IMPLICATIONS OF THE KUIPER BELT STRUCTURE FOR THE SOLAR SYSTEM

Renu Malhotra

Lunar and Planetary Institute, 3600 Bay Area Blvd, Houston, TX 77058, USA.
Electronic mail: renu@lpis39.jsc.nasa.gov

ABSTRACT

The discovery of trans-Neptunian Kuiper Belt objects (KBOs) has opened up new ground for Solar system research. This paper discusses the dynamical structure of the Kuiper Belt and its broader implications for the Solar system. A recent theory for the origin of Pluto predicts that, as a consequence of the orbital evolution of the giant planets during the late stages of their formation, a significant fraction of KBOs may have been swept into a few narrow stable zones located at exterior mean motion resonances with Neptune. Furthermore, these resonant objects are likely to be not in their primordial near-circular orbits, but rather in eccentric orbits, with eccentricities ranging up to Neptune-crossing values. Neptune-crossing, resonant orbits (e.g. Pluto-Charon) are long-lived due to the dynamical protection afforded by libration of their perihelia away from Neptune's longitude. However, some of these orbits do exhibit instability on billion year timescales; this may allow for the delivery of short period comets, and possibly also "Centaurs", from Neptune's resonance zones. Resonant structure of the Kuiper Belt has major implications for the dynamical evolution of the Solar system. The early orbital evolution of the giant planets is poorly constrained by current models of planet formation, but some constraints will be evident in a more complete census of the Kuiper Belt population: the maximum orbital eccentricity and inclination of objects trapped in Neptune resonances is related to the extent and timescale of radial migration of Neptune. Provenance of the KBOs would be strongly correlated with the extent of radial migration of Neptune. The extent of radial migration of the giant planets is determined by mass loss from the outer Solar system, and, hence, the mass of the Oort Cloud can be estimated. A small radial migration of Jupiter can account for the depletion of the outer asteroid belt. Collisional evolution and present-day dust production in the Kuiper Belt would be significantly affected by the non-uniform orbital distribution.

1. INTRODUCTION

There were four review talks on the Kuiper Belt at this conference, not counting the present one. Some would say this is somewhat excessive, considering how little we know about the

[0]Invited review at the "Asteroids-Comets-Meteors Conference" in Versailles, France, July 1996. *Planetary and Space Science*, in press.

Kuiper Belt so far. In fact, as David Jewitt was giving his review, one of the other reviewers expressed alarm that Dave was going to leave nothing to the others! However, I think the attention given to the trans-Neptune region is not misplaced, as it has the potential to significantly increase our knowledge of the early history and evolution of the Solar system. Given that we have heard comprehensive accounts of our present state of knowledge of the basic facts about the Kuiper Belt, I will confine my remarks to filling in some details of my theoretical work that has been mentioned in the earlier reviews. This consists of the "resonance sweeping theory" which provides an economical and perhaps most plausible explanation for the dynamical structure of the Kuiper Belt that is emerging from the observations. This theory reveals deep connections between KBOs and many other aspects of the Solar system. My discussion on these will be mainly speculative and brief, for not much work has been done yet in exploring these connections.

My interest in the trans-Neptune Solar system goes back almost ten years, when I first started my graduate studies and came across the problem of Pluto's peculiar orbit. I was doing my graduate work on resonance dynamics problems in the context of satellite systems of the giant planets, and had just mastered — under Stan Dermott's mentorship — the current ideas about mean motion resonance capture due to slow dissipative processes. The most relevant of these, in the present context, are as follows. A pair of objects whose orbits are initially non-resonant but whose semimajor axes drift closer together due to some external torque will eventually encounter a mean motion resonance and have a non-zero probability of becoming locked in that resonance. For first order resonances, the resonance capture probability depends upon the initial orbital eccentricities — higher probability for smaller eccentricities. If resonance capture does occur, the orbital eccentricity of one (in some circumstances, of both) bodies increases secularly, provided there is no eccentricity damping by the external dissipative agent. [A more detailed review of these concepts may be found in Malhotra (1994a)]. Andrea Milani and Anna Nobili were spending a sabbatical year at Cornell and were working on a paper derived from their LONGSTOP project on the dynamics of Pluto's orbit (Milani *et al.*, 1990). I learned from them the many intriguing properties of Pluto's orbit — its large orbital eccentricity which makes it Neptune-crossing, its mean motion resonance lock with Neptune that protects it from close encounters with that giant planet, its large inclination (the largest amongst the known major planets), as well as a number of other "secular" resonances and near resonances. In thinking about this strange planet, it seemed plausible to me that Pluto had been captured into the orbital resonance with Neptune due to some external dissipative effect that brought the two planets' orbits closer together. We explored the possibility of aerodynamic drag in the solar nebula providing the necessary dissipation. Since aerodynamic drag is a steeply decreasing function of object size, its effect on Neptune would be negligible, but Pluto's orbit could evolve much more, given enough time in the nebular medium. We did a few numerical simulations that included a drag force on a "Pluto" initially further out beyond Neptune in a near-circular, low inclination orbit. As a result of the drag force, "Pluto" was made to evolve inward, approaching Neptune's fixed orbit. In the simulations, we got Pluto captured in the 3:2 Neptune resonance fairly easily. However there are two problems with this mechanism. As I later realized, gas drag damps orbital eccentricities very efficiently, so that "Pluto" would reach an equilibrium eccentricity, $e \sim 0.05$, in the resonance (cf. Malhotra, 1993a); this is much smaller than that observed. (Our initial simulations put in the gas drag incorrectly, so that the eccentricity damping did not occur, and this gave us false hopes of a real solution.) Furthermore, we could not find any significant increase in Pluto's inclination. That is where we left the problem.

More recently, when there was increased activity surrounding Pluto's perihelion passage, I began to think about this problem again. About three years ago, I became convinced that a plausible mechanism for resonance sweeping of Pluto could be found in a different cosmogonic process — related to planetesimal mass loss in the outer Solar system and the formation of the Oort Cloud. This would occur in the later stages of planet formation in a gas-free environment. The main and distinguishing feature of this mechanism is that Neptune's orbit expands and evolves closer to an initially near-circular orbit of Pluto (as well as any other trans-Neptunian bodies); there is no eccentricity damping effect on Pluto, and the magnitude of its eccentricity pumping is directly related to the magnitude of orbital expansion of Neptune subsequent to Pluto's capture into resonance. This idea was published in a short paper (Malhotra, 1993b).

This theory for the origin of Pluto's orbit is testable as it has clear implications for the structure of the Kuiper Belt. The same mechanism that explains the resonance-locking and eccentricity pumping of Pluto applies to all trans-Neptunian minor bodies. This leads to the prediction that a large fraction of Kuiper Belt objects with semimajor axes up to about 50 AU would now be resident in mean motion resonances with Neptune. Numerical simulations done over the last couple of years confirm this picture, and further indicate that the 3:2 and the 2:1 resonances capture the most objects, with the 4:3, 5:3, 7:4 and 8:5 also having non-zero capture probabilities. Equally importantly, the objects captured in mean motion resonances have orbital eccentricities systematically pumped up to values ranging up to about 0.3, significantly higher than their primordial values. These predictions are directly testable by observations of the Kuiper Belt structure that should, hopefully, accumulate towards a reasonably complete census of the trans-Neptunian population over the next few years.

2. PERTURBATION THEORY

In the simplest terms, capture into a mean motion resonance can be pictured as follows. As Neptune's orbit expands, the locations of its mean motion resonances also drift outward. Any small object that is initially on a low inclination near-circular orbit beyond the orbit of Neptune would have a chance of being swept by and captured into a low order mean motion resonance. This is shown schematically in Figure 1. After resonance capture, Neptune would transfer orbital energy and angular momentum to the object in just the correct amounts to maintain the resonance lock even as its orbit continued expanding. One important consequence is that the resonance-locked object would have its eccentricity pumped up during the time that the orbital expansion continued. This can be seen in perturbation theory as follows. In the lowest order, only the mean motion, n, and eccentricity, e, of a massless test particle are perturbed by the gravity of Neptune. The perturbation equations are as follows:

$$\frac{dn}{dt} = 3n^2\mu\frac{\partial R}{\partial \lambda} \simeq 3(j+k)n^2\mu\frac{\partial R}{\partial \phi}, \tag{1}$$

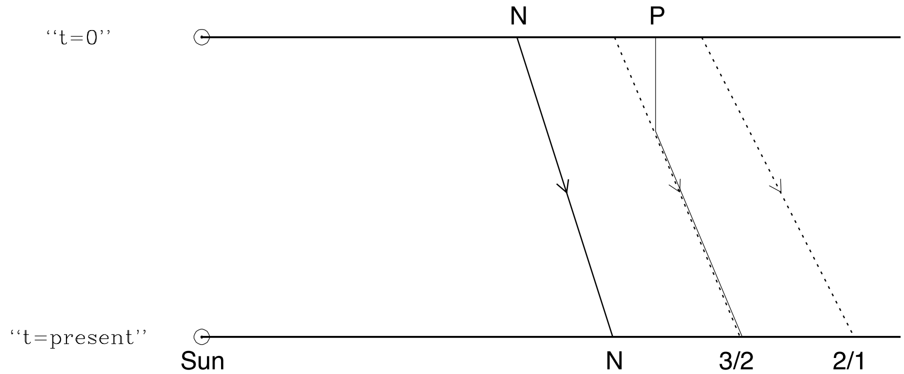

Fig. 1. A schematic diagram illustrating the outward orbital migration of Neptune and its exterior mean motion resonances. For clarity, only two first-order mean motion resonances (3/2 and 2/1) are shown (dotted lines). A "Pluto" in an initially near-circular non-resonant orbit beyond Neptune could have been captured into the 3/2 resonance and would evolve along the solid line path indicated by P—3/2

$$\frac{de}{dt} = \frac{n\mu}{e}\frac{\partial R}{\partial \varpi} \simeq -k\frac{n\mu}{e}\frac{\partial R}{\partial \phi}, \tag{2}$$

where ϕ is the resonance angle for a $(j+k)/j$ resonance given by

$$\phi = (j+k)\lambda - j\lambda_N - k\varpi, \tag{3}$$

λ, ϖ are the the mean longitude and longitude of perihelion of the test particle, λ_N is the mean longitude of Neptune, and $\mu = m_N/M_\odot$ is the mass of Neptune relative to that of the Sun. In Eq. (1) and (2), μR is the disturbing function that describes the gravitational potential of Neptune. The approximate expressions on the extreme right hand sides of Eq. (1) and (2) are obtained by a quasi-averaging that neglects non-resonant terms in R. Eq. (1) and (2) lead directly to the following relationship between the rates of change of mean motion and eccentricity:

$$\frac{de^2}{dt} = -\frac{2k}{3(j+k)}\frac{\dot{n}}{n}. \tag{4}$$

Now, using the resonance condition:

$$\frac{\dot{n}}{n} = \frac{\dot{n}_N}{n_N}, \tag{5}$$

we obtain

$$\frac{de^2}{dt} = -\frac{2k}{3(j+k)}\frac{\dot{n}_N}{n_N} = \frac{k}{(j+k)}\frac{\dot{a}_N}{a_N}, \tag{6}$$

where the last equality follows from Kepler's law: $n^2a^3 = constant$. Eq. (6) can be integrated to yield a relationship between the change in the test particle orbital eccentricity and the magnitude of orbital expansion of Neptune since resonance capture of the particle:

$$\Delta e^2 = \frac{k}{(j+k)}\ln\frac{a_N}{a_{N,0}}, \tag{7}$$

where a_N is Neptune's semimajor axis at the present epoch, and $a_{N,0}$ is its past value at resonance capture.

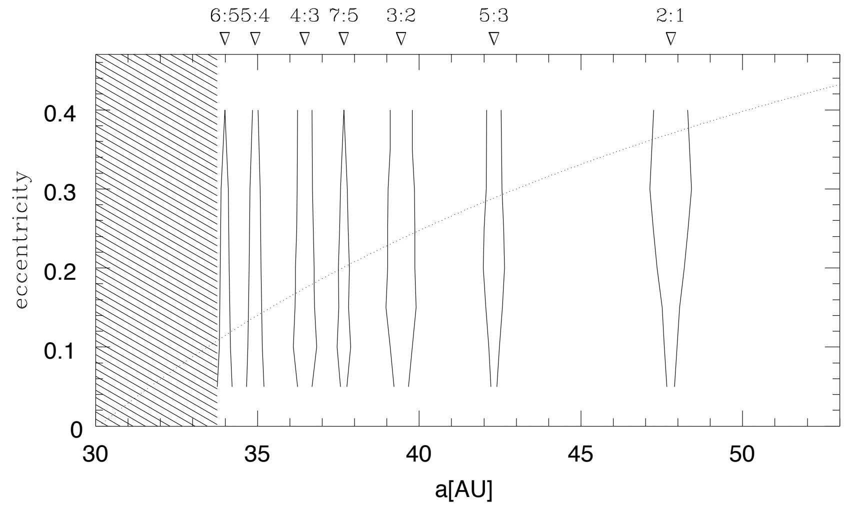

Fig. 2. The locations and widths of first- and second-order Neptune resonances in the Kuiper Belt. The shaded region on the extreme left indicates the "resonance overlap" chaotic zone in the vicinity of Neptune's orbit. In the region above the dotted line, orbits are Neptune-crossing, hence short-lived unless protected by resonance libration.

This result was first applied to the orbit of Pluto which is locked in the 3:2 Neptune resonance and has an eccentricity of 0.25 (Malhotra, 1993b). If Pluto was captured in resonance from a nearly circular orbit, then Eq. (7) shows that Neptune's orbital radius has expanded by about 5 AU since that resonance capture. Of course, it is possible that other objects were captured even earlier; their orbital eccentricities would be pumped up even more (assuming they start with small initial eccentricities). Therefore, the maximum observed eccentricity amongst the objects locked in resonance can be translated into an estimate of the total magnitude of the radial migration of Neptune.

There is a caveat to this conclusion, which is related to the extent of the stable libration regions at low order exterior mean motion resonances of Neptune. The widths of the stable libration zones decrease at moderate-to-large eccentricities. Figure 2 illustrates this concisely in the (a, e) plane (Malhotra, 1996). Using a semi-analytic perturbation theory, Morbidelli *et al.* (1996) have also shown the decrease in stable resonance regions at larger eccentricities, as well as some further complicated dynamics that arises from the secular effects of multiple perturbing planets. For every resonance, there is some $e_{\max}$ beyond which stable resonance librations cannot be sustained for long times. In particular, for the 3:2 resonance, $e_{\max} \approx 0.5$. This means that if 3:2 resonant KBOs are found with eccentricities near $e_{\max}$, Eq. (7) can only provide a lower limit for the net early orbital migration of Neptune. I suspect that we are not in any danger of finding KBOs near this upper limiting eccentricity, and a census of the 3:2 KBOs will in fact yield a meaningful constraint.

3. PHYSICAL MECHANISM FOR RADIAL MIGRATION

3.1 Qualitative Description

Having shown that an outward orbital migration of Neptune can, in principle, account for the resonant, Neptune-crossing orbit of Pluto, as well as the trapping of a significant fraction of KBOs in Neptune's exterior mean motion resonances, I now turn to the physical mechanism that causes the giant planet orbits to have migrated in the early history of the Solar system. The story goes back to the late stages of planet formation, when the outer Solar system had reached a configuration close to what we see at the current epoch, namely four giant planets in well separated near-circular, co-planar orbits, the nebular gas had already dispersed, the planets had acquired a significant fraction of their mass, and there was a residual debris of icy planetesimals and possibly larger planetoids still remaining. (It is possible that the planets may have suffered orbital migration at an even earlier stage, for example, due to nebular torques (cf. Lin and Papaloizou, 1986). This is a highly unconstrained problem, but fortunately it is not of direct concern in the present context.) The late stages of outer planet formation consisted of the gravitational scattering and accretion of this debris. Evidence for this exists in the obliquities of the planets (Lissauer and Safronov, 1991; Dones and Tremaine 1993). Much of the Oort Cloud, the putative source of long period comets, is believed to have formed during this stage by the scattering of outer Solar system planetesimals to wide orbits by the giant planets, and the subsequent action of galactic tidal perturbations and perturbations from passing stars and giant molecular clouds (e.g. Fernandez, 1985). During this era of Oort Cloud formation, the back reaction of planetesimal scattering on the planets could have caused significant changes in their orbital energy and angular momentum. Let us consider this in some detail.

The ejection of large numbers of planetesimals to the Oort Cloud (and almost certainly also into Solar system escape orbits) would result in a net loss of energy and angular momentum from the planetary orbits. Naively, one would conclude that the outer planets' orbits would therefore shrink. However, the energy and angular momentum loss is not extracted proportionately from the four giant planets. It turns out that Jupiter, by far the most massive of the outer planets, provides all of the lost energy and angular momentum, and more; Saturn, Uranus and Neptune actually gain in orbital energy and angular momentum and their orbits expand, while Jupiter's orbit shrinks sufficiently to balance the books. The reasons for this rather non-intuitive result can be understood from the following heuristic picture of the clearing of a planetesimal swarm from the vicinity of Neptune. Suppose that the mean specific angular momentum of the swarm is initially equal to that of Neptune. At first, a small fraction of planetesimals is accreted as a result of physical collisions. Of the remaining, there are approximately equal numbers of inward and outward scatterings. To first order, these cause no net change in Neptune's orbit. However, the subsequent fate of the inward and outward scattered planetesimals is not symmetrical. Most of the inwardly scattered objects enter the zones of influence of the inner Jovian planets (Uranus, Saturn and Jupiter). Of those objects scattered outward by Neptune, some are lifted into wide, Oort Cloud orbits while others return to be accreted or rescattered; a fraction of the latter is again (re)scattered inwards where the inner Jovian planets, particularly Jupiter, control the dynamics. The massive Jupiter is very effective in causing a systematic loss of planetesimal mass by ejection into Solar system escape orbits. As Jupiter preferentially removes the inward scattered planetesimals from Neptune's influence, the planetesimal population encountering Neptune at later times is increasingly biased towards objects with specific angular momentum and energy larger than Neptune's. Encounters with this planetesimal population produce effectively a negative drag on Neptune which results in Neptune experiencing a net gain of angular momentum and energy, hence an increase in its orbital radius. The planets Uranus and Saturn, also being much less massive than and exterior to Jupiter, experience a similar orbital migration, but probably much less in magnitude than Neptune. Note that Jupiter is, in effect, the source of the angular momentum and energy needed for the orbital migration of the outer giant planets, as well as for the planetesimal mass loss. However, owing to its large mass, its orbital radius decreases by only a small amount.

This effect — the inner giant planet Jupiter migrating inward, the outer planet Neptune migrating outward — is reminiscent of the dynamics of a viscous accretion disk, where the inner parts of the disk spread inward, the outer parts spread outward. A similar phenomenon occurs in the spreading of planetary rings due to dissipative collisions amongst the ring particles (e.g. Goldreich and Tremaine, 1982).

The late stages of accretion of Uranus and Neptune have been modeled numerically in computer simulations by Fernandez and Ip (1984, 1996), and their calculations confirm the orbital migration of the giant planets described in the above heuristic model.

3.2 Quantitative Estimates

Putting together the results of perturbation theory and the results of computer models described in the above two sections, we can make some order-of-magnitude estimates of the planetesimal mass involved for the extent of orbital migration of Neptune inferred from the orbit of Pluto. A single scattering event involving a planetesimal of mass m_c and a planet of mass m_p results in a change in the planet's semimajor axis given by

$$\frac{\delta a_p}{a_p} = -\eta \frac{m_c}{m_p}, \tag{8}$$

where η is a numerical factor which is equal to 1 for an isolated planet that ejects a planetesimal from an initially near-circular orbit into a final hyperbolic orbit; η would be less than 1 if the final planetesimal orbit were still bound; η would be negative if the planetesimal were scattered into an interior (smaller) orbit. In a system where there are multiple planets participating in the scattering of a planetesimal swarm, we can write

$$\langle \frac{\delta a_p}{a_p} \rangle = -\eta \langle \frac{m_c}{m_p} \rangle, \tag{9}$$

for the *average* change in the planet's orbit as a result of one scattering event, and where η now represents the numerical factor for an *average* scattering event. Then, the total fractional change in orbital radius of a planet is a fraction η of the total planetesimal mass lost from the system:

$$\frac{\Delta a_p}{a_p} \simeq -\eta \frac{\sum m_c}{m_p}. \tag{10}$$

As discussed above in the heuristic model, we have $\eta > 0$ for Jupiter, and $\eta < 0$ for the other outer planets. In particular for Neptune, from the computer simulations of Fernandez and

Ip, we obtain an estimate of $\eta \approx -0.1$ for that planet. Using this in the previous equation, and together with the estimate $\Delta a_N \approx 5$AU from the resonance sweeping analysis for Pluto, we obtain

$$\sum m_c \approx 2m_N. \tag{11}$$

This represents an order-of-magnitude estimate for the total residual planetesimal mass involved in the late stages of planetary accretion in the outer Solar system. To the best of my knowledge, this is the first such estimate made from observational constraints — that of Pluto's resonant orbit and the magnitude of its orbital eccentricity.

4. COMPUTER SIMULATIONS OF RESONANCE SWEEPING

The resonance sweeping process is most effective in the region beyond Neptune and up to about 48 AU, where the upper limit is the location, in semimajor axis, of the 2:1 Neptune resonance. The 2:1 is the last first-order mean motion resonance given Neptune's current orbit. The upper limit for the detection of ~ 100 km size objects of albedo ~ 0.04 in current ground-based observational surveys is also near that distance, ~ 50 AU (cf. Jewitt and Luu, 1995). Beyond that distance, the strongest resonance is the 3:1 located at $a \simeq 62$ AU.

Computer simulations of resonance sweeping of the trans-Neptune region the general picture developed above (Malhotra, 1995). In these simulations all four giant planets were included, their orbits were integrated self-consistently, together with a small "external" force that was applied to each planet along (or opposite to) its velocity vector to simulate a slow orbital expansion (or contraction). The orbits of hundreds of test particles representing KBOs were also integrated simultaneously, subjected to the perturbations of the giant planets. These simulations were done using a mapping based upon the method of Wisdom and Holman (1991) and modified for non-gravitational forces (Malhotra, 1994b). The mapping is fast and quite accurate for such problems. With a time step of half-a-year, it allows ~ 100 Myr integrations of several hundred test particles in less than a week of CPU time on a modern desktop computer. (Of course, writing the program, testing it, setting up a meaningful set of initial conditions, and finally analyzing the results use a great deal more of human time!)

The results of one such computer simulation are shown in Figure 3. The main points to note are as follows. (1) Most KBOs live in the 3:2 and the 2:1 resonance zones; noticeable numbers are also found in the 5:3 and the 4:3 resonances, as well as non-resonant orbits. (2) Objects in resonant orbits have moderately high eccentricities, up to ~ 0.3; objects in non-resonant orbits remain in nearly circular orbits. (3) Objects in resonant orbits also have significant orbital inclinations; in the 3:2 resonance inclinations range up to $\sim 18°$, while in the 2:1 resonance inclinations range up to $\sim 10°$; even higher inclinations, to $\sim 30°$, are found for objects with $a \lesssim 39$ AU. We note, however, that the latter orbits are generally chaotic, suffering large changes in orbital parameters on timescales of $\sim 10^7$ years. Resonant orbits, on the other hand, are typically stable on 10^8–10^{10} year timescales.

Other computer simulations yield the following additional noteworthy results: the relative numbers of KBOs trapped in the various resonances as well as the distribution of orbital

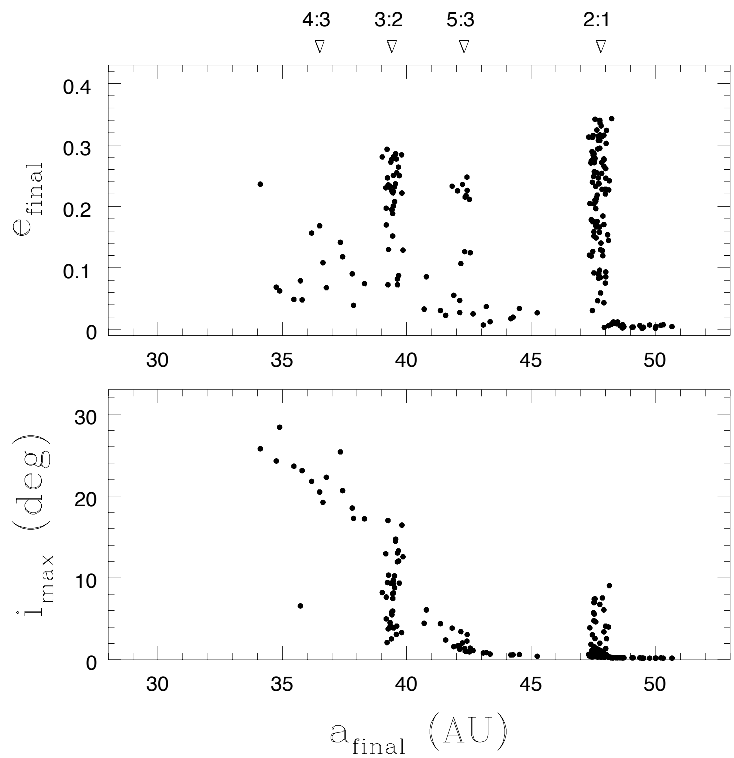

Fig. 3. The orbits of surviving KBOs after resonance sweeping. This is the result from one computer simulation of 300 test particles for 100 Myr; Jupiter, Saturn, Uranus and Neptune migrate radially by amounts $\Delta a = -0.2, 0.8, 3, 7$ AU, respectively, on an exponential timescale of 20 Myr.

inclinations is sensitive to the rate of orbital migration of the planets. Another quantity of interest is the distribution of libration amplitudes. Naively, one might expect a uniform distribution from zero to a maximum amplitude that represents the limit of the stable libration zone. However, as Figure 4 shows, this is not the case for the 3:2 resonance, and, in fact, the distribution peaks near 80°, remarkably close to Pluto's libration amplitude (cf. Malhotra and Williams, 1996). It is worth pointing out a curious property of the 2:1 resonance: objects trapped in this resonance typically exhibit small amplitude, *asymmetric* librations of their perihelia relative to Neptune (cf. Malhotra, 1996), and are stable on timescales comparable to the age of the Solar system.

One note of caution regarding the calculations described here. I have used a highly idealized model with a smooth, adiabatic migration of the planetary orbits, the planetary masses are fixed at their current observed values, collisions and mutual perturbations of KBOs are neglected, initial orbits of KBOs have near-zero eccentricities and inclinations. Relaxation of these assumptions is not likely to affect the main results discussed above in a major way, but may affect the overall efficiency of resonance capture and, to some extent, the relative proportions of resonant and non-resonant objects.

5. OTHER IMPLICATIONS

The resonant structure of the Kuiper Belt that is emerging from recent observational surveys

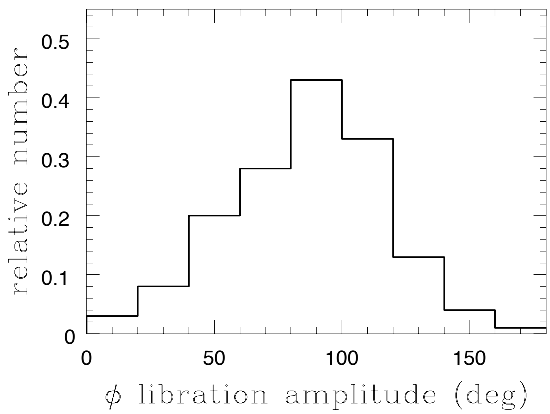

Fig. 4. The distribution of the libration amplitude of KBOs captured in the 3/2 Neptune resonance; $\phi = 3\lambda - 2\lambda_N - \varpi$ is the resonance angle.

(Jewitt *et al.* 1996; Marsden, this conference) supports the resonance sweeping theory. If we consider the fact that the (a, e) plane (see Figure 2) is only a 2-D section in the 6-D phase space of a test particle, and that the resonance zones are severely limited also in angular phases, it is an inescapable fact that the total volume occupied by stable resonance zones is a vanishingly small fraction of the total phase space volume available to KBOs. Thus, only a very efficient dynamical process like resonance sweeping can account for the overpopulation of a resonance zone relative to neighboring non-resonant regions. The orbital eccentricities predicted by the resonance sweeping theory are a further diagnostic of this particular mechanism. The apparent lack of objects at the 2:1 resonance in the observational sample to date may be due to observational incompleteness. We await a more complete census of the Kuiper Belt and a compilation of high-fidelity orbits. This appears to be within reach in the near future. A larger observational data set of the Kuiper Belt in conjunction with more comprehensive modeling will allow us to obtain better constraints on the "free" parameters of the resonance sweeping theory — the magnitude of radial migration of Neptune and the migration timescale. Below is a partial list of significant, outstanding issues in Solar system physics that will be impacted by these Kuiper Belt studies.

Kuiper Belt as a source of SPCs A considerable fraction of the recent theoretical work on the Kuiper Belt has been motivated by the problem of the origin of short period comets (e.g., Duncan *et al.*, 1995). A resonant structure of the Kuiper Belt in the 30–50 AU zone may affect significantly the numerical estimates of the source population of SPCs. Current estimates in the literature neglect the strong non-uniform orbital distribution imposed by resonance sweeping, and the particular characteristics of the dynamical diffusion out of resonant orbits. Similarly, studies of collisional evolution and dust production in the current Kuiper Belt need to be revisited, taking account of the resonance dynamics.

Provenance of KBOs The outward migration of Neptune sweeps trans-Neptunian objects from initial locations as close-in as ~ 27 AU to their present orbits in the Kuiper Belt. This means that KBOs presently resident in Neptune's mean motion resonances at mean heliocentric distances beyond ~ 35 AU may have formed much closer to the Sun. If there were a significant compositional gradient in the primordial trans-Neptunian planetesimal disk, this may be preserved in a rather subtle manner in the orbital distribution in the Kuiper Belt. Because each resonant KBO retains memory of its initial orbital radius in its final orbital eccentricity (cf. Eq. 7), there would exist a compositional gradient with orbital eccentricity within each resonance. On the other hand, non-resonant KBOs in near-circular orbits beyond ~ 35 AU most likely formed at their present locations, and would reflect the primordial composition at those locations.

Mass of the Oort Cloud Past work on this issue has relied on Monte-Carlo models of the delivery of long period comets (LPCs) from the Oort Cloud by random perturbations, and fine-tuning the model parameters to the observed flux and orbital characteristics of LPCs to obtain constraints on the mass of the Oort Cloud. In this fashion, the Oort Cloud mass has been estimated to be between $\sim 14M_\oplus$ and $\sim 10^3 M_\oplus$, with a most-favored estimate being $\sim 50M_\oplus$ (Weissman, 1990). In Section 3, I have described a connection between the dynamical evolution of the Kuiper Belt and the formation of the Oort Cloud. This connection allows the possibility of obtaining an independent estimate of the Oort Cloud from first principles, *i.e.* by modeling the formation of the Oort Cloud in light of the constraints on the late stage evolution of the outer Solar system obtained from the characteristics of the Kuiper Belt.

Depletion of the outer asteroid belt The near-absence of asteroids beyond ~ 3.3AU, except for the concentrations at the 3:2 and the 1:1 Jovian resonances, has long been an outstanding problem in asteroid dynamics. The "gravitational hypothesis" has failed to account for more than about 50% of the depletion (e.g. Lecar *et al.*, 1992). We have recently shown that this puzzle can be neatly resolved by the resonance sweeping that accompanies the small radial migration of Jupiter inferred from the Pluto/Kuiper Belt dynamics (Liou and Malhotra, 1996).

Planet formation The early orbital migration of planets indicated by the resonant structure of the Kuiper Belt suggests the possibility that (i) the outer gas giants, Saturn, Uranus and Neptune may have been born significantly closer to the Sun, and that (ii) at least during the late gas-free stage, the outer planets would have accumulated material from a wide range of heliocentric distances. The significance of this for the timescale of planet formation, and for the composition and evolution of the atmospheres of the giant planets needs to be evaluated. The sizes of trans-Neptunian objects make clear that the formation of large planets was somehow frustrated in that region of the Solar system. The spatial distribution of KBOs may hold clues to the relative timing of the formation of Neptune and of the KBOs themselves because, after formation, Neptune could excite sufficiently large eccentricities in the trans-Neptune planetesimal disk such that accretion of planetesimals would be frustrated (Goldreich, personal communication).

ACKNOWLEDGEMENTS

This research was done while the author was a Staff Scientist at the Lunar and Planetary Institute which is operated by the Universities Space Research Association under contract no. NASW-4574 with the National Aeronautics and Space Administration. Support was also received from NASA's Origins of Solar Systems Research Program grant no. NAGW-4474.

REFERENCES

Dones, L., and S. Tremaine, On the origin of planetary spins, *Icarus* 103(1):67-92 (1993).

Duncan, M., H.F. Levison and S.M. Budd, The dynamical structure of the Kuiper Belt, *Astrophys. J.* 110: 3073 (1995).

Fernández, J.A., The formation and dynamical survival of the comet cloud, in *Dynamics of Comets: their origin and evolution*, eds. A. Carusi and G.B. Valsecchi, IAU Colloquium 83, pp. 45–70, Reidel Publishing Co., Dordrecht, Holland (1985).

Fernández, J.A. and W.-H. Ip, Some dynamical aspects of the accretion of Uranus and Neptune: the exchange of orbital angular momentum with planetesimals, *Icarus* 58:109–120 (1984).

Fernández, J.A. and W.-H. Ip, Orbital expansion and resonant trapping during the late accretion stages of the outer planets, *Planet. Space Sci.* 44(5):431-439 (1996).

Jewitt, D., J. Luu and J. Chen, The Mauna Kea–Cerro Tololo (MKCT) Kuiper Belt and Centaur Survey, *Astron. J.* 112(3):1225-1238 (1996).

Jewitt, D., and J. Luu, The Solar system beyond Neptune, *Astron. J.* 109(4):1867-1876 (1995).

Goldreich, P. and S. Tremaine, Dynamics of planetary rings, *Ann. Rev. Astron. Astrophys.* 20:249-283 (1982).

Lissauer, J.J., and V.S. Safronov, The random component of planetary rotation, *Icarus* 93(2):288-297 (1991).

Lecar, M., F. Franklin and P. Soper, On the original distribution of the asteroids IV: Numerical experiments in the outer asteroid belt, *Icarus* 96:234–250 (1992).

Lin, D.N.C., and J. Papaloizou, On the tidal interaction between protoplanets and the protoplanetary disk. 3. Orbital migration of protoplanets, *Astrophys. J.* 309(2):846-857 (1986).

Liou, J.C., and R. Malhotra, Depletion of the outer asteroid belt, *Science*, in press (1996).

Malhotra, R., Resonances in Solar Nebula: strengths and weaknesses, *Icarus* 106:264-273 (1993a).

Malhotra, R., The origin of Pluto's peculiar orbit, *Nature* 365:819–821 (1993b).

Malhotra, R., Nonlinear resonances in the Solar system, *Physica D* 77:289-304 (1994a).

Malhotra, R., A mapping method for the gravitational few-body problem with dissipation, *Cel. Mech. Dyn. Astron.* 60:373–385 (1994b).

Malhotra, R., The origin of Pluto's orbit: Implications for the Solar System beyond Neptune, *Astron. J.* 110:420–429 (1995).

Malhotra, R., The phase space structure near Neptune resonances in the Kuiper Belt, *Astron. J.* 111:504–516 (1996).

Malhotra, R., and J.G. Williams, Pluto's heliocentric orbit, in *Pluto and Charon*, eds. S.A. Stern and D. Tholen, Univ. of Arizona Press, Tucson, AZ (in press, 1996).

Milani, A., A.M. Nobili and M. Carpino, Dynamics of Pluto, *Icarus* 82:200–217 (1989).

Morbidelli, A., F. Thomas and M. Moons, The resonant structure of the Kuiper Belt and the dynamics of the first five trans-Neptunian objects, *Icarus* 118:322–340 (1995).

Weissman, P.R., The Oort Cloud, *Nature* 344:825-830 (1990).

Wisdom, J., and M.J. Holman, Symplectic maps for the N-body problem, *Astron. J.* 102:1528-1538 (1991).