PTYS558 – May 4, 2020

Particle Acceleration at Shocks

1. Energy change in the Parker equation
2. Conceptual picture of particle acceleration at shocks
3. Derivation of the distribution function of particles accelerated by a shock
4. Acceleration rate, spectral break/rollover, time dependence
5. Applications
    1. Particles associated with CMEs
    2. Supernovae
    3. Termination Shock

Energy change in the Parker Transport Equation:

$$\frac{\partial f}{\partial t} = -V_{w,i}\frac{\partial f}{\partial x_i} + \frac{\partial}{\partial x_i}\kappa_{ij}^{(S)}\frac{\partial f}{\partial x_j} - V_{D,i}\frac{\partial f}{\partial x_i} + \frac{1}{3}\frac{\partial V_{w,i}}{\partial x_i}\frac{\partial f}{\partial \ln p} + Q$$

advection ***diffusion*** ***drift*** ***energy change*** ***source***

- The energy change term involves $\nabla \cdot \boldsymbol{U}$
- When $\nabla \cdot \boldsymbol{U} > 0$ there is deceleration
- $\nabla \cdot \boldsymbol{U} < 0$ gives acceleration
- The rate of deceleration and acceleration is proportional to $\nabla \cdot \boldsymbol{U}$ (note that this has units 1/time, which is a rate).
- Rate $\propto$ to U/L. If L is large, the rate of acceleration or cooling is slow. If L is small, the rate is high.
  - Shocks are very thin, for example, and L is very small, thus, the rate of acceleration is large

Adiabatic Cooling in the Solar Wind

- This occurs when $\nabla \cdot \boldsymbol{U} > 0$
- Since the solar wind moves with (nominally) constant speed beyond a few solar radii out to the termination shock of the solar wind, there is cooling of energetic particles because

$$\nabla \cdot \boldsymbol{U} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 U) = \frac{2U}{r} > 0$$

- The typical cooling time scale at 1 AU, for example, is:

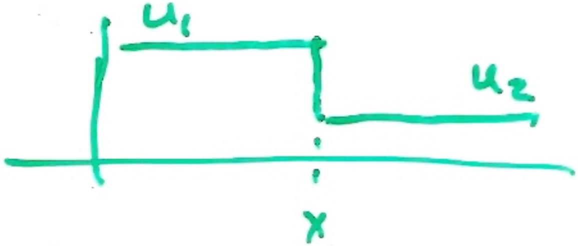
$$\frac{r}{2U} = \frac{1AU}{2(400\ km\ s^{-1})} \approx 2.7\ days$$

- This is considerably shorter than the lifetime of a typical galactic cosmic ray in our heliosphere, indicating that adiabatic cooling is an important effect on galactic cosmic ray transport.
- Although this time is longer than the transport time of high-energy particles from the Sun to get to 1AU, cooling is still important for these particles because the cooling time is considerably shorter near the Sun (40 times shorter at 5 solar radii, or ~ 1.5 hours)
- The above is true for an isotropic distribution of particles. But, It can also be shown from orbit theory that ANY individual charged particle loses energy when it moves within the interplanetary magnetic field and solar wind.

Acceleration at Shocks

- As we discussed previously, shocks are very thin, even in collisionless plasmas. The thickness of a shock is of the order of the "ion inertial length" which is c/ω_p, where c is the speed of light and ω_p is the proton plasma frequency.
- At 1 AU, 1 $c/\omega_p \approx 100$ km.
- Thus, the rate of acceleration, based solely on the argument used previously, is about 100 km / (400 km/s), which is 0.25 seconds.
- But …. this is far shorter than the scattering time of the particles within the waves (or the time over which the distribution is isotropized, which is a fundamental assumption in the Parker equation).
- One can still apply the Parker equation to shocks, however, but the effects of spatial diffusion and advection must be included. One can even derive the acceleration time scale from the time dependent solution to the Parker equation for shocks. This is discussed (not derived) at the end of the lecture today.

* Also note that for shocks



$$u_2 < u_1 \Rightarrow \nabla \cdot \underset{\sim}{u} < 0$$

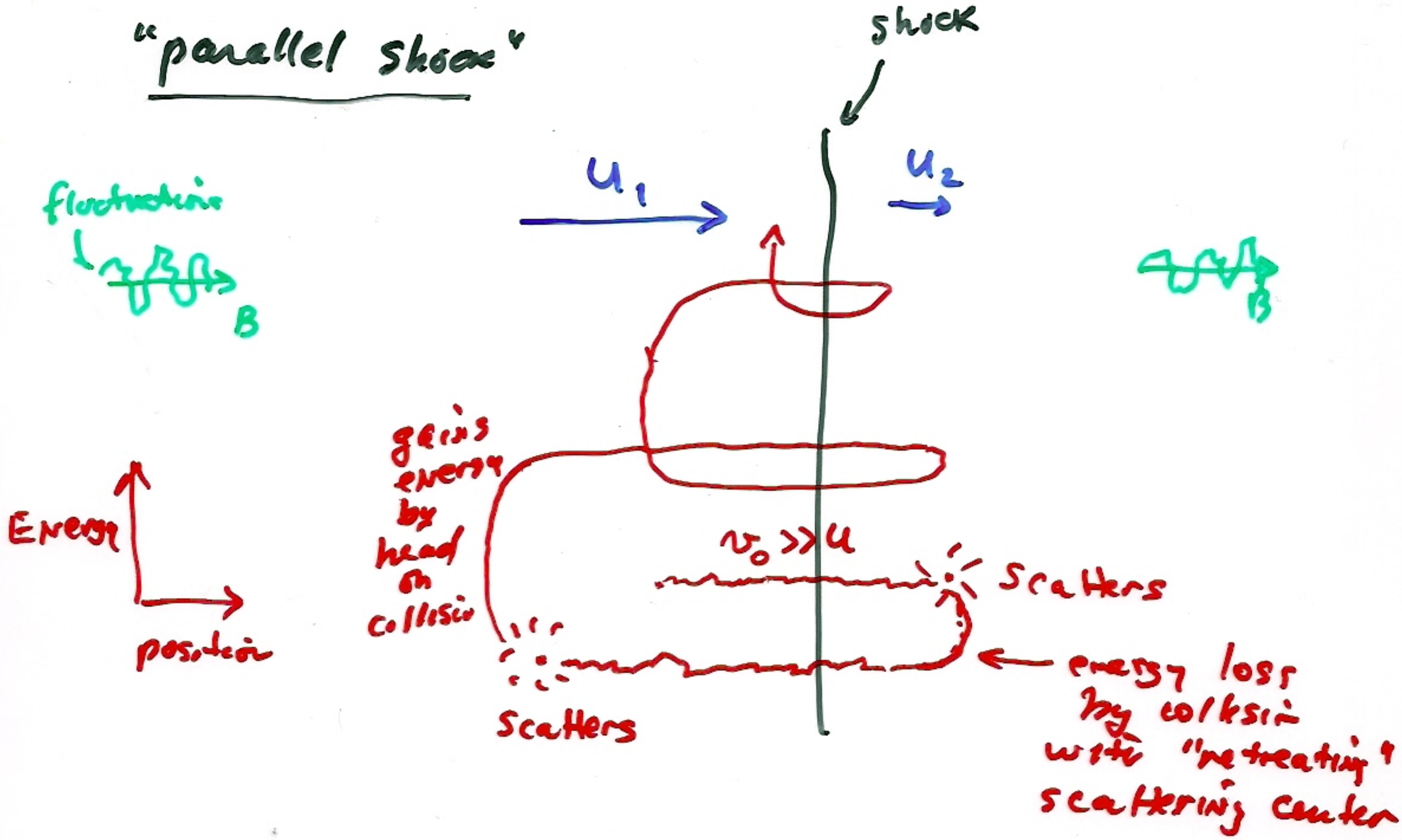
$\therefore$ get acceleration at a shock!

* if $|\nabla \cdot \underset{\sim}{u}|$ is very large, then we get rapid acceleration or cooling

Shock Acceleration

To get an intuitive picture consider the following

"parallel shock"



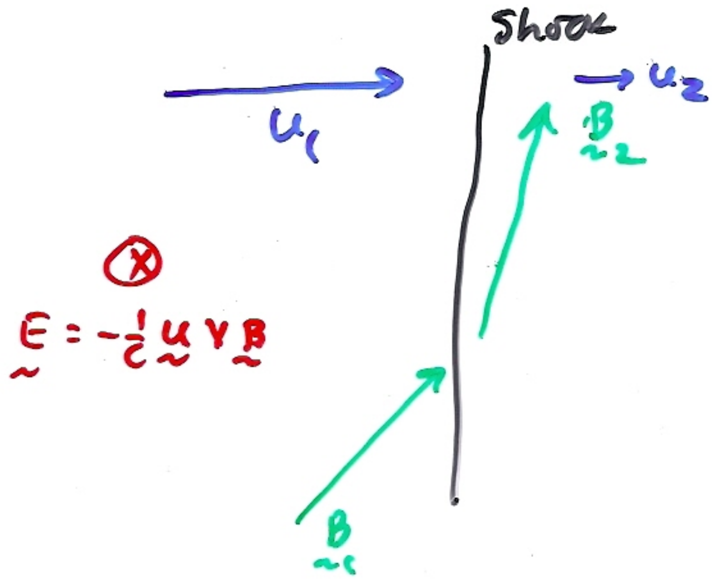
Net energy gain per "cycle" of

$$\underbrace{\frac{\Delta E}{E}}_{\text{change in energy}} \sim \underbrace{\frac{\Delta u}{u}}_{\text{change in flow speed}}$$

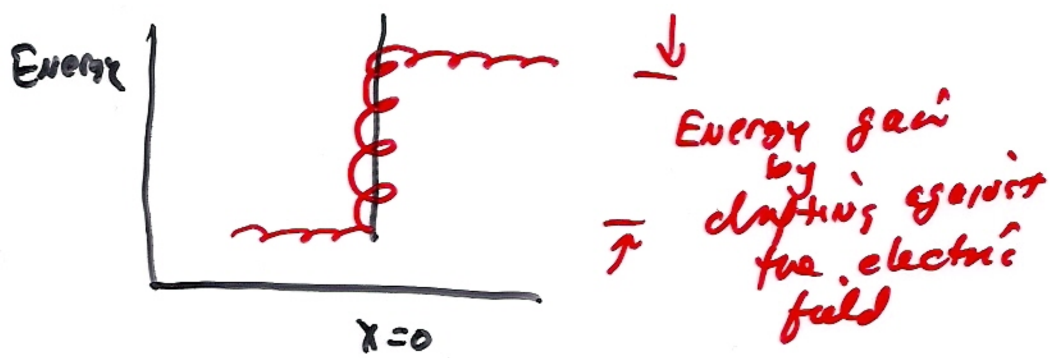
The rate of acceleration depends on the scattering normal to the shock.

$$\frac{1}{\tau_{Acc}} \sim \frac{1}{\kappa_{xx}}$$

For acceleration at an oblique shock



the compression in $\underset{\sim}{B}$ leads to a ∇B drift which is in the same direction as $\underset{\sim}{E}$!!



Acceleration for this case is <u>very</u> rapid (because $K_\perp << K_{\parallel}$)

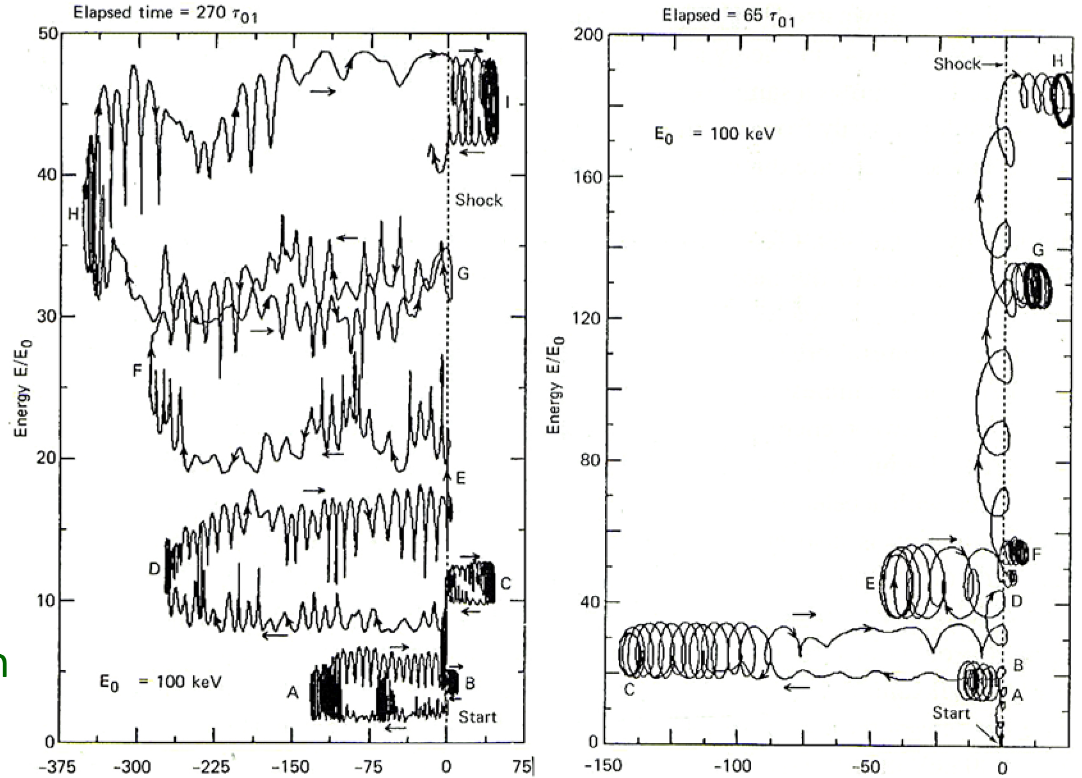
$\therefore$ Perpendicular shocks are faster accelerators than parallel shocks.

Quasi-parallel shock

Quasi-perpendicular shock

Increasing Particle Energy

Slower Acceleration case



More rapid acceleration

Position relative to the shock

Decker, 1988

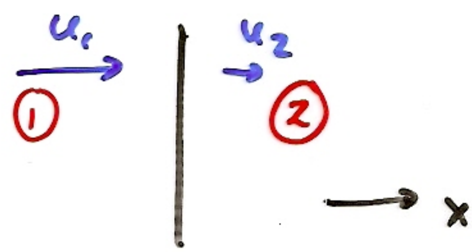
Diffusive Shock Acceleration

To be more quantitative, we solve the Parker transport eq. for a shock geometry

recall (1D Parker eq.)

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\left(\kappa \frac{\partial f}{\partial x}\right) - u\frac{\partial f}{\partial x} + \frac{1}{3}\left(\frac{\partial u}{\partial x}\right) p \frac{\partial f}{\partial p}$$

Consider a shock geometry of the form



$$\frac{\partial f}{\partial t} \rightarrow 0 \quad \text{(steady state)}$$

$$u = \begin{cases} u_1 & x < 0 \\ u_2 & x > 0 \end{cases}$$

$$\kappa(x, p) = \begin{cases} \kappa_1(p) & x < 0 \\ \kappa_2(p) & x > 0 \end{cases}$$

- Note that the diffusion coefficient K in our derivation is that *normal* to the shock
- Since K is a tensor, this corresponds to K_{xx} for our case.
- From the definition of the K tensor, which was written down in slides found in the lectures last week, we have:

$$\kappa_{xx} = \kappa_\perp + (\kappa_\parallel - \kappa_\perp)\frac{B_x B_x}{B^2} = \kappa_\perp + (\kappa_\parallel - \kappa_\perp)\cos^2\theta_{Bn}$$

$$= \kappa_\perp \sin^2\theta_{Bn} + \kappa_\parallel \cos^2\theta_{Bn}$$

- So, our approach is valid for any arbitrary θ_{Bn}. Thus, this applies even for perpendicular shocks .
- The primary issue is whether the Parker equation itself is applicable. All that is assumed is that the distribution is isotropic in the local plasma frame. If there is sufficient scattering to ensure this is the case, the Parker equation is valid.

Boundary Conditions

a. $f(-\infty, p) \rightarrow 0$

b. $f(+\infty, p) \rightarrow$ remain finite

c. $f_1(0, p) = f_2(0, p)$

We solve the transport eq. in each region ① & ② separately and ~~the~~ apply B.C.'s and match sol'n across the shock.

Region 1 upstream

$$\frac{\partial}{\partial x} K_1 \frac{\partial f_1}{\partial x} - u_1 \frac{\partial f_1}{\partial x} = 0$$

Solution is

$$\boxed{f_1 = A_1(p)\, e^{\frac{u_1}{K_1} x} + B_1(p)}$$

where A_1 & B_1 are constants to be determined.

Region 2 downstream

$$\frac{\partial}{\partial x} K_2 \frac{\partial f_2}{\partial x} - u_2 \frac{\partial f_2}{\partial x} = 0$$

$$\Rightarrow \quad \boxed{f_2 = A_2(p)\, e^{\frac{u_2}{K_2} x} + B_2(p)}$$

Apply B.C.'s

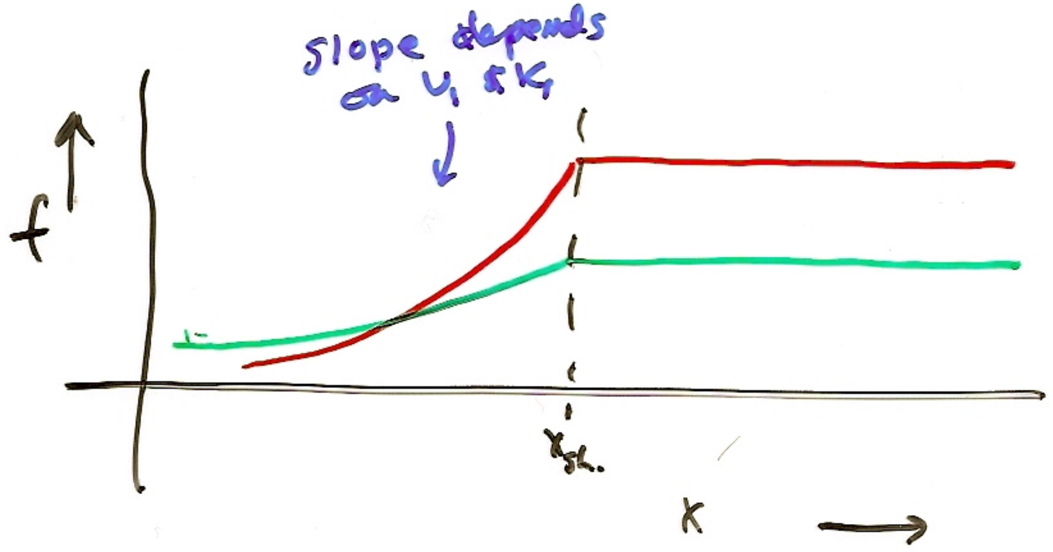
a. as $x \to -\infty$ $f \to B_1$ $\Rightarrow$ $\boxed{B_1 = 0}$

b. as $x \to +\infty$ $f \to \infty$ unless $\boxed{A_2 = 0}$

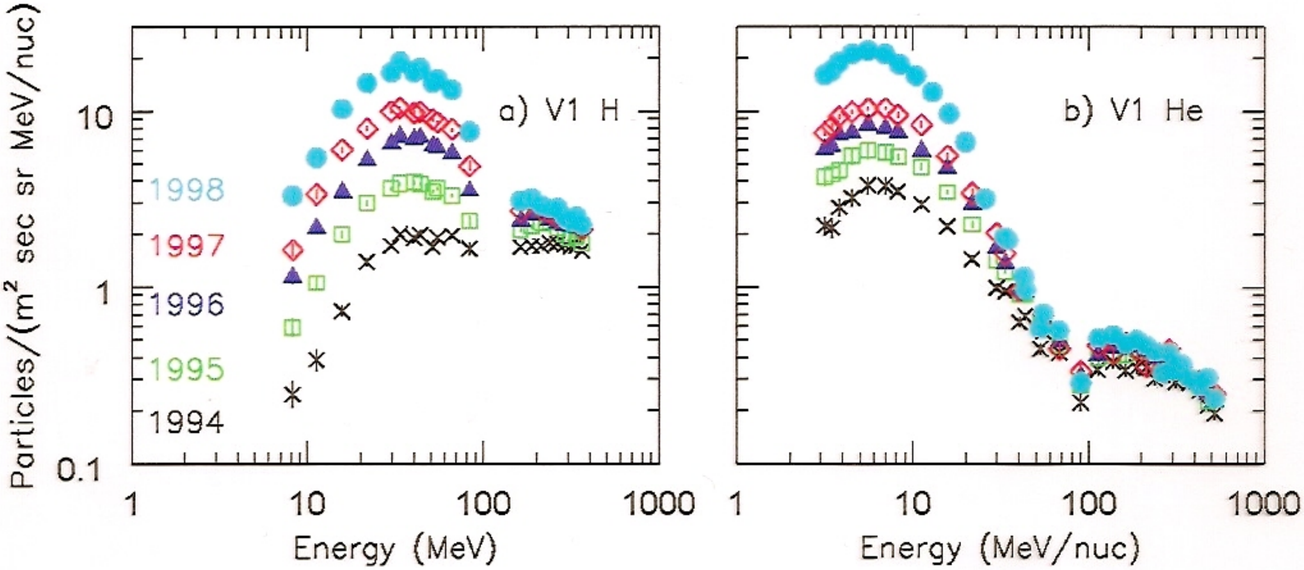
c. $A_1 = B_2$

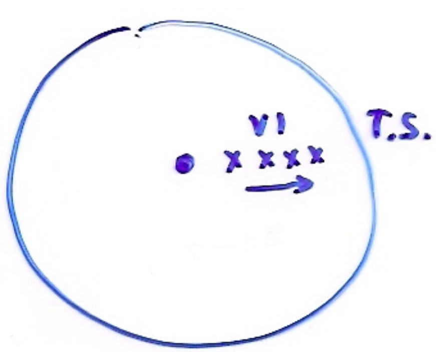
let $A(p) = A_1(p) = B_2(p)$, and we have

$$\boxed{f(x,p) = A(p) \begin{cases} e^{u_1 x / K_1} & x < 0 \\ 1 & x \geq 0 \end{cases}}$$



Voyager Data — approaching termination Shock





To get $A(p)$, we integrate the transport equation from $x = -\epsilon$ to $x = +\epsilon$ and at the end, take the limit $\epsilon \to 0$

Recall

$$\underset{1}{\frac{\partial}{\partial x} K \frac{\partial f}{\partial x}} - \underset{2}{u \frac{\partial f}{\partial x}} + \underset{3}{\frac{1}{3} \frac{du}{dx} p \frac{\partial f}{\partial p}} = 0$$

1 $$\int_{-\epsilon}^{\epsilon} dx \left[\frac{\partial}{\partial x} K \frac{\partial f}{\partial x} \right] = K \frac{\partial f}{\partial x} \Bigg|_{-\epsilon}^{\epsilon}$$

$$= 0 - K_1 \left[A \frac{u_1}{K_1} e^{-\frac{u_1}{K_1}\epsilon} \right]$$

as $\epsilon \to 0$

$$= -u_1 A$$

2 $$\int_{-\epsilon}^{\epsilon} dx \left(u \frac{\partial f}{\partial x} \right) = \int_{-\epsilon}^{0} dx \; u_1 \frac{\partial f_1}{\partial x} + \cancelto{0}{\int_{0}^{\epsilon} dx \; u_2 \frac{\partial f_2}{\partial x}}$$

$$= u_1 \left[f(0) - f(-\epsilon) \right]$$

$$= 0 \quad \text{as} \quad \epsilon \to 0$$

3 $$\int_{-\epsilon}^{\epsilon} dx \ \frac{1}{3}\frac{du}{dx}\frac{\partial f}{\partial(\ln p)} = \frac{1}{3}\int_{-\epsilon}^{\epsilon} dx \ (u_2-u_1)\underbrace{\delta(x)}_{\text{Dirac Delta}}\frac{\partial f}{\partial(\ln p)}$$

$$= \frac{1}{3}(u_2-u_1)\frac{\partial A}{\partial(\ln p)}$$

Thus, adding 1, 2, & 3, we get

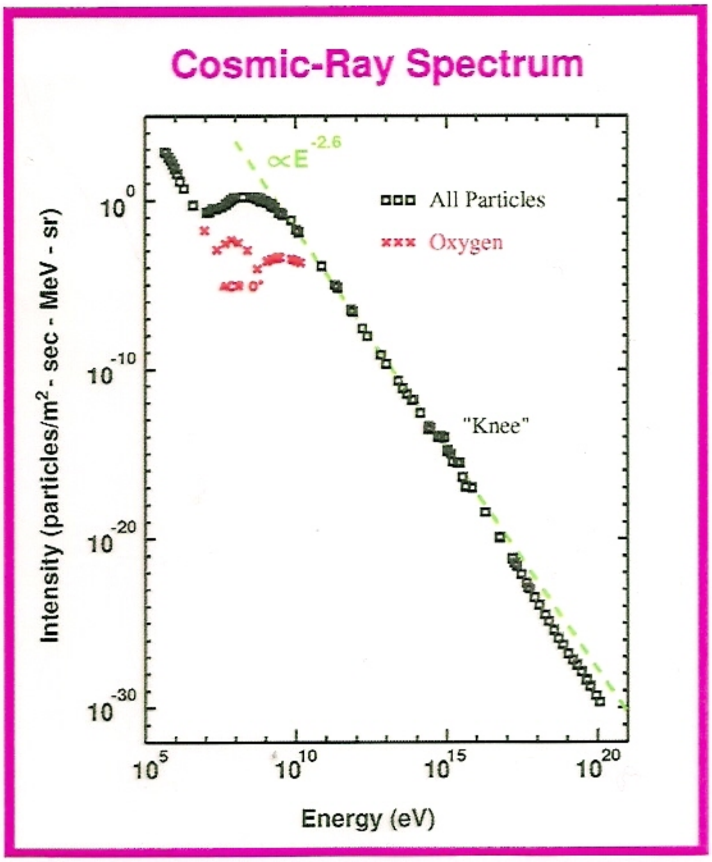
$$-u_1 A + \frac{1}{3}(u_2-u_1)\frac{dA}{d\ln p} = 0$$

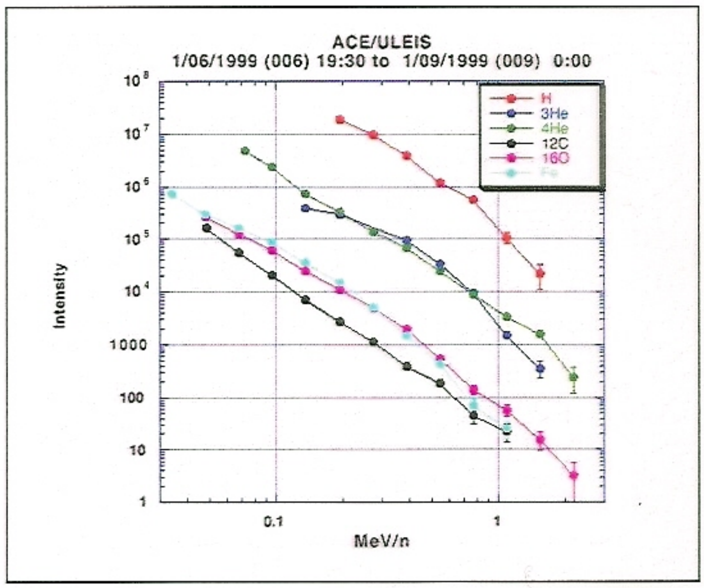
solve

$$\boxed{A(p) = A_0\left(p/p_0\right)^{-\gamma} \quad \text{where} \quad \gamma = \frac{3r}{r-1} \quad r = \frac{u_1}{u_2}}$$

Spectral slope depends <u>ONLY</u> on the density jump across the shock.
Since $r \leq 4 \Rightarrow$ a nearly universal spectrum

Observed Power-law spectra





Mason et al., 1999

"Solar and Space Physics and the Vision for Space Exploration"
Oct. 17, 2005: Wintergreen, VA

$$\frac{dJ}{dE} = p^2 f \propto p^{2-\gamma} \propto E^{2-\gamma}$$ at relativistic energies

$$2-\gamma = -2.6 \Rightarrow \gamma = 4.6$$

$$\Rightarrow r = 2.9$$ (not 4, but still quite strong)

S.E.Ps

These are from the "Halloween" storms of 2003.

Large particle events

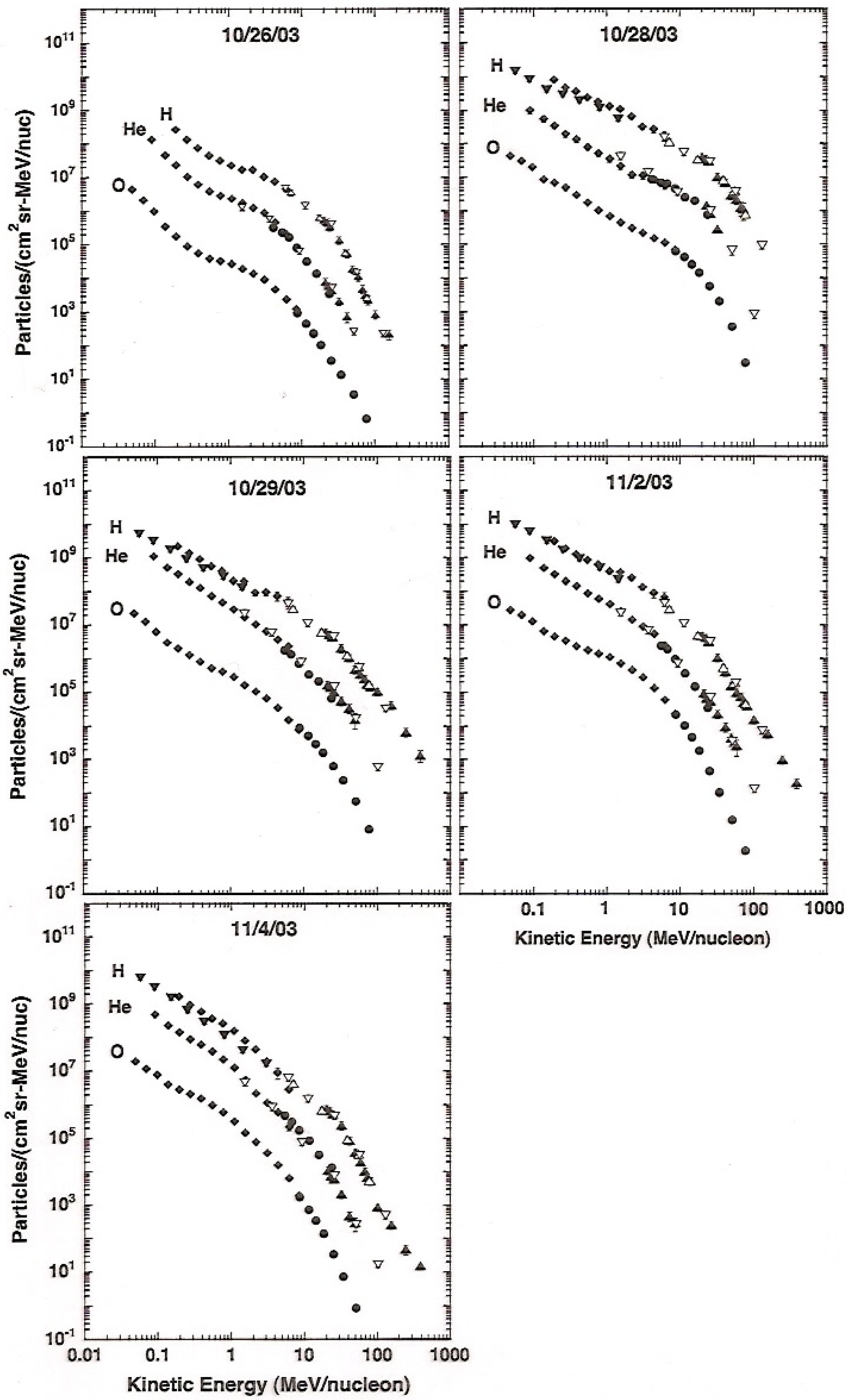
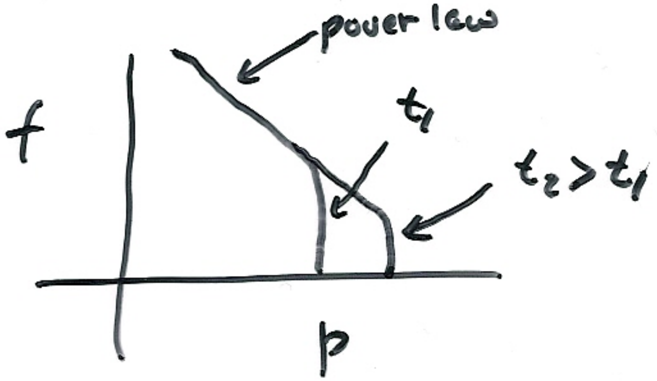


Figure 2. Integrated fluence spectra of H, He, and O for the five SEP events in this study. The data are from ULEIS (filled diamonds), EPAM (downward filled triangles), SIS (filled circles), PET (filled upward triangles), and GOES-11 (upward and downward open triangles). See color version of this figure at back of this issue.

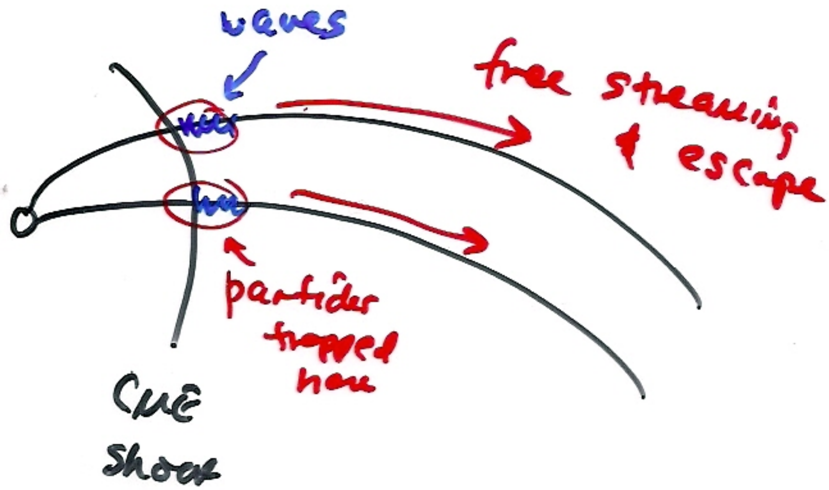
Origin of "spectral breaks"

① time dependence

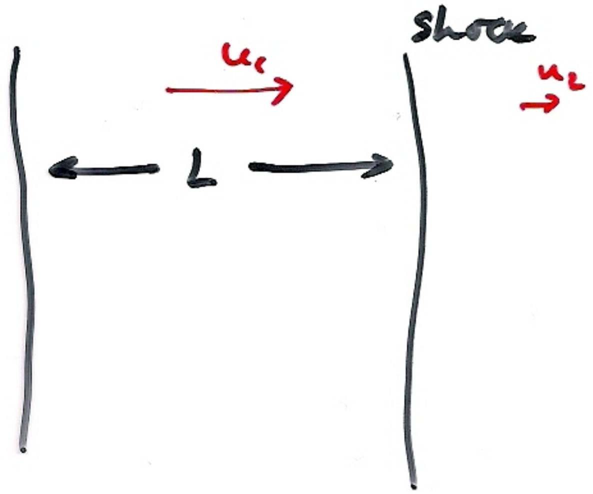
$$\tau_{acc} \simeq 5 \frac{K}{u_1^2} \quad \text{"acceleration time"}$$



② particle escape



To do free escape quantitatively, we adjust the upstream boundary condition



free escape boundary $f \to 0$

Solve as before, but with $f(-L, p) = 0$ we find

$$\boxed{f_2 = A_0 \, e^{-\gamma \int_{p_0}^{p} \frac{dp'/p'}{1 - e^{-\alpha(p') L}}} \qquad \text{where } \alpha(p) = \frac{u_1}{\kappa_1(p)}}$$

Sol'n for free-escape boundary

Acceleration time scale in DSA

For a planar shock, the time to accelerate particles from E_0 to E is (Forman & Drury, 1985)

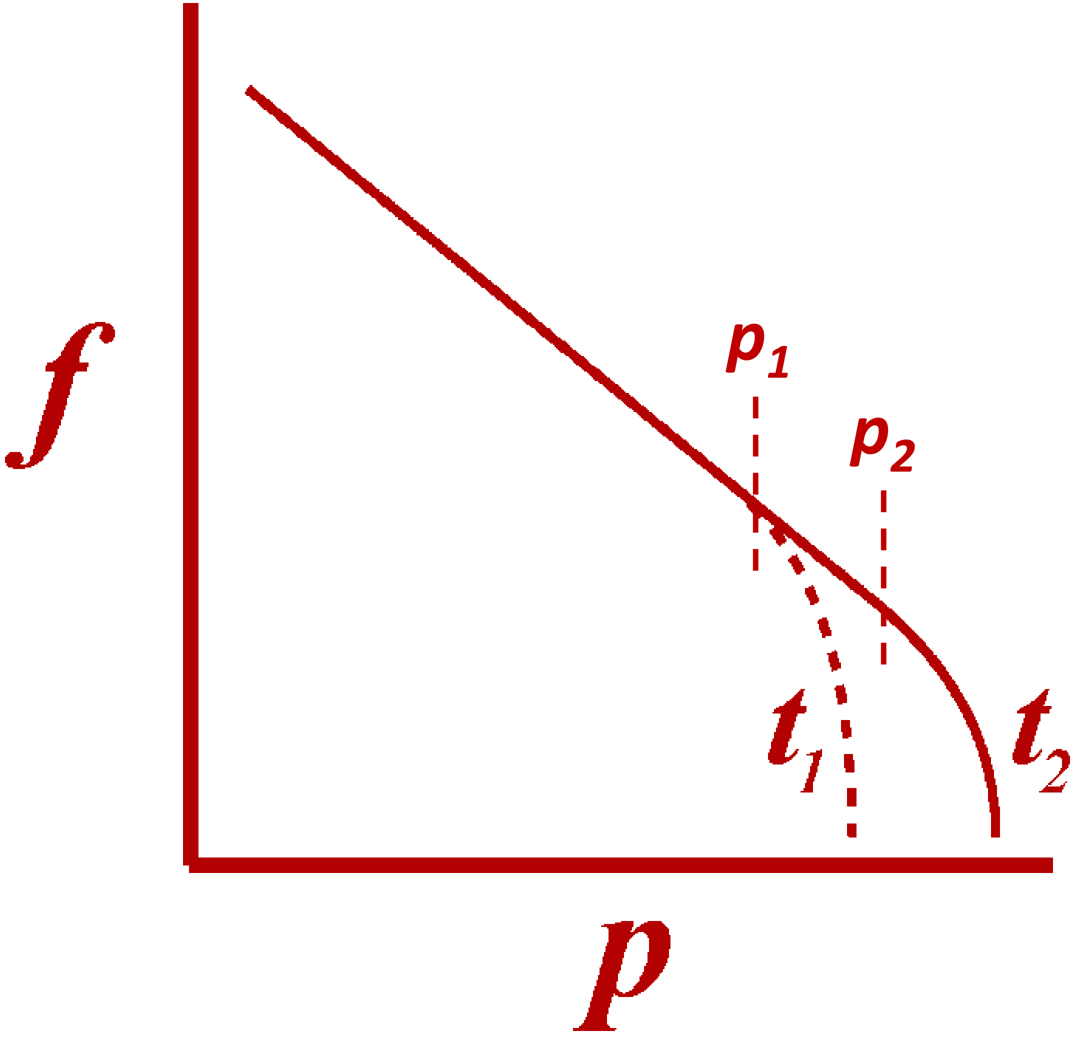
$$\tau_{acc} = \frac{(3/2)}{U_1 - U_2} \int_{E_0}^{E} \left(\frac{\kappa_1(E')}{U_1} + \frac{\kappa_2(E')}{U_2} \right) \frac{dE'}{E'}$$

Where κ is the diffusion coefficient normal to the shock front. The subscripts refer to upstream (1) and downstream (2) of the shock

For an evolving shock, such as one associated with a CME at the Sun, or a that ahead of a supernova blast wave. The spectrum of particles will be a power law below some characteristic energy, *E,* above which the spectrum is steeper.

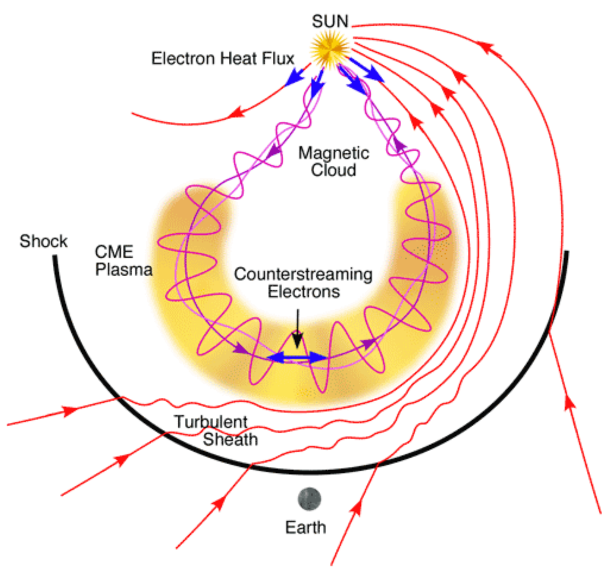
E is known as the "spectral break energy".

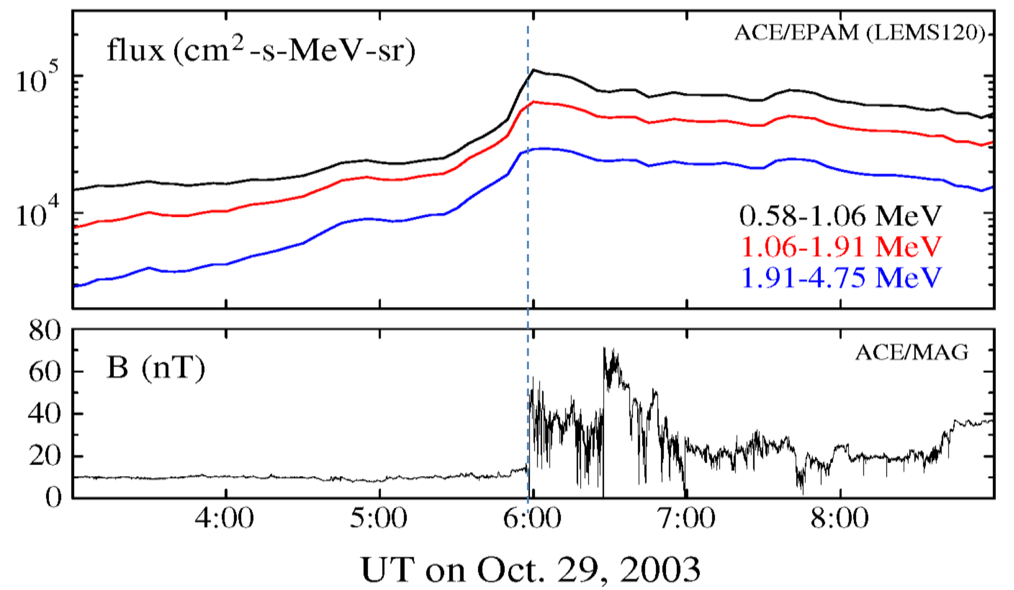
The intensity at the highest energies depends critically on the spectral break energy, and, therefore, on the acceleration time scale (or rate).


f
p
p_1
p_2
t_1
t_2

Energetic particles from CMEs – these are the largest events produced by the Sun

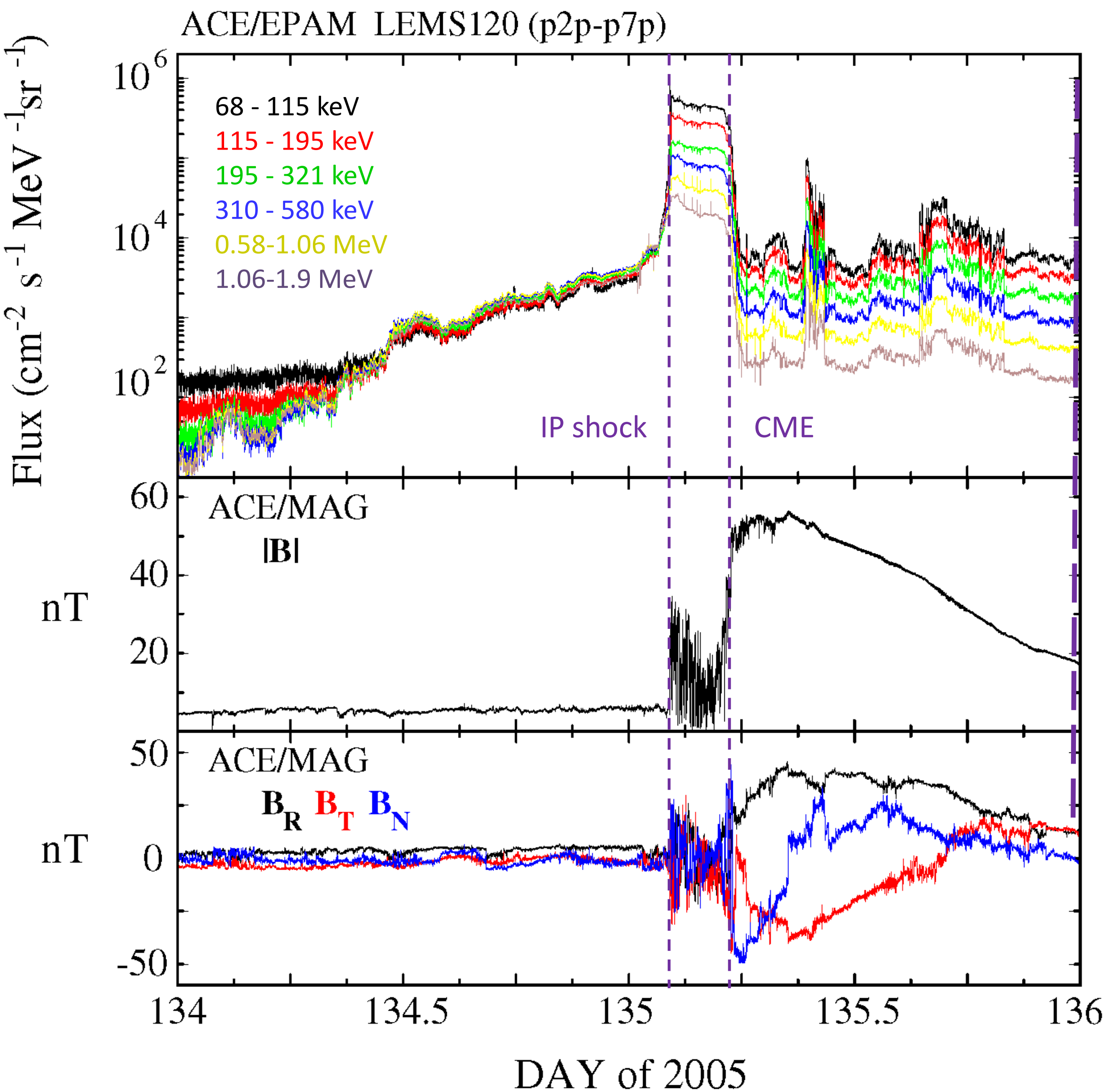
- The largest "solar-proton events" are almost always associated with fast CMEs (usually halo CMEs)
- At 1AU, at energies below several MeV, the peak proton intensity is almost always coincident with the passage of the CME shock.





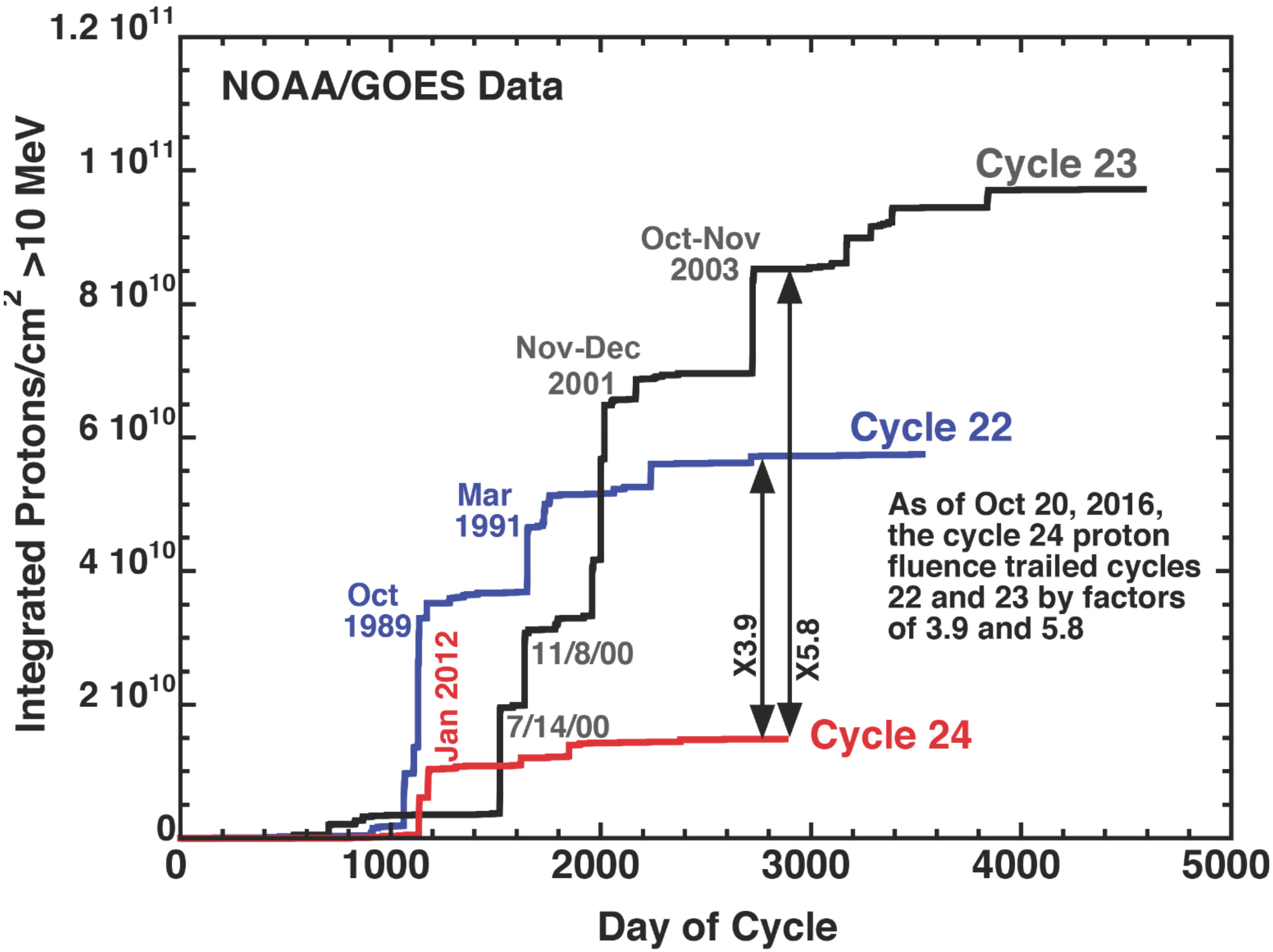
The most widely accepted mechanism is diffusive shock acceleration (DSA) at CME-driven shocks

A large Solar Proton Event associated with a CME



(figure courtesy D. Mewaldt)

Progression of Solar Proton Events For Solar Cycles 22, 23, and 24



There have been few large SPEs this cycle; including only 1 (2?) GLEs this cycle compared to 13 at a similar stage of the previous cycle.

The intensity at the highest energies is very much dependent on the spectral break energy, which depends on the acceleration rate of the particles. This depends on …

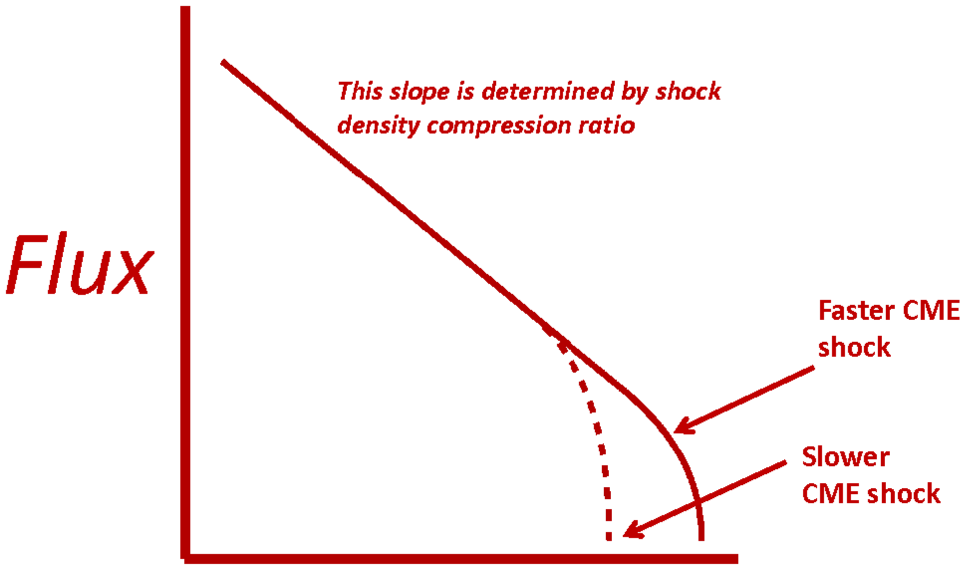
CME speed:

- The acceleration time varies with $1/U_1^2$. Faster CMEs accelerate particles more rapidly, and the spectral break occurs at a higher energy, leading to more particles at high energies.
- For a slower CME, the spectrum will roll over from a power law at a lower energy, and … **the intensity at the highest energies seen at 1 AU will be significantly reduced** compared to a faster CME shock.

Magnetic field strength:

- Generally, a weaker magnetic field will lead to a larger diffusion coefficient along the magnetic field, and a *slower* acceleration rate (longer acceleration time).
- Thus, the spectrum will roll over from a power law at a lower energy for a weak magnetic field, and … **the intensity at the highest energies seen at 1 AU will be significantly reduced** compared to that of a stronger magnetic field.

- A slower CME shock does not create as many high-energy particles as a faster one



- When the solar magnetic field is weaker, shocks do not create as many high-energy particles compared to when it is stronger