{T}}$$

compare ① & ③ term

$$T = L/U$$

$$\frac{B}{L} :: \frac{U}{c} \frac{E}{L} \quad *$$

now consider Faraday's law (4th of Maxwell eq.'s)

$$\frac{E}{L} :: \frac{B}{cT} \Rightarrow \frac{E}{L} :: \frac{U}{c}\frac{B}{L} \quad **$$

Compare ** with *

$$\frac{B}{L} :: \frac{B}{L}\left(\frac{U}{c}\right)^2$$

$\Rightarrow 1 :: (U/c)^2$ last term in Ampere's law is $\mathcal{O}((U/c)^2) \ll 1$ for most applications in heliospheric physics or non-relativistic astrophysical flows

$$\boxed{\nabla \times \underset{\sim}{B} = \frac{4\pi}{c}\underset{\sim}{J}}$$

what is n, $\underset{\sim}{v}$ (and T) ?

depends on applicatin, scale of problem, etc.

determined from particle motion of an ensemble of particles subject to $\underset{\sim}{E}$ & $\underset{\sim}{B}$ fields or other forces.

MHD (Magnetohydrodynamics)

Kinetic

Vlasov

based on conservatin laws, charge, mass, momentum, energy

which form to use depends on scale of problem.

usually this is done a computer!
But, basic eq. can be written down and there are some classic closed-form analytic problems

Another fundamental eq.

$$\frac{d\underset{\sim}{p}}{dt} = \underset{\sim}{F}_{\text{Lorentz}} = q\underset{\sim}{E} + \frac{q}{c}\,\underset{\sim}{v} \times \underset{\sim}{B} \qquad \text{c.g.s.}$$

momentum

single-particle orbit theory

Consider a case in which $\underset{\sim}{E}$ & $\underset{\sim}{B}$ are constant.

$\underset{\sim}{B} = (0, 0, B)$ $\qquad \underset{\sim}{B} \rightarrow$ along $\hat{z}$

$\underset{\sim}{E} = (E_x, E_y, E_z)$

3 eq.'s (non relativistic case) $\qquad \underset{\sim}{p} = m\underset{\sim}{v}$

$$m\dot{v}_x = qE_x + \frac{q}{c} v_y B \qquad \dot{v}_x = \frac{dv_x}{dt}$$

$$m\dot{v}_y = qE_y - \frac{q}{c} v_x B.$$

$$m\dot{v}_z = qE_z$$

look @ last eq. $\quad \dot{v}_z = \frac{q}{m} E_z = \text{constant}$

$$\Rightarrow \quad V_z = V_{z0} + \frac{q}{m} E_z t$$

v_z grows linearly w/ time and will exceed c at some pt. Could do relativistically if we want, but

electrons & ions move in opposite direction; they will create a charge separation, leading to an induced electric field which attempts to cancel the original E_z

no parallel electric field exists for scales larger than "Debye length", λ_D

we consider scales $\gg \lambda_D$, and we have NO parallel electric field

$$\left.\begin{aligned} \underset{\sim}{E} &= (E_x, E_y, 0) \\ \underset{\sim}{B} &= (0, 0, B) \end{aligned}\right\} \text{problem to consider}$$

$$\therefore \quad \dot{v}_x = \frac{q}{m} E_x + \frac{qB}{mc} v_y$$

$$\dot{v}_y = \frac{q}{m} E_y - \frac{qB}{mc} v_x$$

define new variables

$$v_x' = v_x - \frac{cE_y}{B}$$

$$v_y' = v_y + \frac{cE_x}{B}$$

substitute above to get

$$\dot{v}_x' = \Omega v_y'$$

$$\dot{v}_y' = -\Omega v_x'$$

$$\Omega = \frac{qB}{mc} = \text{cyclotron frequency}$$

$$\left(\frac{2\pi}{\Omega} = \text{gyro-period} \right)$$

these equations have the same form as if we had no electric field

the transformation speed to get rid of the electric field is

$$\underset{\sim}{V} = \frac{c\, \underset{\sim}{E} \times \underset{\sim}{B}}{B^2}$$

→ leads to a particle drift motion in lab frame

$$\underset{\sim}{v} = \underset{\sim}{V} + \underset{\sim}{\text{gyromotion part}}$$

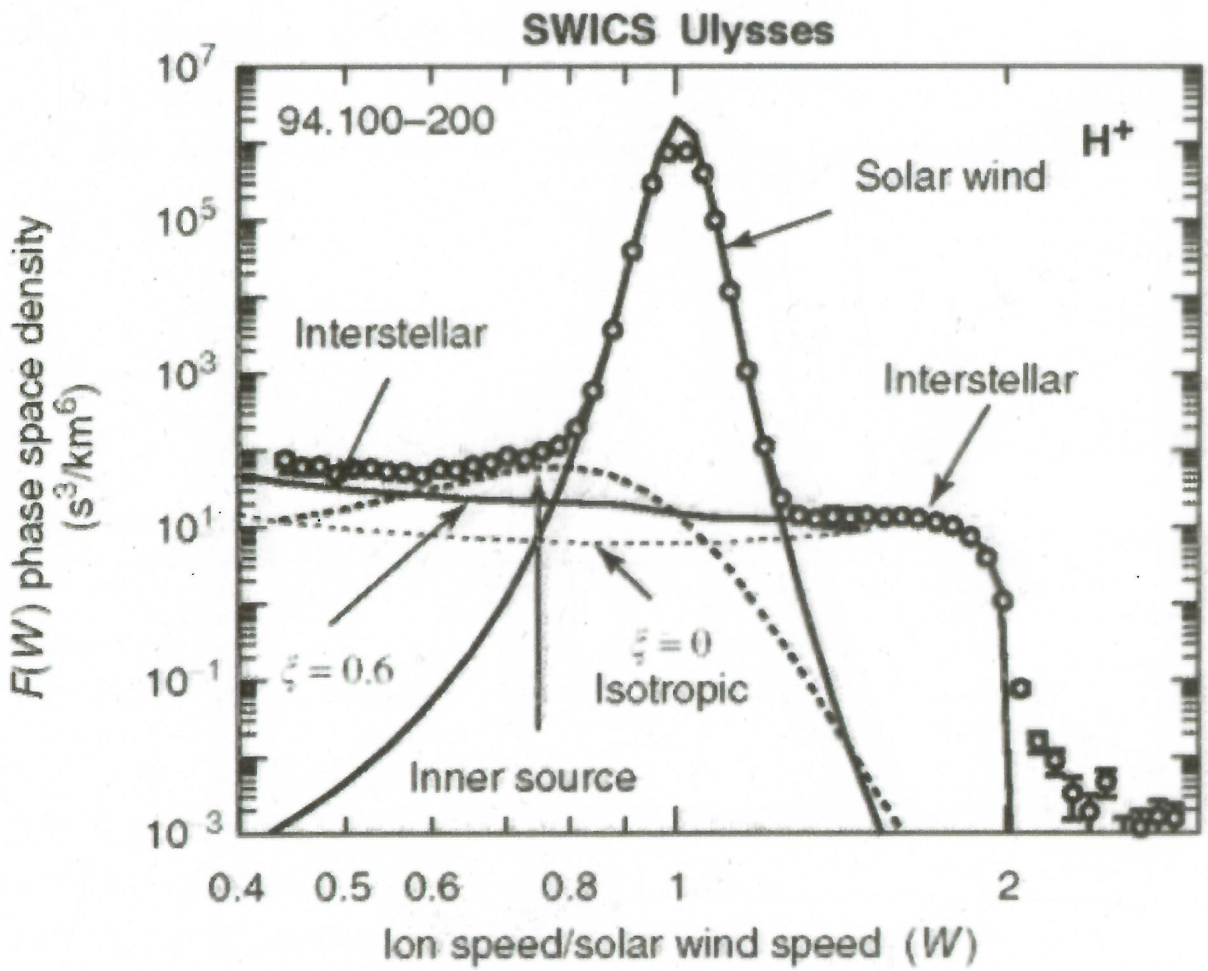
$\underset{\sim}{V}$ does not depend on particle!

bulk velocity of a collection of particles (electrons or protons) $= \underset{\sim}{u} = \langle \underset{\sim}{V} \rangle + \langle \underset{\sim}{\text{gyro. part}} \rangle$

$$= \underset{\sim}{V} = \frac{c\,\underset{\sim}{E} \times \underset{\sim}{B}}{B^2}$$

$$\underset{\sim}{U} = \frac{c\,\underset{\sim}{E} \times \underset{\sim}{B}}{B^2}$$

can show that $\underset{\sim}{E} = -\frac{1}{c}\,\underset{\sim}{U} \times \underset{\sim}{B}$



Gloeckler et al., GRL, 1995

see also Gloeckler et al, Science 1993