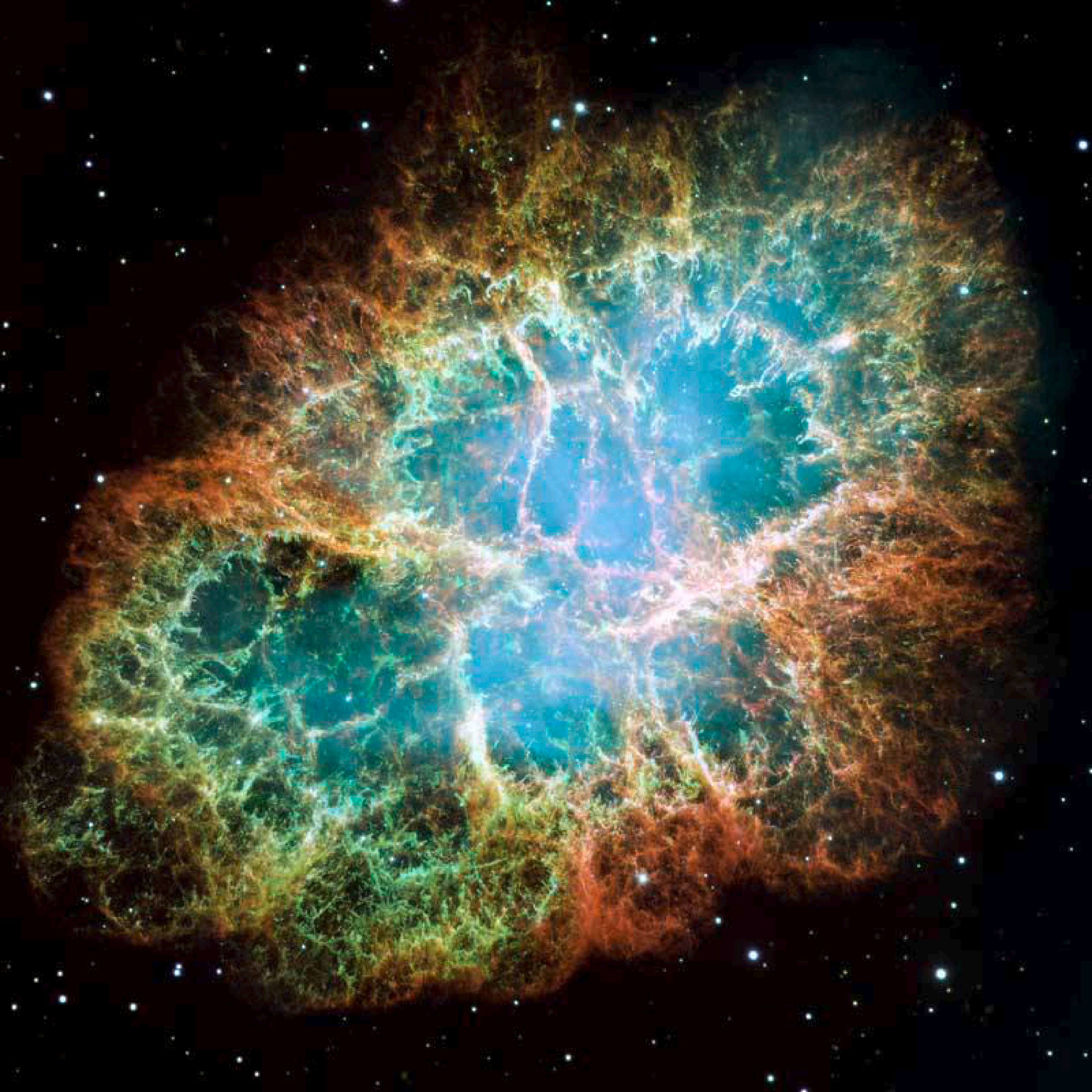
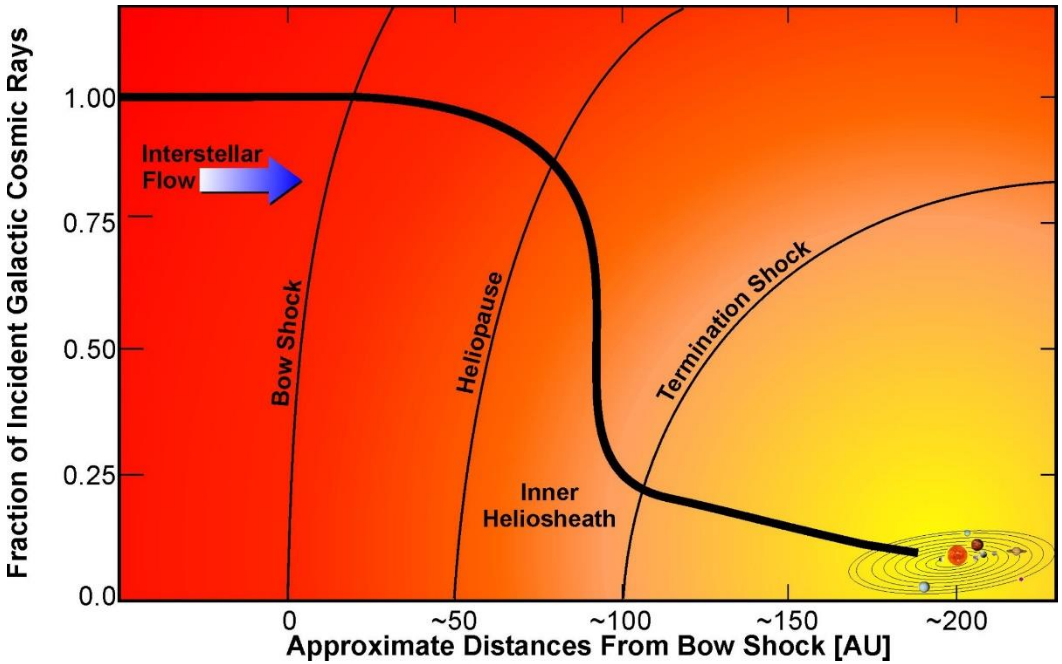
PTYS558 – April 27, 2020

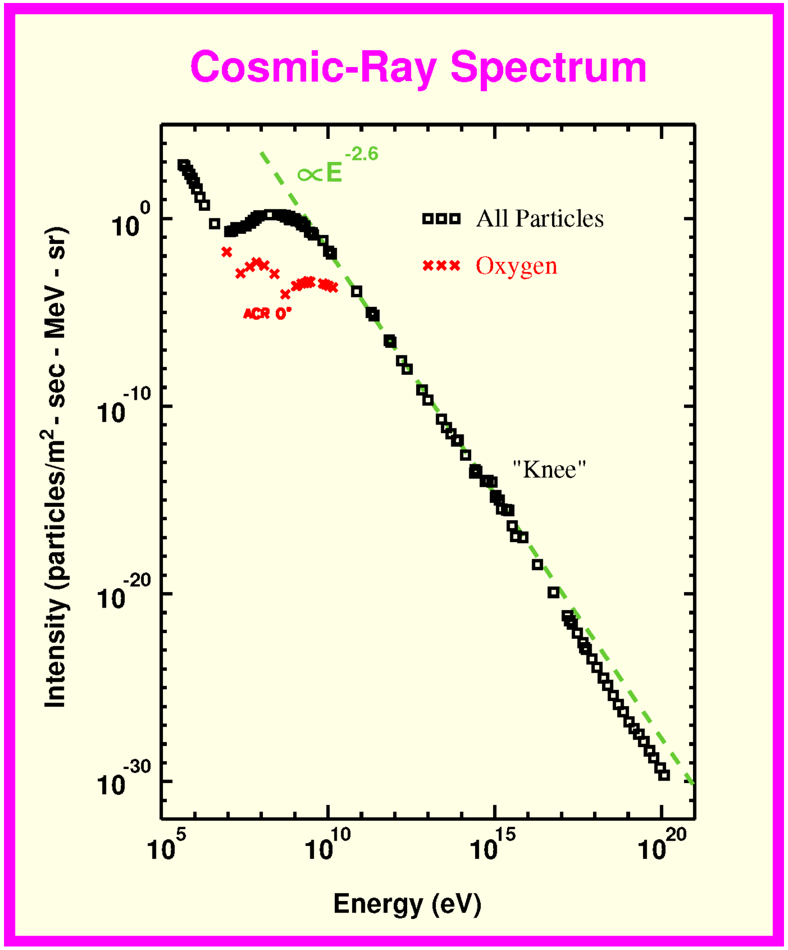
Cosmic Ray Transport: Part 1

1. Cosmic Ray Transport Intro
2. Particle motion in electric and magnetic fields which vary on scales comparable to the particle gyro-radii: pitch-angle scattering
3. Spatial diffusion
4. Energy change
5. The Parker transport equation (also known as the cosmic ray transport equation)



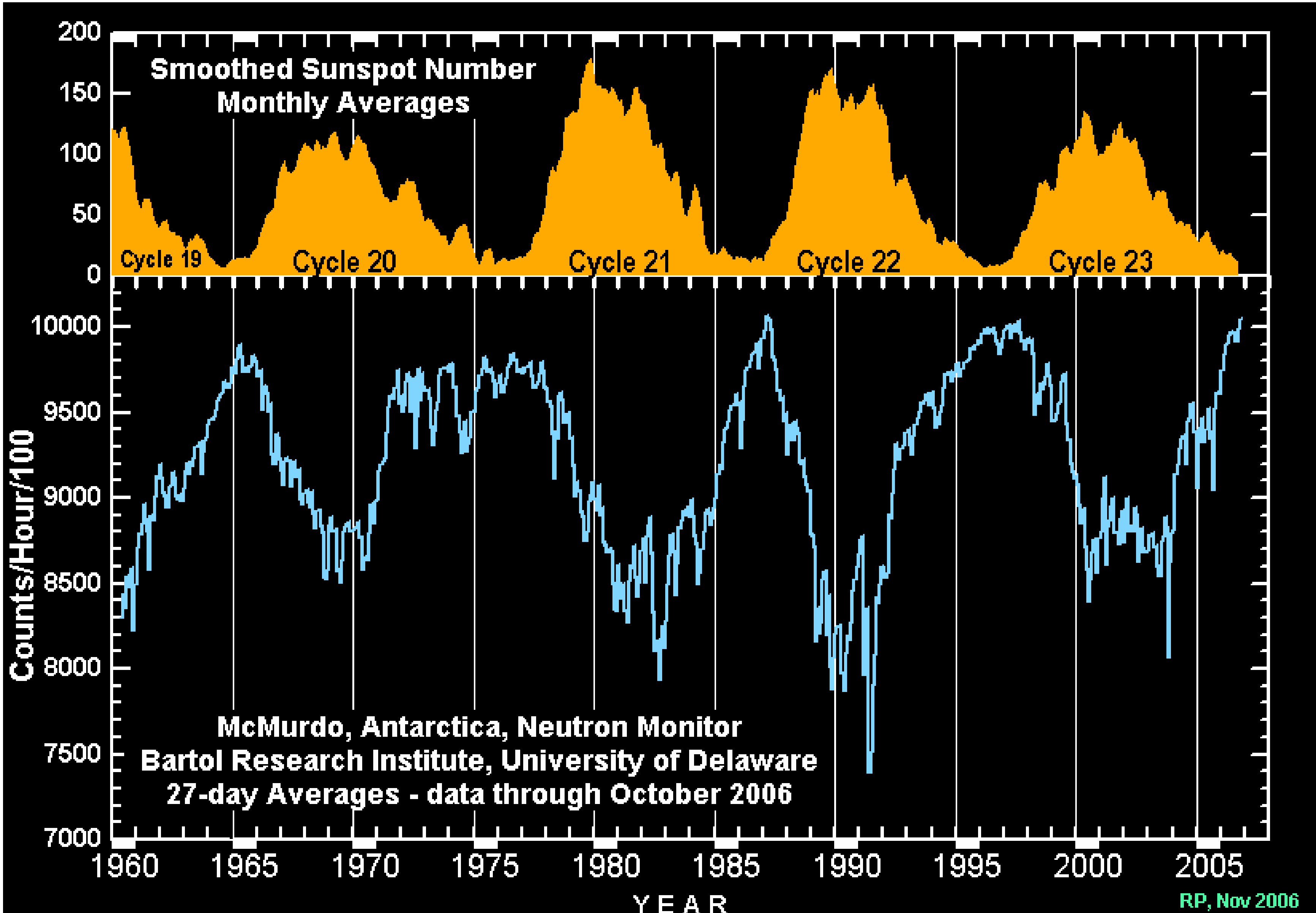


Fraction of Incident Galactic Cosmic Rays
1.00
0.75
0.50
0.25
0.0
Interstellar
Flow
Bow Shock
Heliopause
Termination Shock
Inner
Heliosheath
0
~50
~100
~150
~200
Approximate Distances From Bow Shock [AU]

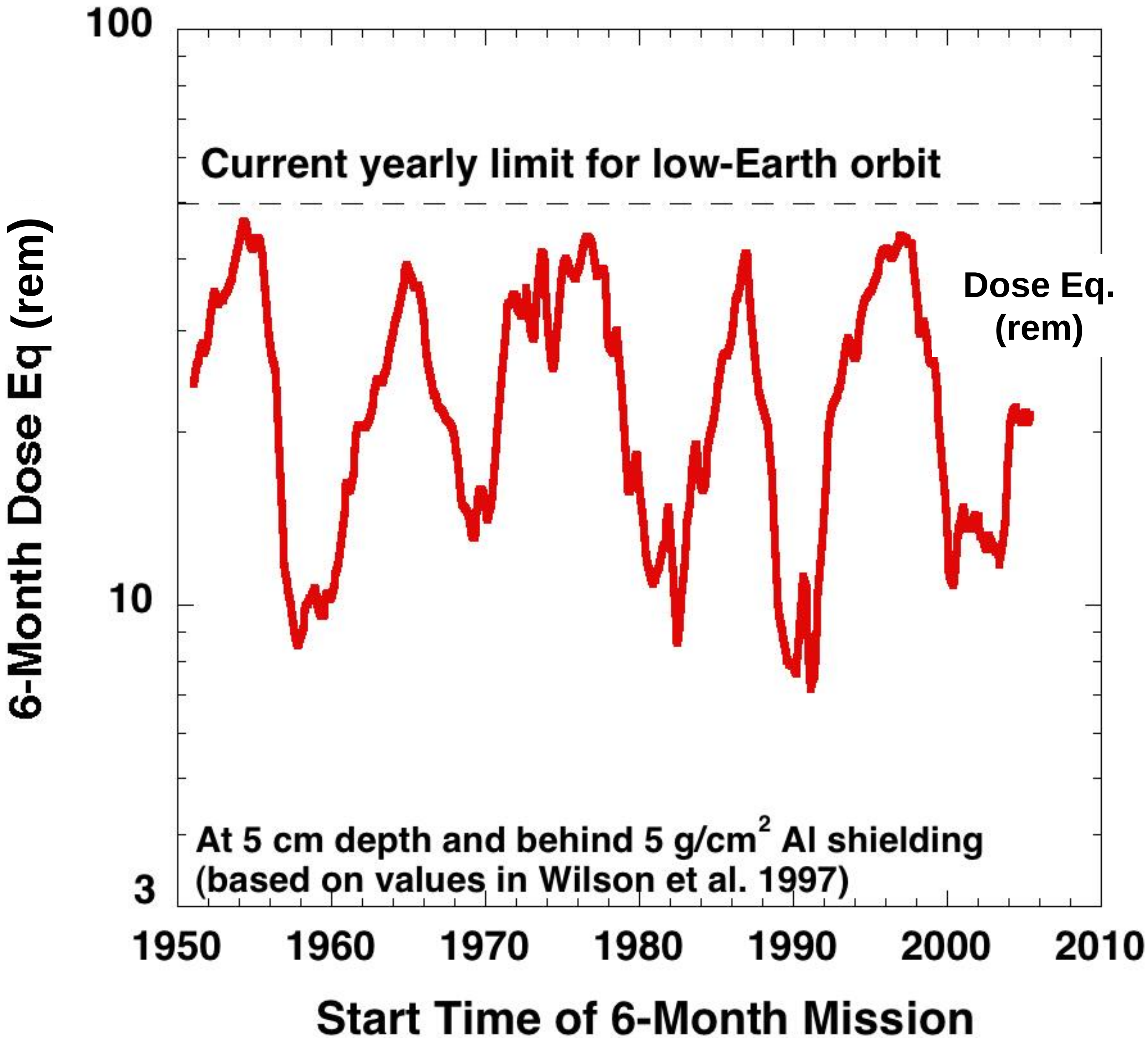


Cosmic-Ray Spectrum
∝E^-2.6
All Particles
Oxygen
ACR O+
"Knee"
Intensity (particles/m2 - sec - MeV - sr)
10^0
10^-10
10^-20
10^-30
10^5
10^10
10^15
10^20
Energy (eV)

Galactic Cosmic Rays and the Solar Cycle



The solar minimum intensity of Galactic cosmic rays (GCRs) is enough to exceed the current radiation limits for astronauts in low Earth orbit

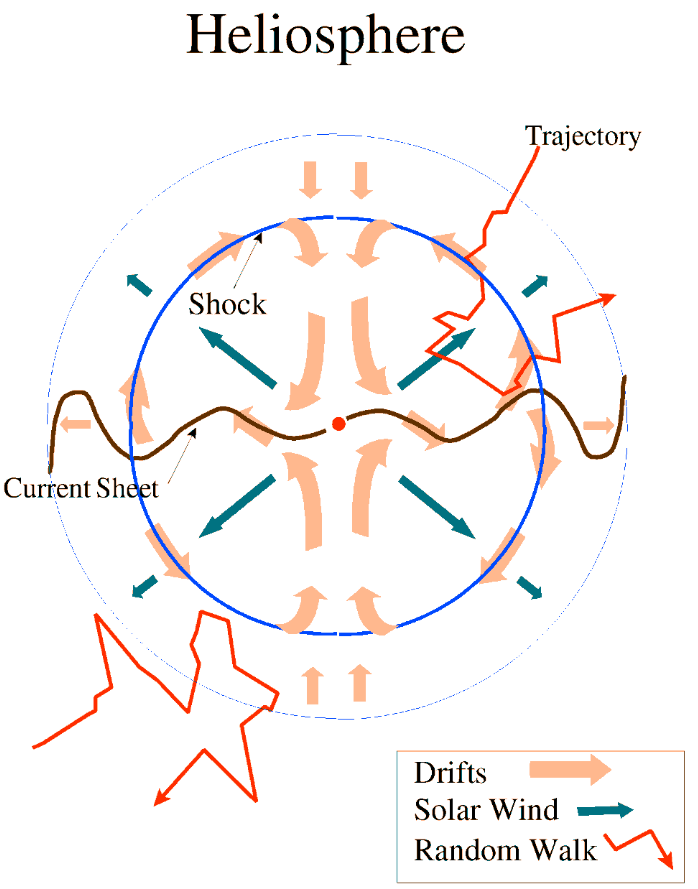


Mewaldt et al. 2006

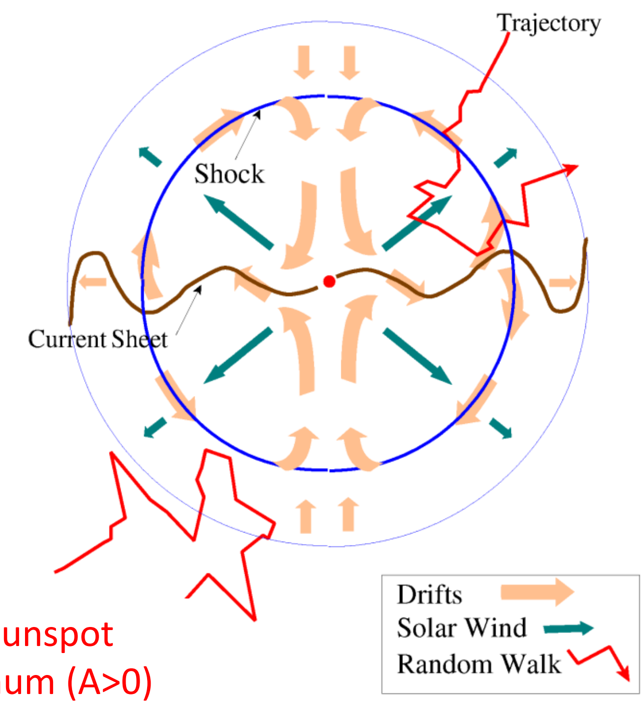
GCRs enter the heliosphere through a combination of diffusion and drift.

These motions are counteracted by outward convection and the associated cooling by the expansion of the wind.

Drift motions are very significant, and depend on the solar magnetic cycle. The pattern at right is for “A positive” (“A>0”, solar **B** is directed outward in northern solar hemisphere)

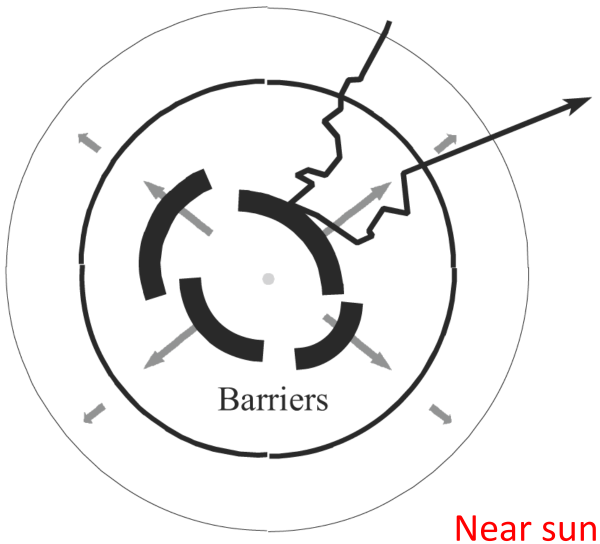


During ***solar minimum***, the interplanetary field is weaker, less solar-wind turbulence, fewer “barriers” (e.g. CMEs, shocks and merged interaction regions) and GCRs have easier access to 1 AU – ***higher GCR intensity***



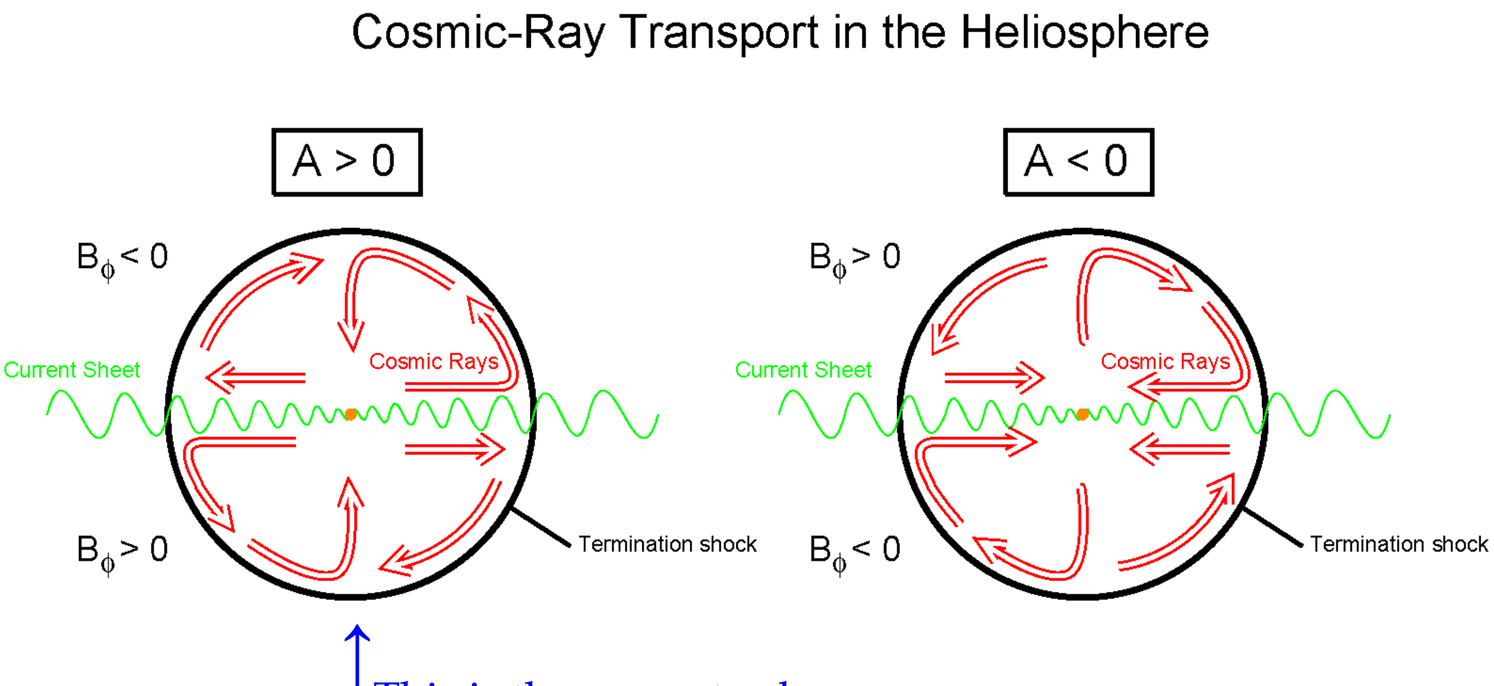
Near sunspot Minimum (A>0)

During ***solar maximum***, the interplanetary field is stronger, more turbulence, numerous large-scale barriers (e.g. CMEs), and GCRs have difficulty entering the solar system – ***lower GCR intensity***



Near sunspot Maximum

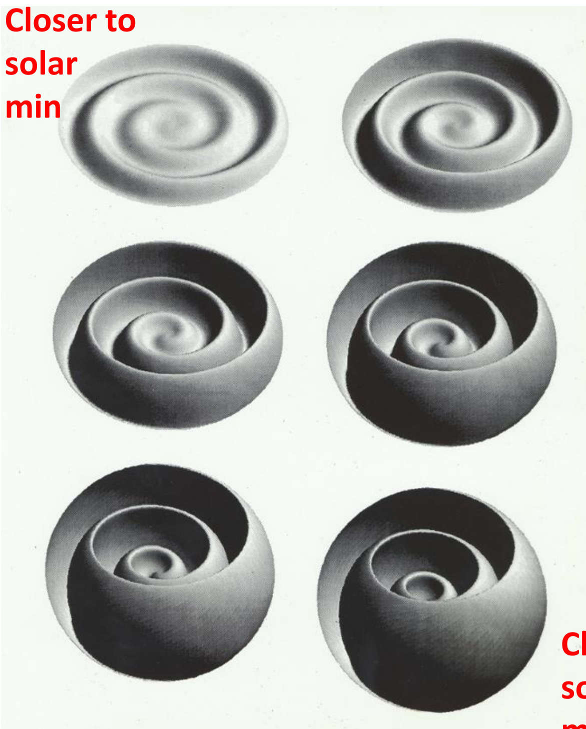
- The sense of the particle drift changes from one sunspot cycle to the next.
- For A>0, as GCRs enter the heliosphere, the drift brings them inward over the poles and out along the current sheet.
- The pattern is reversed for A<0 ("A negative")



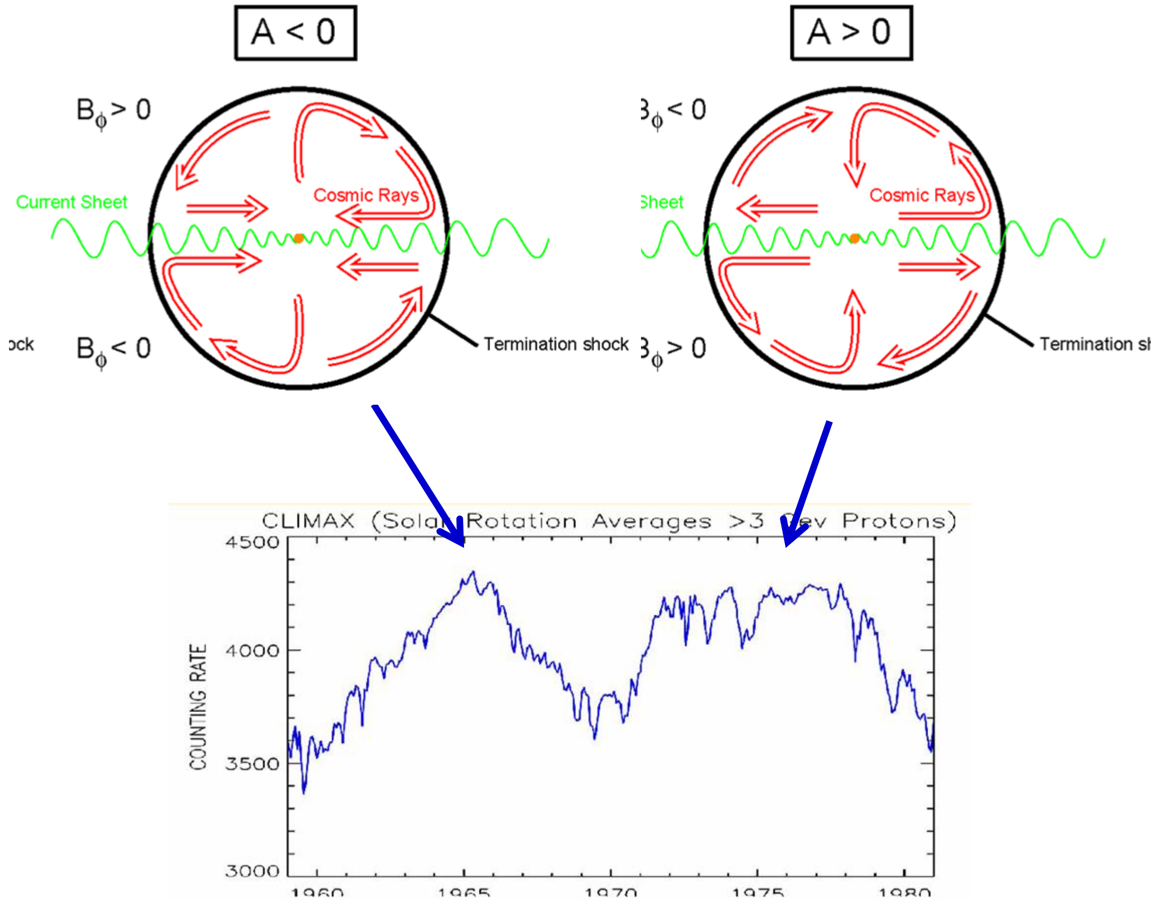
This is the current solar cycle

The current sheet changes from sunspot minimum to sunspot maximum. This effects the GCR intensity observed at 1AU

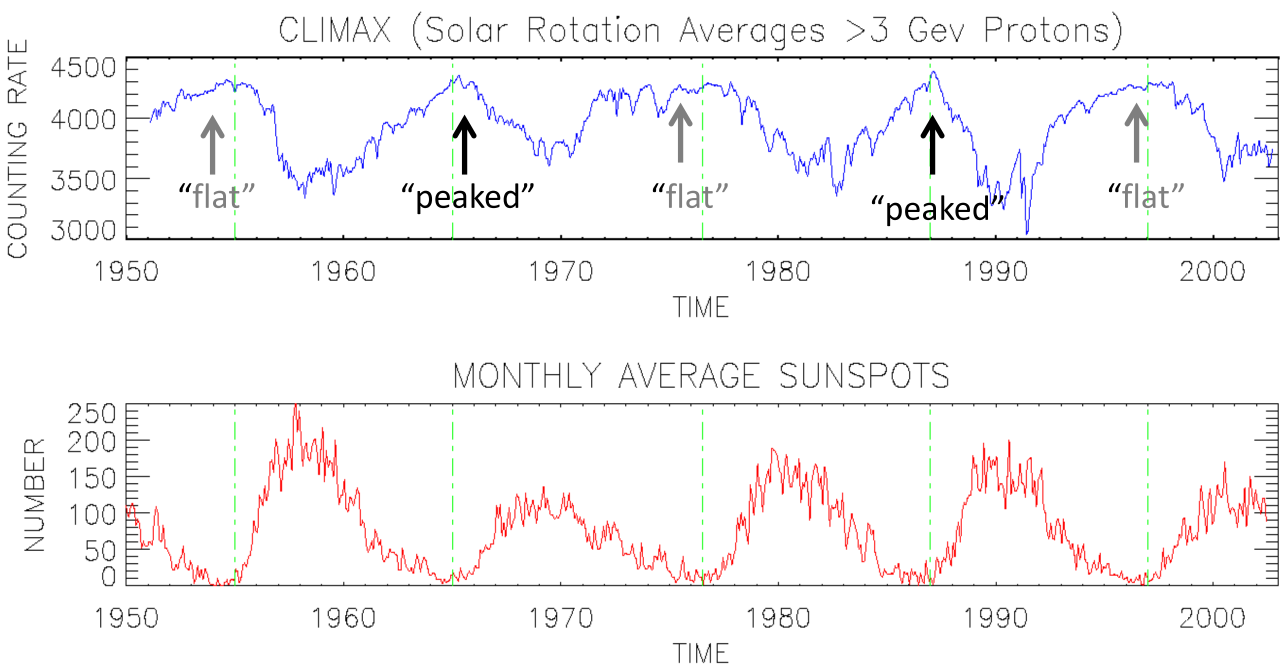
Closer to solar min



Closer to solar max



The 11- and 22-year cosmic-ray cycles



Particle transport in the heliosphere is actually the combination of four physical effects.

- ***Diffusion:*** caused by the scattering of the cosmic rays by the irregularities in the magnetic field. The associated “diffusion” is significantly larger along the magnetic field than normal to it.
- ***Convection:*** with the flow of the plasma.
- ***Guiding-center drifts:*** Such as gradient and curvature drifts, but also arising from interaction with current sheets in the solar wind
- ***Energy Change:*** caused by expansion/compression of the background fluid.

All of these effects play important roles in GCR modulation (it is difficult to isolate the effects – they are all important)

These are combined in Parker’s transport equation, first written down nearly 50 years ago.

The Parker Transport Equation:

$$\frac{\partial f}{\partial t} = -V_{w,i}\frac{\partial f}{\partial x_i} + \frac{\partial}{\partial x_i}\kappa_{ij}^{(S)}\frac{\partial f}{\partial x_j} - V_{D,i}\frac{\partial f}{\partial x_i} + \frac{1}{3}\frac{\partial V_{w,i}}{\partial x_i}\frac{\partial f}{\partial \ln p} + Q$$

advection ***diffusion*** ***drift*** ***energy change*** ***source***

Where the drift velocity due to the large scale curvature and gradient of the average magnetic field is:

$$\mathbf{V_d} = \frac{pcw}{3q}\ \nabla \times \left[\frac{\mathbf{B}}{B^2}\right]$$

And the symmetric part of the diffusion tensor is:

$$\kappa_{ij}^{(S)} = \kappa_{\perp}\delta_{ij} + (\kappa_{\parallel} - \kappa_{\perp})\frac{B_i B_j}{B^2}$$

The Parker Transport Equation is valid whenever the anisotropy is small (as observed for GCRs). It is widely used and remarkably general.

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advection *diffusion* *drift* *energy change* *source*

The focus of today's lecture

Where the drift velocity due to the large scale curvature and gradient of the average magnetic field is:

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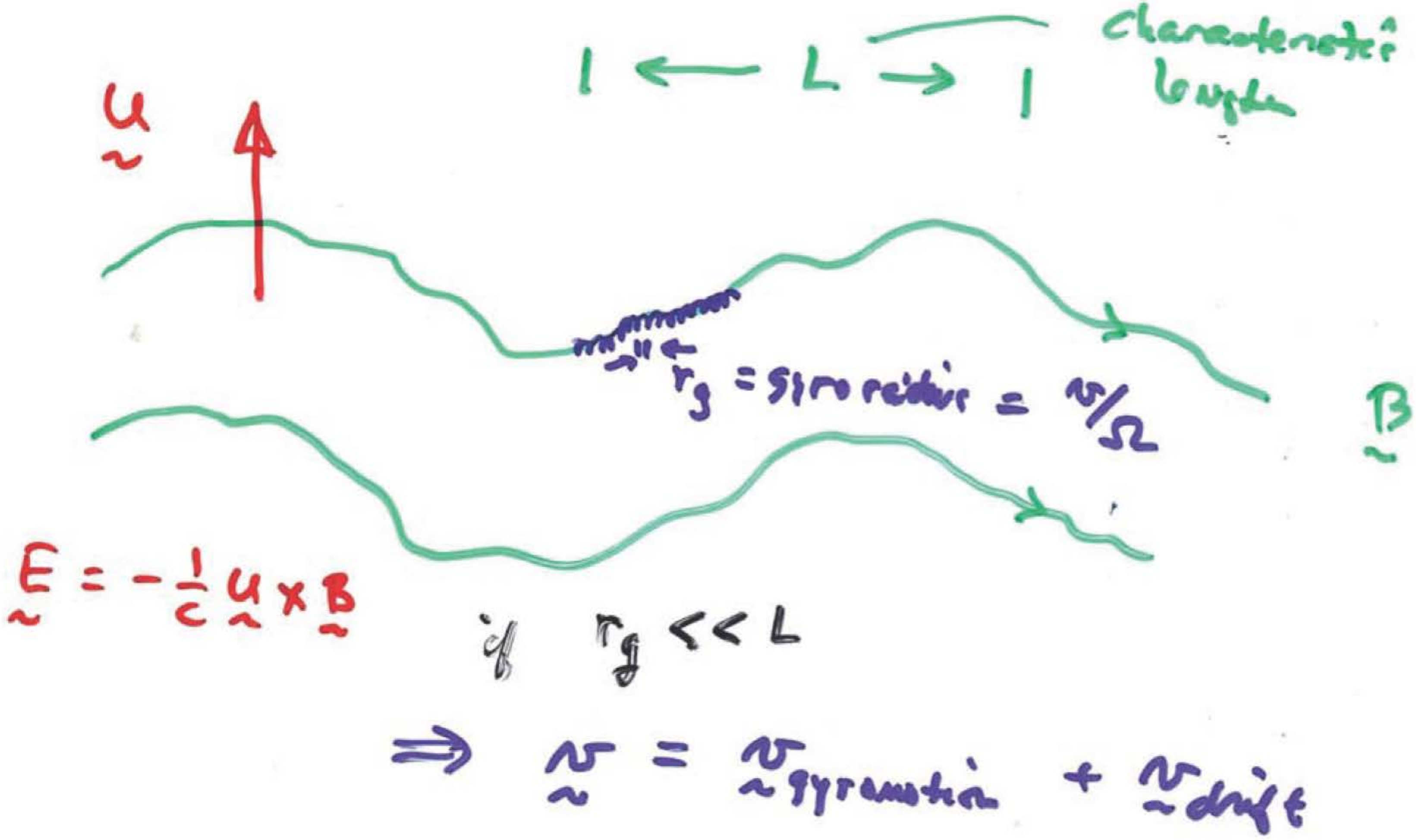
Varying E and B fields
Case 1: Scale of variation >> gyroradius of particles



Then other drifts enter, such as the "gradient" and "curvature" drifts, which, under the approximation of a curl-free magnetic field

$$\mathbf{v}_G = \frac{cW_\perp}{qB^3}\mathbf{B} \times \nabla B$$

$$\mathbf{v}_C = \frac{2cW_\parallel}{qB^3}\mathbf{B} \times \nabla B$$

$W_\perp, W_\parallel$ Are the components of the particle's kinetic energy perpendicular and parallel to the average magnetic field

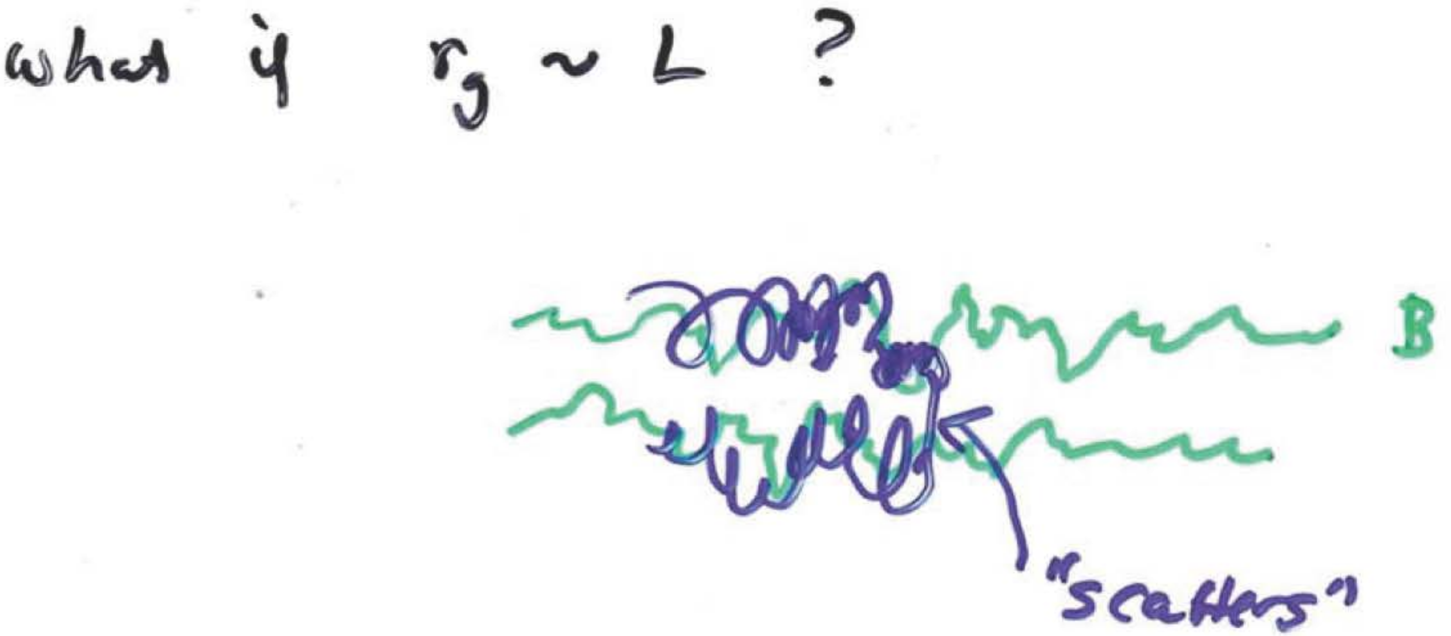
The general expression for motion of the center of gyration of the particle about the field is given by (***b*** is a unit vector along ***B***)

$$\mathbf{v}_{g.c.} = \left[w_\parallel + \frac{cW_\perp}{2qB}\mathbf{b} \cdot (\nabla \times \mathbf{b})\right]\mathbf{b} + \frac{cW_\perp}{2qB}\mathbf{b} \times \nabla B + \frac{cW_\parallel}{qB}\mathbf{b} \times (\mathbf{b} \cdot \nabla)\mathbf{b}$$

That, when averaged over an isotropic distribution of particles gives:

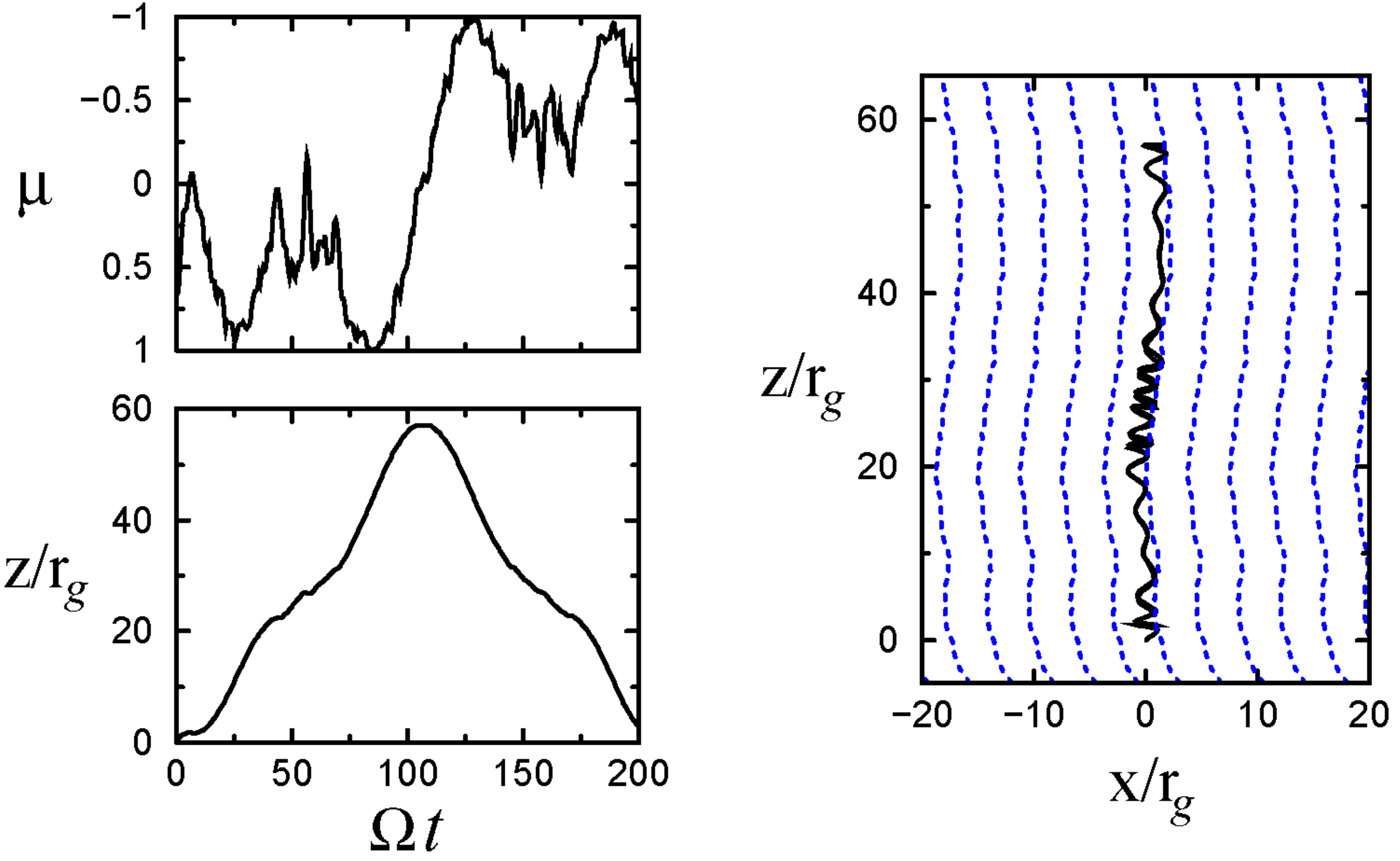
$$\mathbf{V}_d = (cmw^2/q)\nabla \times (\mathbf{B}/B^2)$$

Varying E and B fields
Case 2: Scale of variation ≈ gyroradius of particles



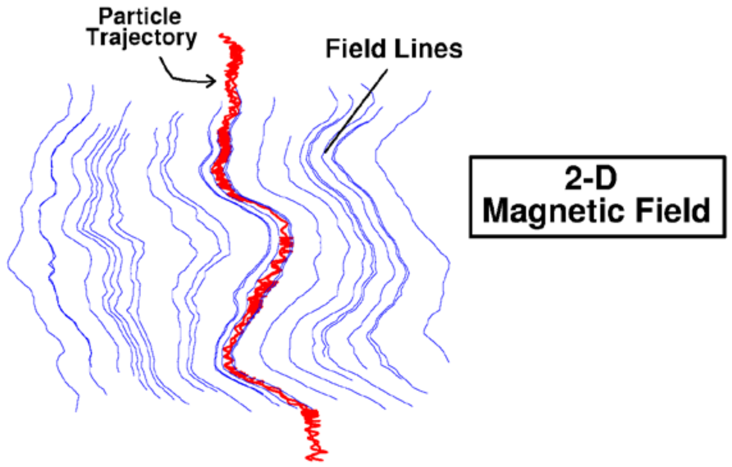
- A "resonance" can occur such that the particles pitch angle is reversed. This is much like a "scattering" event in scattering theory. The resonance condition is $kw\mu = \Omega$, where k is the wavenumber of the fluctuation, w is the particle speed, μ is the cosine of the pitch angle, and Ω is the particle cyclotron frequency.

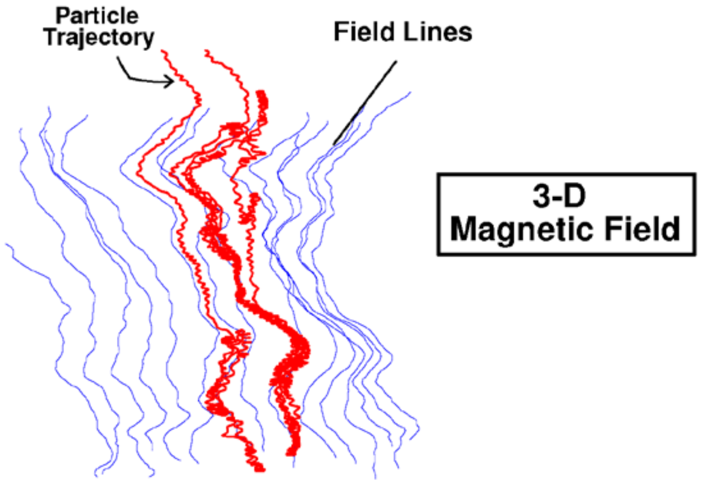
A charged particle moving in a turbulent magnetic field (numerical integration)



Restrictions on particle motions imposed by artificially limiting the dimensionality of the fields

- Charged particles are strictly tied to magnetic lines of force in 1 and 2D electric and magnetic fields
  - This can be proven rigorously and follows directly from the equations of motion
- This is an artificial and unphysical constraint on charged-particle motion!
  - Be aware!





Charged-particle transport : pitch-angle diffusion, spatial diffusion (parallel transport)

Consider a turbulent magnetic field of the form

$$\underset{\sim}{B} = B_0 \hat{z} + \delta B_x(z,t)\,\hat{x} + \delta B_y(z,t)\,\hat{y}$$

generally, $\underset{\sim}{\delta b} << \underset{\sim}{B}_0$

these are Alfven waves, $\underset{\sim}{k} = k\hat{z}$

Consider the equation of motion

$$\frac{dv_x}{dt} = \frac{q}{mc}\left(v_y B_0 - v_z \delta B_y\right)$$

$$\frac{dv_y}{dt} = \frac{q}{mc}\left(-v_x B_0 + v_z \delta B_x\right)$$

$\delta B << B_0$ ⇒ just give the usual oscillatory motion

$$\frac{dv_z}{dt} = \frac{q}{mc}\left(v_x \delta B_y - v_y \delta B_x\right)$$

the last eq. is of interest for us (parallel transport)

re-write $v_z = v\mu$ μ = pitch cosine

v = constant if $\underset{\sim}{\delta B} \neq \underset{\sim}{\delta B}(t)$

(if $v >> v_{phase}$ of waves, the $\underset{\sim}{\delta B}$ will appear static)

⇒ δB is time-independent

~~we~~ because $\delta\underline{B} \ll \underline{B}_0$, we can substitute the zeroth order solution for v_x & v_y.

$$\Rightarrow \quad v\frac{d\mu}{dt} = \frac{q}{mc}\Big(\underbrace{v_\perp\cos(\Omega_0 t-\phi)}_{v_x}\,\delta B_y^{(v\mu t)} + \underbrace{v_\perp\sin(\Omega_0 t-\phi)}_{-v_y}\,\delta B_x^{(v\mu t)}\Big)$$

$$\frac{d\mu}{dt} = \frac{q v_\perp}{vmc}[\ ] \qquad\qquad \Omega_0 = \frac{qB_0}{mc}$$

$$= \frac{q(1-\mu_0^2)^{1/2}}{mc}[\ ]$$

$$= \frac{q(1-\mu_0^2)^{1/2}}{mc}\Big[\cos(\Omega_0 t-\phi)\,\delta B_y(v\mu_0 t) + \sin(\Omega_0 t-\phi)\,\delta B_x(v\mu_0 t)\Big]$$

$\delta B_x, \delta B_y \rightarrow$ sinusoidal functions

$$\sim e^{ikz} \sim e^{ikv\mu_0 t}$$

Solution is

$$\mu = \int_0^t dt'\,\frac{q(1-\mu_0^2)^{1/2}}{mc}[\ ](t')$$

resonance will occur when $kv\mu_0 \sim \Omega_0$

↑ gyroradius ~ wavelength

Jokipii, 1966 noted

$$\langle\mu^2\rangle = \int_0^t dt'\int_0^t dt''\,\frac{q^2(1-\mu_0^2)}{m^2c^2}\langle[\ ](t')[\ ](t'')\rangle$$

the part in < > on the right ~~leads~~ leads to a diffusive process, that is

$\langle \mu^2 \rangle \propto t \rightarrow$ diffusive

↑ proportionality depend on Power spectrum associated with $\delta \underline{B}$ fluctuations

define pitch-angle diffusion coefficient, $D_{\mu\mu}$

$$D_{\mu\mu} = \frac{\langle \Delta \mu^2 \rangle}{2 \Delta t} = \frac{\pi}{4}(1-\mu^2)\,\Omega_0\, \frac{k_r\, P(k_r)}{B_0^2}$$

Jokipii, 1966

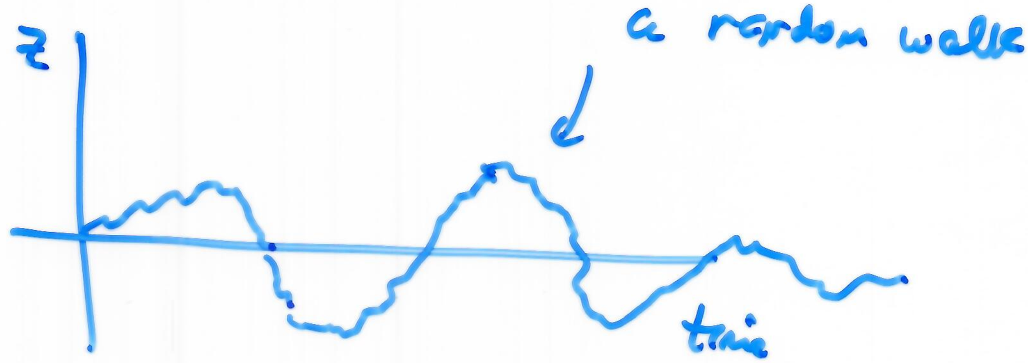
$P(k)$ = Power spectrum of mag. field

$P = P_{xx} = P_{yy}$ in this case

$k_r = \left| \frac{\Omega}{v \mu} \right| \rightarrow$ resonant wave number

What about spatial diffusion?

Consider



$$\frac{dz(t)}{dt} = v_z(t)$$

$$\rightarrow \quad z(t) = \int_0^t dt' \, v_z(t') \qquad \langle z \rangle = 0$$

$$z^2(t) = \int_0^t \int_0^t dt' dt'' \, v_z(t') \, v_z(t'')$$

$$\langle z^2 \rangle = \int_0^t \int_0^t dt' dt'' \, \langle v_z(t') \, v_z(t'') \rangle$$

let $t'' = t' + \tau$

$$\langle z^2 \rangle = \int_0^t \int_{-t'}^{t-t'} dt' d\tau \, \underbrace{\langle v_z(t') \, v_z(t'+\tau) \rangle}$$

$R(\tau) \rightarrow$ does not depend on t'

$$= \int_0^t \int_{-t'}^{t-t'} dt' d\tau \, R(\tau)$$

without loss of generality is

$$\langle z^2 \rangle = \int_0^t \int_{-\infty}^{\infty} dt'\, d\tau\, R(\tau)$$

$$= t \int_{-\infty}^{\infty} R(\tau)\, d\tau$$

$$R(\tau) = R(-\tau)$$

$$\Rightarrow \langle z^2 \rangle = 2t \int_0^{\infty} R(\tau)\, d\tau$$

$\Rightarrow$ $\frac{\langle z^2 \rangle}{2t} \neq$ function of time $\Rightarrow$ $\langle \Delta z^2 \rangle \propto t$
$\Rightarrow$ diffusion

this is spatial diffusion w/ coefficient K

$$K = \frac{\langle \Delta z^2 \rangle}{2 \Delta t} = \frac{1}{3} \lambda v \qquad \lambda = \text{mean-free path}$$

A distribution of particles undergoing diffusion obeys

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial z}\left(K \frac{\partial f}{\partial z}\right) \qquad \text{Diffusion equation}$$

So, how do we relate K to $D_{\mu\mu}$? why do this? because I want to know how K is related to magnetic field power spectrum.

Consider the Boltzmann eq. along the z direction

$$\frac{\partial f}{\partial t} + v\mu \frac{\partial f}{\partial z} + \left(\frac{\underset{\sim}{F}}{m} \cdot \nabla_{\underset{\sim}{v}} f \right)_z = \left(\frac{\partial f}{\partial t} \right)_c$$

do quasi-linear theory (Jokipii, 1966)

we find $\left(\left(\frac{\partial f}{\partial t} \right)_c = 0 \right)$ ← strictly particle-particle collisions

⋮

$$\frac{\partial f}{\partial t} + v\mu \frac{\partial f}{\partial z} = \frac{\partial}{\partial \mu}\left(D_{\mu\mu} \frac{\partial f}{\partial \mu} \right)$$

pitch-angle diffusion equation

we also have

$$\frac{\partial f_0}{\partial t} = \frac{\partial}{\partial z}\left(K \frac{\partial f_0}{\partial z} \right) \quad , \text{ where } f_0 = \int_{-1}^{1} f \, d\mu$$