ICARUS 72, 507–518 (1987)

# The Size, Shape, Density, and Albedo of Ceres from Its Occultation of BD+8°471

R. L. MILLIS,* L. H. WASSERMAN,* O. G. FRANZ,* R. A. NYE,* R. C. OLIVER,* T. J. KREIDL,* S. E. JONES,* W. HUBBARD,† L. LEBOFSKY,† R. GOFF,† R. MARCIALIS,† M. SYKES,† J. FRECKER,† D. HUNTEN,† B. ZELLNER,†,[1] H. REITSEMA,‡ G. SCHNEIDER,§ E. DUNHAM,¶ J. KLAVETTER,¶ K. MEECH,¶ T. OSWALT,‖ J. RAFERT,‖ E. STROTHER,‖ J. SMITH,‖ H. POVENMIRE,‖ B. JONES,‖ D. KORNBLUH,‖ L. REED,‖ K. IZOR,‖ M. F. A'HEARN,** R. SCHNURR,** W. OSBORN,†† D. PARKER,‡‡ W. T. DOUGLAS,‡‡ J. D. BEISH,‡‡ A. R. KLEMOLA,§§ M. RIOS,¶¶ A. SANCHEZ,‖‖,[2] J. PIIRONEN,*** M. MOONEY,††† R. S. IRELAND,††† AND D. LEIBOW†††

*Lowell Observatory, Flagstaff, Arizona 86001
†University of Arizona, Tucson, Arizona 85721
‡Ball Corporation, Boulder, Colorado 80306
§Computer Sciences Corporation, Space Telescope Science Institute, Baltimore, Maryland 21218
¶Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
‖Florida Institute of Technology, Melbourne, Florida 32901
**University of Maryland, College Park, Maryland 20742
††Central Michigan University, Mount Pleasant, Michigan 48859
‡‡Institute for Planetary Research Observatories, South Miami, Florida 33156
§§Board of Studies in Astronomy and Astrophysics, Lick Observatory, University of California, Santa Cruz, California 95064
¶¶Observatorio Astronomico, UNAM-UAZ, Zacatecas, Mexico
‖‖Sociedad Astron. Orion, Nogales, Sonora, Mexico
***Observatory and Astrophysics Laboratory, University of Helsinki, 00130 Helsinki, Finland, and
†††Southern Cross Astronomical Society, Miami, Florida 33186

Received September 2, 1986; revised February 6, 1987

**The occultation of BD+8°471 by Ceres on 13 November 1984 was observed photoelectrically at 13 sites in Mexico, Florida, and the Caribbean. These observations indicate that Ceres is an oblate spheroid having an equatorial radius of 479.6 ± 2.4 km and a polar radius of 453.4 ± 4.5 km. The mean density of this minor planet is 2.7 g/cm³ ± 5%, and its visual geometric albedo is 0.073. While the surface appears globally to be in hydrostatic equilibrium, firm evidence of real limb irregularities is seen in the data.** 

## INTRODUCTION

It has long been known that Ceres is the largest asteroid. However, knowledge of the precise size of this object has been quite uncertain with even modern diameter determinations disagreeing substantially. For example, Brown *et al.* (1982), using infrared radiometry, found the diameter of Ceres to be 953 ± 50 km. Nearly contemporaneous Very Large Array observations at 2 and 6 cm were interpreted by Johnston *et al.* (1982) to indicate a diameter of 818 ± 82 km.

Precise information about the size and shape of Ceres would be valuable for a variety of reasons. Ceres is one of three minor

[1] Present address: Computer Sciences Corporation, Space Telescope Science Institute, Baltimore, MD 21218.

[2] Present address: Instituto Nacional de Astrofisica, Optica y Electronica, Cananea, Sonora, Mexico.



0019-1035/87 $3.00

planets whose mass is accurately known (Schubart and Matson 1979). Given an accurate value of the diameter, the mean density could be computed with sufficiently small uncertainty to be indicative of Ceres' composition. The shape of the asteroid may provide clues to internal structure, while an accurate diameter would extend by nearly a factor of 2 the size range over which the radiometric technique and other indirect methods of size determination can be tested.

In recent years the dimensions of several minor planets have been measured by observation of occultations of stars. Such observations when made with an appropriately distributed network of telescopes typically yield diameters with uncertainties of only 1 or 2% (see, e.g., Millis *et al.* 1981). Application of this technique to Ceres, however, had been thwarted because conventional occultation searches based on star catalogs identified few attractive occultations observable in accessible parts of the world (Taylor 1981). In 1983 a specially designed photographic search at last turned up an occultation of the star BD+8°471 by Ceres occurring on 13 November 1984 (Millis *et al.* 1984). This paper discusses photoelectric observations of that occultation made at 13 sites in Mexico, the United States, and the Caribbean. Preliminary discussions of various small subsets of these data have been published by Hubbard *et al.* (1985), Millis *et al.* (1985), Oswalt *et al.* (1986), and Parker *et al.* (1986).

## OBSERVATIONS

The final predicted ground track for the 13 November occultation based on a plate taken with the 0.5-m Carnegie double astrograph at Lick Observatory is shown in Fig. 1. Given the asteroid's sky-plane velocity of 13.0 km/sec and its estimated diameter from the TRIAD file, a maximum occultation duration of 79 sec was expected. Prior to the occultation, the cross-track uncertainty in the predicted position of the ground track was believed to be no more than one-fourth the width of the track. In fact, the observed track was displaced northward relative to the predicted track by nearly one-third its width.

The geodetic coordinates and altitudes above mean sea level of the observing sites are listed in Table I. Also given are the

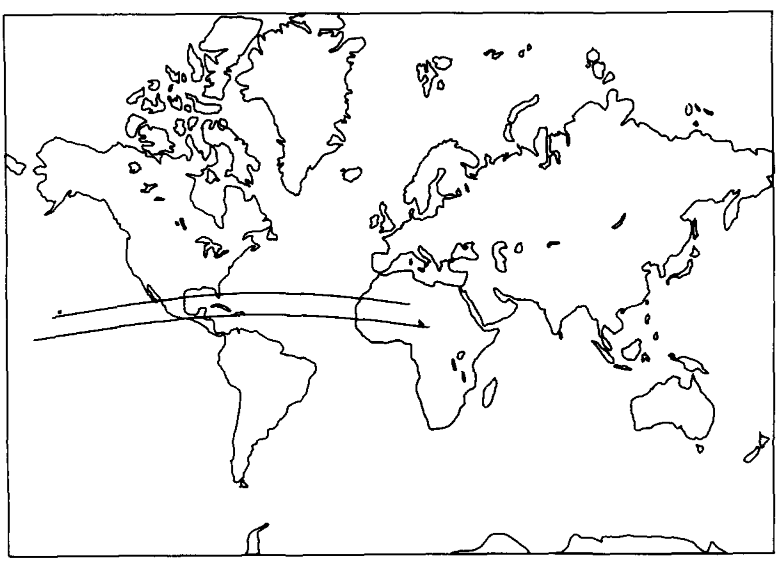

FIG. 1. The predicted ground track of the 13 November 1984 occultation of BD+8°471 by Ceres.

TABLE I

OBSERVING SITES

| | Site No. | Site name | W Longitude | Latitude | Altitude (m) | Observers | Telescope (m) |
|---|---|---|---|---|---|---|---|
| ■ | 1 | Melbourne, Florida | $5^h22^m32^s.71$ | +28°03′30″.7 | 12 | A. Smith<br>D. Kornbluh<br>K. Izor | 0.41 |
| ▶ | 2 | Culiacan, Mexico | 7 09 35.3 | +24 48 10 | 147 | W. Hubbard<br>R. Goff | 0.36 |
| ● | 3 | Chicalahua, Mexico | 7 07 23.38 | +24 11 28.02 | 380 | L. Wasserman<br>R. Nye | 0.36 |
| • | 4 | Burns Lake, Florida | 5 24 55.4 | +25 53 42 | 3 | M. Mooney<br>S. Ireland<br>D. Leibow | 0.28 |
| | 5 | Oasis, Florida | 5 24 8.35 | +25 52 28.4 | 0 | G. Schneider | 0.36 |
| ◆ | 6 | South Miami, Florida | 5 21 12.13 | +25 42 37.0 | 3 | D. Parker<br>W. Douglass<br>J. Beish | 0.36 |
| ✸ | 7 | Mazatlan, Mexico | 7 05 41.5 | +23 11 55.0 | 79 | H. Reitsema<br>B. Marcialis | 0.36 |
| + | 8 | No Name Key, Florida | 5 25 16.45 | +24 41 49.8 | 0 | J. Klavetter<br>H. Povenmire<br>L. Reed | 0.36 |
| ♦ | 9 | Zacatecas, Mexico | 6 50 9.7 | +22 43 56.0 | 2714 | B. Zellner | 0.50 |
| × | 10 | San Blas, Mexico | 7 01 8.5 | +21 32 52 | 1 | R. Millis<br>W. Osborn | 0.36 |
| Z | 11 | Great Exuma, Bahamas | 5 03 6.00 | +23 30 25.2 | 0 | K. Meech<br>E. Strother | 0.36 |
| ✦ | 12 | Chamela, Mexico | 7 00 9.77 | +19 27 8.57 | 40 | M. A'Hearn<br>R. Schnurr<br>S. Jones | 0.36 |
| | 13 | Providenciales, Caicos Islands | 4 49 2.4 | +21 46 35 | 3 | T. Oswalt<br>J. Rafert | 0.36 |

names of the observers and limited information about the instrumentation used for the observations. In most cases, the observations were made with portable telescopes and photometric equipment specifically designed for occultation work. Only at Zacatecas, Mexico; Melbourne, Florida; and South Miami, Florida was the occultation observed at established observatory sites.

Photometric parameters for Ceres and BD+8°471 are listed in Table II along with the expected depths of the occultation lightcurve for different passbands and the expected signal-to-noise ($S/N$) ratios for different observing sites. The magnitude and color indices of Ceres are from the TRIAD file (Bowell *et al.* 1979), while those for the star were measured with the 1.8-m Perkins telescope at Lowell Observatory. The predicted $S/N$ ratios for 1-sec integrations were computed using the relationship given by Millis and Elliot (1979). It is readily evident from Table II that the change in brightness at immersion was too small to be detected by visual or video techniques. However, with photoelectric equipment, the expected $S/N$ ratio for the event was sufficiently high to permit the duration of the occultation to be determined with an uncertainty of 1% or less (see Millis and Elliot 1979). The lightcurve of this occultation observed at San Blas (site 10 in Table I)

TABLE II

PHOTOMETRIC PARAMETERS

| | V | B-V | U-B |
|---|---|---|---|
| Ceres | 7.13 | 0.72 | 0.42 |
| +8°471 | 10.31 | 0.50 | 0.05 |
| | V | U | |
| $\Delta m_{\text{immersion}}$ | $0^m.057$ | $0^m.096$ | |
| Predicted signal-to-noise[a] | 18[b] | 44[c] | |
| Required signal-to-noise[a] | 4 | 4 | |

[a] Quoted signal-to-noise ratios are for 1-sec integrations. See Millis and Elliot (1979).

[b] Assuming a 35-cm-aperture telescope and a sea-level site (i.e., a "worst-case" situation).

[c] Assuming a 35-cm-aperture telescope and an observing site at 2000 m altitude (i.e., a best-case situation).

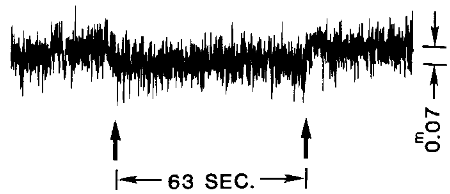


FIG. 2. The lightcurve of the occultation by Ceres observed at San Blas, Mexico (Site 10, Table I). The observations were made through a B filter of the UBV system. Arrows mark the times of immersion and emersion. This lightcurve was reconstructed from digitized data and has a time resolution of 0.1 sec.

is shown in Fig. 2 to illustrate the degree of noisiness typically present in the observations discussed in this paper.

The observed times of immersion and emersion are listed in columns 2 and 3 of Table III. At all sites except South Miami, the photometer signal and WWV radio time signals were simultaneously recorded either on magnetic tape or in a computer memory. At South Miami the data and WWV were simultaneously displayed on a strip-chart recorder. Except at Chamela, as will be discussed later, the limitation to the accuracy of the quoted times of immersion and emersion is imposed by the $S/N$ ratio of the data, not by the mode of data recording. The uncertainties quoted in Table III are estimates by the individual observers. The observations have been given equal weight

TABLE III

OBSERVED TIMES OF IMMERSION AND EMERSION

| Site name | Immersion (UT) | Emersion (UT) | Residuals (km) | | | | Residuals (sec) | |
|---|---|---|---|---|---|---|---|---|
| | | | Circle fit | | Ellipse fit | | Ellipse fit | |
| | | | Imm. | Em. | Imm. | Em. | Imm. | Em. |
| Melbourne, Florida | $4^h42^m6^s.01 \pm 0^s.15$ | $4^h43^m2^s.85 \pm 0^s.15$ | −9.26 | −5.43 | 8.13 | −9.50 | −0.80 | −0.90 |
| Culiacan, Mexico | 4 45 13.5 ± 0.3 | 4 46 20.3 ± 0.5 | −4.42 | 7.83 | 5.19 | 1.53 | −0.44 | 0.12 |
| Chicalahua, Mexico | 4 45 10.6 ± 0.1 | 4 46 20.1 ± 0.1 | −15.14 | 17.13 | −10.42 | 11.32 | 0.83 | 0.88 |
| Burns Lake, Florida | 4 42 05.44 ± 0.13 | — | −4.61 | — | −3.25 | — | 0.25 | — |
| Oasis, Florida | 4 42 03.82 ± 0.04 | 4 43 14.46 ± 0.04 | −2.50 | 10.09 | −1.38 | 5.21 | 0.11 | 0.40 |
| South Miami, Florida | 4 41 58.5 ± 0.1 | (4 43 13.8 ± 0.1)[a] | −4.16 | — | −4.44 | — | 0.34 | — |
| Mazatlan, Mexico | 4 45 07.6 ± 0.5 | 4 46 17.9 ± 0.3 | 6.30 | −8.67 | 3.73 | −11.79 | −0.29 | −0.91 |
| No Name Key, Florida | 4 42 06.53 ± 0.04 | — | 6.43 | — | 0.48 | — | −0.04 | — |
| Zacatecas, Mexico | 4 44 43.0 ± 0.1 | 4 45 52.4 ± 0.2 | 11.62 | 1.41 | 4.46 | 1.84 | −0.35 | 0.15 |
| San Blas, Mexico | 4 45 06.9 ± 0.1 | 4 46 09.9 ± 0.1 | 3.93 | −5.88 | −6.21 | −0.98 | 0.53 | −0.09 |
| Great Exuma, Bahamas | 4 41 28.99 ± 0.01 | 4 42 30.27 ± 0.014 | 13.34 | −3.24 | 5.21 | 2.85 | −0.25 | 0.27 |
| Chamela, Mexico | 4 45 26.20[b] | 4 45 52.48[b] | 6.12 | −12.47 | 3.34 | −1.53 | −0.71 | −0.36 |
| Providenciales, Caicos Islands | 4 41 20.19 ± 0.11 | 4 41 42.0 ± 0.2 (4 41 50.90 ± 0.18)[a] | 6.82 | −17.30 | 4.63 | −7.06 | −1.08 | −1.92 |
| rms residual per degree of freedom (km) | | | 9.84 | | 6.74 | | | |

[a] Originally reported time of emersion. This time has been rejected, as discussed in the text.

[b] The uncertainty in the duration of the occultation at Chamela is ±0.1 sec. However, the uncertainty in the absolute timing at this site may be as much as ±1.0 sec.

in the analysis that follows. Note that all of the observations of this occultation occurred within a 5-min interval, during which time Ceres rotated only 3°.3.

## ANALYSIS

We have analyzed the observations of the 13 November occultation using standard methods which have been described in detail elsewhere (e.g., Wasserman *et al.* 1979). Each observed time of immersion or emersion can be combined with the coordinates and altitude of the corresponding site, the asteroid's ephemeris, and the coordinates of the occulted star to define the position of a point on the apparent limb of Ceres. The points derived from all of the observations in Table III are plotted in Fig. 3. A circle has been fitted to the data by least squares. In this solution the radial residuals have been minimized.

Two serious discrepancies are apparent in Fig. 3. The point corresponding to the emersion timing at South Miami (point A in the figure) deviates from other points near the same chord by 61 km (or 4.7 sec of time), an amount greater than can be explained by any reasonable topography or timing error. Large negative excursions in the recorded signal, subsequently found to be due to electronic problems, are present in the South Miami data for an interval of several seconds around the expected emersion time (Parker *et al.* 1986). We believe that the reappearance of the star occurred while the photometric signal was degraded and that the time originally reported by the observers simply corresponds to the cessation of the electronic difficulties. In any case, one is clearly justified in discarding the emersion timing from South Miami.

The other significant disagreement in the

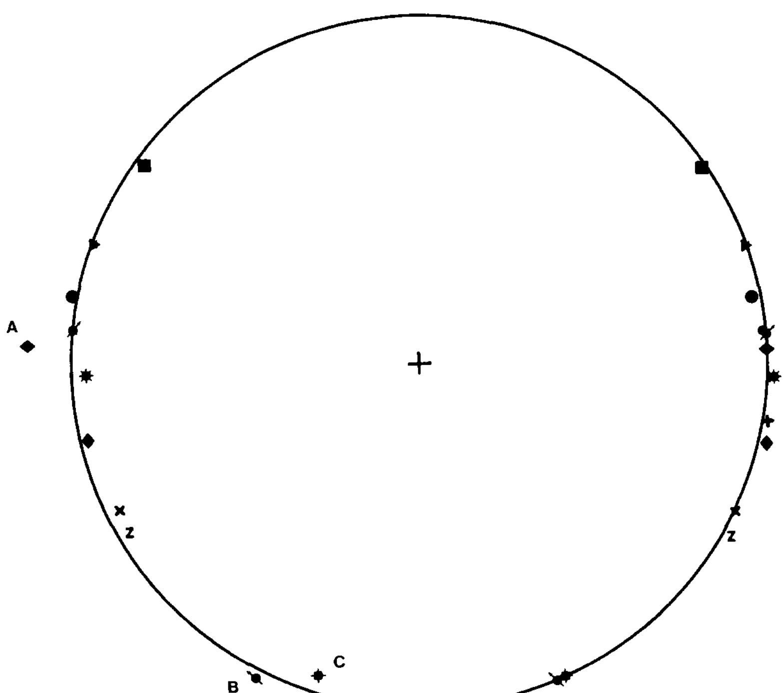


FIG. 3. A least-squares fit of a circular limb profile to the originally reported data set. North is at the top; east is on the left. Emersion observations are those on the left-hand half of the fitted circle. The radius of the fitted limb profile is 476.5 km. Discrepant points are marked A, B, and C. These points are discussed in detail in the text.

total data set concerns the emersion times recorded at Providenciales (point B in Fig. 3) in the Caicos Islands and near Chamela along the western coast of Mexico (point C in Fig. 3). The timing residuals corresponding to points B and C are 2.57 and 4.83 sec, respectively. Both Providenciales and Chamela are near the southern edge of the occultation ground track; as a consequence, these observations are particularly important in distinguishing between a circular and elliptical limb profile. Emersion in the Chamela observations is well defined, and the interpretation of those data seems unequivocal. The Providenciales observers, on the other hand, reported two "flashes" prior to their originally identified time of emersion. During the second and more prominent of these, the signal returned to the preoccultation level for approximately 2 sec. If we identify egress with the onset of this feature, then the Providenciales observations are in good agreement with those from Chamela. Since there is no other obvious way to resolve the discrepancy between the two data sets, we have adopted arbitrarily this identification in the analysis that follows. The alternative would be to discard the Providenciales emersion timing, but that approach would affect the results of the following analysis by amounts that are small compared with the quoted uncertainties.

In Fig. 4 a circle has been fitted to the final data set by least squares, again minimizing the radial residuals. The resulting value for the mean radius of Ceres is listed in Table IV together with corrections to the right ascension and declination of the asteroid indicated by the least-squares solution. (By convention, we have assumed that all position error resides in the aster-

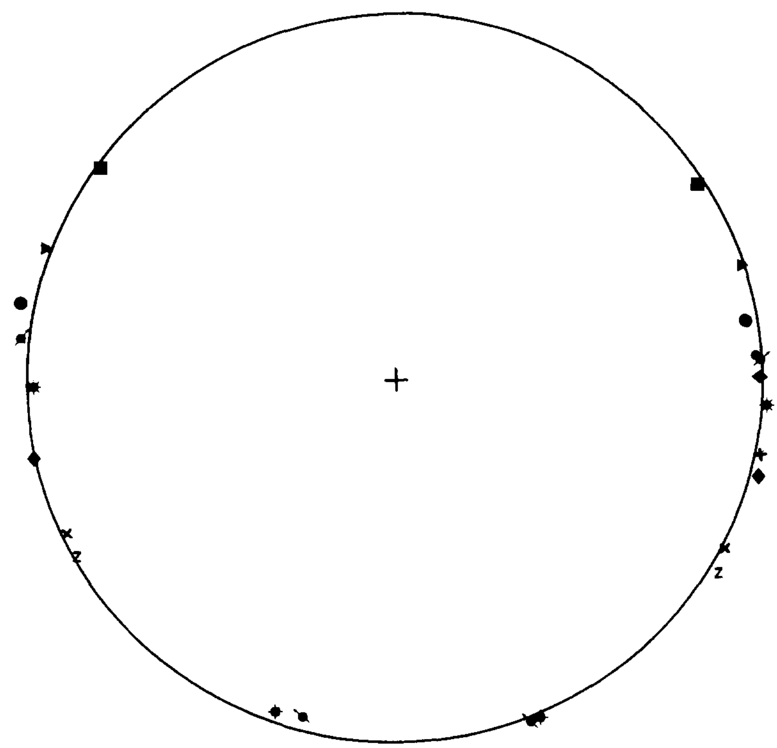

FIG. 4. A circular limb profile fitted by least squares to the adopted data set. Parameters of the solution are given in Table IV. The orientation of this figure is the same as in Fig. 3.

TABLE IV

RESULTS OF LEAST-SQUARES FITS TO THE DATA

| | Radial residuals minimized | Timing residuals minimized |
|---|---|---|
| Circular limb profile | | |
| Mean radius | 471.6 ± 2.2 km | 470.1 ± 2.7 km |
| Right ascension correction | −0.00890 ± 0.00012 sec | −0.00937 ± 0.00013 sec |
| Declination correction | 0″.181 ± 0″.003 | 0″.185 ± 0″.002 |
| Elliptical limb profile | | |
| Semimajor axis | 479.6 ± 2.4 km | 481.6 ± 2.4 km |
| Semiminor axis | 453.4 ± 4.5 km | 450.1 ± 2.0 km |
| Position angle of north pole of semiminor axis | 331°.5 ± 6°.2 | 329°.1 ± 0°.4 |
| Right ascension correction | −0.00884 ± 0.00008 sec | −0.00896 ± 0.00007 sec |
| Declination correction | 0″.176 ± 0″.003 | 0″.176 ± 0″.002 |
| Equivalent radius ($\sqrt{ab}$) | 466.3 ± 2.6 km | 465.6 ± 1.6 km |

oid's ephemeris.) The quoted errors reflect only the uncertainty in fitting a circle to the observed data points and do not include the uncertainty in the observations themselves. Table V contains the ephemeris of Ceres that was used in this analysis. It is based on orbital elements given in the 1975 *Leningrad Ephemeris of Minor Planets*. The radial differences between the observed points and the fitted circle are listed in columns 4 and 5 of Table III. Qualitatively the circular limb profile fits the data reasonably well, but the southernmost and northernmost chords deviate from the circle in, perhaps, a nonrandom way.

TABLE V

ASTROMETRIC, GEOCENTRIC EPHEMERIS OF CERES

| Date (0ʰ ET) (1984) | (1950.0) RA | (1950.0) Dec | $R$ (AU) |
|---|---|---|---|
| Nov. 6 | 3ʰ15ᵐ27ˢ.501 | 8°45′26″.85 | 1.831965 |
| 7 | 3 24 31.484 | 8 44 18.59 | 1.830171 |
| 8 | 3 13 35.126 | 8 43 13.91 | 1.828665 |
| 9 | 3 12 38.488 | 8 42 12.97 | 1.827448 |
| 10 | 3 11 41.629 | 8 41 15.97 | 1.826521 |
| 11 | 3 10 44.610 | 8 40 23.06 | 1.825883 |
| 12 | 3 09 47.492 | 8 39 34.44 | 1.825536 |
| 13 | 3 08 50.336 | 8 38 50.26 | 1.825480 |
| 14 | 3 07 53.205 | 8 38 10.69 | 1.825715 |
| 15 | 3 06 56.161 | 8 37 35.91 | 1.826240 |
| 16 | 3 05 59.264 | 8 37 06.08 | 1.827056 |
| 17 | 3 05 02.580 | 8 36 41.37 | 1.828162 |
| 18 | 3 04 06.169 | 8 36 21.94 | 1.829558 |
| 19 | 3 03 10.097 | 8 36 07.95 | 1.831243 |
| 20 | 3 02 14.426 | 8 35 59.58 | 1.833216 |

Figure 5 shows the same data as in Fig. 4 but fitted with an elliptical limb profile rather than a circle. Also indicated in this figure are constraints placed on the solution by the negative observations by J. F. Barral at Tonantzintla Observatory and M. Frueh at McDonald Observatory. The semimajor and semiminor axes of the fitted ellipse, the position angle of the semiminor axis, and the adjustments to the right ascension and declination of Ceres that result from the least-squares solution are given in Table IV. The radial residuals are listed in columns 6 and 7 of Table III, and the equivalent timing residuals are given in columns 8 and 9. Note that the rms residual per degree of freedom (see Table III) is significantly less in the case of the elliptical fit than for the case of a circular limb profile. Consequently, inclusion of the additional free parameters inherent in the elliptical solution appears to be justified. As in the case of the circular solution, the quoted errors for the elliptical solution are a measure of the uncertainty in the least-square fit without regard for the errors inherent in the observations themselves.

In the elliptical solution just discussed, radial residuals were minimized. A least-

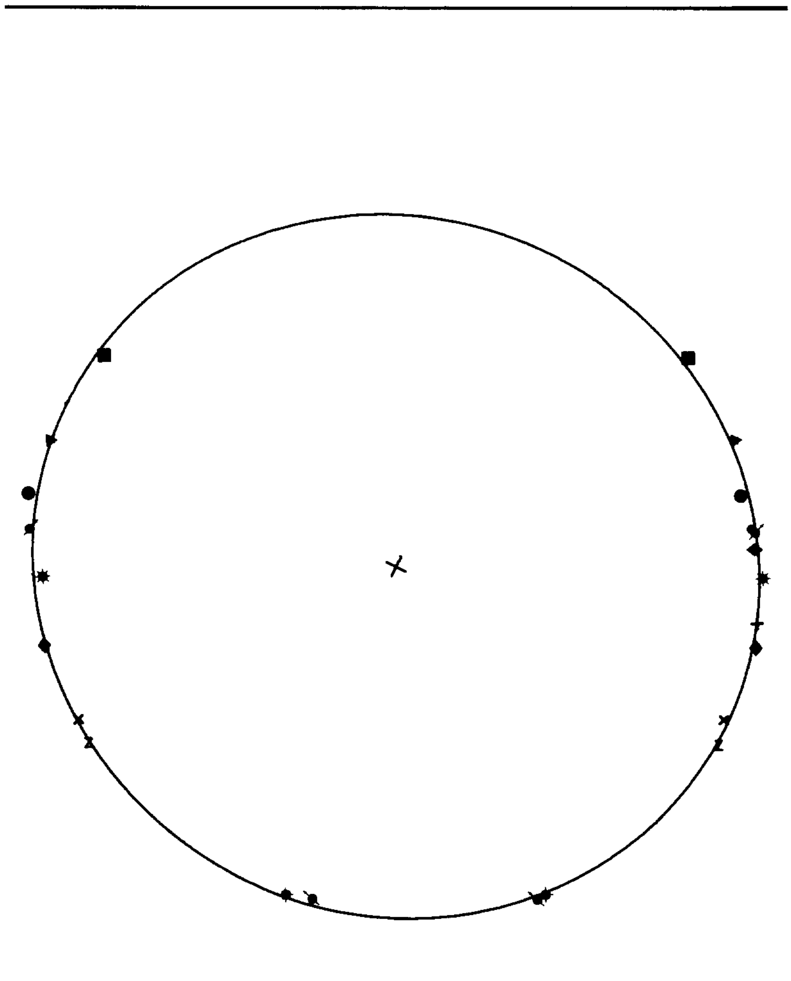

FIG. 5. An elliptical limb profile fitted by least squares to the adopted data set. Parameters of the solution are given in Table IV. As in Figs. 3 and 4, north is at the top, east is to the left. The cross in the center of the figure is aligned with the major and minor axes of the fitted ellipse. Lines at the top and bottom of the figure represent constraints placed on the solution by observations at sites where no occultation occurred.

squares fit to the data was also performed in which the timing residuals were minimized as a further test of the meaningfulness of the elliptical solution. As was called to our attention by a referee, relatively large timing errors near the edges of the track would still produce comparatively small radial residuals. Consequently, the results of a least-squares fit could depend on which parameter was minimized. In actual fact, we found the differences to be insignificant, as can be seen in Table IV. The dimensions and orientation of the elliptical limb profile, within their formal uncertainties, do not depend on which parameter is minimized in the least-squares solution. Moreover, varying the absolute times of immersion and emersion at Chamela within their uncertainty of $\pm 1$ sec while holding the duration constant (see Table III) produces insignificant changes in the results of the least-squares fits.

While the solution minimizing timing residuals gives smaller formal errors for some parameters, we have adopted the results of the other solution because, as will be discussed later, we believe that the departures from a perfect elliptical profile are due primarily to real limb irregularities rather than timing errors. That being the case, minimization of radial residuals is the correct procedure in our opinion.

## DISCUSSION

The occultation observations give the profile of the asteroid in the plane of the sky. However, to compare the occultation results with previous diameter measures and to use the occultation data to derive physical parameters of interest, we must assess the three-dimensional figure of Ceres. Tedesco *et al.* (1983), on the basis of extensive photoelectric photometry of this asteroid, concluded that Ceres is approximately spheroidal and that the obliquity of its pole is small. These conclusions follow from the fact that Ceres displays the same low-amplitude rotational lightcurve regardless of ecliptic longitude. Indeed, in 1984 about 1 week after the occultation, one of us (Piironen) recorded the lightcurve shown in Fig. 6. For all practical purposes, this lightcurve is identical to those published by Tedesco *et al.* (1983) from the 1975–1976 apparition. The arrow in Fig. 6 marks the rotational phase at which the 13 November occultation occurred. Ceres at that phase was at an intermediate brightness, indicating that the effective diameter derived from the occultation observations will be close to the average for all rotational phases. In fact, the total rotational brightness variation can be explained by a variation in apparent effective diameter with rotational phase of less than 2%. It may be less because all or part of the 9-hr-period brightness variation could be produced by differences in albedo across the surface of Ceres.

The assertion that the obliquity of the rotational pole of Ceres is small can be further tested by reference to the occultation observations. Dermott (1979) has noted that the rotational period of Ceres is sufficiently short that an appreciable equatorial bulge should be present. If the rotational axis is perpendicular to Ceres' orbit, then the latitude of the sub-Earth point was within 2°.7 of the asteroid's equator at the time of the occultation. Consequently, we viewed the object essentially equator-on, and the minor axis of the ellipse fitted to the occultation data in Fig. 5 should have very nearly the same orientation as the rotational pole of Ceres. Assuming zero obliquity for the pole, its position angle at the time of occultation would have been 335°.2. The least-squares fit to the data gave 331°.5 ± 6°.2 (see Table IV). This remarkable agreement lends credence not only to the assertion of a small obliquity, but also to the validity of the elliptical limb profile.

One can further ask whether the degree

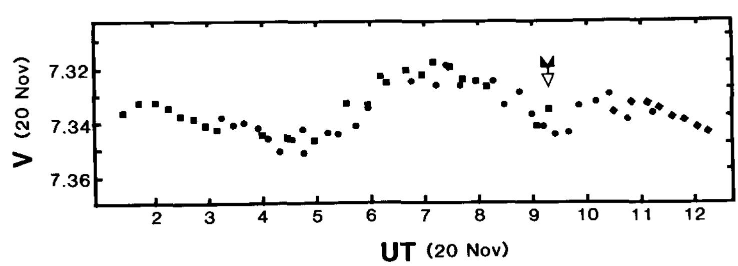


FIG. 6. The composite lightcurve of Ceres observed at Lowell Observatory on 18 and 20 November 1984. The arrow indicates the phase of the lightcurve at which the occultation occurred. Observations from (●) 20 November, (■) 18 November, and (◆) the first eight points of 18 November shifted one period (0ᵈ37812).

of oblateness observed is in line with expectations. It is first necessary, however, to compute the mean density of Ceres. Schubart (1974) has determined the mass of Ceres to be $(5.9 \pm 0.3) \times 10^{-10} M_{\odot}$ based on the gravitational interaction of this asteroid with Pallas. Assuming Ceres to be an oblate spheroid whose polar and equatorial radii are respectively equal to the semiminor and semimajor axes of the fitted ellipse in Fig. 5, the mean density of Ceres is found to be 2.7 g/cm$^3$, with an uncertainty of 5%. The uncertainty in density is due almost entirely to the uncertainty in the asteroid's mass. Our quoted uncertainty in the dimensions of Ceres would have to be increased by nearly a factor of 4 before its contribution to the uncertainty in the density would equal that due to the uncertainty in mass. The value for Ceres' density determined here is identical to within the quoted uncertainties with the density of Pallas (Millis and Elliot 1979), the only other minor planet whose mass and diameter are sufficiently well determined to permit computation of a reliable density. It appears that Ceres, like Pallas, is primarily rocky in composition, although the two differ somewhat in spectral characteristics (Tholen 1984).

Dermott (1979) has noted that for a rotationally distorted body whose surface is in hydrostatic equilibrium, the difference between the equatorial radius, $a$, and the polar radius, $c$, is given by

$$a - c = \tfrac{15}{16} H\beta B. \quad (1)$$

$B$ is effectively the mean radius of the body and for a homogeneous object $H = 1$. The quantity $\beta$ is given by

$$\beta = \Omega^2/\pi G\langle\rho\rangle, \quad (2)$$

where $\Omega$ is the asteroid's rotational angular velocity, $G$ is the gravitational constant, and $\langle\rho\rangle$ is the mean density of the body. Taking Ceres' rotational period of 0.37812 $\pm$ 0.00004 day from Tedesco *et al.* (1983) and the density and mean radius from the present paper, we find the predicted difference between the equatorial radius and the polar radius to be 29 $\pm$ 1 km. The observed difference from this paper is 26 $\pm$ 5 km, which is consistent with the assumption that Ceres is basically homogeneous (i.e., it is not strongly differentiated) and that its surface has achieved, at least on a global scale, a state of hydrostatic equilibrium. The uncertainty in the measured $a - c$, however, is too large to prove that $H = 1$.

On a local scale the residuals listed in Table III indicate definite departures from the mean equilibrium figure of up to about 10 km. To produce the observed maximum residuals through observational error would require that the reported times of immersion and emersion be in error on the order of 1 sec or that the observing site locations be incorrect by as much as 10 km or a combination of the above. While the coordinates of the observing sites were derived from a variety of sources, we are convinced that they are in all cases known to better than 1 km. The timing uncertainty in our observations is typically 0.1 sec, although at three sites it is poorer. In any event, the fact that the timing residuals in the last two columns of Table III are often several times the corresponding observational uncertainties given in columns 2 and 3 of that table (the rms ratio of residual to observational uncertainty is 7.4) argues convincingly, in our opinion, for the presence of sizable real limb irregularities. Limb features of similar magnitude have been detected in other well-observed asteroid occultations (e.g., Millis *et al.* 1981). Dermott (1979) has stated that $C$-type asteroids, if composed of carbonaceous chondritic material, would not be expected to have the structural strength to support topography of greater than about 1 km over a long period of time. However, Ceres is not a typical $C$-type asteroid, belonging as it does to the $G$ subclass defined by Tholen (1984). Asteroids of this subclass have higher albedos and deeper ultraviolet absorption features than classical $C$ types. The existence of significant topographic relief on Ceres may, then,

TABLE VI

PREVIOUS ESTIMATES OF CERES' DIAMETER

| Technique | Diameter (km) | Reference |
|---|---|---|
| Filar micrometer | 781 ± 87 | Barnard (1895) |
| Lunar occultation | 1200 ± 250 | Dunham *et al.* (1974) |
| Polarimetry | 1016 ± 50 | Morrison and Zellner (1979) |
| Radio | 818 ± 82 | Johnston *et al.* (1982) |
| Infrared | 953 ± 50 | Brown *et al.* (1982) |
| Infrared | 962 ± 30 | Lebofsky *et al.* (1984) |

imply a composition that differs markedly from that of the darker classical *C*-type asteroids.

It is of interest to compare the occultation diameter of Ceres (i.e., the effective diameter or the diameter of a circle having the same cross-sectional area as the actual elliptical profile) with previous estimates. The various earlier determinations are given in Table VI. Those values are to be compared with 932.6 ± 5.2 km derived from the occultation observations. Note that the determinations based on infrared radiometry agree with the occultation diameter to within about 3%, while the others differ by 9% or more.

Based on Ceres' absolute *V* magnitude of $3^{m}.61$ from Tedesco *et al.* (1983), the effective occultation radius from Table IV, and apparent *V* magnitudes of the Sun from Gehrels *et al.* (1964), the visual geometric albedo of Ceres is found to be 0.073.

## SUMMARY

The more important physical characteristics of Ceres derived from the observations of the occultation of BD+8°471 are summarized in Table VII. The apparent profile of Ceres seen at the time of the occultation is consistent with that of a homogeneous object of low obliquity whose surface is generally in hydrostatic equilibrium. Irregularities having a vertical scale of a few kilometers were seen on the limb of this minor planet.

TABLE VII

CERES OCCULTATION RESULTS

| | |
|---|---|
| Equatorial radius | 479.6 ± 2.4 km |
| Polar radius | 453.4 ± 4.5 km |
| Effective diameter | 932.6 ± 5.2 km |
| Mean density | 2.7 ± 0.14 g/cm$^3$ |
| Visual geometric albedo | 0.073 |


## ACKNOWLEDGMENTS

The reader no doubt has surmised that the effort devoted to identifying, predicting, and observing the 13 November 1984 occultation by Ceres was substantial. We were assisted in this research project by many individuals whose names do not appear in the author list. Space is not available to acknowledge all of these contributions individually, but they are nonetheless appreciated. However, we would be very remiss indeed if we failed to thank specifically Dr. Luis F. Rodriquez, Director of the Instituto de Astronomia at the Universidad Nacional Autonoma de Mexico. The various expeditions to Mexico would not have been possible without his encouragement and advice and without the direct logistical support of the Institute and the University. Mr. Carlos Sandoval of Televisoras en el Pacifico kindly provided access to appropriate observing sites at Culiacan and Mazatlan. Stanley F. Dermott assisted us with a helpful discussion concerning the expected equilibrium figure of Ceres. J. Piironen thanks the Emil Aaltonen Foundation (Finland) for support. The Caribbean observers gratefully acknowledge the assistance of P. Marx, S. Benjamin, and A. Dean in providing access to observing sites. Finally, we thank Phillip Nicholson and David Tholen for very thorough and helpful reviews of this paper.

This research was supported by NASA Grants NSG-7603, NSG-7500, NSG-7045, NSG-7322, NSG-7114, and NSG 7526; National Science Foundation Grant 84-14142, Grant 2802-84 from the National Geographic Society; the FIT Office of Development; and the American Astronomical Society.


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