Angle subtended by Mars from the spacecraft is ψ:

Use the Law of Sines:

$$\frac{r+z}{\sin\ 90^\circ} = \frac{r}{\sin\ \psi}$$
$$\sin\ \psi = \frac{r}{r+z}$$
$$\psi = 63.^\circ 464$$

Find range of limb. Use the Pythagorean Theorum:

$$(r+z)^2 = L^2 + r^2$$
$$L^2 = (r+z)^2 - r^2$$
$$= 2xr + z^2$$
$$L = \sqrt{2zr + z^2}$$
$$L = 1696.536\,\mathrm{km}$$

Calculate the angle θ, the number of degrees around planet to limb:

$$\psi + \theta = 90^\circ$$
$$\theta = 90^\circ - \psi$$
$$\theta = 26.^\circ 536$$

Case I: When do we lose 100% coverage of Olympus Mons?

This occurs when the far slope becomes co-linear with the line-of-sight from the spacecraft. We assume, for Olympus Mons, a slope $s = 20°$, a height $h = 20$ km, and that the basal diameter (110 km) is small compared to the radius of Mars, $r = 3397$ km. The nominal spacecraft altitude is $z = 400$ km.

Using the Law of Sines:

$$\frac{r+z}{\sin(90° + s)} = \frac{r+h}{\sin\ n}$$

$$\sin\ n = \left(\frac{r+h}{r+z}\right)\cos\ s$$

$$n = \sin^{-1}\left[\left(\frac{r+h}{r+z}\right)\cos\ s\right]$$

$$n = 57.°742, \quad \text{(which is } < \psi, \text{ as it should be.)}$$

Calculate the angle ϕ, the number of degrees around planet to the summit of Olympus. The sum of the angles of a triangle is 180°:

$$180° = 90° + s + n + \phi$$

$$\phi = 90° - s - n$$

$$\phi = 12.°258 \quad \text{and} \quad \theta - \phi = 14.°278$$

Find range of the summit from spacecraft. Again use the Law of Sines:

$$\frac{l}{\sin\ \phi} = \frac{r+h}{\sin\ n}$$

$$l = \left(\frac{r+h}{\sin\ n}\right)\sin\ \phi$$

$$l = 857.925\,\text{km}$$

CASE II: When does Olympus Mons peek above the limb?

This occurs when the summit becomes co-linear with the line-of-sight from the spacecraft, it i.e., when the nadir angle of the summit equals ψ. This calculation is easy, since all our triangles are right triangles:

$$\lambda^2 = (r+h)^2 - r^2$$
$$\lambda = \sqrt{2rh + h^2}$$
$$\lambda = 369.161\,\text{km}$$

So the total range is L_2:

$$L_2 = L + \lambda$$
$$L_2 = 1696.349 + 369.161\,\text{km}$$
$$L_2 = 2069.510\,\text{km}$$

For the angle around the planet when this occurs, use Law of Sines:

$$\frac{\lambda}{\sin\,\theta'} = \frac{r+h}{\sin\,90^\circ}$$
$$\sin\,\theta' = \frac{\lambda}{r+h}$$
$$\theta' = 6.^\circ 202$$
$$\text{and the total}\quad \theta + \theta' = 37.^\circ 738$$

End.

—RLM 12 December 2001
olymons.tex